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Treatment of evacuation time uncertainty using polynomial chaos expansion

Qimiao Xie
State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei, Anhui, PR China

Shouxiang Lu
State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei, Anhui, PR China

Depeng Kong
State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei, Anhui, PR China

Jinhui Wang
College of Ocean Environment and Engineering, Shanghai Maritime University, Shanghai, PR China

Abstract
In order to deal with uncertainties in evacuation time associated with the uncertainty in input parameters at a reasonable computational cost, a probabilistic method based on polynomial chaos expansion is proposed that combines evacuation models with Latin hypercube sampling. Evacuation models enable the prediction of evacuation time; polynomial chaos expansion is used to construct a surrogate model of evacuation time; Latin hypercube sampling is adopted as post-processing of the surrogate model to predict numerically the distribution of evacuation times. Additionally, an Uncertainty Factor is defined to quantify the total effect of the uncertainty of input parameters on evacuation time. To illustrate the proposed probabilistic method, evacuation of a simplified fire compartment typical of large commercial buildings is analyzed while considering uncertain input parameters including occupant density, child-occupant load ratio and exit width. This case study indicates that when exit width is small, the Uncertainty Factor is almost constant with respect to exit width but increases with an

Corresponding author:
Shouxiang Lu, State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei, Anhui, 230027, PR China.
Email: sxlu@ustc.edu.cn

increase in specified (acceptable) reliability level. Furthermore, if exit width exceeds a certain critical value, the Uncertainty Factor will decrease with an increase in exit width and its sensitivity to reliability level will become smaller. Finally, the case study shows that compared with the conventional Monte Carlo simulation, the proposed method can give similar estimations of evacuation time uncertainty at a significantly reduced computational cost.

Keywords
Polynomial chaos expansion, uncertainty analysis, evacuation time, performance-based design, Monte Carlo simulation, safety factor

Introduction

Treatment of the uncertainty of evacuation time is a key consideration in performance-based fire safety design. This uncertainty is generally dealt with by using safety factors [1]. For example, evacuation time is usually calculated as pre-movement time under certain conservative circumstances plus the product of movement time and an empirical safety factor.

In fact, due to the extreme complexity and high uncertainty of the evacuation process and the lack of specialized regulations about the selection of safety factors, it is difficult for fire safety engineers to determine appropriate safety factors in performance-based building fire-safety design. On the one hand, if the selected safety factor is too large, it will not be cost-effective for performance-based design. On the other hand, if the chosen safety factor is too small, the occupants in buildings cannot evacuate successfully in fire situations. From the perspective of life safety, a faulty selection of safety factors may cause serious consequences. Hence, in order to conduct cost-effective performance-based design and ensure occupant life and/or property safety, it is necessary to adopt probabilistic methods to obtain evacuation time at an acceptable level of uncertainty.

The uncertainty of evacuation time in buildings has been widely investigated through probabilistic methods. From data on 66 verified incidents, Mills et al. [2] suggested that evacuation time fit well with a log-normal distribution. In addition, based on the STEPS and EXIT89 models, Meacham et al. [3] and Lord et al. [4] employed the Monte Carlo simulation (MCS) method to investigate the distribution of evacuation time and obtained evacuation time at a certain acceptable level of uncertainty. Moreover, in order to quantify the uncertainty of evacuation time at a reasonable computational time, Spearpoint [5] proposed a probabilistic network-based model to obtain the distribution of evacuation time. Furthermore, due to the significant computational cost of the MCS with complex evacuation models, Song et al. [6] proposed an evacuation model coupling the lattice-gas model and the mean field model to quantify the uncertainty of evacuation time by running a single simulation, which is applicable to a large number of occupants following normal distributions. Averill et al. [7] also developed a Modified-Markov Modeling method to obtain the distribution of evacuation time resulting from the uncertainty in the number of occupants following the normal distribution through a single simulation.

From the above discussion, it appears that at present the distribution of evacuation time in a building environment is mainly investigated by experiments and computer simulations. In fact, it is extremely expensive to obtain evacuation time data from a large number of experiments [8]. Moreover, due to computational cost and time, it is also difficult to perform the MCS method with complex computer evacuation models in practical performance-based design and risk assessment. Meanwhile, improved simulation methods having low computational cost are focused on the uncertainty associated only with the number of occupants following a normal distribution. In order to obtain the distribution of evacuation times resulting from the uncertainty of several input parameters at an acceptable computational cost while determining appropriate safety factors, a probabilistic method based on polynomial chaos expansion (PCE) is proposed that combines evacuation models with Latin hypercube sampling (LHS).

The evacuation process and evacuation time

The evacuation process

The evacuation process is divided into the pre-movement and the movement processes [9,10]. The terms related to the evacuation process have been defined in the literature [9–11]. From the perspective of an individual occupant, the pre-movement process consists of the recognition process and the response process, and pre-movement time is the sum of recognition time and response time. In the recognition process, the activity of each occupant is not influenced by the alarm or cues of fire. However, it is affected by the fire growth rate, detector type, alarm type, occupant characteristics, occupancy type, etc. [9]. Due to the complexity and uncertainty in the above factors, recognition time is considered a stochastic variable. Moreover, during the response process, occupants may collect valuable things, gather family members, call firefighters or evacuate immediately [12]. Thus, response time can also be considered as a random variable, affected by many stochastic events. From the perspective of a population, pre-movement time is different for different occupants due to the difference in human behavior. For the distribution of pre-movement times of a population, MacLennan et al. [13] revealed that the log-normal distribution is the most appropriate and other distributions may also be suitable. Purser et al. [11] suggested that the pre-movement time distribution tends to be skewed and the observed pre-movement time data from a retail store with food hall and mixed occupancies fit well with the log-normal distribution. Bensilum et al. [14] revealed that for most cases, the uncertainty of pre-movement time would be appropriately characterized by using certain uni-modal, positively skewed distributions, such as normal, log-normal and Weibull distributions. Due to the inconsistency of the available data on pre-movement time [15], the distribution type and numeric range of pre-movement time have not been clear for life safety evaluation of buildings. Furthermore, PD 7974-6 [16] took into account two pre-movement times, i.e. the 1st percentile and 99th percentile pre-movement times, which are determined according to occupancy type, alarm system, building complexity and safety management system.

For the movement process, there are many influencing factors, such as the choice of exits, the selection of egress routes, walking speed and the interaction between occupants, buildings and fire environment, etc. Additionally, walking speed related to occupant type is influenced by occupant density, which changes during the movement process. Since human behavior and decision-making are stochastic during the movement process, these factors above are significantly uncertain. Thus, movement time of an individual occupant and/or a population is considered to be uncertain during the movement process.

From the analysis of the evacuation process, it can be seen that the evacuation process is random. It is important to note that when occupants are informed of a fire occurring, occupant density and occupant type are random. Moreover, it is difficult to identify the characteristics of occupants, such as the body dimension, the unimpeded walking speed, the initial location of evacuees, the familiarity with buildings, etc.

Evacuation time

According to the analysis of the evacuation process, it is found that every occupant can be modeled as an agent. His/her motion can be determined by the interaction between occupants, buildings and the fire. The evacuation time of each occupant is determined when he/she reaches a safe place. In order to ensure that the occupants can evacuate successfully, evacuation time for a population is defined as the maximum value of evacuation time of each individual occupant in a building. For a given evacuation scenario, Spearpoint [17] revealed that the maximum potential evacuation time, which is defined as the sum of the maximum pre-movement time and movement time, is larger than the value of evacuation time obtained for the situation of pre-movement time having a certain probabilistic distribution. Thus, in order to obtain a reasonably conservative estimation of evacuation time for a given evacuation scenario, evacuation time can be calculated as the sum of the conservative pre-movement time determined according to the building codes and movement time, which is denoted as follows:

$$t_e = t_p + t_m \tag{1}$$

where t_e is evacuation time; t_p is pre-movement time for a population, which is associated with the occupancy type, alarm system, building complexity and safety management system as well as determined based on building codes; t_m is movement time, which is determined by occupant density, occupant type, occupant velocity, exit width etc.

Furthermore, due to the randomness of the evacuation process, evacuation time will be different each time an identical evacuation scenario is repeated. This kind of uncertainty of evacuation time is regarded as aleatory uncertainty [18]. Aleatory uncertainty of evacuation time is not considered here. Rather, the uncertainty of evacuation time resulting from the uncertainty in input parameters is the focus of this article.

Method for uncertainty quantification of evacuation time

PCE in mathematical terms is based on the homogeneous chaos theory proposed by Wiener [19] and consists of the conventional PCE, the generalized polynomial chaos and the abitrary polynomial chaos. It can be used to construct a surrogate model of the output associated with uncertain input parameters. Thereafter, numerical or analytical methods are used as post-processing of the surrogate model to estimate the uncertainty of the output resulting from a defined distribution of input parameters at a reduced computational cost [20]. The uncertainty quantification method based on PCE has been successfully applied in various fields [21]. For evacuation time involving uncertainty in input parameters, the PCE method combining evacuation models with LHS is shown in Figure 1.

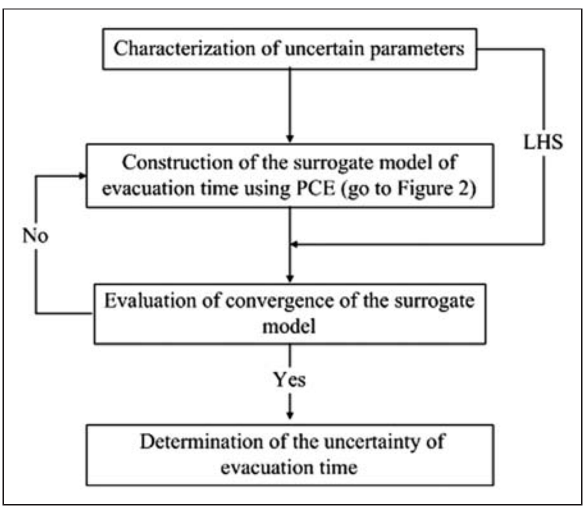

Figure 1. Flowchart of the iterative procedure for quantifying evacuation time uncertainy using polynomial chaos expansion (PCE).

Characterization of uncertain parameters

Due to the randomness of fires and occupant characteristics, input parameters of evacuation models, such as occupant density, occupant type, the familiarity of each evacuee with the building, etc., exhibit a considerable level of uncertainty. In prescriptive-based or current performance-based building fire safety design, these input parameters are treated as deterministic variables, which are taken as mean values or conservative characteristic values. It is clear that the use of deterministic methods may have serious shortcomings in the evaluation of building design. Thus, uncertain input parameters of evacuation models should be characterized by their probabilistic distributions and numeric ranges. These distribution types and ranges are usually determined based on empirically measured values, statistical data, controlled experiments, reputed expert judgment or from the literature [22,23].

Construction of the surrogate model of evacuation time using PCE

The procedure for the construction of the surrogate model of evacuation time using PCE, which relates to the uncertainty in input parameters, is presented in Figure 2.

Selection of orthogonal polynomial chaos bases. Conventional PCE constructs a surrogate model of the output of interest using Hermite polynomals, which are orthogonal with respect to the standard normal distribution. For arbitrary stochastic processes with finite second-order moments, the Hermite-Chaos expansion converges [24]. Meanwhile, its convergence rate is exponential for the normal stochastic process [25]. Thus, it is applicable to solve the uncertain parameters following normal distributions or other specific probabilistic distributions that can be transformed to the standard normal distribution by the iso-probablistic transform. However, for this situation of input parameters following non-normal

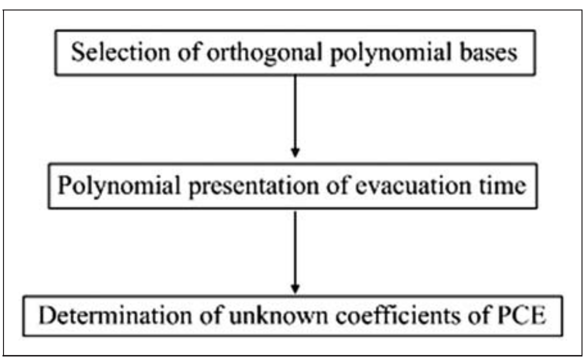

Figure 2. Procedure for the construction of the surrogate model of evacuation time using polynomial chaos expansion (PCE).

distributions, the convergence rate of the Hermite-Chaos expansion may be slower [26]. To solve this problem, the generalized polynomial chaos was developed by Xiu and Karniadakis [26] using the Wiener–Askey polynomials, which are orthogonal with respect to the standard normal distribution or some specific non-normal distributions. Due to limited applicability of the methods above, the arbitrary polynomial chaos [27] was developed, which can be used to deal with arbitrary distributions having known moments.

According to the PCE method described above, the orthogonal polynomial basis of the surrogate model of evacuation time is determined based on the distribution type of input parameters. If uncertain input parameters of evacuation models follow the normal distribution, Hermite polynomials are selected as the orthogonal basis. For input parameters following some specific non-normal distributions, Wiener–Askey polynomials can be chosen as the orthogonal basis. Additionally, if these non-normal distributions can be transformed to normal distributions, the Hermite polynomials can also be chosen as the orthogonal basis. For other distribution types, arbitrary PCE can be adopted to build the orthogonal basis needed.

Polynomial presentation of evacuation time. According to the PCE method, evacuation time associated with uncertain input parameters is represented in a series form, as shown in equation (2).

$$Y = \sum_{i=1}^{p} \alpha_i \psi_i \left(\vec{\varepsilon} \right), \quad \vec{\varepsilon} = \{\varepsilon_1, \ldots, \varepsilon_m\}. \tag{2}$$

where Y is the output variable (corresponding to evacuation time); α_i are the unknown coefficients of PCE of order n; ψ_i are the univariate orthogonal polynomials and/or multivariate orthogonal polynomials that are the product of the one-dimension orthogonal polynomials with different variables; $\vec{\varepsilon}$ is the multi-dimension random vector, each element of which is associated with uncertain input parameters of the evacuation process; m is the number of input variables; P is the number of unknown coefficients of PCE of order n, which equals $\frac{(m+n)!}{m!n!}$.

Determination of unknown coefficients of PCE. The methods for determining the unknown coefficients of PCE are generally classified into two types, intrusive and non-intrusive [21]. Due to intrusive methods requiring the adaptation of the original model [21,28], the regression non-intrusive method (typically including the projection method) is chosen to calculate the unknown coefficients of PCE for evacuation time here. Thereafter, sample points should be determined. According to the optimal design [29], the combination of the roots of the orthogonal polynomial of order $n + 1$ is selected as the sample space of $(\varepsilon_1, \ldots, \varepsilon_m)$. For multiple uncertain input parameters of the evacuation process, the number of the available samples of $(\varepsilon_1, \ldots, \varepsilon_m)$ may be much larger than the number required.

In order to reduce computational cost, the minimum number of samples of $(\varepsilon_1,\ldots,\varepsilon_m)$ should be determined. According to the efficient collocation method [30], the sample point of $(\varepsilon_1,\ldots,\varepsilon_m)$, which corresponds to the region of high probability, should be given the preferred choice. Once the rank of the matrix, which is composed of the ranked sample points of $(\varepsilon_1,\ldots,\varepsilon_m)$, equals the number of unknown coefficients, the unknown coefficients of PCE can be solved. Based on the method above, the minimum number of simulations can be determined. Meanwhile, the input parameter samples of evacuation models can be obtained based on the selected sample points of $(\varepsilon_1,\ldots,\varepsilon_m)$. Thereafter, the corresponding output samples of Y (evacuation time) can be calculated using evacuation models. Once the required samples are obtained, the unknown coefficients can be estimated using the singular decomposition method.

Evaluation of convergence of the surrogate model

When the surrogate model of evacuation time is constructed by PCE of order n, its convergence should be evaluated [31]. The aim is to quantify the uncertainty of evacuation time with consideration of the uncertainty in input parameters. Thus, the convergence of the surrogate model can be examined by comparing the uncertainty results of evacuation time based on the surrogate model and PCE with order $n+1$. The uncertainty of the model output can be characterized by the probability distribution function, the cumulative distribution function (CDF) or the inverse CDF. Each one of them can fully characterize the uncertainty in the output data set [32]. In addition, the level of uncertainty of evacuation time is an issue of concern for performance-based fire safety design. For simplicity, the CDF is used to describe the uncertainty of evacuation time. The CDF for the surrogate model of evacuation time, which is associated with the uncertain input parameters, can be estimated using computationally inexpensive numerical methods [31]. Compared with other numerical sampling methods, LHS, which was first developed by McKay, et al. [33], is an effective stratified sampling method [34]. So, LHS is adopted as a post-processing technique of PCE of evacuation models to estimate the CDF of evacuation time. For m variables, the range of every variable is divided into i intervals of equal probability based on the LHS method [35]. One value is selected randomly from every interval. Consequently, i values are obtained for every variable. Thereafter, the i values for the first variable are paired randomly with the i values for a second variable. Then, these i pairs are combined randomly with the i values of a third variable to constitute i triplets. The same procedure is performed until the i m-tuplets are formed. Finally, the i samples of m variables are determined, which is the same as these i-tuplets.

If the CDF of evacuation time estimated by the surrogate model with PCE of order n has little deviation from that estimated by PCE of order $n+1$, then the surrogate model of evacuation time is regarded as convergent. If there is a significant difference, PCE of order $n+2$ should be constructed. Meanwhile, it is necessary to repeat the evaluation of the convergence until the surrogate model is convergent.

Determination of the uncertainty of evacuation time

When the surrogate model of evacuation time is convergent, its CDF can be used to describe the uncertainty of evacuation time associated with defined uncertain input parameters. Evacuation time at the level of uncertainty of interest, which is obtained from the CDF of the convergent surrogate model, can provide efficient decision support for performance-based fire safety design.

Case study and analysis

Case description

In order to illustrate the above probabilistic method, a fire compartment in a large commercial building is analyzed in this section. The fire compartment considered is a simplified 2500 m^2 square commercial room without obstacles, as shown in Figure 3. Its area is chosen in accordance with the Chinese code for one fire compartment of civil buildings up to 2500 m^2 [36]. Fire compartments larger than 2500 m^2 will require an automatic fire extinguishing system or a smoke exhaust system, etc. This case is regarded as a representative scenario for uncertainty analysis of the evacuation process. The fire compartment is designed with two exits that are denoted as Exit A and Exit B, along with the center of the walls. In the designed uncertain evacuation scenario, Exit B is assumed to be not available for evacuees when fire occurs. The fire compartment studied is installed with an automatic detection system. Additionally, trained staff are not always present in this fire compartment. Furthermore, it is assumed that the occupants in this fire compartment can move freely without mobility impairments. Finally, occupant density and the child to adult occupant load ratio are considered to be random variables.

It is generally difficult to determine appropriate values of exit width during the process of building design optimization. On the one hand, if exit width is insufficient, congestion and queuing may occur, which reduce evacuation efficiency or even lead to the occurrence of casualties. On the other hand, large exit width will reduce the usable area of the building, though it increases evacuation efficiency. Moreover, when exit width exceeds a certain value, it has little influence on evacuation time [37]. Hence, in order to determine a reasonable value of exit width, it is important to examine the relationship between exit width and evacuation time under uncertain conditions. Furthermore, exit width is assumed to be a stochastic variable following the uniform distribution in this case.

The distribution type and range of uncertain input parameters related to commercial buildings are presented in Table 1. Meanwhile, these uncertain parameters are assumed to be independent.

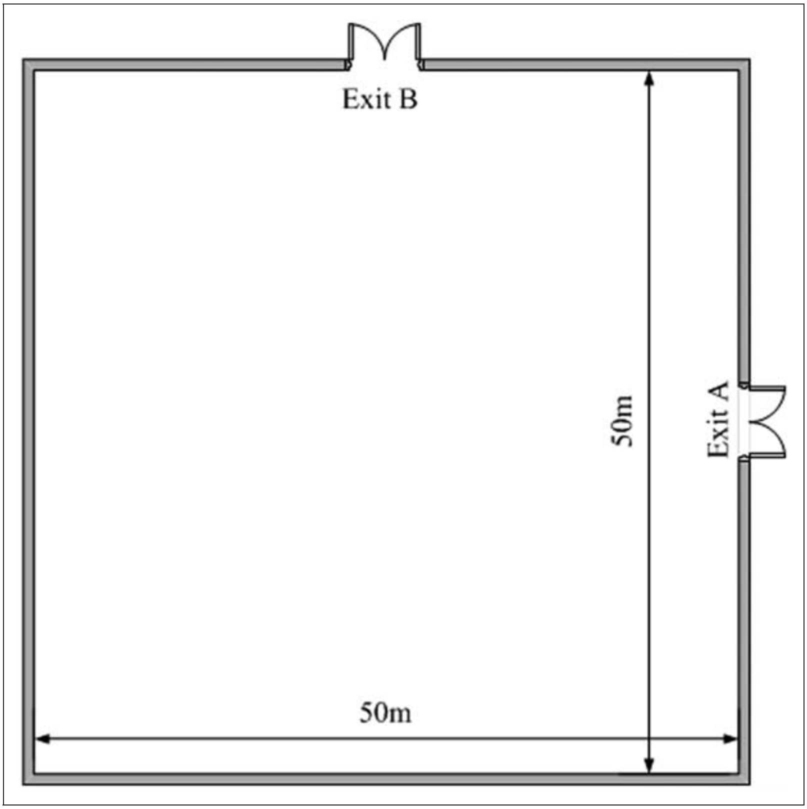

Figure 3. A simplified fire compartment for commercial buildings.

Table 1. Input parameters for the fire compartment case study.

Variable	Distribution type	Mean Value	Mean Base on	Standard deviation	Range	Unit
Occupant density	Normal	0.85	[36]	0.085	0.595–1.105	person/m^2
Child-occupant load ratio	Normal	0.2	[38]	0.02	0.0–1.0	—
Exit width	Uniform				1–21.2[a]	m

[a]21.2 is calculated as the product of 2500 m^2 (area of the fire compartment), 0.85 person/m^2 (occupant density) and 0.01 m/person (1 m exit width for every 100 persons), which are chosen according to the Chinese code [36].

Determination of evacuation time based on PCE

Based on the definition of evacuation time in equation (1), t_p is deterministic and its value is in accordance with building codes. The value of t_m is affected by the uncertainty of input parameters of evacuation models. Thus the polynomial presentation of t_e can be obtained by the sum of t_p and the PCE of t_m. For the above described case, the 99th percentile pre-movement time is 180 s according to PD 7974-6. Thus, t_p is taken as 180 s for the fire compartment studied. Additionally, t_m associated with the uncertainty of the input parameters (Table 1) can be estimated by coupling PCE with an evacuation model.

As shown in Table 1, occupant density and occupant type follow normal distributions while exit width follows the uniform distribution, which can be transformed to the standard normal distribution in terms of the isoprobabilistic transform. Thus, Hermite polynomials are chosen as the orthogonal basis here. Then, the PCE of order n will be used to represent t_m. The initial order 2 is chosen. This second-order polynomial representation of t_m is shown in equation (3).

$$\begin{aligned} t_m = a_1 + a_2\varepsilon_1 + a_3\varepsilon_2 + a_4\varepsilon_3 + a_5\varepsilon_1\varepsilon_2 + a_6\varepsilon_1\varepsilon_3 + a_7\varepsilon_2\varepsilon_3 \\ + a_8(\varepsilon_1^2 - 1) + a_9(\varepsilon_2^2 - 1) + a_{10}(\varepsilon_3^2 - 1) \end{aligned} \tag{3}$$

where t_m is movement time; ε_1, ε_2, ε_3 are standard normal random variables determined according to the Hermite polynomial basis; $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ are the unknown coefficients of the 2nd order PCE. In order to estimate the 10 unknown coefficients, the samples of $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ and the corresponding samples of t_m should be determined. The sample space of $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is the combination of the roots of the third-order Hermite polynomial $(\varepsilon^3 - 3\varepsilon)$. Every sample of $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is ranked according to the distance from the origin based on the efficient collocation method. Due to 10 unknown coefficients, the rank of the matrix of the selected ranked samples of $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ should be 10 with the minimum number of samples 19. The corresponding sample values of t_m can be obtained from evacuation models. Due to the randomness of the evacuation process, FDS + Evac [39], a stochastic evacuation model is used to predict the sample values of t_m. Occupant density, child-occupant load ratio and exit width, which are used as input parameters for evacuation models, can be determined from equation (4).

$$\begin{aligned} x_1 &= 0.085\varepsilon_1 + 0.85 \\ x_2 &= 0.02\varepsilon_2 + 0.2 \\ x_3 &= 1 + (21.2 - 1)\left(\frac{1}{2} + \frac{1}{2} erf\left(\varepsilon_3 \Big/ \sqrt{2}\right)\right) \end{aligned} \tag{4}$$

where x_1 is occupant density; x_2 is child-occupant load ratio; x_3 is exit width.

Once the samples of $(\varepsilon_1, \ldots, \varepsilon_m)$ and the corresponding samples of t_m are obtained, the 10 unknown coefficients can be calculated by the singular value decomposition method.

The same procedure is adopted to build the PCE of order 3 or higher until the constructed PCE of t_m is convergent. Thereafter, t_e can be estimated by the sum of t_p (180 s) and t_m that is determined by the convergent PCE.

Results and discussion

The CDFs of t_m, which are estimated by coupling the 2nd, 3rd and 4th order PCE with LHS, are presented in Figure 4. The goodness of fit between two curves can be measured using the coefficient of determination, R^2, which is in the range of 0 to 1. A larger R^2 indicates that the goodness of fit is better. The value of R^2 between the two CDF curves of t_m, estimated from the 2nd and 3rd order PCE, is 0.938. In order to obtain more accurate approximations, the 4th order approximations are performed. The R^2 value between the two CDF curves estimated from the 3rd and 4th order PCE is 0.986, which shows that the convergence of the third-order approximation is good. In order to verify the accuracy of the surrogate model, the CDF of t_m estimated by MCS with the FDS + Evac model is also obtained, as shown in Figure 4. The values of R^2 between the CDF curves estimated from the second-, third- and fourth-order PCE and MCS are, respectively, 0.953, 0.972 and 0.989. The R^2 values show that the third-order approximation agrees well with the fourth-order approximation and with the MCS approximation. Moreover, the number of samples of the third-order approximation is 49, which is far fewer than that for the fourth order approximation.

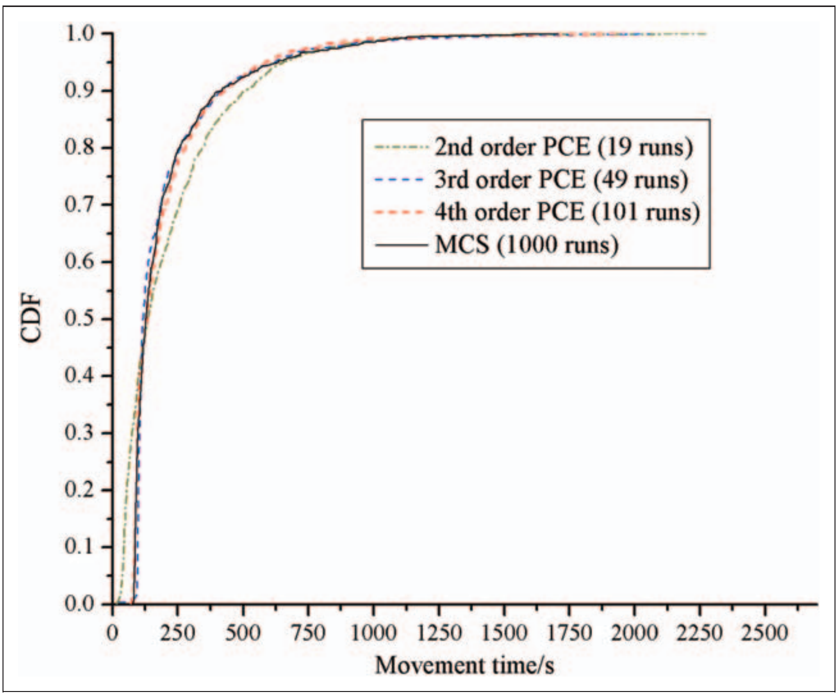

Figure 4. Cumulative distribution functions (CDFs) of movement time associated with three uncertain parameters based on polynomial chaos expansion (PCE) and Monte Carlo simulation (MCS).

Exit width is a design parameter, and it will not be varied once the building is built. However, occupant density and occupant type change stochastically due to the randomness of fire occurring. In order to conduct a reasonable performance-based design, the influence of exit width on evacuation time should be examined. In order to reduce computational cost, the third-order PCE is adopted here to quantify evacuation time uncertainty resulting from the uncertainty of occupant density and child-occupant load ratio (Table 1) for a certain deterministic exit width. The relationship between evacuation time and exit width under uncertain conditions is shown in Figure 5. For example, for a 1 m exit width, evacuation time at reliability levels, $\beta = 90\%$ and 99.9%, are 1809.2 s and 2042.6 s. If the acceptable performance criterion is the reliability level, $\beta = 99.9\%$ in building evacuation design and fire risk assessment, evacuation time will be chosen as 2042.6 s for this condition of 1 m exit width. Additionally, for the condition of 8 m exit width, evacuation time at reliability levels $\beta = 90\%$ and 99.9% are 367.0 s and 394.1 s. From Figure 5, it can also be seen that the difference among evacuation times at different reliability levels is significant when exit width is small. However, the difference is small when exit width is large.

In practical engineering applications, the safety factor method is generally adopted to deal with the uncertainty in the evacuation process [10]. A safety factor applied to the base movement time is chosen from 1.5 to 2.0 for a given building

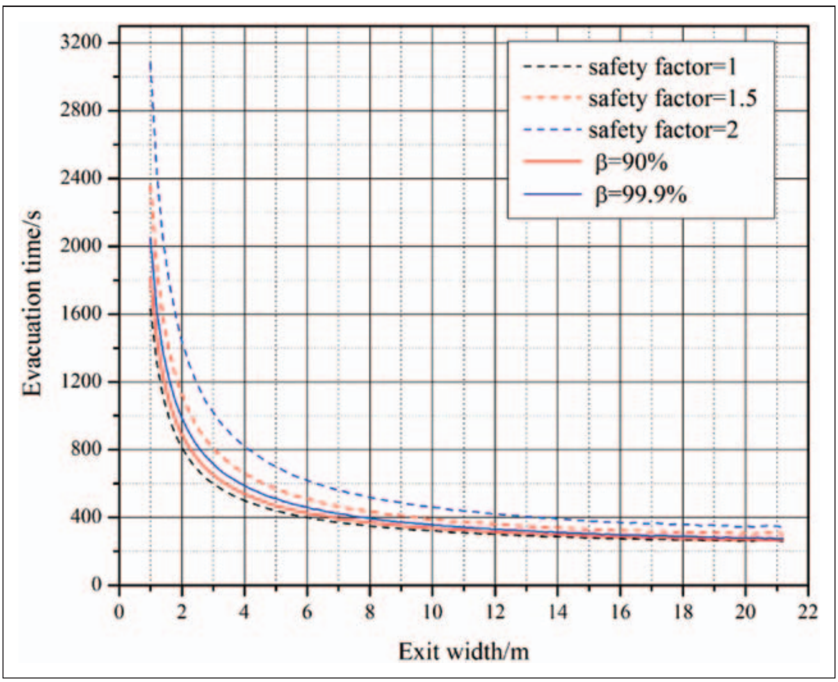

Figure 5. The relationship between evacuation time and exit width.

in performance-based design and risk assessment [40]. The base movement time is generally determined from the evacuation model with mean values of uncertain input parameters, and here it is calculated by the FDS + Evac model with the mean values of occupant density and child-occupant load ratio, which are 0.85 and 0.2, respectively. Evacuation time obtained by using safety factors of 1.0, 1.5 and 2.0 applied to this base movement time is shown in Figure 5. This figure indicates that evacuation time calculated by using safety factors of 1.5 and 2 is larger than evacuation time calculated at reliability levels of $\beta = 90\%$ and 99.9% for a given building geometry. Additionally, Figure 5 also shows that evacuation time using a safety factor of 1.0 is smaller than evacuation time at reliability levels $\beta = 90\%$ and 99.9%. Furthermore, when exit width is small, evacuation time decreases rapidly. However, when exit width increases to a certain value, evacuation time is almost constant.

The choice of performance criteria [23] and the acceptable reliability level, β, for evacuation time need to be investigated for performance-based design. An Uncertainty Factor is introduced to examine the total effect of uncertain parameters on evacuation time at a certain reliability level, β. The Uncertainty Factor at a certain reliability level β is defined as the ratio of movement time at a certain reliability level β to the base movement time.

$$U_\beta = t_{m,\beta}/t_{m,b} \tag{5}$$

where U_β is the Uncertainty Factor at the reliability level β; $t_{m,\beta}$ is movement time at the reliability level β; $t_{m,b}$ is the base movement time that can be calculated from evacuation models using the mean values of the uncertain parameters. For the fire compartment of commercial buildings studied here, the relationship between the Uncertainty Factor and exit width is shown in Figure 6.

It is seen in Figure 6 that for small exit width, the Uncertainty Factor (which is greater than 1 but less than 1.5) at the reliability level β in the range of 90% to 99% is almost constant with increasing exit width. However, when exit width exceeds a certain value, the Uncertainty Factor will decrease with an increase of exit width. Additionally, it is deduced that Uncertainty Factors at reliability levels $\beta = 90\%$, 95%, 99% and 99.9% are approximately 1.0 when exit width is large enough. This can be accounted for by the fact that occupant type and occupant density have little effect on evacuation time when exit width is large enough [41]. Furthermore, Figure 6 shows that when exit width is 1 m, the Uncertainty Factor at the reliability level $\beta = 90\%$ to 99.9% ranges from 1.12 to 1.28. When exit width is 21.2 m, the Uncertainty Factor at the reliability level $\beta = 90\%$ to 99.9% is in the range of 1.05 to 1.14. Through the above analysis, it can be deduced that when exit width is large enough, the uncertainty factor will be insensitive to the reliability level β.

Conclusions and future work

In order to deal with the uncertainty of evacuation time resulting from the uncertainty in input parameters at a reasonable computational cost and at a specified (acceptable) reliability level, the probabilistic method based on PCE combining

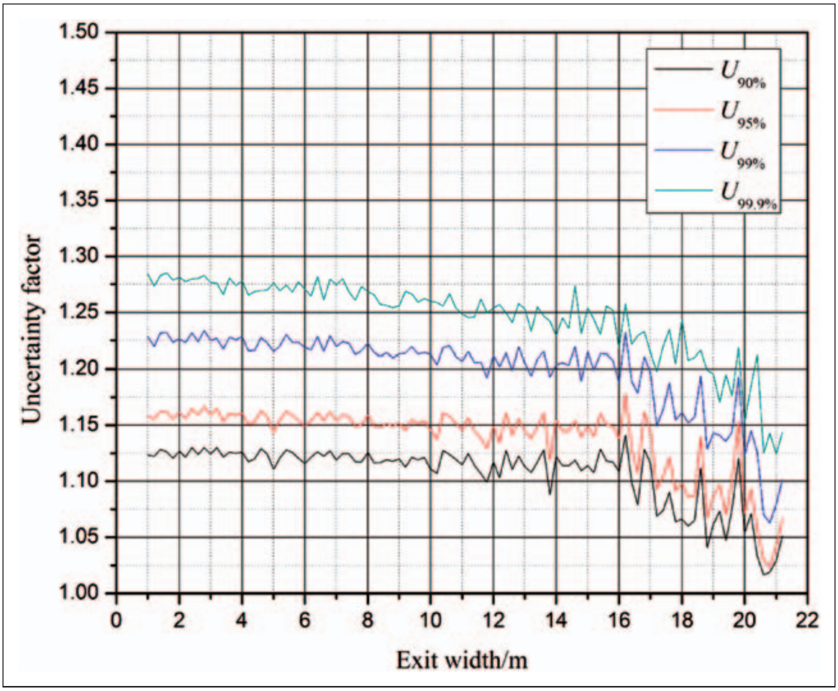

Figure 6. The relationship between the Uncertainty Factor and exit width.

evacuation models with LHS has been proposed and applied to the uncertainty analysis of evacuation time in a simplified fire compartment of a commercial building. Through the results of this case study, several conclusions can be drawn, as follows.

First, the probabilistic method based on PCE combining evacuation models with LHS can effectively deal with the uncertainty of evacuation time resulting from the uncertainty in input parameters at a low computational cost.

Second, an Uncertainty Factor varies with exit width or reliability level. For small exit width, the Uncertainty Factor is almost constant with exit width but increases with an increase of specified reliability level. However, when exit width exceeds a certain value, the Uncertainty Factor will decrease with a continuing increase of exit width, such that when exit width is large enough, the Uncertainty Factor at a specified reliability level is approximately 1.0.

In practical performance-based design and risk assessment, the probabilistic method based on PCE combining evacuation models with LHS and the deterministic method of using safety factors should be combined together to obtain a conservative and acceptable evacuation time.

However, there are some limitations, as follows. First, the probabilistic distribution type and range of uncertain input parameters are hypothetical based on the building codes. Hence, more data related to the evacuation process should be collected, such as pre-movement time, occupant density, occupant type, walking velocity and so on. In addition, the uncertainty in occupant mobility impairment during a fire is not examined.

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Nomenclature

CDF	Cumulative distribution function
LHS	Latin hypercube sampling
m	Number of input variables
MCS	Monte Carlo simulation
n	Order of polynomial chaos expansions
P	Number of unknown coefficients of polynomial chaos expansion
PCE	Polynomial chaos expansion
t_e	Evacuation time (s)
t_m	Movement time (s)
$t_{m,b}$	Base movement time (s)
t_p	Pre-movement time (s)
U	Uncertainty Factor
x_1	Occupant density (person/m^2)
x_2	Child-occupant load ratio;
x_3	Exit width (m)
Y	Output variable
α_i	Unknown coefficients of a certain order polynomial chaos expansions
β	Reliability level
ε	Multi-dimension random vector
ε_1	Standard normal random variable corresponding to occupant density
ε_2	Standard normal random variable corresponding to child-occupant load ratio
ε_3	Standard normal random variable corresponding to exit width
ψ_i	Univariate polynomials or multivariate polynomials that are the product of the one-dimension orthogonal polynomials with different variables

Subscript

β	Reliability level

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