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# A Monte Carlo analysis of the effect of heat release rate uncertainty on available safe egress time

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## Abstract

Available safe egress time is an important criterion to determine occupant safety in performance-based fire protection design of buildings. There are many factors affecting the calculation of available safe egress time, such as heat release rate, smoke toxicity and the geometry of the building. Heat release rate is the most critical factor. Due to the variation of fuel layout, initial ignition location and many other factors, significant uncertainties are associated with heat release rate. Traditionally, fire safety engineers prefer to ignore these uncertainties, and a fixed value of heat release rate is assigned

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based on experience. This makes the available safe egress time results subjective. To quantify the effect of uncertainties in heat release rate on available safe egress time, a Monte Carlo simulation approach is implemented for a case study of a single hypothetical fire compartment in a commercial building. First, the effect of deterministic peak heat release rate and fire growth rate on the predicted available safe egress time is studied. Then, the effect of uncertainties in peak heat release rate and fire growth rate are analyzed separately. Normal and log-normal distributions are employed to characterize peak heat release rate and fire growth rate, respectively. Finally, the effect of uncertainties in both peak heat release rate and fire growth rate on available safe egress time are analyzed. Illustrations are also provided on how to utilize probabilistic functions, such as the cumulative density function and complementary cumulative distribution function, to help fire safety engineers develop proper design fires.

**Keywords**
Time-squared fire growth, peak heat release rate, heat release rate uncertainty, Latin hypercube sampling, probability-based available safe egress time

## Introduction

Occupant safety is the main objective of fire risk assessment and performance-based fire protection design of buildings. The safety level of occupants is usually determined by comparing two timelines, i.e. the available safe egress time (ASET) and the required safe egress time (RSET) as shown in Figure 1. If ASET is greater than RSET, occupants are considered to be safe; otherwise, occupants cannot evacuate to safe places before the onset of untenable conditions and casualties occur. Traditionally, ASET is estimated through fire dynamics models with a user-specified set of input parameters, such as fire loads, fire spread velocity and heat release rate (HRR). Due to the complexity of fire dynamics, uncertainties are always associated with these input parameters. However, the traditional approach is usually applied using deterministic values of uncertain variables and there are

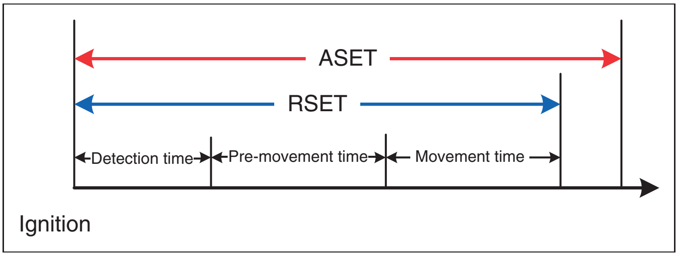


**Figure 1.** Evacuation timeline in building fires.

even recommendations on national levels [1] of using certain deterministic values of some variables, such as fire growth rate and peak HRR. As a result, fire safety engineers tend to ignore these uncertainties and assign a fixed value to the uncertain parameters according to their judgment or recommendations. Such assignment is hard to prove right or wrong and modeling results may vary from engineer to engineer. If the engineers' estimated values of parameters are on the conservative side, the design will end up with overly safe and expensive solutions [2]. On the other hand, if non-conservative parameters are selected, the solutions will be unsafe. This variation makes authorities having jurisdiction dissatisfied with the results. Therefore, more advanced methods, such as probability-based methods, should be employed to quantify the parameter uncertainties in order to obtain more accurate results.

Recently, the influence of the above-mentioned uncertainties on fire phenomena has drawn the attention of many researchers. Notarianni [3] analyzed the effect of uncertainty related to the critical criteria determining untenable time in the model CFAST using a Monte Carlo simulation. Lundin [4] compared the predicted results using CFAST with the experimental results under the same conditions and discussed the uncertainty of smoke layer temperature predicted by CFAST. Uncertainty analysis is time-consuming as it usually requires many runs of fire models. In order to improve the calculation efficiency, Au et al. [5] proposed subset simulation to analyze the influence of uncertainty related to input parameters in CFAST on the maximum smoke layer temperature in a compartment. Quadrature Method of Moments (QMOM) was employed by Upadhyay and Ezekoye [6] to analyze the uncertainty of smoke height at a given critical time in a single compartment when HRR follows a generalized beta distribution. In addition, a fire risk analysis tool called Probabilistic Fire Simulator (PFS) [7], which combines Monte Carlo simulation and CFAST, was developed to predict the probabilities of smoke filling in an electronics room fire. Sensitivity analysis results from PFS indicate that fire growth rate has the largest influence on smoke filling probability.

Despite the differences in the aforementioned methods, all of them analyze the effect of input parameter uncertainties on the output and agree that the major source of parameter uncertainties is associated with the prescription of HRR. However, the influence of the uncertainty associated with HRR on ASET has not been studied in depth previously. In this article, a Monte Carlo simulation approach is applied to quantify such influence. Latin Hypercube sampling (LHS) is employed to generate the required samples. Smoke movement is modeled using the zone model, CFAST, where a time-squared fire growth is assumed. Uncertainties associated with the peak HRR and fire growth rate are considered. First, the effects of uncertainty in peak HRR and fire growth rate are studied separately, and then the combined effect is studied. Then, the probability density distribution of ASET is determined when the peak HRR and fire growth rate follow certain specifications. Several suggestions are also made for fire scenario design that can be employed in fire risk assessment and performance-based fire protection design.

## Fire modeling for ASET

### Fire modeling

Fire models can be categorized as computational fluid dynamics (CFD) models, zone models and hand-calculation models [8]. Among these, CFD models are the most sophisticated because of having high resolution and the ability to describe more complicated fire phenomena. Compared with CFD models, the computation time of zone models is short, but these models represent a greater simplification of reality. Even so, zone models are still widely used in fire safety engineering design because of having acceptable accuracy. Since many separate runs of a fire model are required to conduct an uncertainty analysis and because flexibility and ease of implementation is also a requirement, the two-zone model, CFAST, is adopted here to predict fire and smoke movement.

CFAST was developed by National Institute of Standards and Technology (NIST) to calculate the evolving distribution of smoke, fire gases and temperature throughout compartments of a building during a fire. Details can be found in [9] and [10].

In performance-based fire protection design and analysis, the onset of untenable conditions must be defined to determine ASET. Tenability criteria can be based on smoke layer height, temperature, visibility and toxicity. For example, the Fire Engineering Guidelines [11] recommend that the threshold for the onset of untenable conditions is an average upper layer temperature of 200°C or an optical density of $0.1\,\mathrm{m}^{-1}$ with an upper layer interface position 2.1 m above the floor.

From the standpoint of hazard, time of smoke descent to a chosen level may be a reasonable criterion, according to Peacock et al. [12]. Hence, the time when smoke descends to 2.1 m is used to determine ASET in this study.

### Heat release rate

From an occupant safety perspective, the early stages of a fire are the most important period [13]. Examination of data from fire tests and real fires [14] indicates that the fire growth rate in the early stages may reasonably be approximated by a time-squared growth curve and becomes constant after it reaches the peak HRR, as follows [15]

$$\dot{Q}(t) = \begin{cases} \alpha t^2, & \text{if } t < t_g \\ Q_{\max}, & \text{if } t \geq t_g \end{cases} \tag{1}$$

where,
$\dot{Q}(t)$, kW is the HRR at time $t$ (s); $\alpha$ is the fire growth rate (kW/s$^2$); $Q_{\max}$ is the peak HRR (kW); $t_g$ is the time when HRR reaches the peak HRR (s).

The peak HRR is either ventilation controlled or fuel controlled [8]. For fire scenarios in a single enclosure, the peak HRR is the minimum of ventilation-controlled HRR and fuel-controlled HRR [16]

$$Q_{\max} = \min\{Q_{\max f}, Q_{\max,v}\} \tag{2}$$

The peak HRR for a fuel-controlled fire may be calculated by the HRR per unit area as follows [15]

$$Q_{\max f} = Q'' A_f \tag{3}$$

where, $Q''$ is the HRR per unit area (kW/m$^2$) and $A_f$ is the burning area of the fuel (m$^2$).

For a fully developed ventilation-controlled fire, the peak HRR may be calculated based on the flow of air into the enclosure [8]

$$Q_{\max,v} = 1500 A_o \sqrt{H_o} \tag{4}$$

where, $A_o$ is the area of ventilation openings (m$^2$); and $H_o$ is the height of ventilation openings (m). For a specified peak HRR, equation (4) can be employed to determine whether the ventilation limit is reached.

Since CFAST only works for the preflashover stage, it is important to ensure that flashover will not occur by limiting the HRR. The HRR required for flashover in a compartment can be calculated with the Thomas equation as follows [17]

$$Q_{fo} = 7.8 A_t + 378 A_o \sqrt{H_o} \tag{5}$$

where, $Q_{fo}$ is the HRR required for flashover (kW); $A_t$ is the total surface area of the compartment (m$^2$).

## Uncertainty analysis

### *Monte Carlo simulation*

The main task of uncertainty analysis is to evaluate the influence of inputs on the outputs of a model, i.e. study the propagation of uncertainties. A large number of uncertainty analysis methods have been developed, including the response surface method [18], fuzzy arithmetic [19], probability-bound analysis [20] and the Monte Carlo simulation-based sampling approach [21].

Among these uncertainty analysis approaches, the Monte Carlo simulation-based sampling approach is probably the most widely used. Considering the criteria of easy implementation and flexibility, this approach is generally suitable and quite often the best [21]. Monte Carlo simulation is a method of performing numerical experiments using random numbers. With the advent of modern computers, Monte Carlo simulations have been employed in many fields, such as physics [22], risk analysis [23] and fire safety engineering [24]. In uncertainty analysis, a mapping from analysis inputs to results may be developed by using a sampling procedure in the simulation [25]. An introduction to Monte Carlo simulations is presented by Madras [26].

## Latin hypercube sampling

In Monte Carlo simulations, the sampling method that is used plays a crucial role. Many sampling methods have been proposed, such as simple random sampling and LHS. LHS is a stratified sampling scheme, which can cover the input parameter space with a minimal number of samples and allows for the extraction of a large amount of uncertainty information from this relatively smaller number of samples [27]. Moreover, LHS can generate random samples from the ranges of all possible values, thus giving insight into the tails of the probability distributions. Because thousands of runs of CFAST are to be conducted in this study, it is highly advantageous to have efficient sampling. Thus, LHS is employed.

An example of generating five samples ($N = 5$) for two input variables $U$ and $V$ in Kong et al. [24] is employed to illustrate the LHS generation procedure: 1. the random space of $U$ and $V$ is divided into five non-overlapping intervals based on equal probability. The lines that originate at 0.2, 0.4, 0.6 and 0.8 on the ordinates extend horizontally to the cumulative distribution function (CDF) curves of $U$ and $V$, respectively. Five intervals are produced by lines dropped vertically to the abscissas. 2. The required five pairs of random probability values $PU(1), PU(2), \ldots, PU(5)$ and $PV(1), PV(2), \ldots, PV(5)$ are then sampled from the above intervals. $PU(1)$ and $PV(1)$ are sampled from a uniform distribution on [0, 0.2]; similarly, $PU(2)$ and $PV(2)$ are sampled from a uniform distribution on [0.2, 0.4], and so on. The corresponding $U$ and $V$ values, i.e. the samples $U(1), U(2), \ldots, U(5)$ and $V(1), V(2), \ldots, V(5)$, can then be identified with the CDF curves. 3. The five generated samples are paired randomly without replacement. The generation of samples by LHS for more than two variables proceeds in a similar manner.

## The product limit estimate

The CDF and complementary cumulative distribution function (CCDF) of the output provide a complete representation of uncertainty for the output. In many estimation problems, it is inconvenient or impossible to make complete measurements on all members of a random sample. In this case, the CDF and CCDF can be estimated from the product limit estimate, which is also referred to as the empirical CDF or Kaplan–Meier estimate [28].

The $N$ calculated ASETs are sorted in order of increasing magnitude and denoted by $ASET_1 \leq ASET_2 \leq \cdots \leq ASET_N$. The empirical CDF can be estimated as follows

$$\hat{P}(ASET) = \prod_r [(N - r)/(N - r + 1)] \tag{6}$$

where $r$ runs through those positive integers for which $ASET_r \leq ASET$. Equation (6) indicates that the empirical CDF is a step-function, which changes its value at the observed ASET, where it is discontinuous. The empirical CCDF can also be determined, which is simply one minus the CDF.

## Analysis of HRR uncertainty effects on ASET

### *Uncertainty analysis procedure*

The procedure for analyzing the influence of HRR on ASET using Monte Carlo simulation can be summarized as follows (Figure 2)

1. Characterization of uncertainties in HRR: according to the assumption of a time-squared fire, two uncertain parameters, i.e. peak HRR and fire growth

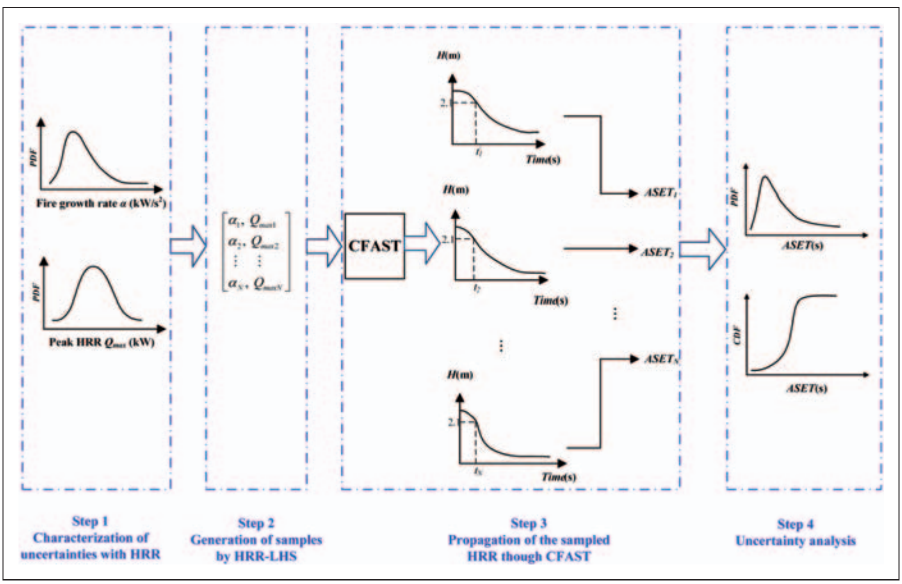


**Figure 2.** Schematic of uncertainty analysis for ASET.
ASET: available safe egress time.

rate, are considered. The distribution form and range of these two uncertain parameters is identified.
2. Generation of samples from the defined distribution of HRR: once the distribution and range of peak HRR and fire growth rate are determined, samples of HRR can be generated by LHS. A computer script named 'HRR-LHS' was written in Matlab® to generate the required HRR samples.
3. Propagation of the sampled HRR though CFAST: this procedure involves little more than a reiterated calculation. For each simulation, the sample input generated by HRR-LHS is inserted into CFAST. The corresponding output is obtained for a run of CFAST. The results are stored for later uncertainty analysis. As thousands of runs of CFAST have to be made, CFAST is used in batch mode to improve efficiency.
4. Uncertainty analysis: after performing thousands of simulation runs, the uncertainty characterization of ASET, such as the probability density function (PDF), empirical CDF and CCDF are described.

## *Case description*

In order to demonstrate the procedure for performance-based fire protection design, a case study with a hypothetical single fire compartment in a one-story commercial building based on Chinese fire protection regulations is presented. According to a Chinese building code [29], installation of a sprinkler system is only required for a fire compartment larger than 2500 m$^2$. The floor area in this case study is set at 2500 m$^2$ with no sprinklers installed. Geometric details are presented in Table 1 and a schematic is shown in Figure 3.

For convenience, the predicted ASET is simplified as

$$ASET = f(Q_{\max}, \alpha) \tag{7}$$

where, $f$ is the calculation in the two-zone model, CFAST, $Q_{\max}$ is the peak HRR and $\alpha$ is the fire growth rate.

**Table 1.** Geometric detail for the single fire compartment

| Parameter | Value |
|---|---|
| Area of the compartment (m$^2$) | 2500 |
| Height of the compartment (m) | 5 |
| Height of the door (m) | 3 |
| Width of the door (m) | 5 |

## Characterizing uncertainties with HRR

The selection of the fire growth rate in a performance-based fire protection design, often from NFPA's ultrafast, fast, medium, slow fires, is based on expert judgment. However, as Morgan [30] points out, for a given occupancy, there will actually be a distribution of possible fire growth curves depending on factors such as variation in fuel layout and location of the ignition, etc. There are two methods to design the distribution of the fire growth rate curve. One relies on the available data on common materials, which can be used in cases where the exact fuel involved in a fire is known [31]. An example of this method is a set of values for $\alpha$ and $t_g$ generated from a series of furniture calorimeter tests, such as those in Stroup et al. [32]. However, in most cases, the exact fuel is not known, and the fire growth rate has to be estimated from fire statistical data, which represent actual fire conditions from real fires. Holborn et al. [33] suggest that the observed fire growth rates for a range of different occupancy types are reasonably well approximated by log-normal distributions, based on Greater London Area data from real fire incidents. Deguchi et al. [34] analyzed data for fire growth time and burned area in national fire statistics from Japan and obtained distributions for fire growth rate in real fire situations. They also found that the distributions for fire growth rate in four kinds of buildings, i.e. offices, residences, restaurants and stores, follow a log-normal distribution. The log-normal distribution is therefore employed to characterize uncertainty associated with fire growth rate in this study.

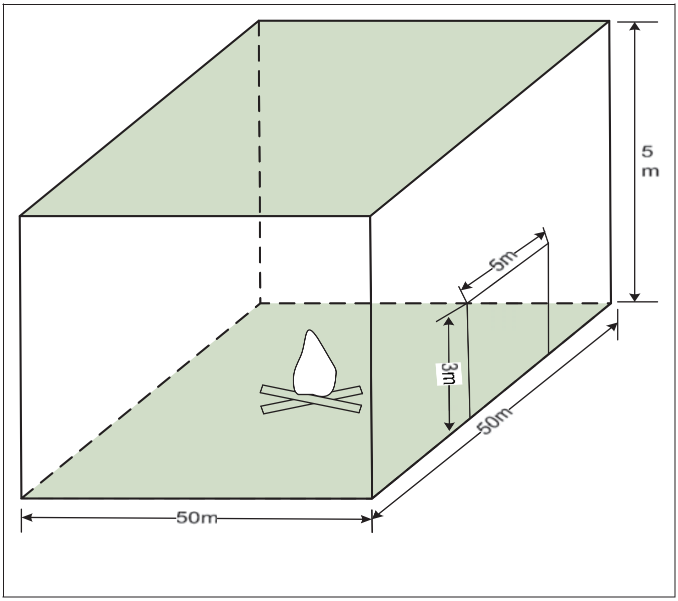


**Figure 3.** Compartment geometry for the case study (not scaled).

The peak HRR is controlled by ventilation conditions, sprinkler installation and the building geometry, etc. It also varies depending on the conditions involved [34]. Although some recommendations are available for peak HRR, it is argued that the peak HRR should be a stochastic variable rather than a deterministic one. Similar to fire growth rate, the distribution of peak HRR should be estimated based on fire statistics. Some research results have confirmed that peak HRR is a variable following a specific probability distribution; but the specific probabilistic distribution form is still unknown. Previous studies indicate that a generalized beta distribution [6], a beta distribution [35] or a normal distribution [5] may be suitable for characterizing peak HRR. In this study, a normal distribution is employed to characterize the peak HRR.

## Generation and propagation of samples

The procedure for generating samples involves sampling and pairing using LHS as described above. With the aid of the script HRR-LHS, the samples are generated

$$\begin{bmatrix} Q_{\max 1}, & \alpha_1 \\ Q_{\max 2}, & \alpha_2 \\ \vdots & \vdots \\ Q_{\max N}, & \alpha_N \end{bmatrix} \tag{8}$$

where $N$ is the sample size for individual variables.

The procedure for propagation of the sample HRR is conducted according to the above description. The accuracy of Monte Carlo simulation depends on the sample size. The sample size is estimated by the statistical tolerance limits ($a\%$, $b\%$) [36]. In this case, 99.7% is used as the statistical tolerance, i.e. $a = b = 99.7$, which indicates one may be 99.7% confident that the samples are not underestimates of the desired 99.7% fractal. The corresponding sample size is 1933. In order to ensure adequate accuracy, a sample size $N = 2000$ was used in the subsequent analysis.

## Analyzed cases

In order to analyze the effect of uncertainties in HRR on ASET, four cases are studied as shown in Table 2:

Case 1: Both peak HRR and fire growth rate are first considered as deterministic;
Case 2: Uncertainties in peak HRR are studied while a deterministic value is employed for the fire growth rate;
Case 3: Uncertainties in fire growth rate are studied while a deterministic value is used for the peak HRR;
Case 4: Uncertainties in both peak HRR and fire growth rate are studied.

**Table 2.** Four cases used in case study

| Case number | $Q_{max}$ | $\alpha$ |
|---|---|---|
| 1 | D[a] | D |
| 2 | N[b] | D |
| 3 | D | logN[c] |
| 4 | N | logN |

[a]Deterministic value.
[b]Normal distribution.
[c]Log-normal distribution.

## Results and discussion

### *Case 1: the effect of deterministic peak HRR on ASET*

For a commercial building with sprinklers, Morgan and Gardner [37] suggest 5000 kW as the value for peak HRR. Similar national recommendations are also available. For example, the Swedish National Board of Housing, Building and Planning [1] recommends that HRR be kept constant after sprinkler activation if the HRR exceeds 5000 kW. Since no sprinklers are considered in the case study and because large quantities of goods are usually stored in commercial buildings because of high rental prices, the peak HRR could be larger than 5000 kW. For buildings without sprinklers, the Swedish National Board of Housing, Building and Planning [1] recommends a peak HRR of 5000 kW for dwellings, offices, schools, hotels and health care facilities while 10,000 kW is recommended for assembly halls. The Chinese specification for building smoke control [38] suggests 8000 kW as the peak HRR for public buildings without sprinklers. As defined by Office of the Federal Register [39] ('open to the public during normal business hours'), supermarkets and retail stores are considered to be public buildings. Therefore, the upper limit of peak HRR here is set at 8000 kW. On the other hand, due to the variation of the type, quality and position of materials stored, small fires are also likely to occur. In order to cover the uncertainty of peak HRR, the peak HRR of 1000 kW for a fire similar to burning a soft toy mountain [40] is considered as the lower limit for the peak HRR. In order to investigate the influence of deterministic peak HRR on ASET, seven deterministic values of peak HRR are selected from 1500 to 8000 kW: 1500, 2000, 2500, 3000, 4000, 6000 and 8000 kW. The peak HRR needed for flashover for the geometry used in this case is well above the selected peak HRR set according to the Thomas equation [17], nor is there any risk of ventilation-controlled burning based on equation (4) [8].

For the fire growth rate, three common kinds of fires are considered: one with the mean value of a log-normal fire growth rate in commercial buildings according to Holborn et al. [33], a fast fire and an ultrafast fire. The resultant fire growth rates are 0.027, 0.047 and 0.188 $kW/s^2$, respectively.

Figure 4 shows the predicted ASET plotted as a function of the peak HRR and fire growth rate. It indicates that, for the same type fire, the larger the peak HRR, the shorter ASET will be. For the same peak HRR, ASET decreases with an increase of the fire growth rate. Furthermore, when the peak HRR is small, ASET is less influenced by fire growth rate. However, ASET becomes more dependent on the fire growth rate when the peak HRR increases. As a result, for a large peak HRR, an untenable condition is reached before the fire grows to the peak HRR, which indicates ASET is much more dependent on the fire growth rate than peak HRR. However, for a small peak HRR, an untenable condition may be reached after the fire reaches the peak HRR. Under such conditions, ASET will be independent of the fire growth rate and the peak HRR has a more significant influence on ASET.

### *Case 2: the effect of uncertain peak HRR on ASET*

The analysis in Case 1 reveals that ASET is less influenced by fire growth rate when the peak HRR is small, especially when the peak HRR is smaller than 3000 kW for the case study. The influence of the uncertainty associated with peak HRR is analyzed for two situations: one is the situation where the peak HRR ranges

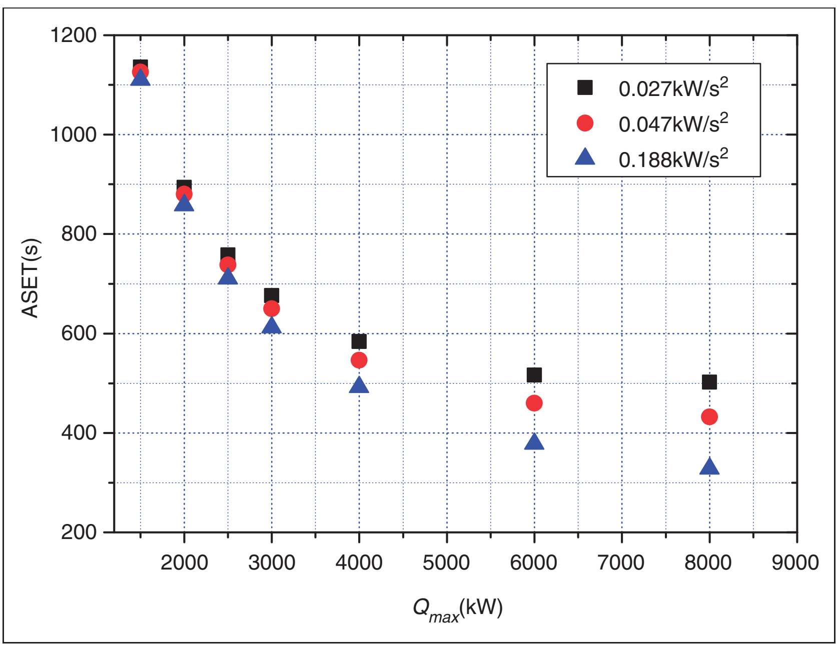


**Figure 4.** ASET calculated by deterministic peak HRR and fire growth rate.
ASET: available safe egress time; HRR: heat release rate.

from 1000 to 3000 kW, denoted as $S_1$; the other one is where the peak HRR ranges from 3000 to 8000 kW, denoted as $S_2$. Since only the uncertainty in the peak HRR is considered, the fire growth rate is treated as a deterministic parameter, so the three growth rates used in the preceding section are investigated. A normal distribution is used to characterize uncertainties in the peak HRR. The mean and variance of the peak HRR are determined by the 'three σ' rule [41], which suggests that the probability of a normal uncertain variable in the interval $[\mu - 3\sigma, \mu + 3\sigma]$ is 0.9774.

Figure 5 shows a box plot of the predicted ASET values for three fires. It can be seen that, for each kind of $S_1$ fire, due to the variation in peak HRR, the predicted ASET ranges from smaller than 700 s to greater than 1370 s, which represents a large variation. Thus, a cautious selection of peak HRR should be made during fire scenario design for this case. Moreover, the shape and range of the three boxes in Figure 5(a) have minor differences, which reveal that the variation of fire growth rates has little effect on ASET for the normal peak HRR ranging from 1000 to 3000 kW. Therefore, in the case of fire scenarios with a peak HRR varying from 1000 to 3000 kW, the selection of peak HRR should be done cautiously while the selection of fire growth rate is relatively less restrictive. For results with the $S_2$ fire, conclusions are different. It can be seen that the standard deviation of predicted ASET for each kind of fire is much smaller than the corresponding ones for the $S_1$ fire. The smaller standard deviation implies that the variation of predicted ASET induced by the variation of peak HRR is not as large as that for the $S_1$ fire. Furthermore, the difference in the range of these boxes in Figure 5(b) is clear, which indicates that the influence of different fire growth rates on the predicted ASET is significant. Therefore, the influence of fire type on ASET is much more important than that of the peak HRR when the normal peak HRR is between 3000 and 8000 kW. The selection of the proper fire type should be done with caution in performance-based fire protection design and analysis when a high peak HRR can be expected.

Since large uncertainties are associated with both ASET and RSET, the conventional method is to assign a safety factor to ASET or RSET to account for these uncertainties [42]. Here, a probabilistic method is provided with the aid of CDF and CCDF. Taking the predicted ASET for the fast fire as an example, the probability of ASET exceeding 706 s is 0.975, which indicates the probability that ASET is smaller than 706 s is only 0.025, as shown in Figure 6(a). The value of 706 s can be considered as characteristic of ASET, denoted as $ASET_c$. Similarly, using CDF for RSET, the characteristic value of RSET, $RSET_c$, which is the corresponding probability of RSET smaller than is 0.975, can be obtained. If $ASET_c$ is greater than $RSET_c$, occupants can be considered safe. Otherwise, measures should be taken to reduce RSET, for example, by improving the evacuation exit width and exit signals; or to increase ASET by installing mechanical smoke exhaust systems and sprinklers, etc.

Another issue that should be discussed is the differences among the three CCDF curves for $S_1$ and $S_2$. The differences among the three CCDF curves for the $S_1$ fire,

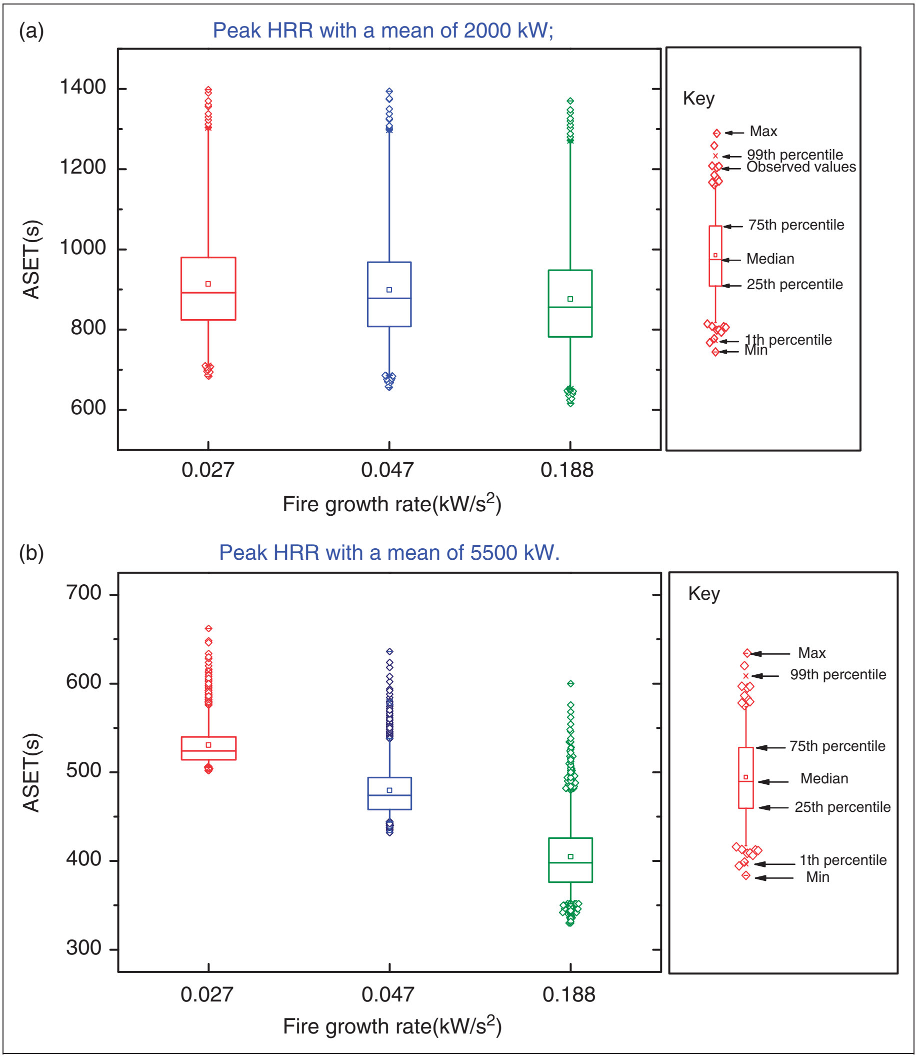


**Figure 5.** Box plot of predicted ASET with normal distribution of peak HRR and deterministic fire growth rates: (a) normal peak HRR with a mean of 2000 kW; (b) normal peak HRR with a mean of 5500 kW.
ASET: available safe egress time; HRR: heat release rate.

(Figure 6(a)) are insignificant; so, these possibly can be ignored. The minor differences indicate that the influence of various fire growth rates on the predicted ASET is negligible when the peak HRR varies from 1000 to 3000 kW. In addition, the 95% confidence intervals of ASET for the three fire growth rates are from about 650 to 1130 s. The large range reveals that there is a major variation in ASET due to the uncertainty associated with peak HRR. Compared to the differences among

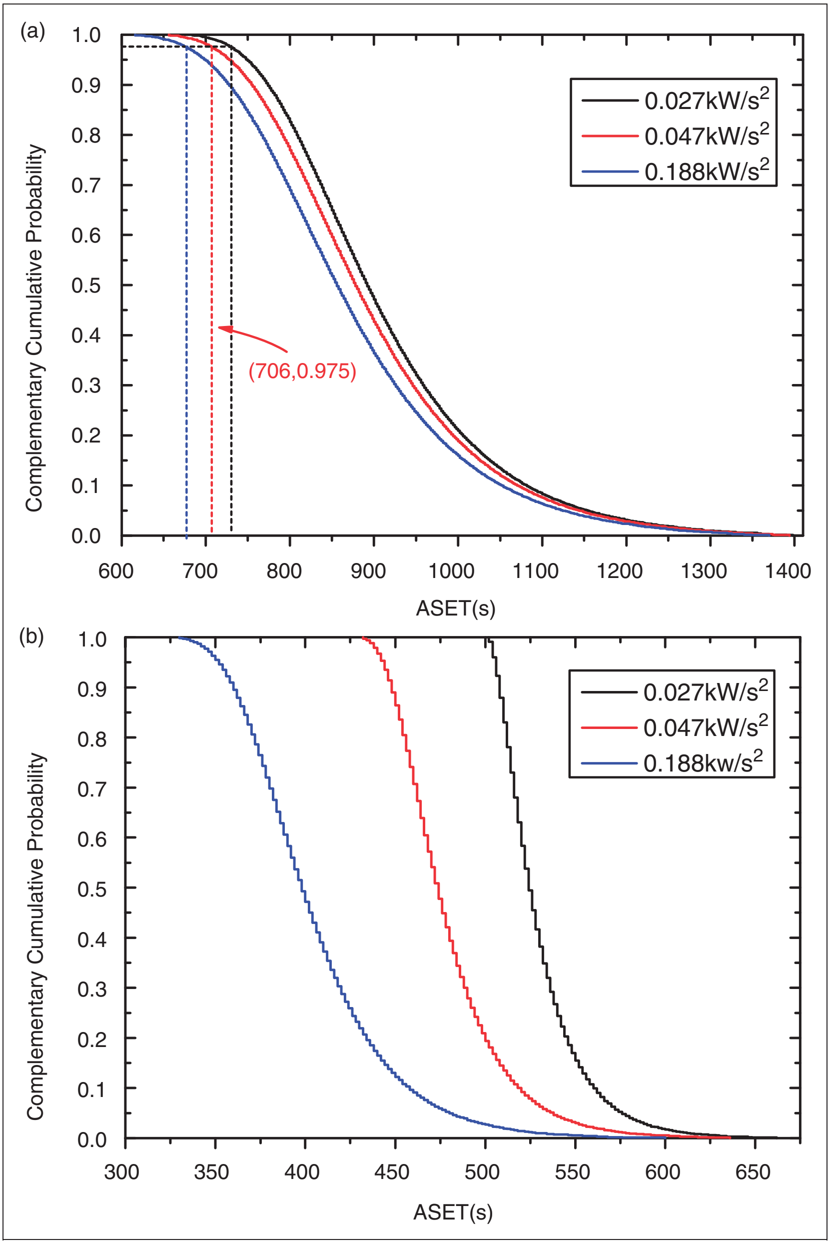


**Figure 6.** Complementary cumulative distribution of predicted ASET with normal distribution of peak HRR for three kinds of fire growth rate: (a) normal peak HRR with a mean of 2000 kW; (b) normal peak HRR with a mean of 5500 kW.
ASET: available safe egress time; HRR: heat release rate.

the three CCDF curves for the $S_1$ fire, the differences among the three kinds of $S_2$ fires are significant as shown in Figure 6(b). However, the 95% confidence intervals among the three kinds of fires do not vary to the same degree as in Figure 6(a). These phenomena indicate that the influence of fire growth rate plays a more important role than that of the peak HRR on the predicted ASET for the $S_2$ fire. These conclusions are consistent with those drawn from Figure 5.

### *Case 3: the effect of uncertain fire growth rate on ASET*

As shown in Case 2, the fire growth rate has a large influence on the predicted ASET even if it is a deterministic value when the peak HRR is uncertain and varies from 3000 to 8000 kW. In this section, the influence of uncertainty associated with fire growth rate on the predicted ASET is discussed when only fire growth rate is the uncertain parameter. The fire growth rate is characterized by a log-normal distribution, logN ($-5.4$, $1.9^2$) [33]. Six deterministic values of peak HRR are selected, i.e. 3000, 4000, 5000, 6000, 7000 and 8000 kW. The same reason for selecting the peak HRR is employed as for Case 1.

The main statistical parameters for the predicted ASET with six different peak HRRs are given in Table 3. The box plots of the predicted ASETs are given in Figure 7. It can be seen that, due to the uncertainty associated with fire growth rate, the predicted ASET has a variation for each peak HRR. As peak HRR increases from 3000 to 8000 kW, this variation range increases from 128 to 310 s. It can be concluded that the influence of uncertainty associated with fire growth rate gradually becomes more important as the peak HRR increases.

When the peak HRR varies from 3000 to 6000 kW, the differences in the range of predicted ASET as well as the shape of the four box plots are much larger than that in the case when the peak HRR varies from 6000 to 8000 kW. In the latter case, the differences are so small that they possibly can be ignored. It can be concluded that, when fire growth rate is considered uncertain, the peak HRR has a significant influence on the distribution of the predicted ASET even when

**Table 3.** Statistics of the predicted ASET from 6 peak HRRs

| | Main statistics for ASET | | | | | |
|---|---|---|---|---|---|---|
| Peak HRR (kW) | Mean (s) | SD (s) | Min (s) | Median (s) | Max (s) | Range (s) |
| 3000 | 674.4 | 35.8 | 612 | 674 | 740 | 128 |
| 4000 | 581.2 | 50.1 | 492 | 580 | 672 | 180 |
| 5000 | 534.6 | 62.6 | 424 | 534 | 648 | 224 |
| 6000 | 510.5 | 74.0 | 378 | 510 | 642 | 264 |
| 7000 | 498.3 | 82.8 | 348 | 498 | 640 | 292 |
| 8000 | 491.3 | 88.3 | 328 | 494 | 638 | 310 |

ASET: available safe egress time; HRR: heat release rate.

it is considered deterministic. As the peak HRR increases, an untenable condition may be reached before the peak HRR is attained, thus making the predicted ASET almost independent of peak HRR above a threshold value, as shown in Table 3 and Figure 7.

## Case 4: the effect of both uncertain peak HRR and fire growth rate on ASET

In the above analysis, the influence of uncertainties associated with fire growth rate and peak HRR on the predicted ASET is discussed separately. In this section, the influence of uncertainties in both parameters is analyzed simultaneously. The distribution of fire growth rate is the same as that in Case 3 and the distribution of peak HRR is the same as that in Case 2.

The probability distribution of the predicted ASET is given in Figure 8, where best-fit log-normal distributions are plotted. The log-normal curve fits the distribution of the predicted ASET well when the peak HRR is between 1000 and 3000 kW (Figure 8(a)). However, the log-normal distribution curve does not fit so well for peak HRR varying from 3000 to 8000 kW. More than 80% of the predicted ASET lies within the range 600–950 s for peak HRR varying from 1000 to 3000 kW; the range is 400–630 s for peak HRR varying from 3000 to 8000 kW. These are the ranges where an untenable condition is most likely to occur. Based on this, fire safety engineers may judge whether the selected peak HRR and fire growth rate can meet requirements in the performance-based fire protection design.

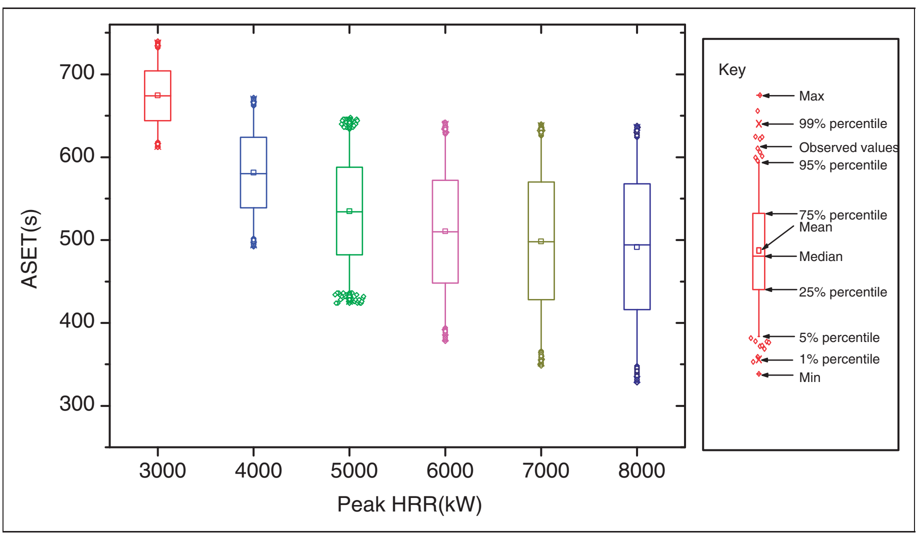


**Figure 7.** Box plot of the predicted ASET with six different peak HRRs.
ASET: available safe egress time; HRR: heat release rate.

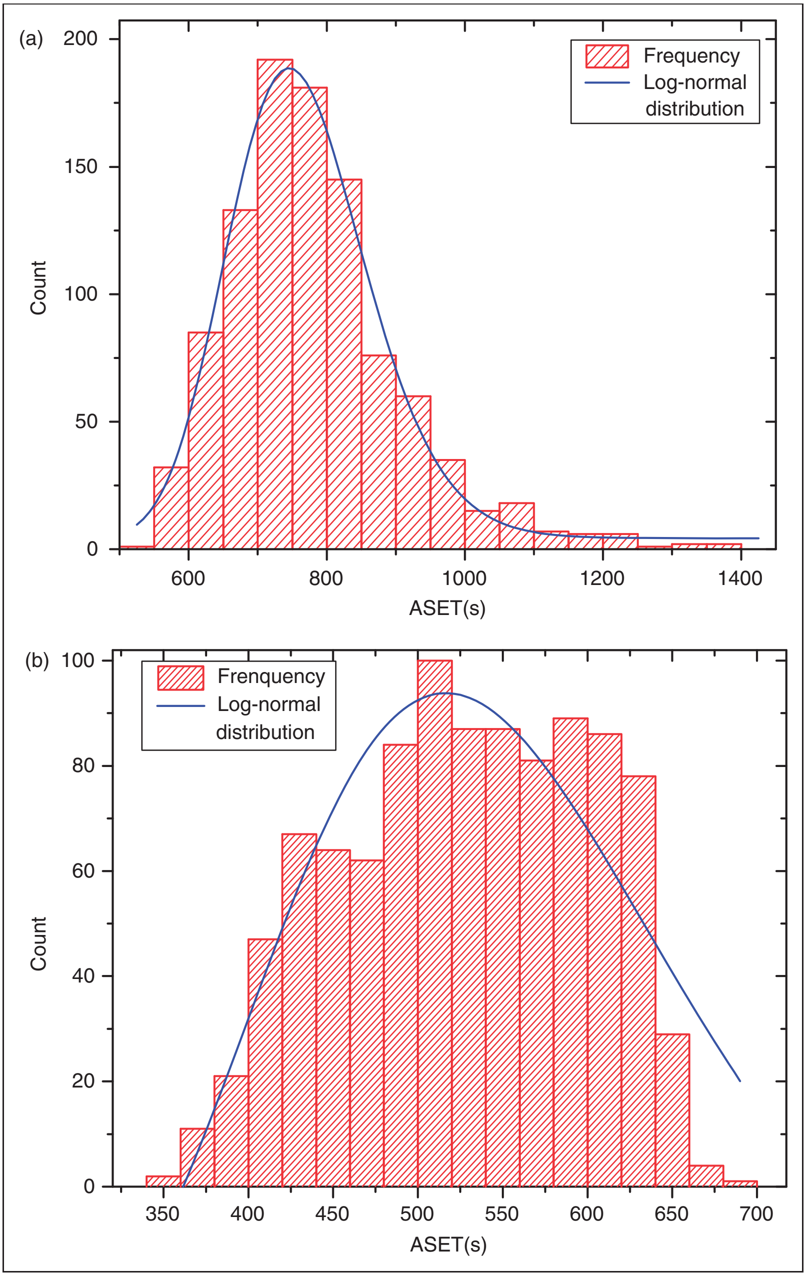


**Figure 8.** Probability distribution of predicted ASET and theoretical log-normal distribution curve fit: (a) normal peak HRR with a mean of 2000 kWand log-normal fire growth rate; (b) normal peak HRR with a mean of 5500 kW and log-normal fire growth rate.
ASET: available safe egress time; HRR: heat release rate.

With the help of CDF and CCDF of the predicted ASET, fire safety engineers can effectively develop a design and jurisdiction authorities can better make regulatory decisions. For example, from the curve of CDF shown in Figure 9(a), the probability that the predicted ASET is not more than 900 s is 84.8% (i.e. $P[\mathrm{ASET} \leq 900\,\mathrm{s}] = 0.848$). With this probability, engineers cannot only judge whether the current design needs improvement but also provide a probabilistic assessment of the uncertainty in the selection of the peak HRR and fire growth rate and then give a rational basis for analysis.

Fire safety engineers can also determine whether or not a selected most-likely fire scenario is proper as a representative fire scenario. Taking this case as an example, the predicted ASET for the most-likely fire scenario is 526 s. The corresponding probability that the predicted ASET is smaller than 526 s is 48.6%. This indicates that nearly half of the fire scenarios are worse than the most-likely fire scenario. Therefore, in this case, the most-likely fire scenario is not appropriate as the representative fire scenario.

In order to rank the influence of uncertain peak HRR and fire growth rate, a sensitivity analysis is performed. The partial correlation coefficient (PCC) and partial rank correlation coefficient (PRCC) are employed to measure the sensitivity of the predicted ASET on the uncertain peak HRR and fire growth rate. PCC measures the linear relationship between one input parameter and the output after the linear effects of other input parameters have been removed; similarly, PRCC provides a measurement of the non-linear but still monotonic relationship between one input and the output after removing the effect of other input parameters. A detailed description of PCC and PRCC can be found in the studies by Frey and Patil [43] and Iman and Helton [44].

The calculated PCCs and PRCCs for the case study are given in Figure 10. It can be seen that the signs for both PCCs and PRCCs for peak HRR and fire growth rate are negative, which suggests that the larger the peak HRR and fire growth rate are, the smaller ASET will be. Both PCCs and PRCCs in Figure 10(a) indicate that the peak HRR has a larger influence than the fire growth rate on the predicted ASET when the peak HRR ranges from 1000 to 3000 kW. However, when the peak HRR increases to the range of 3000 to 8000 kW, the sensitivity results in Figure 10(b) suggest that the influence of fire growth rate on predicted ASET becomes more significant than that of the peak HRR.

## *Conclusions for the analyzed case study*

For a normal distribution of peak HRR with a low mean value, the predicted ASET varies over a large range. Variation in fire growth rate has little effect on the distribution of predicted ASET for small normal peak HRR. However, for normal peak HRR with a large mean value, the range of predicted ASET becomes small. The effect of different values for the fire growth rate becomes more important than that for the normal peak HRR with a low mean value.

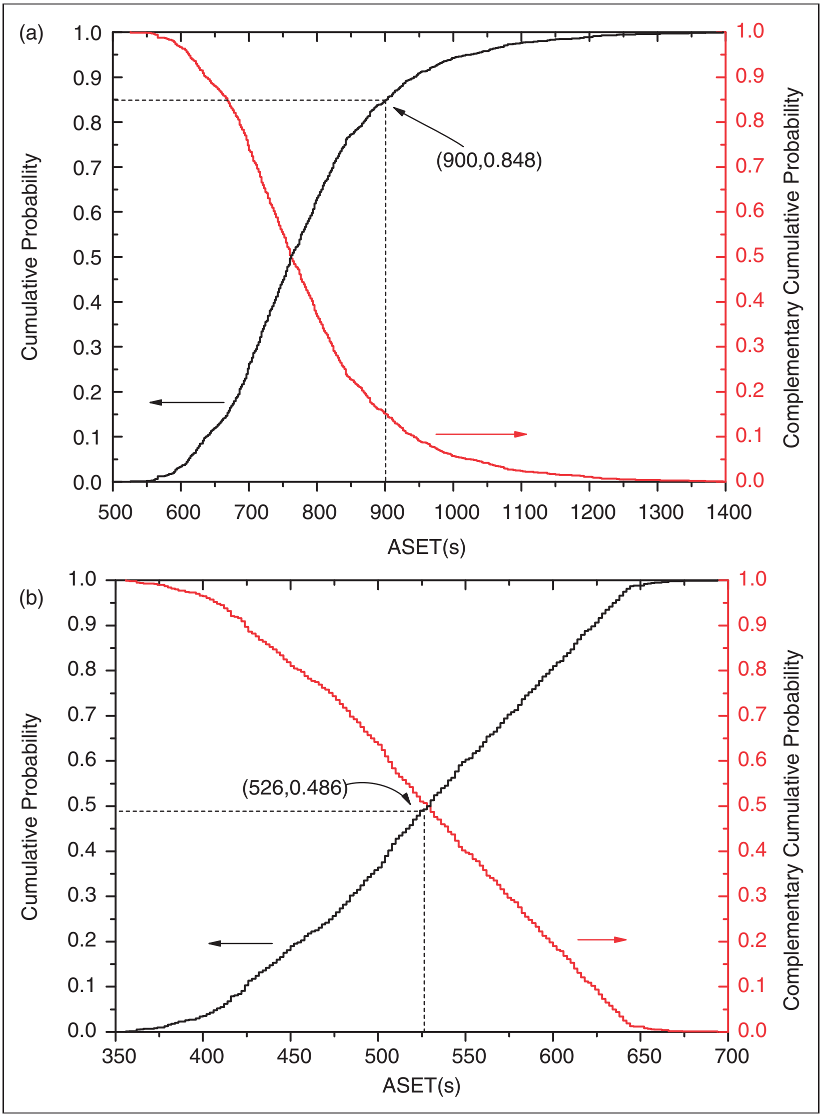


**Figure 9.** Cumulative probability curve and complementary cumulative probability curve of predicted ASET: (a) normal peak HRR with a mean of 2000 kW and log-normal fire growth rate; (b) normal peak HRR with a mean of 5500 kW and log-normal fire growth rate. ASET: available safe egress time; HRR: heat release rate.

For a log-normal fire growth rate and six different deterministic peak HRRs, it is not useful to analyze the effects of a higher peak HRR for the studied building larger than a certain threshold peak HRR if the fire growth rate suggested by Holborn et al. [33] is used. This is an important conclusion,

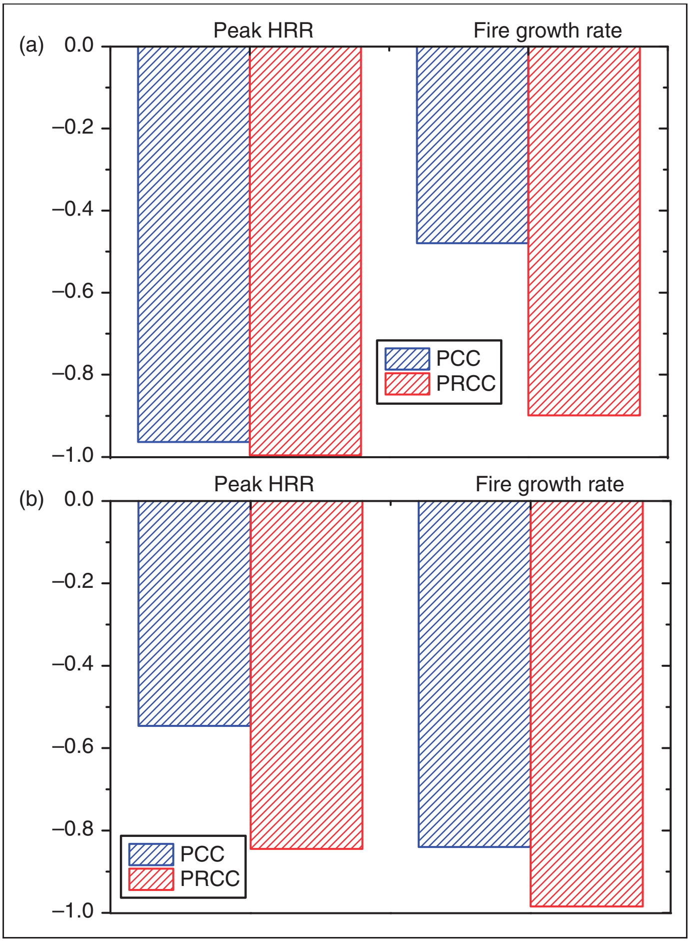


**Figure 10.** Sensitivity results based on PCC and PRCC for the predicted ASET: (a) normal peak HRR with a mean of 2000 kW and log-normal fire growth rate; (b) normal peak HRR with a mean of 5500 kW and log-normal fire growth rate.
ASET: available safe egress time; HRR: heat release rate; PCC: partial correlation coefficient; PRCC: partial rank correlation coefficient.

since as mentioned earlier, little data are available on peak HRR in compartment fires.

## Limitations

An approach to quantify the effect due to uncertainties associated with peak HRR and fire growth rate on predicted ASET is presented. It is shown that, for a specific case study, there are threshold peak heat release rates for certain fire growth distributions. This conclusion is arrived at with certain assumptions.

The distribution of peak HRR is arbitrarily assumed to follow a normal distribution due to the limited available statistical data and knowledge about uncertainties in the peak HRR. Assuming an arbitrary probabilistic distribution to the peak HRR would increase the overall uncertainty of the model.

The influence of the geometry of the compartment such as the height and position of the doors on ASET is not considered. These influences are expected to be significant. Nevertheless, the uncertainty analysis approach presented in this study can be applied for any building. Considering the effect of uncertainties in HRR for various building geometries is a topic for future studies.

A fire scenario without sprinklers was considered in the study. Sprinklers have a significant influence on fire development. In addition, the smoke layer height was employed as the tenability criterion for ASET. Some other criteria, such as the smoke layer temperature, visibility and toxicity, could influence the calculation of ASET. Hence, the influence of uncertainties in HRR on ASET in a sprinkler-installed scenario as well as with other calculated criteria for ASET should be studied.

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# Appendix

## *Notation*

$a$ = fractal
$\alpha$ = fire growth rate (kW/s$^2$)
$\alpha_1, \alpha_2, \ldots, \alpha_N$ = sampled fire growth rate (kW/s$^2$)
$A_f$ = burning area of the fuel (m$^2$)
$A_o$ = area of the ventilation openings (m$^2$)
$ASET_c$ = characteristic value of ASET (s)
$A_t$ = total surface are of the compartment (m$^2$)
$b$ = confidence level of a fractal

$f$ = simplified presentation of two-zone model used in CFAST
$H_o$ = height of ventilation openings (m)
$N$ = number of Latin Hypercube Samples
$\hat{P}(ASET)$ = empirical cumulative distribution probability of ASET
$PU(1), PU(2), \ldots, PU(5)$ = cumulative probability of the sampled values of $U$
$PV(1), PV(2), \ldots, PV(5)$ = cumulative probability of the sampled values of $V$
$Q''$ = heat release rate per unit area (kW/m$^2$)
$Q_{fo}$ = heat release rate required for flashover (kW)
$Q_{\max}$ = peak heat release rate (kW)
$Q_{\max1}, Q_{\max2}, \ldots, Q_{\max N}$ = sampled peak heat release rate (kW)
$Q_{\max,\mathrm{f}}$ = peak heat release rate for fuel-controlled (kW)
$Q_{\max,\mathrm{v}}$ = peak heat release rate for ventilation-controlled (kW)
$\dot{Q}(t)$ = heat release rate at time $t$ (kW)
$r$ = subscript for which $ASET_r \leq ASET$
$RSET_c$ = characteristic value of RSET (s)
$t_g$ = time when heat release rate reaches the peak heat release rate (s)
$U(1), U(2), \ldots, U(5)$ = sampled values of variable $U$
$V(1), V(2), \ldots, V(5)$ = sampled values of variable $V$
$\mu$ = mean value of a uncertain parameter
$\sigma$ = standard deviation of a uncertain parameter