# A Simple Approximation to Predict the Transition from a Balcony Spill Plume to an Axisymmetric Plume

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**ABSTRACT:** All existing simple entrainment calculation methods for the three-dimensional (3D) balcony spill plume assume that the mass flow rate of gases produced increases linearly with height of rise. It is recognized that at very large heights of rise the balcony spill plume will eventually behave like an axisymmetric plume in terms of entrainment behavior. The transition from a balcony spill plume to axisymmetric has not been rigorously studied and current guidance can only be considered to be an estimate. To remedy this situation, a general approach similar to that used in previous work is utilized to develop a simple approximation that predicts the height of transition in entrainment behavior. This type of analysis is further supported with a limited number of simulations using numerical modeling. The proposed simple approximation provides an upper height limit for which linearly based 3D balcony spill plume entrainment formulae can successfully be applied.

**KEY WORDS:** balcony spill plume, axisymmetric plume, smoke management, entrainment.

## INTRODUCTION

THE DESIGN OF smoke management systems requires appropriate entrainment calculation methods to predict the volume of smoky gases produced in a fire in order to determine the required exhaust fan capacity or ventilator area for a design clear layer height. Consideration is often given to entrainment of air into a smoke flow from a compartment opening that subsequently spills at a balcony edge and then rises into an adjacent atrium void. This type of thermal plume is commonly known as

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a balcony spill plume (Figure 1). The smoke flow will spill at the balcony edge (i.e., the spill edge) and rise into the atrium space with a large surface area over which entrainment of air occurs. The amount of entrainment can be reduced by restricting the ability of the smoke flow to spread laterally with the use of channeling screens (otherwise referred to as draft curtains) beneath the balcony to 'channel' the flow to the balcony edge. Spill plumes that do not include entrainment into the ends of the plume are known as two-dimensional (2D) plumes and those that include end entrainment are

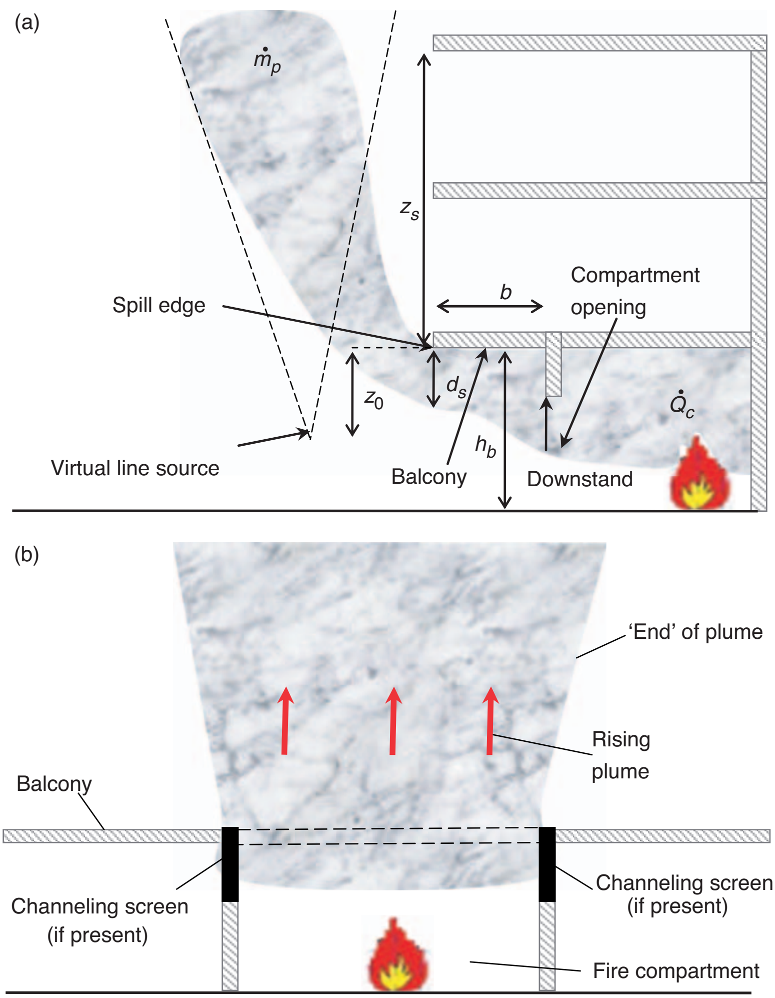


***Figure 1.*** *A typical 3D balcony spill plume: (a) section, (b) front view.*

known as three-dimensional (3D) plumes. The plume behavior shown in Figure 1 assumes that the plume has sufficient buoyancy for it to rise into the atrium void. Clearly the impact of sprinklers in the fire compartment may cause sufficient cooling of the gases to affect plume behavior and entrainment. However, if the fire is partially shielded from the sprinkler spray there can be a sufficient convective flow of hot gases for the plume to rise into the void. In existing guidance for the spill plume (e.g., [1]) sprinklered design fires assume fires are shielded as a conservative approach.

All current simple entrainment calculation methods for the 3D balcony spill plume assume that the mass flow rate of gases increases linearly with height of rise. Where the design scenario involves a spill plume which rises unhindered into a tall atrium void, it is recognized that entrainment into the free ends of the plume will eventually cause it to behave like an axisymmetric plume [2] (i.e., the mass flow rate will increase according to a power law beyond the transition height). This can potentially lead to unsafe designs if the designer extrapolates the linear relationship between mass flow rate and height of rise beyond the height of transition. Therefore, simple guidance to predict the height of transition is of use to fire engineers as an upper limit to which linearly based spill plume formulae can be applied and above which an axisymmetric entrainment formula should be used instead.

Very limited and differing guidance on the location of the height of transition is given by CIBSE [3] and previously by NFPA 92B [4]. The transition from a 3D balcony spill plume to an axisymmetric plume has not been rigorously studied and current guidance can only be considered to be an estimate. Thomas et al. [5] have proposed that the height of transition in entrainment behavior ($z_{trans}$) will be dependent upon the ratio between the height of rise of the plume ($z_s$) and the lateral extent of the plume below the spill edge ($W_s$). Existing guidance assumes that the spill plume rises unhindered into an atrium void without any interaction of the plume with balconies that may exist above the spill edge. Recent work by Tan et al. [6] has examined spill plume trajectories and interaction between the plume and balconies above the spill edge. They showed that the plume trajectory is dependent on factors such as the spill plume width, balcony breadth, and fire size, and demonstrated that the plume can sometimes rise unhindered into the void without interaction with balconies above (if such are present).

The analysis in this article assumes a simple case of a spill plume that rises unhindered into an atrium void and begins with an empirical entrainment formula for a channeled 3D balcony spill plume recently proposed by Harrison and Spearpoint [7]. The entrainment formula, derived from reduced-scale experimental data, is used to obtain a simple approximation to predict the height at which a 3D balcony spill plume and an axisymmetric plume give equivalency in terms of entrainment behavior. This analysis is supported with a limited number of numerical simulations using Computational Fluid Dynamics modeling to extrapolate the entrainment analysis to much greater heights of rise than were possible in the previous [7] experiments.

## PREVIOUS ANALYSIS

Thomas [2] first presented an analysis to determine a general expression for $z_{trans}$ by matching design equations for a balcony spill plume and an axisymmetric plume to give equivalency in terms of entrainment behavior at high heights of rise. To describe balcony spill plume entrainment, Thomas used the Lee and Emmons line plume model [8] given by:

$$\begin{aligned} \dot{m}_p &= 0.58\left(\frac{\rho_{amb}^2 g}{c_{p,air} T_{amb}}\right)^{1/3} \dot{Q}_c^{1/3} W^{2/3}(z + z_0) \\ &\Rightarrow \dot{m}_p = 0.21\dot{Q}_t^{1/3} W^{2/3}(z_s + z_0) \end{aligned} \tag{1}$$

To describe axisymmetric plume entrainment, Thomas used the following formula given by Zukoski [9]:

$$\begin{aligned} \dot{m}_p &= 0.21\left(\frac{g}{\rho_{amb} c_{p,air} T_{amb}}\right)^{1/3} \dot{Q}_c^{1/3}(z + z_0)^{5/3} \\ &\Rightarrow \dot{m}_p = 0.071\dot{Q}_c^{1/3}(z + z_0)^{5/3} \end{aligned} \tag{2}$$

Thomas determined that at high heights of rise, Equations (1) and (2) become equivalent when:

$$W = W_s + 0.22(z + 2z_0) \tag{3}$$

This analysis was used to develop a formula for 3D balcony spill plume entrainment given by:

$$\dot{m}_{p,3D} = 0.21\dot{Q}_c^{1/3}(z_s + z_0)(W_s + 0.22(z_s + 2z_0))^{2/3} \tag{4}$$

Thomas states that Equation (4) exhibits linearity in entrainment for small $z_s/W_s$ but becomes axisymmetric in nature (i.e., dependent on $z_s^{5/3}$) beyond $z_s/W_s \approx 5$. Therefore:

$$z_{trans} = 5W_s \tag{5}$$

Heskestad [10] employed a similar approach to that of Thomas but did not use the Lee and Emmons model to describe balcony spill plume entrainment. Instead, Heskestad used the 3D balcony spill plume formula by Law [11] given by:

$$\dot{m}_{p,3D} = 0.34\left(\dot{Q}_t W_s^2\right)^{1/3}(z_s + 0.15h_b) \tag{6}$$

As Equation (6) is dependent on $\dot{Q}_t$, it was assumed that $\dot{Q}_c = 0.7\dot{Q}_t$ so that it becomes:

$$\dot{m}_{p,3D} = 0.38\left(\dot{Q}_c W_s^2\right)^{1/3}(z_s + 0.15h_b) \tag{7}$$

To further simplify the analysis, Heskestad assumed that the terms describing the location of the virtual line source in Equations (2) and (7) (i.e., the $z_0$ and $0.15h_b$) could be neglected as these become insignificant at high heights of rise. Thus, Equation (2) for the axisymmetric plume becomes:

$$\dot{m}_p = 0.071\dot{Q}_c^{1/3} z_s^{5/3} \tag{8}$$

(assuming $z = z_s$) and Equation (7) for the 3D balcony spill plume becomes:

$$\dot{m}_{p,3D} = 0.38\dot{Q}_c^{1/3} W_s^{2/3} z_s \tag{9}$$

Heskestad then expressed Equation (9) in a form compatible with Equation (8) given by:

$$\dot{m}_{p,3D} = 0.071\dot{Q}_c^{1/3} z_s^{5/3} / (0.08 z_s / W_s)^{2/3} \tag{10}$$

Therefore, Equation (10) reduces to Equation (8) when:

$$0.080 z_s / W_s = 1 \Rightarrow z_s / W_s = 12.5 \tag{11}$$

Hence:

$$z_{trans} = 12.5 W_s \tag{12}$$

This criterion is essentially the same as that given within an earlier version of NFPA 92B [4] given by:

$$z_{trans} = 13 W_s \tag{13}$$

Clearly, there are significant differences in the general expressions to describe $z_{trans}$ from the Thomas and Heskestad analyses which is also reflected in differing guidance given by CIBSE and NFPA 92B. The only significant difference between each analysis is the entrainment coefficient assumed for the 3D balcony spill plume (i.e., the 0.21 in Equation (1) and the 0.38 in Equation (7)). Since the analysis methods were essentially the same, it is unsurprising that different expressions for $z_{trans}$ were obtained if different entrainment coefficients were assumed for the 3D balcony spill plume. The 0.21 value assumed by Thomas was for line plume entrainment, yet the 0.38 assumed by Heskestad was determined from empirical 3D balcony spill plume data where end entrainment was more significant than in a line plume. This analysis suggests that expressing $z_{trans}$ simply as a factor of $W_s$ may not be entirely appropriate as it will not give a general expression that applies for a variety of 3D balcony spill plumes where the contribution of end entrainment varies. The analysis described by Harrison and Spearpoint [7] demonstrates that entrainment into 3D balcony spill plumes is dependent upon the width and depth of the layer flow below the spill edge (i.e., $W_s$ and $d_s$) and it appears that a general expression to describe $z_{trans}$ should include both of these terms.

## MATCHING 3D BALCONY SPILL PLUME AND AXISYMMETRIC PLUME ENTRAINMENT

A similar analysis to that described above can be carried out to determine $z_{trans}$ by matching the simplified entrainment formula for the 3D balcony spill plume proposed by Harrison and Spearpoint [7] (Equation (14)) with the entrainment formula for an axisymmetric plume:

$$\dot{m}_{p,3D} = 0.16\dot{Q}_c^{1/3}\left(W_s^{2/3} + 1.56d_s^{2/3}\right) z_s + 1.34\dot{m}_s \tag{14}$$

Equation (14) was determined from an extensive series of 1/10th physical scale modeling experiments for plumes generated from a range of fire compartment geometries, fire sizes, and heights of rise of plume, and applies more generally than existing simplified design formulae. Equation (14) is a linearly based entrainment formula as the data did not exhibit axisymmetric behavior over the heights of rise of plume that were possible to be measured experimentally.

The analysis can be simplified by only considering the rate of entrainment above the spill edge (consistent with the approach by Heskestad), as the entrainment below the spill edge is relatively insignificant in the overall entrainment process at large heights of rise. As it is not possible to specify entrainment above the spill edge in Equation (14) in a form that is compatible with Equation (8), the last term in Equation (14) (i.e., the $1.34\dot{m}_s$) is neglected as a simplifying assumption to give:

$$\dot{m}_{p,3D} = 0.16\dot{Q}_c^{1/3}\left(W_s^{2/3} + 1.56d_s^{2/3}\right) z_s \tag{15}$$

Equation (15) can be expressed in a form compatible with Equation (8) (for the axisymmetric plume) such that:

$$\dot{m}_{p,3D} = \frac{0.071\dot{Q}_c^{1/3}\left(W_s^{2/3} + 1.56d_s^{2/3}\right) z_s^{5/3}}{0.44z_s^{2/3}} \tag{16}$$

Therefore, Equation (16) reduces to Equation (8) when:

$$\frac{\left(W_s^{2/3} + 1.56d_s^{2/3}\right)}{0.44z_s^{2/3}} = 1 \tag{17}$$

Hence, $z_{trans}$ is given by:

$$z_{trans} = 3.4\left(W_s^{2/3} + 1.56d_s^{2/3}\right)^{3/2} \tag{18}$$

Equation (18) is dependent on $W_s$ and $d_s$ (rather than $W_s$ alone) which seems reasonable following the analysis described by Harrison and Spearpoint [7]. Table 1 shows the prediction of $z_{trans}$ (m) using Equation (18) for the range of $W_s$ and $d_s$ examined in the Harrison and Spearpoint [7] experiments. Table 1 also shows the transition in entrainment expressed in a nondimensional form consistent with Thomas and Heskestad (i.e., in terms of $z_{trans}/W_s$).

Table 1 indicates that, in general, the absolute value of $z_{trans}$ tends to decrease as $W_s$ decreases. This is expected as narrower plumes will tend to become axisymmetric in nature at lower heights of rise compared to wider plumes due to end entrainment being more significant in the overall entrainment process. This is consistent with the analysis made by Harrison and Spearpoint [7], which shows that entrainment is dependent on the characteristics of the layer flow below the spill edge in terms of $W_s$ and $d_s$, such that plumes generated from narrow, deep layer flows entrain air at a greater rate with respect to height compared to plumes generated from wide, shallow layers.

However, Table 1 gives a small increase in $z_{trans}$ as $d_s$ increases due to the increase in $\dot{Q}_t$ (and hence $\dot{Q}_c$) for each $W_s$ examined. This is somewhat

***Table 1. Values of $z_{trans}$ for the range of $W_s$ and $d_s$ examined in the experiment.***

| $\dot{Q}_t$ (kW) | $Q_c$(kW) | $W_s$ (m) | $d_s$ (m) | $z_{trans}$(m) | $z_{trans}/W_s$ |
|---|---|---|---|---|---|
| 5.0 ± 0.3 | 3.6 ± 0.2 | 1.0 | 0.100 ± 0.005 | 5.3 | 5.3 |
| 10.0 ± 0.3 | 8.0 ± 0.6 | 1.0 | 0.115 ± 0.005 | 5.5 | 5.5 |
| 15.0 ± 0.3 | 12.2 ± 1.0 | 1.0 | 0.125 ± 0.005 | 5.6 | 5.6 |
| 5.0 ± 0.3 | 3.7 ± 0.3 | 0.8 | 0.105 ± 0.005 | 4.5 | 5.6 |
| 10.0 ± 0.3 | 7.8 ± 0.6 | 0.8 | 0.115 ± 0.005 | 4.7 | 5.9 |
| 15.0 ± 0.3 | 12.8 ± 0.8 | 0.8 | 0.135 ± 0.005 | 4.9 | 6.1 |
| 5.0 ± 0.3 | 3.9 ± 0.2 | 0.6 | 0.110 ± 0.005 | 3.8 | 6.3 |
| 10.0 ± 0.3 | 7.7 ± 0.5 | 0.6 | 0.120 ± 0.005 | 3.9 | 6.5 |
| 15.0 ± 0.3 | 12.2 ± 0.7 | 0.6 | 0.140 ± 0.005 | 4.1 | 6.8 |
| 5.0 ± 0.3 | 3.6 ± 0.2 | 0.4 | 0.115 ± 0.005 | 3.0 | 7.5 |
| 10.0 ± 0.3 | 7.2 ± 0.4 | 0.4 | 0.125 ± 0.005 | 3.1 | 7.8 |
| 15.0 ± 0.3 | 10.9 ± 0.6 | 0.4 | 0.145 ± 0.005 | 3.3 | 8.3 |
| 5.0 ± 0.3 | 3.5 ± 0.2 | 0.2 | 0.135 ± 0.005 | 2.2 | 11.0 |
| 10.0 ± 0.3 | 6.6 ± 0.4 | 0.2 | 0.155 ± 0.005 | 2.4 | 12.0 |
| 15.0 ± 0.3 | 9.9 ± 0.6 | 0.2 | 0.170 ± 0.005 | 2.5 | 12.5 |

counter-intuitive as it is expected that as $d_s$ approaches $W_s$ then $z_{trans}$ should decrease, as the aspect ratio of the layer flow below the spill edge becomes more square than rectangular in nature (hence, a greater contribution of end entrainment above the spill edge). This is most likely due to the assumption of neglecting the term describing the entrainment below the height of the spill edge in Equation (14) (i.e., the $1.34\dot{m}_s$) which should ideally be subtracted from $\dot{m}_{p,3D}$ when considering the entrainment above the spill edge. The value of $1.34\dot{m}_s$ will increase as $\dot{Q}_t$ increases and neglecting this subtraction probably explains the increase in $z_{trans}$. However, as these differences are small and Equation (18) is an approximate solution, it seems reasonable that these differences can be ignored in the analysis.

Table 1 also shows that if the transition in entrainment is expressed in nondimensional form dependent only upon $W_s$, then $z_{trans}/W_s$ varies between approximately 5 and 13. It is encouraging to note that for spill plumes where $W_s \gg d_s$ (i.e., for spill plumes similar in nature to line plumes) then $z_{trans}/W_s \approx 5$ which is consistent with the analysis by Thomas, and for plumes where $W_s \approx d_s$ then $z_{trans}/W_s \approx 13$ similar to that determined by Heskestad. This variation in $z_{trans}/W_s$ with respect to $W_s$ demonstrates that it is not appropriate to generally describe the transition in entrainment in terms of $W_s$ alone. This gives further support in the use of Equation (18) to describe $z_{trans}$, which is dependent on $W_s$ and $d_s$ which then reconciles differences between existing guidance and applies more generally. Equation (18) describes an upper height limit that Equation (14) (or any linear based entrainment formula) can successfully be applied, and at a height beyond $z_{trans}$ the entrainment in the plume should be determined using Equation (8) for an axisymmetric plume.

## FDS VERSION 5 MODELING

Harrison [12] describes numerical modeling of the Harrison and Spearpoint [7] experiments using Fire Dynamics Simulator (FDS), version 5 [13]. It is shown in [12] that FDS provides a good prediction of 3D balcony spill plume entrainment when compared against the experimental results. This gives confidence in the further use of FDS to examine entrainment from identical plumes to that produced in the Harrison and Spearpoint experiments, but over a much greater height of rise than was possible experimentally. Table 2 shows the series of four FDS simulations carried out to examine plume entrainment up to a height of rise of 5.0 m on model scale (i.e., 50 m full scale), for plume widths varying from 0.2 to 1.0 m on model scale (2–10 m full scale). The fire size was assumed to be 10 kW on model scale (3.2 MW full scale equivalent). The analysis will also enable the performance of Equation (18) to predict the height in transition behavior to be assessed against the FDS predictions.

The modeling procedure and assumptions made were essentially the same as that described by Harrison [12] and therefore a description is not repeated here. The only significant difference was the increased size of computational domain used to encompass the rising plume. The domain had dimensions of $4.0 \times 4.0 \times 6.0$ m high (all dimensions are 1/10th scale), encompassing 10 separate domains (each $4.0 \times 4.0 \times 0.6$ m high), which were assigned for parallel processing purposes. This allowed heights of rise of plume of up to 5.0 m to be examined due to a shallow smoke layer forming under the ceiling of the domain.

Figure 2 shows the modeled geometry, the computational domain and a typical plume produced showing a velocity contour (0.5 m/s) for Simulation STR3. A uniform grid size of 25 mm was used for each domain encompassing a total of approximately 6.1 million grid elements.

**Table 2. The series of FDS simulations.**

| Simulation | $\dot{Q}_t$ (kW) | $W_s$ (m) | $z_s$ (m) |
|---|---|---|---|
| STR1 | 10.0 | 0.2 | 0–5.0 |
| STR2 | 10.0 | 0.4 | 0–5.0 |
| STR3 | 10.0 | 0.6 | 0–5.0 |
| STR4 | 10.0 | 1.0 | 0–5.0 |

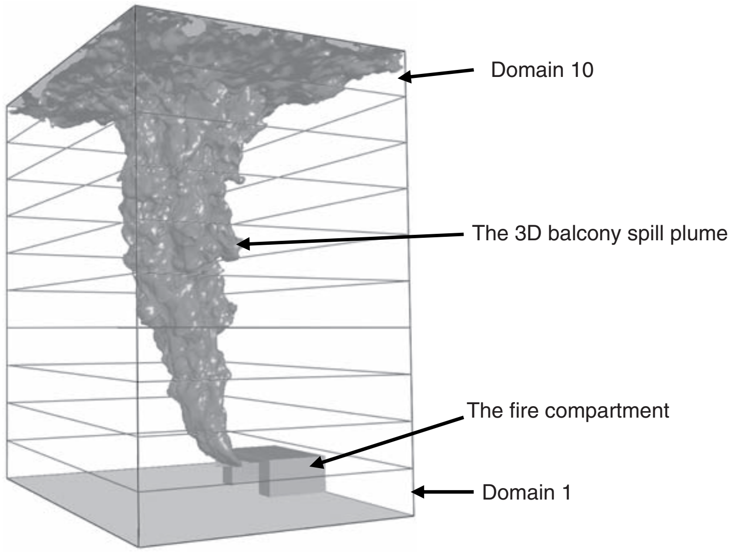


***Figure 2.*** *The geometry, computational domain and plume (Simulation STR3).*

Each simulation took approximately 30 days to complete. Predictions of $\dot{m}_{p,3D}$ were obtained from specified horizontal planar areas occupied by the plume every 0.1 m above the spill edge up to a maximum height of 5.0 m.

## Plume Behavior

Figure 3 shows a plan view of horizontal velocity slice files through the plume with increasing height above the spill edge for Simulation STR3 (i.e., $W_s = 0.6$ m). These slice files highlight the shape of the plume cross-section with increasing height.

Figure 3 shows an important change in the shape of the cross-sectional area of the plume as it rises. Close to the balcony (Figure 3(a)), the plume can be considered to be wide and narrow and similar in nature to a line plume. However, as $z_s$ increases (Figure 3(b) and (c)), it appears that the action of entrainment into the ends of the plume causes the cross section to become more circular in nature, similar to an axisymmetric plume. The velocity tends to a maximum close to the plume centerline with increasing height. The cross-sectional area shown in Figure 3(c) was generally typical for the plume at $z_s > 1.0$ m. This change in the cross-sectional area gives further support to the hypothesis that a 3D balcony spill plume will eventually behave like an axisymmetric plume due to end entrainment. This analysis assumes that the plume rises unhindered into the atrium void.

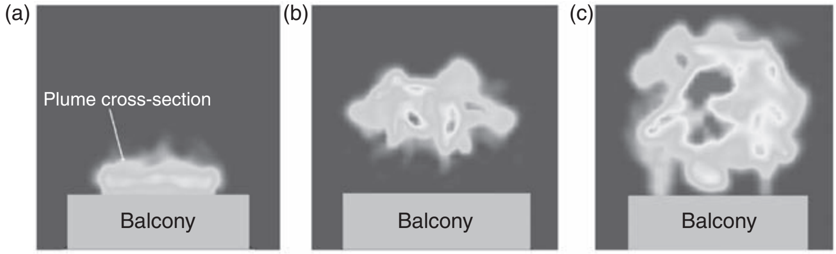


***Figure 3.*** *Horizontal cross-sectional area of the plume with height (Simulation STR3): (a) $z_s = 0$ m, (b) $z_s = 0.5$ m, (c) $z_s = 1.0$ m.*

The behavior of the plume at higher heights of rise could be influenced by the presence of bounding walls such that it becomes confined.

## Entrainment Analysis

In order to examine the entrainment behavior at high heights of rise, the FDS predictions of $\dot{m}_{p,3D}$ are plotted against $z_s$ on a log-log scale, as changes in the characteristic slope of the predictions will demonstrate if and when a transition in entrainment behavior occurs.

Figures 4–7 show plots of the predictions of $\dot{m}_{p,3D}$ with respect to $z_s$ for Simulations STR1 to STR4 respectively. A line representing $z_{trans}$ determined from Equation (18) is shown to enable an assessment to be made of its applicability to predict the transition in entrainment behavior. A line representing the new simplified entrainment formula proposed by Harrison and Spearpoint [7] (Equation (14)) is also shown to assess the height limit beyond which this equation should not be applied. A line with a slope representing axisymmetric plume entrainment (Equation (8)) is also shown.

Figure 4 shows that for Simulation STR1 (i.e., $W_s = 0.2$ m) the FDS predictions of $\dot{m}_{p,3D}$ generally obey the linearly based relationship given by Equation (14) up to a height of rise of ~2.0 m. However, Figure 4 shows that above this height the FDS prediction departs from the predictions using Equation (14) characterized by a line with a steeper slope (i.e., entrainment according to a power law). This indicates that FDS predicts a transition in entrainment behavior due to entrainment into the ends of the spill plume. The height at which the transition in entrainment behavior occurs is close to the prediction of $z_{trans}$ using Equation (18) (i.e., 2.4 m). It is encouraging to note that the predicted value of $z_{trans}$ coincides with point of intersection between the lines representing Equations (8) and (14). The FDS predictions approach that of an axisymmetric plume beyond the point of intersection. Figures 5 and 6 shows that for Simulations STR2 and STR3 (i.e., $W_s = 0.4$

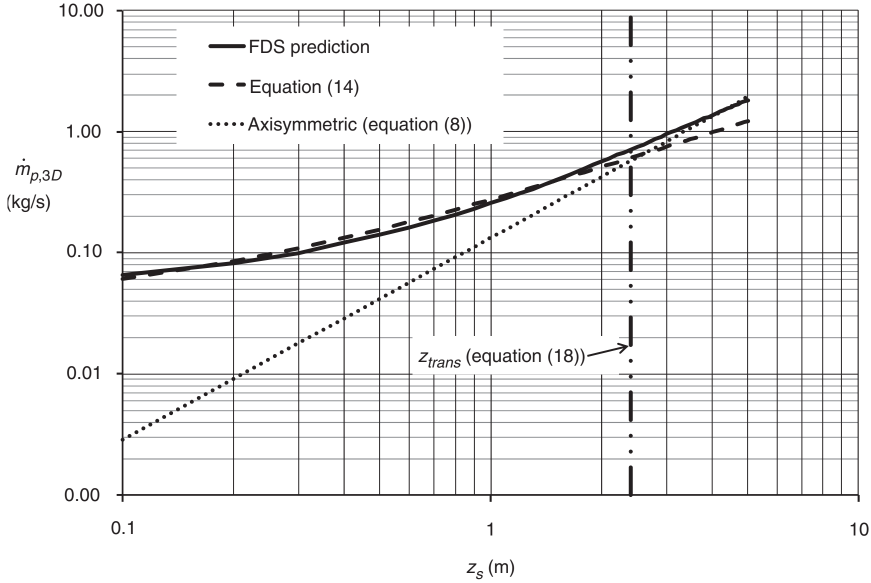


***Figure 4.*** *Predictions of $\dot{m}_{p,3D}$ with respect to $z_s$ (Simulation STR1).*

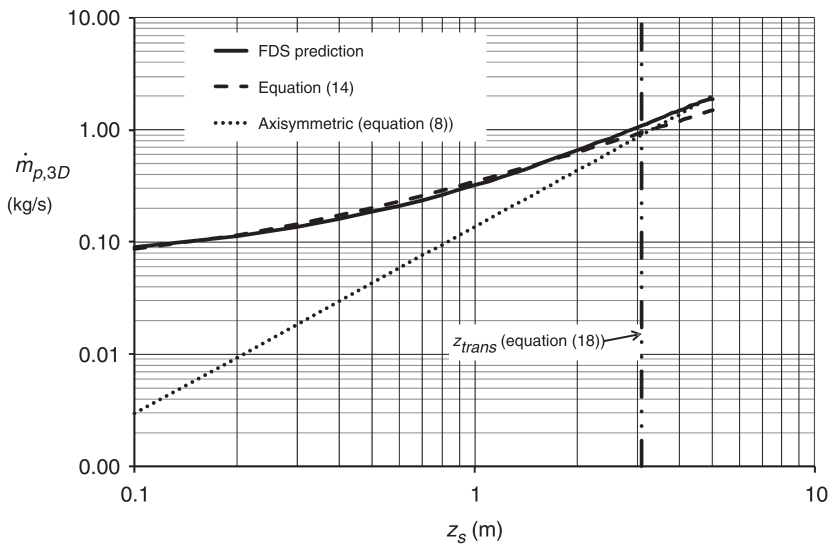


***Figure 5.*** *Predictions of $\dot{m}_{p,3D}$ with respect to $z_s$ (Simulation STR2).*

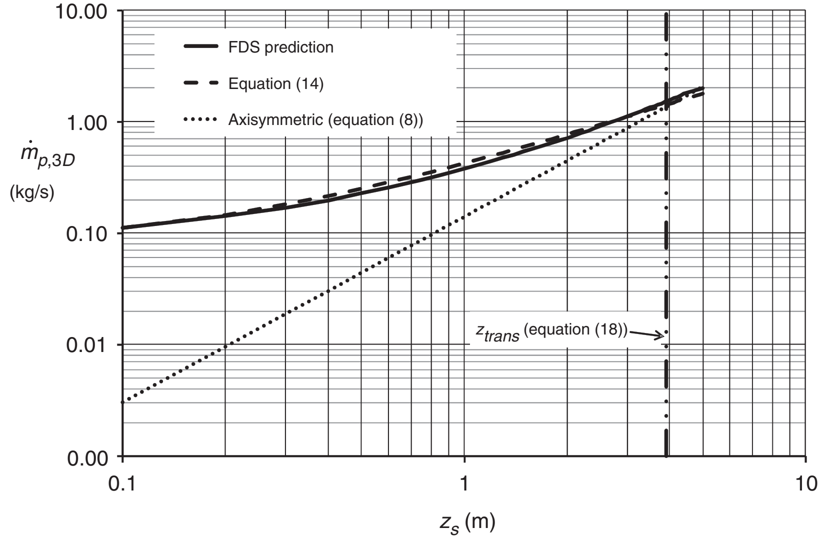


***Figure 6.*** *Predictions of $\dot{m}_{p,3D}$ with respect to $z_s$ (Simulation STR3).*

and 0.6 m) the predictions of $\dot{m}_{p,3D}$ with respect to height of rise demonstrate similar behavior to that described above, but with the transition in entrainment behavior occurring at a greater height of rise due to end entrainment being less significant for wider plumes. Again the prediction of $z_{trans}$ using Equation (18) gives a good approximation of the location of the height of transition for each simulation.

Figure 7 shows that for Simulation STR4 (i.e., $W_s = 1.0$ m) the predictions of $\dot{m}_{p,3D}$ with respect to $z_s$ are broadly in line with Equation (14) over the full height of rise examined without any significant difference in entrainment behavior. This is because the width of the plume was such that the transition in entrainment behavior does not occur over the height of rise examined due to end entrainment being less significant. Equation (18) predicts a value of $z_{trans}$ of ~5.5 m, which is higher than maximum height of rise examined in the simulations. Therefore, it is not surprising that no transition in entrainment is observed. It would have been useful to extend the height of the domain for this simulation; however, the execution time would have been excessive. The predicted plume temperature close to $z_{trans}$ was ~2°C above ambient. Therefore, it is expected that stratification of smoke at high levels is more likely to be an issue for wide plumes rather than a change in the rate of entrainment.

The above analysis indicates that FDS provides a prediction of $\dot{m}_{p,3D}$ that is broadly in line with Equation (14) for plumes generated from a variety of

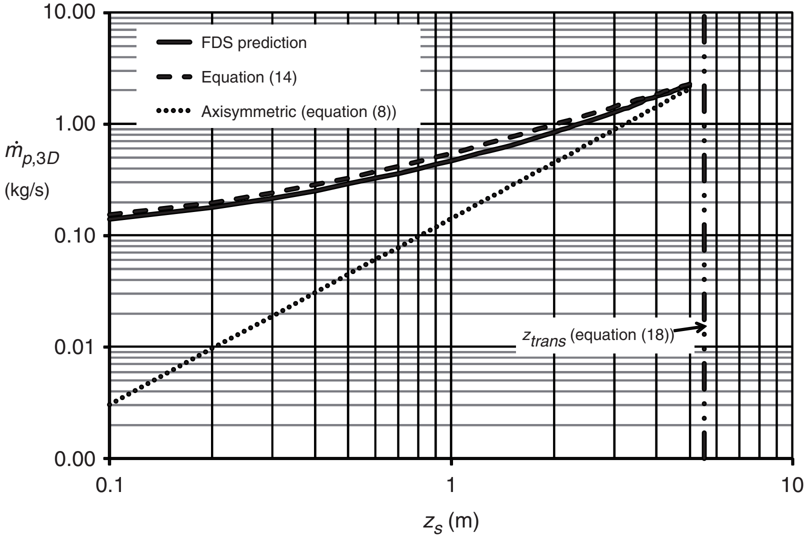


***Figure 7.*** *Predictions of $\dot{m}_{p,3D}$ with respect to $z_s$ (Simulation STR4).*

$W_s$ up to a height of $z_{trans}$. Beyond $z_{trans}$, FDS appears to predict a rate of entrainment that is greater than described by Equation (14) and approaches that for an axisymmetric plume (for the conditions studied). Therefore it seems reasonable to use Equation (18) to provide the upper height limit for which Equation (14) should be applied (i.e., $z_{trans}$). For design scenarios where the height of rise of the plume is greater than $z_{trans}$ the entrainment beyond this height should be according to an axisymmetric plume formula.

Hence, if, $z_s \leq z_{trans}$ then $\dot{m}_{p,3D}$ can be predicted using Equation (14), such that:

$$\dot{m}_{p,3D} = 0.16\dot{Q}_c^{1/3}\left(W_s^{2/3} + 1.56d_s^{2/3}\right) z_s + 1.34\dot{m}_s$$

If $z_s > z_{trans}$ then Equation (8) should be used, such that:

$$\dot{m}_{p,3D} = 0.071\dot{Q}_c^{1/3} z_s^{5/3}$$

## CONCLUSIONS

This article provides an analysis of 3D balcony spill plume entrainment for much greater heights of rise than were possible in the Harrison and Spearpoint [7] experiments. The following approximation is proposed to describe the height of transition in entrainment behavior from a balcony spill plume to an axisymmetric plume. This was determined by matching the new design formula proposed by Harrison and Spearpoint [7] with an axisymmetric plume formula so that the two become equivalent at a sufficiently great height of rise. The formula is given by:

$$z_{trans} = 3.4\left(W_s^{2/3} + 1.56d_s^{2/3}\right)^{3/2}$$

where $z_{trans}$ is dependent on $W_s$ and $d_s$ (rather than $W_s$ alone), which seems reasonable following the entrainment analysis described by Harrison and Spearpoint [7]. This equation reconciles differences between existing guidance and applies more generally.

The absolute value of $z_{trans}$ tends to decrease as $W_s$ decreases. This is expected as narrower plumes will tend to become axisymmetric in nature at lower heights of rise compared to wider plumes, due to end entrainment being more significant in the overall entrainment process.

The predicted value of $z_{trans}$ coincides with the point of intersection between the relationships describing the rate of entrainment for the 3D balcony spill plume and that for an axisymmetric plume for the range conditions studied. The proposed formula to determine $z_{trans}$ describes the upper limit that Equation (14) (or any linear based entrainment formula) should be applied and that beyond $z_{trans}$, entrainment should be determined using a formula for the axisymmetric plume. Hence, if, $z_s \leq z_{trans}$ then entrainment can be predicted using Equation (14), such that:

$$\dot{m}_{p,3D} = 0.16\dot{Q}_c^{1/3}\left(W_s^{2/3} + 1.56d_s^{2/3}\right) z_s + 1.34\dot{m}_s$$

However, if $z_s > z_{trans}$ then Equation (18) should be used, such that:

$$\dot{m}_{p,3D} = 0.071\dot{Q}_c^{1/3} z_s^{5/3}$$

The analysis is supported by a limited number of FDS simulations which demonstrate a transition in entrainment behavior that departs from a linear relationship. For the conditions studied, the FDS predictions approach that for an axisymmetric plume beyond the point of transition. The predicted height of transition in entrainment from the FDS simulation broadly coincides with the predicted value of $z_{trans}$ for the range of fire compartment opening widths examined. This gives further confidence in use of the approximate method for design purposes. There remains scope for further FDS modeling to examine plumes generated from a greater range of fire sizes, compartment geometries, and heights of rise than were possible in this study. The FDS predictions highlight that stratification of smoke at a high level is more likely to be a design issue for wide spill plumes rather than a change in the rate of entrainment.

## NOMENCLATURE

$c_p$ = Specific heat (J/(kg K))
$d$ = Depth of gas layer (m)
$g$ = Acceleration due to gravity (m/s$^2$)
$h_b$ = Height of balcony above the floor (m)
$\dot{m}$ = Mass flow rate of gases (kg/s)
$\dot{Q}_c$ = Convective heat flow of gases below the spill edge (kW)
$\dot{Q}_t$ = Total heat output of the fire (kW)
$T$ = Absolute gas temperature (K)
$W$ = Width or lateral extent of line plume (m)
$W_s$ = Lateral extent of gas flow below the spill edge (m)
$z$ = Height of rise of plume (m)
$z_0$ = Height of virtual source from the spill edge or base of fire (m)
$z_s$ = Height of rise of the plume above the spill edge (m)
$z_{trans}$ = Height of rise of plume above the spill edge where there is a transition in the rate of entrainment to that of an axisymmetric plume (m)

**Greek symbol**
$\rho$ = Density (kg/m$^3$)

**List of subscripts**
*amb* = An ambient property
*air* = A property of air
*p* = Variable evaluated in the plume at an arbitrary height of rise
*s* = Variable evaluated in the layer flow below the spill edge
*3D* = Property of the 3D balcony spill plume

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