Linking Safety Factor and Failure Probability for Fire Safety Engineering

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ABSTRACT: In response to the call for the development of risk informed, performance-based building fire regulations, a literature review was conducted to link the traditional safety factor approach in fire safety engineering to failure probability. It can be demonstrated that a safety factor alone is an insufficient measure, or is only a first-order measure of risk or failure probability. To achieve a higher order estimate of failure probability, an α-percentile method is proposed. As part of this methodology, it is also proposed that the fire engineering briefing process involve nominations of percentile values for design fire scenarios as well as other parameters that define the scenarios.

KEY WORDS: risk, safety factor, failure probability, performance parameter, timeline analysis, uncertainty.

INTRODUCTION

DISCUSSIONS OF RISK informed performance-based codes and standards for fire safety design have been in progress for more than a decade [1–4]. Risk or probability-based regulations are not an entirely new concept. In the 1960s, Cornell [5] proposed a framework for a probability-based structure code to improve consistency in the code treatment of uncertainty. This concept has already been adopted in some building codes and standards [6,7] for structural design. In contrast, the incorporation of probabilistic concepts in fire regulations has been very slow [3], although the idea of basing the criteria for structural fire resistance design on failure probability has been suggested for nearly 30 years [8]. Some of the impediments to this incorporation, as highlighted by Cornell more than four decades ago in regard to structure codes, still exist in the fire

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engineering and regulatory communities. One impediment is the belief that the probability analysis calculations do not promise the benefit of simplicity. This belief is not without any grounds, for many professionals take the traditional engineering approach and regard probability analysis as uncharted territory.

It has long been recognized that design parameters are associated with uncertainties due to the complex nature of reality. As a consequence, these parameters become random variables [5,9–12]. To deal with such uncertainties, engineering design analyses can be carried out on three levels, as suggested by Frantzich et al. [13]. A Level 1 approach is based on the use of one characteristic value for each uncertain, or random, parameter. A Level 2 approach involves the description of the random parameters by a mean and a standard deviation, and Level 3 requires full descriptions of probability distributions for all random parameters involved in the design.

A Level 1 approach is adopted when there is insufficient knowledge of the behavior of the design parameters. In fact, multiple values of design parameters may be used for sensitivity analysis [14]. This approach is characterized by the analysis of a limited number of design scenarios using deterministic methods, though the number of possible scenarios in the real world may be enormous or even infinite [15]. The selected design scenarios are deemed to be representative of most likely and/or most severe conditions. To deal with the uncertainties associated with the design parameters, safety factors are introduced as a design criterion [14]. Because of the lack of knowledge, the probability of failure and risk cannot be quantified in this level of approach.

If additional knowledge about the mean and standard deviation of the design parameters are known, it is possible to evaluate failure probability approximately or, in special cases when the forms of the probability distribution functions of the design parameters are determined by the mean and standard deviation only, precisely. This can be achieved in Levels 2 and 3 approaches.

Fire safety engineering can be considered a form of risk management, the objective of which is to minimize the risk to a level acceptable by the general community. It is impossible and not cost-effective for engineering designs to aim at absolute safety or risk-free solutions. All engineering designs have associated risk as expressed in terms of failure probability and potential consequences [16,17]. It is beyond the scope of the current article to discuss what an acceptable level of risk is. For this, the reader is referred to Bukowski [2] and Meacham [3]. The aim here is instead to relate the safety factor approach to failure probability or to relate the Level 1 approach to the other two levels. More specifically, this article aims at bridging the traditional engineering approach with the probabilistic approach.

In the following sections, a review of failure probability and its evaluation is presented first. Then the relationship between safety factor and failure probability is discussed for the cases where the probability density distributions are known. For the case where the precise distribution function is not known, a method is proposed followed by an application example.

FAILURE PROBABILITY AND SAFETY FACTOR

Failure Probability

The engineering design and risk management process often involves the analysis of a pair of variables, such as load and strength, demand and supply. In fire safety engineering, the pairs of parameters of interest include required safe egress time and available safe egress time and/or fire severity and fire resistance levels [14,18]. The relationships between these pairs of parameters can all be cast into that between demand and supply. The engineering design objective is to ensure that there is sufficient supply to meet the demand. The event of supply not meeting the demand is regarded as failure.

The pair of demand and supply parameters is generally a function of other parameters. For example, in fire safety engineering, the required safe egress time is a function of detection time, building occupant response time and movement time. The available safe egress time is a function of fuel load, fire growth rate, fire suppression capacity, smoke control capacity, fire resistance capacity, and/or any other fire safety measures. In this article, the pair of parameters for engineering evaluation is referred as the 'evaluation parameters'. The parameters that influence the evaluation parameters are referred as design parameters.

The overall objective of fire safety design is to protect life or, sometimes, both life and property [14,18]. This objective is satisfied by ensuring that the available safe egress time (ASET) for the building occupants is greater than the required safe egress time (RSET), which is determined by the physical processes of the fire event, or in the case of fire resistance design, the fire resistance level of building structures is greater than the fire severity [14]. Inadequate fire safety occurs when the required safe egress time exceeds the available safe egress time:

$$\mathrm{RSET} > \mathrm{ASET}$$

Or symbolically, failure occurs when

$$R > A. \tag{1}$$

The safety margin in fire safety design is defined as follows [19]:

$$W = A - R. \tag{2}$$

Failure occurs when

$$W < 0. \tag{3}$$

The safety margin, W, defined in Equation (2) is sometimes referred as the response parameter, and the equation

$$W = 0, \tag{4}$$

is referred as the state equation [20]. Because safety margin is used by fire safety engineers to evaluate the fire safety system performance, it is referred to here as the performance parameter.

The probability of failure is a primary parameter in risk analysis [4,17]. In the risk-based approach, risk is defined as the expected loss and is the product of the probability of an event and the consequence of the event. Failure probability or reliability theory and risk analysis methods have been well established for many engineering and social disciplines. Elishakoff's book [21] gives a comprehensive account of reliability theory in the field of structural engineering. Applications in other engineering disciplines and economics can be found in [9,22,23]. In these references, system reliability and failure probability are extensively addressed.

Because both the design parameters, R and A, are random variables with associated probability density distribution functions, the safety margin is also a random variable. The likelihood of failure exists if the distribution functions for the two design parameters overlap, or more precisely, if the probability density function for R overshadows that for A [10] (Figure 1). Number of approaches can be found in the literature for the probabilistic or stochastic timeline analysis.

General Case

Generally, if the joint probability density distribution, $h(R, A)$,[1] for random variables R and A exists, the probability distribution function of W is given by the following [24]:

$$F_W(W) = \iint_{R-A \leq W} h(R, A) \mathrm{d}R \mathrm{d}A, \tag{5}$$

and the failure probability is simply

$$P_f = F_W(0) \tag{6}$$

[1]In the current discussion, all random variables and their realizations are denoted by the same symbols.

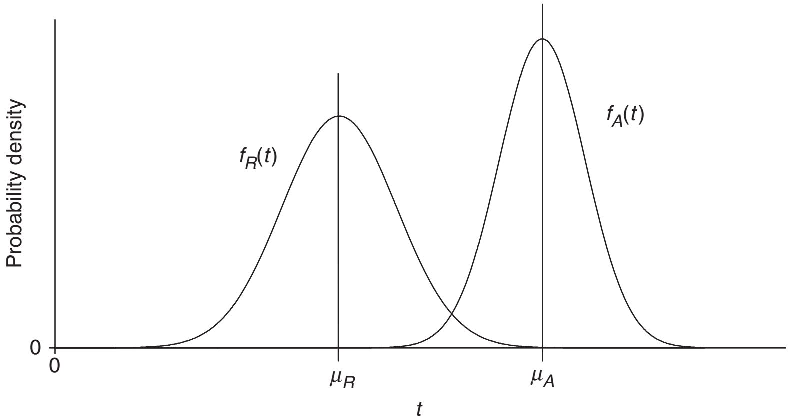

Figure 1. *Hypothetical (normal) probability density distributions of ASET and RSET, and failure probability.*

Specifically, if R and A are independent and have the corresponding probability density distribution $f_R(t)$ and $f_A(t)$, the probability of failure can be evaluated from the following:

$$P_f = 1 - \int_0^\infty f_R(t) F_A(t) \mathrm{d}t, \tag{7}$$

where $F_A(t)$ is the cumulative probability distribution function of A.

Equations (6) and (7) are applicable for any type of distributions. Examples of their applications can be found in [10,12,25,26].

Normal Distribution and Safety Index

The safety index (or the Beta reliability index) method, which was used in structural safety analysis in the 1960s [5] and systematically formulated later by Hasofer and Lind [27], was introduced to fire safety analysis toward the end of the last century [8,13]. Hasofer and Beck [28] applied this method to analyze occupant safety in building fires. In essence, if random variables R and A have normal distributions, their difference, W, is also a normally distributed random variable with the mean and standard deviation

$$\mu_W = \mu_A - \mu_R \quad \text{and} \quad \sigma_W = \sqrt{\sigma_A^2 + \sigma_R^2 - 2\rho\sigma_A\sigma_R}, \tag{8}$$

where ρ is the correlation between A and R. The safety index, β, is defined as follows:

$$\beta = \frac{\mu_W}{\sigma_W} \tag{9}$$

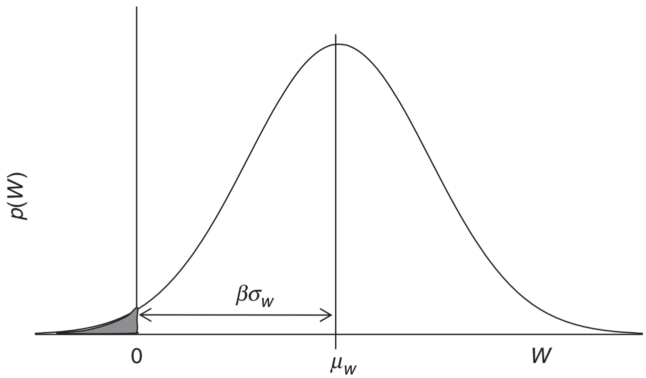

Figure 2. *Normal distribution for W and failure probability.*

It is a measure of how far away the mean of W is from zero in terms of its standard deviation (Figure 2). Note that the β index is the reciprocal of the coefficient of variation

$$\beta = \frac{1}{\alpha} \tag{10}$$

Let $\Phi(x)$ denote the probability distribution function of the standard normal variable. The probability of failure is the probability of $W \leq 0$ and is evaluated from the following:

$$P_f = \text{Prob}\{\text{W} \leq 0\} = \Phi(-\beta) = 1 - \Phi(\beta). \tag{11}$$

The probability of failure is indicated by the shaded area in Figure 2.

Equations (8), (9), and (11) reveal that in order to evaluate the probability of failure, a knowledge of the mean, standard deviation, and the correlation coefficient of R and A is required.

Safety Factor and Failure Probability

Because both R and A are random variables, for any nominated values of these two parameters, there are always associated uncertainties. The uncertainty in fire safety engineering design is traditionally addressed by introducing a safety factor, λ, which is defined as the ratio of the evaluated ASET to RSET for a given design scenario

$$\lambda = \frac{A_s}{R_s} \tag{12}$$

The safety factor in its traditional sense is not a random parameter. The subscript, s, in Equation (12) denotes that R and A values are for a specific design scenario.

Critical values of the safety factor, λ_c, for design assessment can be determined as a result of the fire engineering briefing process [14], and the determination of the critical safety factor value is, by and large, an empirical process. The design acceptance criterion is

$$\lambda \geq \lambda_c \tag{13}$$

Some questions may then be asked about the safety factor approach. For any given design, how safe is the safety factor? For a given value of safety factor, what is the potential risk in terms of probability of failure?

SAFETY FACTOR OF A SYSTEM DESIGN

In fire safety system design, engineers usually deal with the expected values of design parameters and the expected values of the evaluation parameters. Hence, the safety factor of a system design is defined as the ratio of the expected values, μ_R and μ_A, of the evaluation parameters R and A

$$\lambda = \frac{\mu_A}{\mu_R} \tag{14}$$

Under the assumption of normal distribution (Figure 1), a relationship can be established between safety index, β, and the system design safety factor, λ (Appendix)

$$\beta = \beta_R \frac{\lambda - 1}{\sqrt{(\gamma\lambda)^2 + 1 - 2\rho\gamma\lambda}}, \tag{15}$$

where β_R ($=\mu_R/\sigma_R$) is the safety index of the R distribution, ρ is the correlation coefficient, and γ is the ratio of the variation coefficient for A distribution to that for R distribution (α_A/α_R). See the Appendix for the derivation of Equation (15) and the associated parameters.

Equations (11) and (15) establish a relationship between safety factor and probability of failure. For given values of β_R, γ, and ρ, the safety index β is a function of safety factor λ, and the corresponding probability of failure can be obtained from Equation (11). This relationship is depicted graphically in Figure 3 for a special case.

Equations (15) and (11) reveal that the safety index, and hence the failure probability, is not a function of safety factor alone. It also depends on the 'safety index' β_R, the ratio of variation coefficients γ, and the correlation coefficient ρ as well. For any given value of safety factor λ, the safety index β may still be small if either β_R is small or γ is large or both. This means that specifying a large design safety factor value, λ_c, may not necessarily guarantee a sufficiently low failure probability.

Conversely, if the target failure probability is selected, then

$$\beta = \Phi^{-1}(1 - P_f), \tag{16}$$

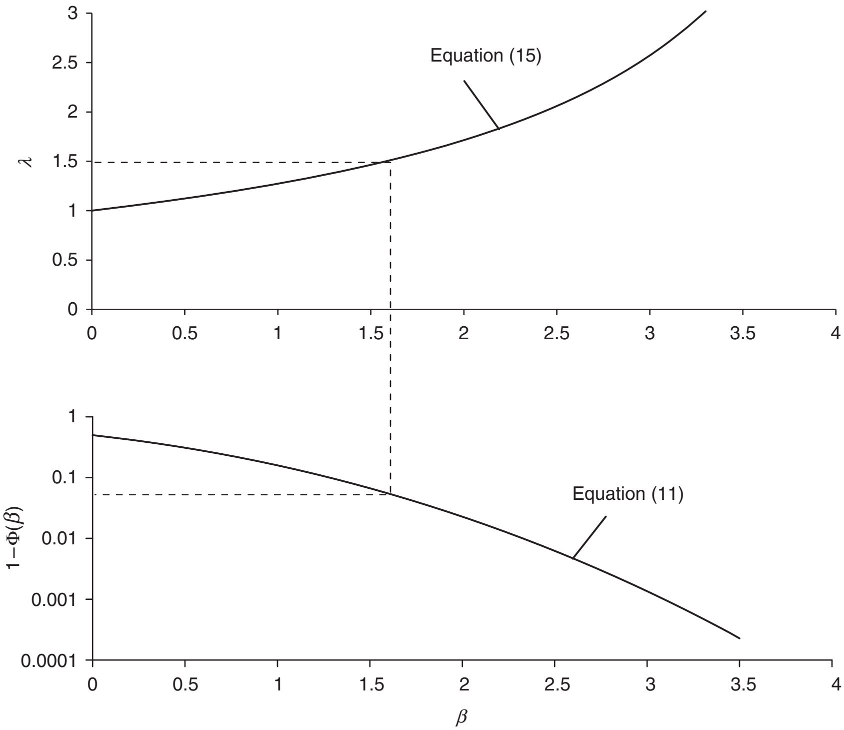

Figure 3. *Relationship between safety factor and probability of failure for a special case ($\beta_R = 5$, $\gamma = 1$, and $\rho = 0$).*

where $\Phi^{-1}(.)$ denotes the inverse function of $\Phi(.)$. The corresponding design safety factor can be determined by solving Equation (15) for λ with the known value of β from Equation (16) (Appendix).

THE LOGNORMAL DISTRIBUTION

Because the random variables R and A reside in positive time domain, the likely form of the governing probability density distribution functions is the lognormal distribution [28]. In such case, the definition of safety factor can be extended beyond its traditional deterministic sense, and the safety factor can be treated as a random variable

$$\lambda = \frac{A}{R}. \tag{17}$$

Take log of both sides of Equation (17) and define

$$Z = \ln \lambda, \;\; U = \ln A \text{ and } V = \ln R, \tag{18}$$

then

$$Z = U - V. \tag{19}$$

Now, the variable Z is a performance parameter and is normally distributed. Its safety index

$$\beta_Z = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_U - \mu_V}{\sqrt{\sigma_U^2 + \sigma_V^2 - \rho\sigma_U\sigma_V}} \tag{20}$$

can be used in Equation (11) to evaluate failure probability. Again, knowledge of σ_U, σ_V, and ρ, as well as of μ_U and μ_V, is required.

Conversely, for a given target probability of failure, P_f

$$\beta_Z = \Phi^{-1}\left(1 - P_f\right) \tag{21}$$

and

$$\mu_Z = \beta_Z\sigma_Z \tag{22}$$

The corresponding mean of the safety factor, or the system design safety factor, is as follows [29]:

$$\mu_\lambda = \exp\left(\mu_Z + \frac{1}{2}\sigma_Z^2\right) = \exp\left(\beta_Z\sigma_Z + \frac{1}{2}\sigma_Z^2\right) \tag{23}$$

THE α-PERCENTILE APPROACH

Definition of Scenarios with Statistical Attributes

More often than not, the probability distributions or even the standard deviations of the design parameters are not known *a priori* for any specific design project. Even if the probability distribution functions for some of the design parameters, such as fire growth rate, fan capacity of smoke control system, or fire detector response time, are known, it is still difficult to derive distribution functions for evaluation parameters such as ASET and RSET. To obtain an estimate of failure probability, one may consider engineering judgment and assumptions.

It is not an uncommon practice in the fire safety engineering design process to select one or a few of 'worst credible scenarios' (sometimes referred as 'worst-case scenario' or simply 'worst scenario') as part of a sensitivity study for design assessment of a particular system. The deterministic timeline analysis is then employed to obtain a set of ASET and RSET values for those scenarios. If the corresponding safety factors are greater than one or a number of predetermined values, then the design is considered acceptable [30]. The probability concept is embedded in this approach in the selection of the scenarios. If the likelihood of a 'worst scenario' is smaller than a 'typical' scenario, a smaller than typical safety factor may be accepted [14]. This approach seemingly makes sense. However, the most challenging step in this approach is, perhaps, an agreement by all stakeholders on the selection of the 'worst scenario'. A clear definition of 'worst scenario' is not found in the literature [31]. The risk ranking method as described in ISO technical reference [15] requires the knowledge of the consequences of a range of identified scenarios, which should be the outcome of the fire engineering assessment. A definition of fire scenarios on a statistical basis is warranted before a relationship between the risk and the safety factor based on the worst scenario can be established.

The worst scenario can be defined on a statistical basis at different levels. The term 'worst credible scenario' may be interpreted in a number of ways. It may mean the scenario with the highest risk ranking [15,32], or the scenario with credible likelihood (non-negligible probability) and significant consequences. Such interpretations have focused on particular types of events. From a probabilistic risk analysis point of view, a more meaningful definition may be the one that is based on percentile, or cumulative, distribution.

Let α denote the percentile value and X_α the corresponding limit value that α percent of the realization of the random variable X will exceed. In other words, X_α is the lower percentile limiting value and α is the cumulative probability for $X > X_\alpha$.

Equipped with the percentile definition of fire scenarios, it is now possible to explore a method for determining failure probability and safety factor.

Normal Distribution

If variables A and R are governed by normal distributions, it is easy to work with the safety margin and the percentile case of the safety margin can be defined from

$$W_\alpha = (A - R)_\alpha, \tag{24}$$

where subscript α denote the values of parameters corresponding to α-percentile.

In the traditional fire engineering approach, one needs to demonstrate that $W_\alpha > 0$ (or $\lambda_\alpha = (A/R)_\alpha > 1$) for the identified design scenarios as nominated in the fire engineering briefing process (Figure 4).

The question remains as to what the associated failure probability is for a given design. As stated earlier, knowledge of probability density distribution function for the performance parameter is generally required in order to evaluate the failure probability. For normal distributions, the knowledge of the mean and the standard deviation will suffice. However, such information

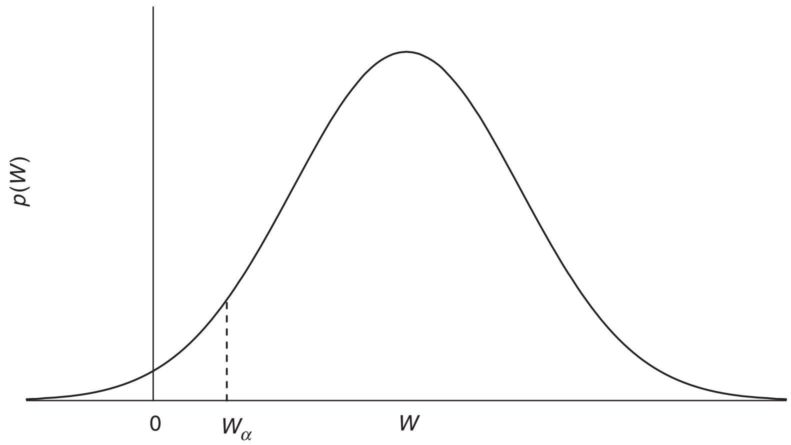

Figure 4. *W value corresponding to an α-percentile.*

may not be readily available. On the other hand, the stakeholders in the fire engineering briefing process may have some idea of what percentile a particular design fire scenario may represent based on expert judgment. Then, under the condition that the distribution function for W can be approximated by a normal distribution function, the failure probability can be estimated. The process is explained in the following.

Assume that the mean and standard deviation of W are unknown. Suppose that two design fire scenarios have been selected with estimated percentile representations, α_1 and α_2, and the timeline analysis have yielded two safety margins W_1 and W_2. Define the standard Gaussian variable

$$Y = \frac{W - \mu_w}{\sigma_w}. \tag{25}$$

In view of the percentile values being cumulative probabilities, two values of y corresponding to the two values of α can be obtained

$$Y_1 = \Phi^{-1}(1 - \alpha_1) \quad \text{and} \quad Y_2 = \Phi^{-1}(1 - \alpha_2). \tag{26}$$

Substituting W_1, W_2, Y_1, and Y_2 into Equation (25), one gets

$$Y_1 = \frac{W_1 - \mu_w}{\sigma_w} \quad \text{and} \quad Y_2 = \frac{W_2 - \mu_w}{\sigma_w} \tag{27}$$

The above set of equations can be solved for μ_w and σ_w. The failure probability is, according to the safety index method,

$$P_f = \Phi\left(-\frac{\mu_w}{\sigma_w}\right) \tag{28}$$

It can be discerned from Equations (25) and (28) that the safety index method is a special percentile case, where $W_\alpha = 0$ ($\lambda_\alpha = 1$) and the complement of the corresponding α-percentile, namely, $1 - \alpha$ is the failure probability.

Lognormal Distribution

If variables A and R are governed by lognormal distributions, the percentile case can be defined from the log value of the safety factor (see Equation (18))

$$Z_\alpha = (U - V)_\alpha \tag{29}$$

Define the standard Gaussian variable

$$Y = \frac{Z - \mu_z}{\sigma_z} \tag{30}$$

The process of evaluating the failure probability is the same as that described in the preceding section. In addition, one could estimate the expected safety factor, or the system design safety factor, from the mean and standard deviation of Z.

The application of the α-percentile approach is illustrated in the following hypothetical example.

Example

In a fire safety engineering assessment of a building design, it was decided during the fire engineering briefing process to evaluate the safety factors for the 50-percentile design scenario and another design fire scenario that was deemed to be a 95-percentile case (i.e., 95% of all possible scenarios will yield better or safer results, or 5% of all possible scenarios will be worse off). The timeline analysis yielded two pairs of ASET and RSET values of 30 and 15 min for the 50-percertile scenario and 22.5 and 14.7 min for the 95-pecentile scenario. What is the failure probability of the assessed fire safety system? Here are two solutions.

NORMAL DISTRIBUTION CASE

If W is governed by a Gaussian distribution, it follows that $\alpha_1 = 0.5$, $Y_1 = 0$, $\mu_w = W_1 = 30 - 15 = 15$ (min), $W_2 = 22.5 - 14.7 = 7.8$ (min), $\alpha_2 = 0.95$ and

$$Y_2 = \Phi^{-1}(1 - 0.95) = -1.645. \tag{31}$$

From Equation (25),

$$\sigma_w = \frac{W_2 - \mu_w}{Y_2} = \frac{7.8 - 15}{-1.645} = 4.377. \tag{32}$$

Therefore,

$$\beta = \frac{\mu_w}{\sigma_w} = \frac{15}{4.377} = 3.427, \tag{33}$$

and

$$P_f = 1 - \Phi(\beta) = 1 - \Phi(3.29) = 3.05 \times 10^{-4}. \tag{34}$$

For normally distributed W, the 50-percentile scenario is also the expected scenario and, therefore, the safety factor is

$$\lambda = \left(\frac{A}{R}\right)_{0.5} = \frac{30}{15} = 2 \tag{35}$$

LOGNORMAL DISTRIBUTION CASE

Assume $\ln \lambda$ is governed by Gaussian distribution. It follows that $\mu_z = Z_1 = \ln \lambda_1 = \ln(30/15) = 0.693$, $Z_2 = \ln \lambda_2 = \ln(22.5/14.7) = 0.426$.

From Equation (30),

$$\sigma_z = \frac{Z_2 - \mu_z}{Y_2} = \frac{0.426 - 0.693}{-1.645} = 0.163. \tag{36}$$

Therefore,

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{0.693}{0.163} = 4.26, \tag{37}$$

and

$$P_f = 1 - \Phi(\beta) = 1 - \Phi(4.26) = 1.01 \times 10^{-5}. \tag{38}$$

The corresponding mean or expected design safety factor is

$$\mu_\lambda = \exp\left(\mu_Z + \frac{1}{2}\sigma_Z^2\right) = \exp\left(0.693 + \frac{1}{2} \times 0.163^2\right) = 2.03. \tag{39}$$

REMARKS

It happens in this particular example that the 50-percentile scenario is given and, therefore, the design safety factor can be determined for the case of normal distribution. Had any other percentile scenario been nominated instead of the 50-percentile scenario, it would not be possible in general to evaluate the design safety factor, even though μ_w and σ_w can be worked out.

On the other hand, the design safety factor can be evaluated for lognormally distributed A and R from any given pair of scenarios.

For the same pair of percentile scenarios, the normal distribution yielded a failure probability that is an order of magnitude higher than that associated with the lognormal distribution. This is because the normal distributions extend the domains of the evaluation parameters R and A into negative territory.

DISCUSSION AND CONCLUSION

The probability-based approach to fire safety engineering design and assessment is one of the corner stones of the fire safety engineering discipline. The use of safety factor is a traditional way of dealing with uncertainties in design for safety or for meeting design objectives. However, a safety factor is only a partial or first-order measure of risk in terms of failure probability. Knowledge of the safety factor alone is insufficient for evaluating failure probability. Any attempt to set a safety factor criterion for acceptance of an engineering design will automatically or intrinsically introduce a large degree of empiricism into the design.

To achieve a higher order estimate of failure probability, more characteristics of the probability distribution functions of the performance parameter are required. The traditional safety factor approach to fire safety engineering design can be linked to a probability-based approach, provided that the probability distributions for the performance parameters are known.

In cases where precise probability distribution functions for the performance parameters are not known, it may be plausible to assume the form of the distribution to be normal or lognormal. Engineering judgment can be employed as part of the fire engineering briefing process to determine the percentile values of a limited number of specific design fire scenarios. The timeline analysis can then be conducted to obtain safety factors for those design fire scenarios and the failure probability can finally be evaluated.

In principle, the analysis of a single fire scenario will suffice in the safety factor method, and the multiple scenario analysis is not a requirement of the method itself, although a sensitivity study is recommended. In comparison, the α-percentile method entails multiple scenario analysis as an essence and not only the conditions that define the design fire scenarios but also the statistical attribute of the selected scenarios must be determined.

The challenging task in the α-percentile method is the determination of the α-percentile values for specific fire scenarios. Similar to risk indexing [17] and risk ranking [33] approaches, the process of determining the α-percentile value will be a heuristic process and can be based on experience and expert judgment.

The basis for expert judgment should be the statistics of fire scenarios and design fire parameters such as type of fires, fire growth rate, fuel load, ventilation factor, building occupant characteristics, and other parameters that influence the evaluation parameters such as ASET versus RSET and fire resistance versus fire severity. It is essential that the reliabilities of the fire safety measures, or the subsystems [14] involved in the fire engineering design be available for the determination of the likelihood of the design fire scenarios and their positions in the cumulative probability distribution. A database may be developed to aid the expert judgment and to minimize the uncertainty in determining design fire scenarios and their percentile representations. Some useful work has been done in the recent past [34–36] and more is needed.

An alternative and quantitative method to determine the percentile representation of fire scenarios may be to employ risk assessment models, such as FIRECAM [37] and CESARE Risk [38,39], to compute the scenario percentile distribution for a generic building of a building class. Then a finite number of scenarios with the associated α-percentiles can be selected for the specific design solution in question.

No engineering analysis can be completely devoid of empiricism. The proposed α-percentile method does rely on empirical input in the form of expert judgment of the percentile representation of selected design fire scenarios, which may be based on general statistics. Nevertheless, it represents one step forward from the empirical basis of the safety factor approach and many steps forward from the empirical basis of the prescriptive building code approach.

NOMENCLATURE

A = available safe egress time
$F(t)$ = cumulative probability distribution function
$f(t)$ = probability density distribution function
P_f = failure probability
$p(t)$ = failure probability density distribution function
R = required safe egress time
T = time
$U = \ln A$
$\mathrm{V} = \ln R$
W = safety margin
Y = standard Gaussian variable
$Z = \ln \lambda$

Greek

α = percentile value
β = safety index
$\Phi(y)$ = probability distribution function of the standard normal variable
γ = ratio of variation coefficients
λ = safety factor
μ = mean
σ = standard deviation

Subscript

A = pertaining to A
R = pertaining to R
s = of a scenario
α = pertaining to α-percentile

APPENDIX

The relationship between the safety index and the safety factor based on the mean values of normally distributed A and R is explained in this appendix.

From Equation (8),

$$\mu_W = \mu_R\left(\frac{\mu_A}{\mu_R} - 1\right) = \mu_R(\lambda - 1) \tag{40}$$

and

$$\begin{aligned}\sigma_W &= \sigma_R\sqrt{\left(\frac{\sigma_A}{\sigma_R}\right)^2 + 1 - 2\rho\frac{\sigma_A}{\sigma_R}} \\ &= \sigma_R\sqrt{\left(\frac{\alpha_A\mu_A}{\alpha_R\mu_R}\right)^2 + 1 - 2\rho\frac{\alpha_A\mu_A}{\alpha_R\mu_R}},\end{aligned} \tag{41}$$

where α_A and α_R are coefficients of variation. Define

$$\gamma = \frac{\alpha_A}{\alpha_R}. \tag{42}$$

Using Equations (14) and (42), Equation (41) can be written as follows:

$$\sigma_W = \sigma_R \sqrt{(\gamma\lambda)^2 + 1 - 2\rho\gamma\lambda}. \tag{43}$$

Substituting Equations (40) and (43) into Equation (9) yields

$$\beta = \beta_R \frac{\lambda - 1}{\sqrt{(\gamma\lambda)^2 + 1 - 2\rho\gamma\lambda}}, \tag{44}$$

where

$$\beta_R = \frac{\mu_R}{\sigma_R} = \frac{1}{\alpha_R}. \tag{45}$$

Rearrange Equation (44)

$$\left(\beta^2\alpha_R^2\gamma^2 - 1\right)\lambda^2 - 2\left(1 - \rho\beta^2\alpha_R^2\gamma\right)\lambda + \left(\beta^2\alpha_R^2 - 1\right) = 0. \tag{46}$$

The solution of the above quadratic equation establishes the relationship between safety factor and the probabilistic characteristics of the evaluation parameters

$$\lambda = \frac{\left(1 - \rho\beta^2\alpha_R^2\gamma\right) \pm \sqrt{\left(1 - \rho\beta^2\alpha_R^2\gamma\right)^2 - \left(\beta^2\alpha_R^2\gamma^2 - 1\right)\left(\beta^2\alpha_R^2 - 1\right)}}{\beta^2\alpha_R^2\gamma^2 - 1}. \tag{47}$$

For a nominated failure probability, P_f, hence safety index (see Equation (16)), the required design safety factor can be worked out from Equation (47), provided that all other parameters on the left-hand side of the equation are known.

Note that the ratio of coefficients of variation can also be expressed as the ratio of safety indices

$$\gamma = \frac{\alpha_A}{\alpha_R} = \frac{\beta_R}{\beta_A}. \tag{48}$$

Notwithstanding that 'safety index' may not be semantically appropriate for either β_R or β_A.

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