A Risk-based Equivalence Approach to Fire Resistance Design for Buildings

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ABSTRACT: This article presents a risk, or probability-based method for determining the residual fire resistance that is required when sprinklers are installed supplementary to building regulatory requirements. The concept of equivalence of a building design to a code-complying design on the basis of risk is introduced. The probability of failure is formulated, taking into account the probability density distributions for fire resistance of building structures and for fire severity. The sprinkler system is treated as a dual-exclusive-status system which has a bi-valued probability distribution. The probability density distribution for fire severity is moderated by the discrete reliability for sprinklers. A parametric study is conducted to demonstrate the sensitivity of the residual fire resistance to sprinkler reliability and fire severity distribution.

KEY WORDS: risk analysis, fire severity, fire resistance, structural failure, compartmentation failure, prescriptive requirement, alternative design solution.

INTRODUCTION

ALL BUILDINGS HAVE some form of inherent risk associated with them, even those constructed to the prescriptive requirements of building codes. The prescriptive fire resistance requirements in building codes are generally based on the building's classification (usage category), rise in stories, and the size of compartments, but independent of whether sprinkler protection is provided [1].

From a fire safety engineering perspective, the fire resistance that is required of the building structure and compartment boundaries depends on

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the performance expectations for those building elements in case of a range of fire scenarios. The required fire resistance for a low-rise building adequately separated from its neighbors may be quite low as the building may only need to survive long enough to permit safe evacuation. On the other hand, a high-rise building in an urban area may need to withstand the burn-out of a fire compartment.

Fire resistance levels (FRL) in building regulations have generally been developed on an empirical basis and calibrated through a test of time. If a history of inadequate performance is evident then regulations are amended (usually following a disaster) to rectify the perceived shortcoming. Thus the prescriptive requirements of the building regulations can be considered to be a *de facto* community standard that is deemed to satisfy the objectives and performance requirements of building codes. However, the deemed-to-satisfy (DTS) provisions are quite constrictive and may hamper the design of modern buildings or refurbishment of existing or heritage buildings. The performance-based building regulations, such as the Building Code of Australia [1], accept alternative solutions so long as it can be verified that the performance requirements are satisfied. One of the verification methods is the equivalence of the alternative solution to the DTS provisions in terms of adequate safety.

A code-complying building may be required to have a prescribed fire resistance rating but not sprinkler protection. An alternative solution is to install a sprinkler system and have the FRL reduced. The question arises, as illustrated in Figure 1, by how much can the FRL be reduced such that fire safety in the sprinkler-protected building is at least at the same level as the code-complying building.

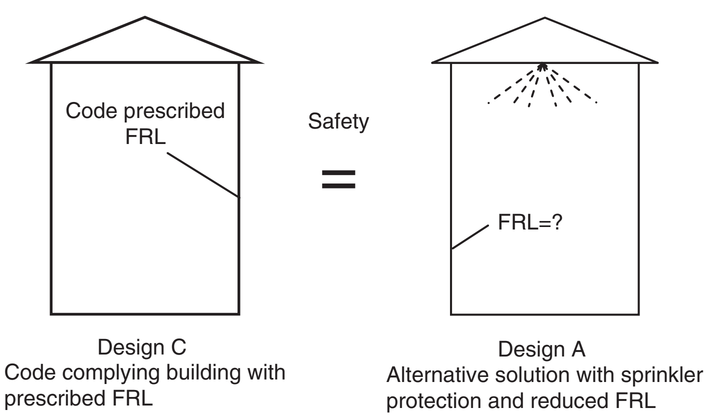

Figure 1. *Two building design options.*

The fire resistance design criterion is that fire rated building elements should endure the expected fire severity. The determination of adequate FRL for building elements is then reduced to the evaluation of the severity of the fire should it occur in the building. Correlations exist in the literature for the evaluation of fire severity in buildings with or without sprinklers [2]. These correlations constitute the empirical type of approaches. A risk-based method for evaluating equivalent fire resistance of buildings with sprinkler protection was presented by Grubits and Tung [3]. The method for determining the residual fire resistance required is based on the concept of equivalence with the DTS requirements of the regulations. The two fire safety strategies (one relying on passive fire resistance alone and the other incorporating a sprinkler system) are considered equivalent if the probability of failure is the same. Thus the residual fire resistance could be determined from a comparison of the probabilities of failure. In this article, the approach in [3] is further formulated explicitly. Presented in the following sections are a review of risk analysis in fire safety engineering, the formulation of the concept of risk-based equivalence, followed by a general discussion on the form of the probability density distribution functions for fire severity and FRL. A parametric study is then presented with discussions. Finally, conclusions and recommendations are drawn in the last section.

FIRE RESISTANCE ASSESSMENT AND PROBABILITY ANALYSIS

Building fire resistance design involves two basic parameters, namely, fire severity and FRL. Fire severity is a measure of fire intensity and is expressed in terms of the equivalent time of exposure, S, to the standard fire test [2,4,5]. FRL is a measure of a building elements' capability to withstand fire attack and is a property of building elements. It is expressed in terms of survival time, R, of an element in the standard fire test without losing its stability, integrity, and thermal insulation capability [5].

The overall objective of fire safety design is to protect life or, sometimes, both life and property [6,7]. This objective is translated to fire resistance design by ensuring that the FRL of building structures is greater than the fire severity. Inadequate fire resistance of a structural member or failure of compartmentation occurs when the severity of a fire exceeds the fire resistance provided by the member to support the load or to contain the fire:

$$\text{Fire severity} > \text{Fire resistance level}$$

Or symbolically, failure occurs when:

$$S > R. \tag{1}$$

At this point, the concept of safety margin, in an analogy to that in life safety analysis [8], is introduced. The safety margin, W, in fire resistance design is defined as:

$$W = R - S. \tag{2}$$

Failure occurs when:

$$W < 0. \tag{3}$$

In deterministic approaches, the FRL of building elements and the fire severity are treated as definite variables and the impact of sprinkler installation on fire severity is simply dealt with by introducing a modification factor. For example, the fire severity in a building with sprinklers is reduced to half of the value for the same building if no sprinkler is installed [2]. Therefore, it may be argued that the FRL of a building with sprinkler protection can be reduced to half of the value required for the same building without sprinkler protection. Such an approach is very much empirical and does not explicitly consider the possibility of sprinkler failure [9].

To take into account the reliability of sprinkler systems as well as the reliability of fire resistance building elements, Hui [10] used the method of event tree analysis to study the equivalence between buildings with sprinklers and buildings without sprinklers. A bifurcating event tree is constructed and both the sprinkler system and the fire resistant building elements are treated as dual-status systems, that is, their status can be either operational or failure. In this approach, as in many other documents in the published literature [9,11], the reliability of the fire barrier or fire resistance building elements is regarded as a predetermined property of the building structure and is independent of fire events. It was concluded by Hui [10] that the reliability of the sprinkler system in the alternative solution should be at least equal to that of the fire resistant building elements.

Fire severity is related to fuel load density, compartment height and floor area, ventilation condition, and room-lining material [12]. These parameters are associated with uncertainties. Even for a compartment of specified geometry and lining material, the fuel load density and vent opening condition at the time of a fire incident cannot be determined with certainty. The resulting fire severity is, therefore, a random variable. Similarly, FRL of building elements is also a random variable due to field variations (quality of construction materials, construction workmanship, number and extent of penetrations, accidental damage, normal wear and tear, etc.). Therefore, the probability of building element failure is dependent on the probability distributions that govern the two parameters concerned.

The relationship between FRL and fire severity is analogous to that between supply and demand or strength and load [13]. What is more relevant, this relationship is very similar to the relationship between the available safe egress time and the required safe egress time in the analysis of building occupant life safety in fire emergencies [6]. The analysis of the latter pair is often referred to as the timeline analysis.

A number of approaches can be found in the literature for the probabilistic or stochastic timeline analysis. The safety index (or the Beta reliability index) method, which was used in structural safety analysis in the 1960s [14] and systematically formulated later on by Hasofer and Lind [15], was introduced to fire safety analysis towards the end of the last century [16,17]. Hasofer and Beck [18] applied this method to analyze occupant safety in building fires. In essence, the difference of any two normally distributed random variables is also a normally distributed random variable:

$$W = U - V. \tag{4}$$

The mean and standard deviation of W are related to those of U and V by:

$$\mu_W = \mu_U - \mu_V \quad \text{and} \quad \sigma_W = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}, \tag{5}$$

where ρ is the correlation between U and V. The safety index, β, is defined as:

$$\beta = \frac{\mu_W}{\sigma_W}. \tag{6}$$

It is a measure of how far away from zero the mean of W is in terms of the number of its standard deviations. Let $\Phi(x)$ denote the probability distribution function of the standard normal variable. The probability of failure is the probability of $W \leq 0$ and is evaluated from:

$$P_{\text{failure}} = \text{Prob}\{W \leq 0\} = \Phi(-\beta) = 1 - \Phi(\beta). \tag{7}$$

An application of this method and a Monte Carlo simulation technique was described by Hasofer et al. [19] for determining failure probability of a timber stud wall.

The safety index method offers a simple way to evaluate failure probability, provided that the random variables, U and V, are governed by normal distributions. If the random variables are governed by distributions other than normal, Equation (7) is no longer valid or accurate and the evaluation of probability of failure will require further manipulation of the governing probability density distribution functions as demonstrated by He et al. [20] in a stochastic analysis of life safety in building fire situations. In their study, the building occupant evacuation process was assumed to be a Poisson process and the probability density for the available safe egress time was approximated with a uniform distribution. The equivalence concept presented by Grubits and Tung [3] and the analytical method employed in [20] form the basis of the analysis in the current study.

THEORETICAL BASIS

Probability of Failure

In the risk-based approach outlined in this article, both the FRL, R, of the building structure and the fire severity, S, are regarded as continuous random variables with associated probability density distribution functions $f(t)$ and $g(t)$, respectively. The safety margin is also a random variable. Figure 2(a) illustrates the probability density distributions for fire severity, $g_C(t)$, and FRL, $f_C(t)$, where only passive fire protection is adopted. Similar to the probabilistic/stochastic timeline analysis for life safety [20], the overlapping of probability density distribution functions for R and S gives rise to probability of failure of building elements. Diagrammatically, failure is represented in Figure 2(a) by the region where the fire severity probability density distribution curve overlaps (or overshadows) the probability density distribution curve for FRL. Generally, under the assumption of R and S being independent, the cumulative probability function of failure is:

$$P(t) = F(t) - \int_0^t f(r)G(r)\mathrm{d}r, \tag{8}$$

and the probability of failure can be evaluated from

$$P = P(\infty) = 1 - \int_0^{\infty} f(r)G(r)\mathrm{d}r, \tag{9}$$

where $F(t)$ and $G(t)$ are probability distribution functions of R and S respectively. A detailed explanation of Equation (8) is given in Appendix A. In the following discussion, the distribution functions $f(t)$, $g(t)$, $F(t)$, and $G(t)$ are referred to as the source distribution functions while the probability distribution function $P(t)$ and its corresponding density distribution

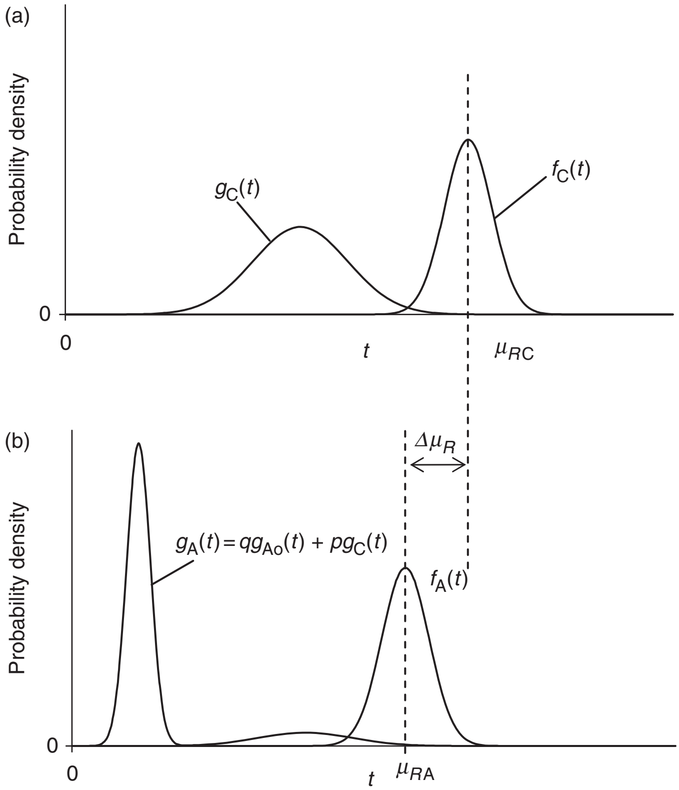

Figure 2. *Probability density functions for fire severities and FRL in (a) a code-complying design without sprinklers and (b) an alternative solution with sprinklers.*

function $p(t)$ are referred as resulting distribution functions. In addition, all the notional values of random parameters, such as the code required FRL, the residual FRL and the notional fire severity, are regarded as the expected, or the mean values of the corresponding random variable.

The design objective of building fire safety systems is to minimize the probability of failure by managing either the shapes or the relative positions of the two probability density curves. Narrowing the two curves in Figure 2(a) (or decreasing the standard deviations of the two probability density functions) or shifting the two curves apart (or increasing the difference in the means of the two density functions) will result in reductions in failure probability. To evaluate the probability of failure, one would need the knowledge of source distributions.

In the case of the code-complying design, the source distribution functions may be relatively simple. The notional, or the code required FRL, can be regarded as the mean of the FRL, μ_{RC}, and the notional fire severity as the mean, μ_{SC}, for fire severity distribution (Figure 2(a)).

In the case of sprinkler protected buildings, the fire severity probability density distribution function will be altered and is a superposition of two density functions – one is associated with the case when the sprinklers control the fire and the other with the case of sprinkler failure. The resulting curve has, therefore, two peaks as shown in Figure 2(b). The first peak from the left is skewed to the low end of the scale, reflecting the fact that the fire severity level of sprinkler controlled fires is much less than that of noncontrolled fires. The first peak also exhibits a higher value than the second, due to the high reliability and effectiveness of sprinklers [21,22]. The second peak is similar in shape to the case without sprinklers but moderated by the sprinkler failure probability to reflect the much lower frequency of nonsprinkler controlled fires.

Mathematically, the fire severity probability density distribution curve, $g_{\mathrm{A}}(t)$, in Figure 2(b) for the alternative solution with sprinklers can be expressed as:

$$g_{\mathrm{A}}(t) = qg_{\mathrm{Ao}}(t) + pg_{\mathrm{C}}(t), \tag{10}$$

where q is the reliability of the sprinkler system, $g_{\mathrm{Ao}}(t)$ denote the probability density function for fire severity when the sprinkler system operates, $g_{\mathrm{C}}(t)$ denote that when there is no sprinkler or sprinkler has failed and p is the probability of sprinkler failure. In words, a building with sprinkler protection has a bimodal fire severity probability density distribution, which is determined by sprinkler reliability and sprinkler impact. The derivation of Equation (10) is given in Appendix B.

The Concept of Equivalence

When a sprinkler system is installed as an alternative solution in a building where it is not required by the building code, a reduction in the realized FRL of the building construction may be adopted on the basis that the alternative solution of sprinkler installation presents a risk equivalent to that of the DTS design. Risk is defined as the expected losses and is expressed as the product of the probability of an undesirable event and the consequence of that event [23]. Assuming that the consequences of fire resistance failure in both the DTS design and the alternative solution are the same, then the equivalence in risk is determined by the equivalence in probability of failure. The premise behind the risk-based approach is that there exists an equal probability of failure for two fire safety strategies – one based on passive fire protection systems only (Design C), and the other incorporating both passive and active fire protection systems (Design A).

The basic concept in terms of equivalence between fire safety strategies for a building with and without sprinklers is illustrated in Figure 2. The fire severity curve in Figure 2(b) is modified by the installation of sprinklers in the building and it is possible to shift the fire resistance curve towards the left with the same or less failure probability of the counterpart without sprinklers whose failure probability is depicted in Figure 2(a). The fire resistance curve now has a new mean, or the new notional level, $\mu_{R\mathrm{A}}$, and the corresponding probability density distribution, $f_{\mathrm{A}}(t)$. The difference between the original code required notional FRL and the new notional level is the reduction in FRL afforded by the installation of sprinklers:

$$\Delta\mu_R = \mu_{R\mathrm{C}} - \mu_{R\mathrm{A}}. \tag{11}$$

The reduction in FRL can be determined by equating the failure probability of the alternative solution to that of the code-complying solution. In other words, $\mu_{R\mathrm{A}}$ is the solution of the following equation:

$$P_{\mathrm{A}} = P_{\mathrm{C}}, \tag{12}$$

where P_{A} and P_{C} are failure probabilities of the alternative design and the code-complying design, respectively. They are obtained by substituting g_A (Equation (10)), f_{A}, g_{C}, and f_{C} into Equation (9).

In summary, the code-complying design is established as a reference design in the risk equivalence approach. The risks associated with the reference design and an alternative design solution are analyzed using the same established assessment method. The alternative solution is said to be equivalent to the reference design if the former has the same or less associated risk than that of the latter. If the consequences of failure are the same, the equivalence is evaluated by comparing the failure probabilities. For the case of fire resistance design discussed in the current study, the acceptable or equivalent alternative solution involving a residual FRL plus sprinkler protection should yield no greater risk or probability of failure than the code-complying design which involves a much higher FRL.

Remarks

To end the discussion of this section, the following observations are made:

(1) It can be discerned from Equation (9) that the probability of a fire barrier or building element failure is not a property of the element. It is related to the properties of the building element, overall building design and usage (fuel load) via probability density distributions for fire severity and FRL.

(2) Whatever forms the distribution functions $g_{A_o}(t)$ and $g_C(t)$ take, the probability density distribution function for fire severity in a sprinkler protected building, $g_A(t)$, as expressed in Equation (10), is by no means normal.

FORM OF DISTRIBUTIONS AND PARAMETER RANGE

Fire Resistance Levels

Although the purpose of standard fire tests [4,5] is to determine FRL of building elements but not necessarily their probability density distributions, it may be possible to obtain estimates of such distributions from the test results. However, the fact that the standard tests usually terminate at or soon after the notional FRL is reached makes the estimate not straight forward. Moreover, as discussed in previous sections, FRL of building elements are influenced by many parameters that have attached uncertainties. Although an element or an assembly may pass the standard fire resistance test with the prescribed, or notional, FRL as the minimum level, there is a likelihood of degradation caused by imperfect quality control during manufacturing and construction, accidental damage, and wear and tear during usage [24]. This degradation contributes to deviations in the FRL likely to be realized in the event of fire from the notional value. Hence any estimate of the probability distribution for the FRL, based on the standard fire resistance test, needs to be modified.

Since FRL is a positive random variable, the likely form of the governing probability distribution function is lognormal [25]. If the standard deviation is much smaller than the mean, the distribution can be approximated by a normal function.

An estimate of the standard deviation may be obtained from considerations of factors that influence the fire resistance. For example in case of a concrete column, estimates of variation in concrete cover may be translated to variations in fire resistance by considering the concrete cover thickness required for various fire resistance ratings. Variations in loading conditions also influence the fire resistance of load-bearing members such as columns. The ultimate load-bearing capacity of reinforced concrete columns is an approximately linear function of temperature of the reinforcing steel. Thus variations in load during the fire manifest themselves as a change in the critical temperature of the reinforcing steel. The reinforcing steel temperature, however, is in turn dependent upon the cover to the reinforcing elements [26].

However, information about the impact of accidental damage and wear and tear on the fire resistance distribution is scarce. As such, assumptions have to be made for risk analysis.

Fire Severity

The fire severity of a compartment is influenced by the fire load, geometry, ventilation conditions, and thermal properties of the compartment's construction material. A number of correlations have been developed to estimate fire severities [12,27]. As there are uncertainties in the contributing factors, fire severity manifests itself as a random variable.

Generally, better insulated compartments have greater fire severity for a given geometry, fuel load, and ventilation condition. In this sense, fire severity is dependent on FRL of the building structure. In the case of sprinkler-protected buildings with reduced FRL, the fire severity is likely to decrease. The assumption of independence between fire severity and FRL will lead to an overestimate of the fire severity for the building, though the error may not be significant. From a fire safety point of view, this is a conservative approach and is, therefore, accepted in the current study.

The dependence of fire severity on FRL is also manifested in the consequence of fire resistance failure. The breaching of a fire barrier may result in the creation of additional ventilation openings, which will lead to a reduction in fire severity. However, this dependence is of no concern since the variation in fire severity occurred after the barrier failure and there will be no impact on the probability of failure in the same compartment. See further discussion in Appendix A.

Thus an estimate of the probability distribution of fire severity may be obtained by considering the distribution characteristics of the contributing factors.

Variations in compartment geometry and openings may be obtained from surveys of the building designs. The nature of the lining materials can be simplified into discrete probability estimates of whether the linings are likely to be lightweight. Estimates of the distribution of fire load density may be obtained from historical fire load density surveys [2] and field research [28].

In the case of the reference design, distribution characteristics may be either the likely variation in characteristics of the specific reference building, or more broadly, the likely variation in buildings of similar regulatory classification.

PARAMETRIC STUDY

Parametric studies are conducted to verify the failure probability evaluation against the safety index method and to demonstrate an application of the risk equivalence approach to determining the residual FRL in a sprinkler protected building and the sensitivity of the residual fire resistance to the reliability of the sprinkler system. The reliability of the fire resistance systems and time of failure are also discussed.

Verification Against Normal Distributions

If the source functions $f(t)$ and $g(t)$ are normal distributions, Equation (9) will produce the same result as Equation (7). Figure 3 presents the failure probability distribution for identical $f(t)$ and $g(t)$ with $\mu = 60$ min and $\alpha = 0.1$, where α is coefficient of variation and is defined as the ratio of the standard deviation to the mean, σ/μ. Indeed for this case, $\beta = 0$ and $P = P(\infty) = \Phi(0) = 1 - \Phi(0) = 0.5$.

Sensitivity Study

Numerical analysis was conducted to reveal sensitivity of the residual fire resistance design in the alternative solution to two parameters: (1) the form of the source probability distribution function; and (2) the sprinkler reliability.

The sensitivity study takes a 120 min rated compartment as the typical code-complying reference case. The notional fire severity is taken to be 90 min. This leads to a safety factor of 1.33 ($=\mu_{RC}/\mu_{SC} = 120/90$). The expected fire severity associated with the situation when the sprinkler is operational in the alternative solution is assumed to be 20 min.

Among all the influential parameters, fuel load is the most significant contributor to fire severity [12,27]. The uncertainty associated with the fuel load is also the largest. According to Hadjisophocleous and Zalok's [28] study, the fuel load distributions for commercial type of buildings deviated in a wide range. This will lead to significant deviation in fire severity. In the current study, the coefficient of variation for fire severity probability density function is assigned a larger value than that for FRL. Table 1 lists the parameters of

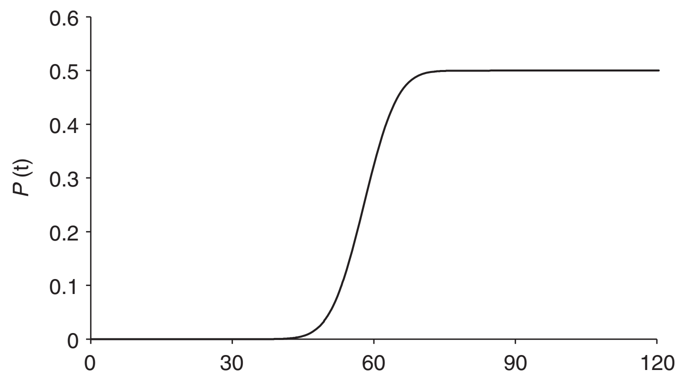

Figure 3. Failure probability distribution for identical f(t) and g(t) with $\mu = 60$ min and $\alpha = 0.1$.

probability density functions for fire severities and FRL. The mean, or the notional, FRL of Design A is to be determined in the analysis.

To analyze sensitivity of the required notional fire resistance to the form of the probability density distribution functions, three forms, namely normal, triangular, and lognormal, were used for all distributions listed in Table 1. The sprinkler reliability was varied from 0.7 to 0.99.

Figure 4 shows the variation of the required FRL for Design A with sprinkler reliability for the three forms of distributions. It can be discerned that the overall variation of μ_{RA} versus q is insensitive to the form of probability density distributions. Nonetheless, significant differences, particularly that between the normal and lognormal distributions, exist in the region $0.91 < q < 0.925$. For example, at $q = 0.915$, the residual FRL by normal and lognormal source distributions may differ by 20 min.

The chart can be divided into three regions as shown in Figure 4. In the left region where the sprinkler reliability is relatively low ($q < 0.85$), the residual FRL is predominantly determined by the fire severity. The residual

Table 1. Parameters of source probability density functions.

Variable	Distribution	μ (min)	α
R_C	$f_C(t)$	120	0.1
S_C	$g_C(t)$	90	0.2
S_{Ao}	$g_{Ao}(t)$	20	0.2
R_A	$f_A(t)$	μ_{RA}	0.1

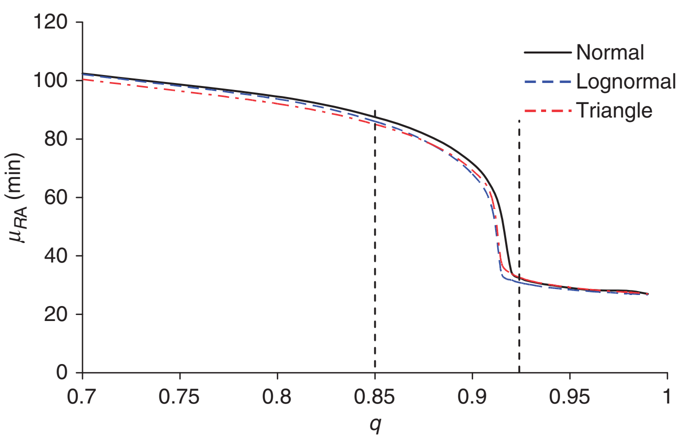

Figure 4. *Variation of the required FRL with sprinkler reliability. (The color version of this figure is available online.)*

FRL asymptotically approaches the code-prescribed FRL as the sprinkler reliability q approaches zero. On the other hand in the right-side region where reliability is high ($q > 0.925$), the survival of the structure is predominantly determined by the fire severity associated with the sprinkler system. The residual FRL asymptotically approaches 26.7 min, which is equal to the product of the expected fire severity of sprinkler-controlled fires (20 min) and the same safety factor (1.33) as in the case of code-complying design, or the reference case. In the middle region ($0.85 < q < 0.925$), both the fire barrier and the sprinkler are at work.

The μ_{RA} versus q chart can also be used to determine the maintenance requirement for the sprinkler system. For example at a designed FRL of 60 min, the sprinkler reliability should be maintained above 0.92 or 92%. If in an existing building, which has an expected intrinsic FRL of 90 min, a sprinkler system is proposed to upgrade its fire resistance capability to the equivalence of the current code requirement of 120 min, only 85% sprinkler reliability is needed (see Figure 4). It is interesting to note that not much can be gained, in terms of reduction in residual FRL, by increasing the sprinkler reliability above 92.5%.

Reliability and Expected Time of Failure

Presented in Figure 5 are fire resistance failure probability density and probability functions for three forms of source distributions. In this case, the sprinkler reliability is $q = 0.92$ and the residual FRL for the alternative solution, μ_{RA}, is 60 min. All other parameters of the source distributions are the same as in Table 1.

The alternative solution is shown to be able to achieve lower failure probability or higher reliability than the code-complying solution. However, if failure does occur, it occurs earlier with the alternative solution than with the reference solution as shown in Table 2 where the fire resistance system reliability, Q, and the expected failure time t_f, of the code-complying design and two alternative solutions with different sprinkler reliability, q, and residual FRL, μ_{RA}, are compared.

In the alternative solution, the expected failure time is determined by the residual, or the expected, FRL of the building element, since the expected fire severity is greater than the residual resistance when the sprinkler fails. This is further evident in the results of another analysis with $q = 0.92$ and $\mu_{RA} = 60$ min (Table 2). In the reference design, the expected fire severity (90 min) is smaller than the FRL (120 min), the expected failure time is, therefore, somewhere in between.

The difference in the expected failure time may lead to the difference in the failure consequences. For example, an earlier breach of a fire barrier will

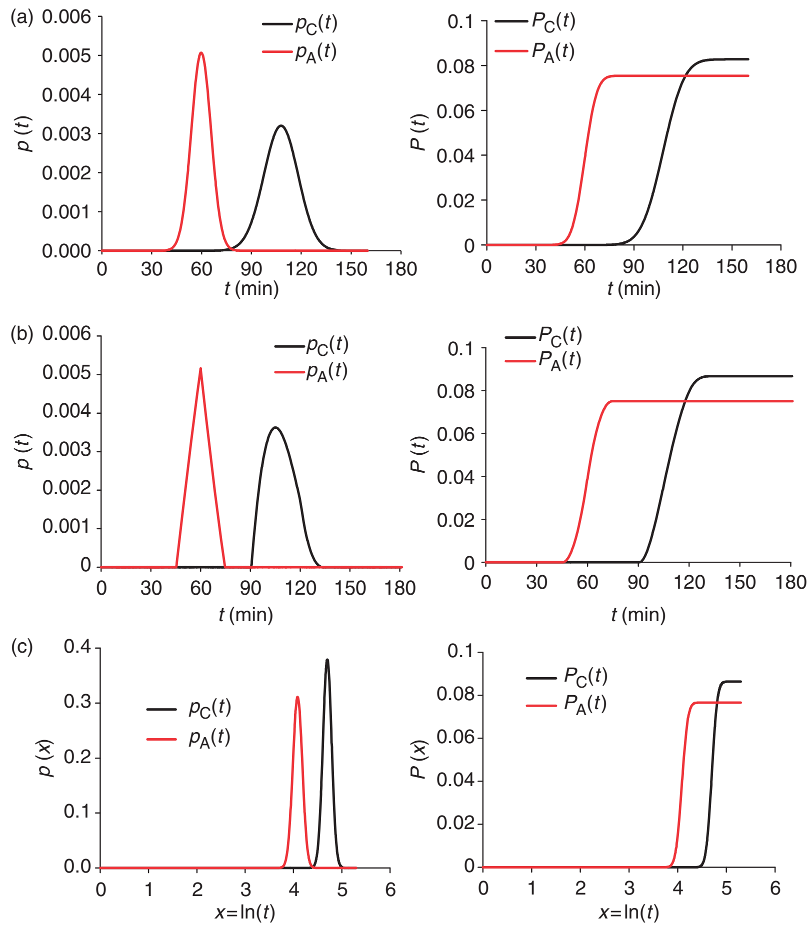

Figure 5. *Fire resistance failure probability density and probability functions for three types of source distributions. ($q = 0.92$, $\mu_{RA} = 60$ min): (a) normal distribution, (b) triangle distribution, (c) lognormal distribution. (The color version of this figure is available online.)*

Table 2. Reliability, Q, and expected time of failure, t_f.

Source distribution	Code-complying design		Alternative solution A1 $q = 0.92$, $\mu_{RA} = 60$ min		Alternative solution A2 $q = 0.85$, $\mu_{RA} = 90$ min	
	Q_C	t_{fC} (min)	Q_A	t_{fA} (min)	Q_A	t_{fA} (min)
Normal	0.917	108	0.925	59.7	0.925	86.7
Triangle	0.913	107	0.925	59.6	0.925	86.7
Lognormal	0.914	110	0.923	59.3	0.929	86.1

result in an earlier arrival of untenable conditions in the neighboring compartments. Hence, other fire safety measures including an efficient evacuation strategy will be warranted to ensure life safety of the building occupants. A detailed analysis of the consequences and a comprehensive assessment of building fire safety system performance are beyond the scope of this article.

CONCLUSION AND RECOMMENDATIONS

This article presents a risk or probability-based analytical approach for determining the residual fire resistance that is required when sprinklers are installed supplementary to code requirements. This methodology is not limited to sprinklers and can be applied to assessments of other alternative solutions, which may involve a pair of random variables analogous to supply and demand. The methodology relies upon the following key principles:

– The random variables can be described by probability density distribution functions
– The failure criteria can be established
– A reference system can be established
– Probabilities of failure exist for both the reference system and the alternative system
– If the consequences of failure are the same, the alternative system can be accepted if its failure probability is no greater than that of the reference system.

For the particular case of fire resistance design studied in this article, it has been demonstrated that the reliability of fire resistant building elements is not an intrinsic property of the elements. It is a function of the properties of building elements and the environment in which they operate.

Parametric studies have been conducted using three types of probability distribution functions for fire resistance and fire severity. Although the overall relationship between the residual FRL and the sprinkler reliability is insensitive to the form of source distributions, there exists a critical region where the dependence on the distribution form is pronounced.

The parametric analysis conducted in the present study was based on simplified probability distribution forms for fire severity and FRL. Building fire severity and FRL of building construction are influenced by many parameters, such as fuel load, ventilation factor, materials properties, and even the manufacturing and construction processes. It would be desirable to obtain comprehensive probabilistic descriptions of these parameters. Data for such descriptions can be obtained by laboratory experiment and field research for various types of materials, building elements, structures, and building classes. A database may be developed for risk-based analysis.

To obtain a reasonable estimate of intrinsic FRL probability distribution, it will be necessary to extend the fire resistance tests to the time of failure.

Although the sprinkler plus residual fire resistance approach has not been common practice in fire resistance design, it is desirable to collect field data of this kind of design for reliability and risk analysis. The field data may include the time of structure or element failure when the sprinkler system failed to operate and the designed residual FRL of the failed element.

If the governing probability density distribution functions for the influential parameters are known, then numerical, or the Monte Carlo method can be used to obtain the probability distribution functions for fire resistance and fire severity. Future research may focus on numerical methods that will be capable of handling more complicated systems.

The current study is limited to the reliability analysis of fire resistance design. Fire resistance design is only one of a wide range of issues that need to be considered in building fire safety system design. The consequence of fire resistance failure should also be considered in the overall risk assessment.

NOMENCLATURE

$f(t)$ = probability density function for fire resistance level
$g(t)$ = probability density function for fire severity
$h(r,s)$ = joint probability density distribution function
P = probability of failure
$P(t)$ = failure probability distribution function
$p(t)$ = failure probability density distribution function
$p = 1 - q$
Q = reliability of fire resistance design $= 1 - P$
q = reliability of sprinkler system
R = fire resistance level (min)
r = realization of R (min)
S = fire severity (min)
s = realization of S (min)
t = time (min)
t_f = expected failure time (min)
U = random supply variable with normal distribution
V = random demand variable with normal distribution
W = safety margin
X = subset of sample space
$x = \ln(t)$

α = coefficient of variation
β = safety index (Beta reliability index)
μ = mean
σ = standard deviation
Φ = probability distribution function of the standard normal variable

Subscript

A = that of the alternative solution with sprinkler protection
C = that of the code-complying design without sprinkler protection
o = that when sprinkler operates

APPENDIX A

The domains of the two probability density functions for fire severity and the realized FRL can be either finite or infinite. Without losing generality, the infinite time domain is assumed in the following discussion.

The fire resistance failure of a building structure is defined as the loss of structure stability or integrity or heat resistance capability. Notionally, such an event occurs when the fire severity in a compartment exceeds the FRL of the building elements of which the compartment is constructed. Therefore, probability of failure is the probability that the fire severity in a compartment is greater than the FRL of a building construction, or Prob$\{S \geq R\}$.

For independent R and S, the joint probability density is:

$$h(r, s) = f(r)g(s). \tag{13}$$

The probability density that $S \geq R$ at a given $R = r$ is:

$$\begin{aligned} p(r) &= \text{Prob}\{S \geq r\} \\ &= \int_r^\infty h(r, s)\text{d}s \\ &= f(r)\int_r^\infty g(s)\text{d}s \\ &= f(r)\left[1 - \int_0^r g(s)\text{d}s\right] \\ &= f(r)[1 - G(r)], \end{aligned} \tag{14}$$

where $G(r)$ is the cumulative probability function of fire severity up to time r. The cumulative failure probability function is the integration of the above equation up to time t:

$$P(t) = \int_0^t p(r)\mathrm{d}r$$
$$= \int_0^t f(r)[1 - G(r)]\mathrm{d}r$$
$$= F(t) - \int_0^t f(r)G(r)\mathrm{d}r \tag{8}$$

The probability of failure is the integration of Equation (14) over the entire time domain, or the value of $P(t)$ at $t = \infty$:

$$P = P(\infty) = 1 - \int_0^\infty f(r)G(r)\mathrm{d}r. \tag{9}$$

Note:

(1) P is not the area of overlapping of the two probability density distribution curves, $g(s)$ and $f(r)$.
(2) Since fire severity is a function of ventilations condition, the overall fire severity in a compartment can be dependent on the FRL of the structural elements of the compartment. For example, the integrity failure of a fire rated door will create additional vent opening to the compartment, hence affecting the fire severity. However, the resulting change in fire severity will not affect the evaluation of failure probability since the integration in Equation (14), namely $G(r) = \int_0^r g(s)\mathrm{d}s$, terminates at the moment of failure and what happens afterwards is irrelevant. In other words, failure occurs before the change in fire severity.

APPENDIX B

The event of fire severity, S, reaches a given time in a sprinkler protected building can be viewed as the intersection with the union of two mutually exclusive subsets, X_1 and X_2, which represent the two statuses of the sprinkler system: operating or failure.

$$S = S \cap (X_1 \cup X_2)$$
$$= (S \cap X_1) \cup (S \cap X_2). \tag{15}$$

According to probability theory [29]:

$$P(S) = P(S \cap X_1) + P(S \cap X_2)$$
$$= P(X_1)P(S|X_1) + P(X_2)P(S|X_2). \quad (16)$$

If X_1 represents sprinkler operating and X_2 failure, then:

$$P(X_1) = q \quad \text{and} \quad P(X_2) = p. \quad (17)$$

For continuous distributions, $P(S|X_1)$ and $P(S|X_2)$ can be expressed as:

$$P(S|X_1) = g_{\mathrm{Ao}}(t),$$

and

$$P(S|X_2) = g_{\mathrm{C}}(t). \quad (18)$$

Then, according to Equation (16), the probability density distribution function for fire severity in a sprinkler protected building is:

$$g_{\mathrm{A}}(t) = q g_{\mathrm{Ao}}(t) + p g_{\mathrm{C}}(t). \quad (10)$$

Note that Equation (10) satisfies:

$$\int_0^\infty g_{\mathrm{A}}(t)\mathrm{d}t = 1. \quad (19)$$

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