

# THE PROBABILITY OF DEATH IN THE ROOM OF FIRE ORIGIN: AN ENGINEERING FORMULA

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## ABSTRACT

The paper presents a partial safety factor approach to the problem of evaluating competing designs for fire safety in the room of fire origin in a building. A partial safety factor is defined as the ratio of the design value to the characteristic value for a load type variable and its inverse for a resistance type variable. The safety criterion considered is the expected number of deaths in the room or, alternatively, the probability of any death. A death is assumed to occur when the time,  $X$  between the occurrence of the alerting cue and the onset of untenable conditions is shorter than the time,  $Y$  to evacuation.

First, a safety index,  $\beta$  is obtained, based on the means and standard deviations of the logarithms of  $X$  and  $Y$ . Partial safety factors for  $X$  and  $Y$  can then be calculated.

It is further shown that there are theoretical reasons as well as empirical evidence for assuming that  $X$  and  $Y$  both have approximately a lognormal distribution. There is then a direct connection between the probability of death and the safety index  $\beta$ , which leads to a rationale for selecting appropriate values of  $\beta$ . A discussion of the choice criteria for characteristic values and code calibration procedure is also included.

## INTRODUCTION

In the 1970s, a number of structural design codes, for example the CEB code<sup>1</sup> in Europe and the National Building Code of Canada,<sup>2</sup> adopted the so-called "partial safety factors" approach to reflect uncertainties in the data and/or model inaccuracies. This approach involves two steps:

1. A characteristic value is determined for each of the variables which appear in the model.
2. Each characteristic value is multiplied or divided by a "partial safety factor," depending on whether the variable is a load variable or a resistance variable, to obtain the values of the design variables.

The use of the partial safety factor approach in fire safety design has been forcefully advocated in a number of publications (see e.g. Ramachandran<sup>3</sup>). More recently, it was introduced in an invited lecture at the Fifth International Symposium on Fire Safety Science in 1997<sup>4</sup> under the name of "Level 1 design method." Further details and an example are given in a lecture by Frantich et al at the same Conference.<sup>5</sup>

In this paper we look at the problem of evaluating competing designs for fire safety in the room of fire origin in a building. The criterion we wish to examine is the expected number of deaths in that room. Another possible criterion will be examined in Distributional Assumptions. In the simplified model we are setting up, there are just two variables: the time to onset of "untenable conditions,"  $X$  and the time to evacuation,  $Y$ . The partial safety factor approach requires the determination of characteristic values,  $X_C$  and  $Y_C$ , for these two variables and then the determination of two partial safety factors  $SF_X$  and  $SF_Y$ . Since an increased evacuation time decreases safety, evacuation time behaves like a load variable. On the other hand, the time to onset of untenable conditions, which when decreased decreases safety, behaves like a resistance variable. Thus the appropriate formulae for the design variables  $X_{des}$  and  $Y_{des}$  are

$$X_{des} = X_C / SF_X \quad (1)$$

$$Y_{des} = Y_C \cdot SF_Y \quad (2)$$

In this paper we will address the problem of relating the characteristic values,  $X_C$  and  $Y_C$ , and the

partial safety factors,  $SF_x$  and  $SF_y$ , to the expected number of deaths as derived from a probabilistic risk analysis. The analysis can be carried out at the code drafting stage, based on statistical data and fire modeling. The resulting characteristic values and partial safety factors can then be tabulated in the code for the various types of occupancies, fuel loads, barrier design and any other relevant factors.

## THE MODEL

The central reason for advocating probabilistic modeling as a basis for code construction is the fact that there cannot be accurate knowledge in advance of the exact conditions under which a fire will start and develop. Nevertheless, we can subsume data from statistics of fires, experiments and computer simulations in the form of probability distributions. In particular, we can estimate central values and spread for the key design parameters. As Ramachandran<sup>6</sup> has written: "Probability modeling is concerned with final outcomes rather than the detailed knowledge of the processes that make it." For example, in the case of tossing a coin, it is theoretically possible to calculate the outcome for each coin, provided we have full knowledge of the initial conditions. But without this detailed knowledge, which is not normally available, we can still make accurate statements about the mean and standard deviation of the proportion of heads in a large number of coin tosses, based only on the symmetry of the coin.

Simple probabilistic models have been used successfully in a number of areas in fire risk analysis. For example, it is generally accepted that fire loss follows a lognormal distribution and that the number of fire insurance claims per year follows a Poisson distribution (see Ramachandran<sup>7</sup>). In this paper, the idea of using a simplified probabilistic model, rather than a fully detailed deterministic model whose parameters will remain unknown, is extended to the study of safety in the compartment of fire origin.

In the first instance, the proposed calculations will involve only means, standard deviations and correlations. This is known as the "second moment" level of representation. In a subsequent

section, distributional assumptions will be introduced and justified.

Consider a particular occupant of the room. When a fire starts, it may take a short or a long time until it causes a cue to be given to that occupant. This may be an alarm, smoke, flame, window glass breaking, direct observation of ignition or the arrival of the fire brigade. To each of those cues is attached a probability that it will be the first to alert the considered occupant. For the purposes of the present model, we shall assume that there is only one cue, and that this cue occurs before the onset of untenable conditions.

More than one cue can be accommodated as follows: the conditional probability of death of the considered occupant is calculated for the case of each of the considered cues being the first to alert the occupant. Then, the obtained values are combined, using the theorem of total probability, from the knowledge of the probabilities attached to the various cues. In addition, there may be a non-zero probability that the onset of untenable conditions will occur before the considered occupant is alerted. In our model, this will result in the death of the occupant. This additional case can be incorporated into the conditional probability algorithm described in the preceding paragraph.

Starting from the point of time when the alerting cue occurs, we denote by  $X$  the random time until the onset of "untenable conditions" in the room of fire origin. By "untenable conditions" we mean a room condition which will result in the death of the considered occupant if they have not evacuated by then. The distribution of  $X$  can be estimated by running a Monte Carlo simulation, using the appropriate program for compartment fires. The stochastic variation is introduced by sampling the parameters of the fire program from suitable probability distributions. The cumulative effect of mechanisms affecting the considered occupant, the most important being toxic fumes, radiation, oxygen depletion and heat stress, can be monitored by the Monte Carlo simulation. There are also more rapid mechanisms of fatal injury that can occur for individuals who are "intimate with ignition", such as the rapidly fatal burns associated with clothing fires.

The mechanisms affecting the occupants can sometimes impair movement without causing total incapacitation which would prevent escape. In principle, it would be possible to take account of their influence on the speed of the escape. However, in most circumstances, if the design is sufficiently conservative, such a detailed evaluation is not justified. For the purposes of design it will be assumed that the movement of the occupants is unaffected by the fire until untenable conditions are achieved, at which time movement ceases.

Ensuring that the design is conservative is part of the code calibration process which needs to be used to determine an appropriate level of safety. There is a fairly broad consensus about the essential steps of the code calibration process. A short presentation is given in Appendix C. For full details and references see Melchers.<sup>8</sup>

Let us recollect that we deal separately with the case when the onset of untenable conditions occurs before the considered occupant is alerted, resulting in certain death. Thus, in the following analysis,  $X$  will be taking only positive values. It is then convenient to write  $\ln X = U$ , and to work with  $U$  rather than  $X$ . In view of the impossibility of having accurate knowledge of the conditions that will prevail in an actual fire, it is not unreasonable, for engineering design purposes, to subsume our knowledge of  $X$  by just two parameters:

$$E(U) = \mu_U \quad (3)$$

$$\text{Var}(U) = \sigma_U^2 \quad (4)$$

Also, starting from the point of time when the alerting cue occurs, we denote by  $Y$  the time until the considered occupant evacuates the room. This is made up of a number of components: a waking up time component (mainly at night), a cognitive process time component, an investigation component and an evacuation preparation component. The distribution of  $Y$  should be derived from statistical surveys. For the purpose of engineering design, we lump together all these components, write  $\ln Y = V$  and subsume our knowledge of  $Y$  by just two parameters:

$$E(V) = \mu_V \quad (5)$$

$$\text{Var}(V) = \sigma_V^2 \quad (6)$$

There are two basic simplifying assumptions for the model.

1. We do not explicitly consider the interaction between occupants. The interaction can be taken into account by using conservatively large values for the evacuation time.
2. Occupants will incur death if, and only if, their evacuation time,  $Y$  is longer than the time of onset of untenable conditions,  $X$ .

It follows from these assumptions that if  $p = P(X < Y)$  is the probability that the evacuation time is longer than the time of onset of untenable conditions, and there are  $N$  occupants in the room of fire origin the expected number of deaths will be  $Np$ . Thus if the expected number of deaths must be kept below some given value  $N_0$  the allowable probability of death will be inversely proportional to the number of occupants. How this will affect the design will be discussed at length in what follows.

## CALCULATION OF A SAFETY INDEX

The simplest way of quantifying the probability of death is through the safety index  $\beta$  (see Melchers).<sup>8</sup> It follows from the basic assumptions of the model that the event of the considered occupant not dying is identical to the event  $X > Y$ . But, in view of the monotonicity of the logarithmic transformation, this event is identical to the event  $U > V$ .

We now need to address the problem of the correlation between  $X$  and  $Y$ . It stands to reason that this correlation will be positive. Indeed, if the fire is severe, untenable conditions will occur early. At the same time, the fire will supply more compelling cues to the considered occupant, so that we can assume that evacuation time will tend to be shorter.

In fact, perusal of the available literature (see Brennan<sup>9</sup> and the references quoted therein), yielded no evidence that a more severe fire could lead to an extended time of evacuation for any reason. Even if this did occur in isolated cases, the sign of the correlation would not be affected, as it is a statistical property of the whole population of fires considered and is not significantly

affected by isolated outcomes. We conclude that there will be some positive correlation between  $X$  and  $Y$  and, therefore, also between  $U$  and  $V$ . Let the correlation between  $U$  and  $V$  be denoted by  $\rho$ .

Let now  $W = U - V$ ,  $E(W) = \mu_w$ ,  $\text{Var}(W) = \sigma_w^2$ . Then

$$\mu_w = \mu_U - \mu_V \quad (7)$$

$$\sigma_w^2 = \sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V \quad (8)$$

By definition, the safety index  $\beta$  is the threshold value of  $\beta$  for which  $\mu_w \geq \beta\sigma_w$ . In other words, safety occurs when the mean of  $W$  is further away from zero than  $\beta$  times its standard deviation.

Thus, in order to achieve a safety index of  $\beta$ , it is necessary to choose the parameters of the onset of untenable conditions in such a way that

$$\mu_U > \mu_V + \beta\sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V} \quad (9)$$

The appropriate value of the safety index depends on the severity of the risk to life and property. For example, it would be appropriate to take  $\beta = 2.33$  if there are 10 occupants at risk but  $\beta = 3.09$  if there are 100 occupants at risk. The rationale for these recommendations will be given in Distributional Assumptions.

## SIMPLIFICATIONS

The above result can be greatly simplified if we are prepared to make some conservative assumptions.

Firstly, it should be noted that there is very scant knowledge of the value of the correlation  $\rho$  in real situations. Fortunately, since, as pointed out above,  $\rho$  will normally be positive, it is a conservative assumption to assume it to be zero, as can be seen from Eq. (9).

Secondly, suppose that we define two “design values” for  $U$  and  $V$  as follows:

$$U_{des} = \mu_U - \beta^*\sigma_U \quad (10)$$

$$V_{des} = \mu_V - \beta^*\sigma_V \quad (11)$$

We choose  $\beta^*$  in such a way that when  $V_{des}$  is less than  $U_{des}$ , the safety index is greater than  $\beta$ . Then it is easy to see that the appropriate value of  $\beta^*$  is given by

$$\beta^* = \beta \frac{\sqrt{\sigma_U^2 + \sigma_V^2}}{\sigma_U + \sigma_V} \quad (12)$$

From this, it follows that  $\beta^*$  will vary from  $0.707\beta$  (when the two standard deviations are equal) to  $\beta$  (when one of the standard deviations is zero).

Let us now estimate “characteristic values” for  $U$  and  $V$  as follows:

$$U_C = \mu_U - k\sigma_U \quad (13)$$

$$V_C = \mu_V + k_1\sigma_V \quad (14)$$

where  $k$  and  $k_1$  are appropriate values which depend on the reliability of our estimation. Recommended values are given in Distributional Assumptions.

It then turns out that

$$U_{des} = U_C - (\beta^* - k)\sigma_U \quad (15)$$

$$V_{des} = V_C + (\beta^* - k_1)\sigma_V \quad (16)$$

Finally, we can revert to the original values of the variables  $X$  and  $Y$  as follows:

$$X_C = \exp(U_C) \quad (17)$$

$$Y_C = \exp(V_C) \quad (18)$$

Furthermore,;

$$SF_X = \exp[(\beta^* - k)\sigma_U] \quad (19)$$

$$SF_Y = \exp[(\beta^* - k_1)\sigma_V] \quad (20)$$

Note that  $X_C$  and  $Y_C$  are simply the quantiles of  $X$  and  $Y$  corresponding to  $U_C$  and  $V_C$ . As for  $SF_X$  and  $SF_Y$ , they are the partial safety factors corresponding to the variables  $X$  and  $Y$ , respectively.

The design values for  $X$  and  $Y$ ,  $X_{des}$  and  $Y_{des}$ , will then be given by the standard formulae:

$$X_{des} = X_C/SF_X \quad (21)$$

$$Y_{des} = Y_C \cdot SF_Y \quad (22)$$

Thus, in each design class, the partial safety factors can be calculated in terms of the standard deviations of  $U$  and  $V$ .

## DISTRIBUTIONAL ASSUMPTIONS

In the previous sections, no distributional assumptions have been made and attention has been focussed on means, standard deviations and correlations.

While this approach provides a convenient way to compare the safety of different designs, it does not yield actual probabilities of death. To obtain the latter, it is unavoidable to make distributional assumptions.

The two most frequently used criteria for safety of a fire design are the expected number of deaths and the probability of any death. If the probability of death is denoted by  $p$  and the number of occupants by  $N$ , then the expected number of deaths is given by  $Np$  and the probability of any death (i.e. at least one death) by

$$1 - (1 - p)^N \quad (23)$$

However, in the range of values of  $p$  and  $N$  considered in this paper, namely  $Np \leq 0.02$  (see next paragraph) there is hardly any difference between the two criteria. For example, if  $N = 10$  and  $p = 0.002$  we have  $Np = 0.02$  and  $1 - (1 - p)^N = 0.0198$ .

A very convenient and robust assumption when dealing with non-negative variables such as  $X$  and  $Y$  above is to assume that they have a lognormal distribution. This, of course, implies that  $U$  and  $V$  are normally distributed. Some basic properties of the lognormal distribution are given in Appendix A and a justification of the assumption of lognormality for  $X$  and  $Y$  is given in Appendix B.

The rationale behind the recommended values of  $\beta$  in the previous section then becomes apparent. Since  $W$  is normally distributed, the probability of death for each occupant corresponding to  $\beta = 2.33$  is 0.001 and the expected number of deaths in one fire among 10 occupants is 0.01. Similarly, the probability of death for each occupant corresponding to  $\beta = 3.09$  is 0.0001 and the

expected number of deaths in one fire among 100 occupants is again 0.01.

We now discuss the rationale behind the choice of  $k$  and  $k_1$ . They represent the discrepancy in standard deviations between the means of  $U$  and  $V$  and the corresponding "characteristic values," which are supposed to lie somewhere in the tails of the distributions.

There is little information available about the methodology of choice of characteristic values. In the CEB code<sup>1</sup> the characteristic values of the loads and resistances are considered as their 95 or 5 percentile values as appropriate, or their currently accepted values in lieu. This approach may be sufficient for static design in a structural code context, but it needs more flexibility in the context of fire engineering, where the availability of data is, in general, poor and the variability of parameters large. If the reliability of the estimation is poor, we cannot expect to accurately estimate the tails of the distributions. We, therefore, propose the following methodology: If the reliability is "excellent," the characteristic values will be taken as the 5th and 95th percentiles of the distribution. For a "good" reliability, they will be taken as the 10th and 90th percentiles, for a "reasonable" reliability, they will be taken as the 20th and 80th percentiles and, for a "poor" reliability, they will be taken as the 40th and 60th percentiles. The corresponding values of  $k$  and  $k_1$  for a lognormal distribution of  $X$  and  $Y$  are given in Table 1.

In addition, it should be pointed out that under the lognormal assumption the standard deviations  $\sigma_U$  and  $\sigma_V$  can be directly calculated from the mean and standard deviation of each of  $X$  and  $Y$ , using the well-known exact relations (see Appendix A):

**Table 1. Values of  $k$  for a Lognormal Distribution of  $X$  and  $Y$**

Reliability	Percentiles	$k$ or $k_1$
Excellent	5-95	1.64
Good	10-90	1.29
Reasonable	20-80	0.84
Poor	40-60	0.25

$$\sigma_U = \sqrt{\ln(1 + CV_X^2)} \quad (24)$$

$$\sigma_V = \sqrt{\ln(1 + CV_Y^2)} \quad (25)$$

where  $CV_X$  and  $CV_Y$  are the coefficients of variation of  $X$  and  $Y$ , respectively.

These relations are, however, very robust and insensitive to distributional assumptions, particularly for small coefficients of variation.

## EXAMPLES

### Example 1

In the analysis of a particular compartment, it was determined that the mean evacuation time was 1.25 minutes with a coefficient of variation of 0.4.

A Monte Carlo simulation of the fire spread was carried out and the time from various cues until the onset of untenable conditions was evaluated. The means and coefficients of variation are given in Table 2.

**Table 2. Means and Coefficients of Variation for Times to Untenable Conditions**

Cue	$\mu_X$	$CV_X$
Light smoke	7.859	0.521
Medium smoke	7.099	0.525
Heavy smoke	4.397	0.483
Automatic smoke detector	7.351	0.526
Heat detector	3.288	0.541

**Table 3. Values of  $\beta$ ,  $p$ ,  $\beta^*$ ,  $SF_X$  and  $SF_Y$**

Cue	$\beta$	$p$	$\beta^*$	$SF_X$	$SF_Y$
Light smoke	2.877	0.002	2.049	1.222	1.34
Medium smoke	2.700	0.0035	1.923	1.15	1.276
Heavy smoke	2.050	0.0202	1.455	0.919	1.066
Automatic smoke detector	2.752	0.003	1.961	1.172	1.295
Heat detector	1.434	0.0759	1.023	0.731	0.902

Let us assume that the reliability of estimation for the time to untenable conditions is “excellent” so that we can use  $k = 1.64$ , while the reliability of estimation of the evacuation time is no better than good, with  $k_1 = 1.29$ . Table 3 gives the values of the reliability index  $\beta$ , the probability of death  $p$ , and the corresponding values of  $\beta^*$ ,  $SF_X$  and  $SF_Y$ .

If it is required that the probability of death be less than 0.01, then an acceptable degree of safety can be achieved only if the occupants are alerted as soon as there is a medium amount of smoke in the compartment. This level of safety can be achieved with the installation of an automatic smoke detector. However, a heat detector does not react quickly enough to ensure acceptable safety in the scenario under investigation.

### Example 2

Suppose that the time,  $X$  to untenable conditions has a coefficient of variation of 0.5 and that the time to evacuation,  $Y$  has a coefficient of variation of 0.4. Suppose further that it is required to achieve a safety index,  $\beta = 2.7$ . Under the assumptions detailed in the previous sections, it is possible to calculate the appropriate partial safety factors under various assumptions relating to the reliability of the estimation of the characteristic values. Using the formulae derived earlier in the paper, we find that the partial safety factor for the time to untenable conditions,  $SF_X$ , and the partial safety factor for the time to evacuation,  $SF_Y$ , are as in Table 4.

It should be noted that the value of the partial safety factor depends on the reliability of estimation of the variable to which it relates, but not on the reliability of estimation of the other variables in the model. The interaction between the

**Table 4. Values of  $SF_X$  and  $SF_Y$  for Various Values of  $k$  and  $k_1$**

Reliability	Percentiles	$k$ or $k_1$	$SF_X$	$SF_Y$
Excellent	5–95	1.64	1.141	1.113
Good	10–90	1.29	1.346	1.274
Reasonable	20–80	0.84	1.665	1.515
Poor	40–60	0.25	2.2	1.902

variables is accounted for by the values of  $\beta$  and  $\beta^*$ .

## CONCLUSION

Characteristic values of the time to onset of untenable conditions  $X$  and the evacuation time,  $Y$  have been formulated, based on their variability and on the reliability of their estimation. Furthermore, formulae for the two partial safety factors,  $SF_X$  and  $SF_Y$  have been obtained, based on the coefficients of variation of  $X$  and  $Y$  and on the number of occupants in the room of fire origin. The derivation of these formulae requires only a conservative assumption about the correlation between  $X$  and  $Y$  and the mild distributional assumption that  $X$  and  $Y$  are approximately lognormal.

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**Appendix A: The lognormal distribution**

We say that  $X$  has the lognormal distribution if  $U = \ln(X)$  is normally distributed. Suppose  $U$  has mean  $\mu_U$  and standard deviation  $\sigma_U$ . Furthermore, denote the mean of  $X$  by  $\mu_X$  and its coefficient of variation by  $CV_X$ . We then have the following two pairs of relations:

$$\mu_U = \ln\left(\frac{\mu_X}{\sqrt{1 + CV_X^2}}\right) \tag{A-1}$$

$$\sigma_U = \sqrt{\ln(1 + CV_X^2)} \tag{A-2}$$

and reciprocally:

$$\mu_X = \exp\left(\mu_U + \frac{1}{2}\sigma_U^2\right) \tag{A-3}$$

$$CV_X = \sqrt{\exp(\sigma_U^2) - 1} \tag{A-4}$$

**Appendix B: Lognormality of  $X$  and  $Y$**

First, here is a justification of the assumption that the time from the alerting cue to untenable conditions has a lognormal distribution.

Computer models of compartment fires usually consist of a set of differential equations which allow the time to untenable conditions to be evaluated as a function of the input parameters of the model. When the input parameters are sampled from appropriate probability distributions, Monte Carlo simulations yield estimators of the distribution of the time to untenable conditions.

It has been observed by many workers in the field that the time to untenable conditions,  $X$  can be approximately expressed in terms of the input parameters  $A_1, A_2, \dots, A_n$  by a formula of the form

$$X = \kappa A_1^{\alpha_1} A_2^{\alpha_2} \dots A_n^{\alpha_n} \tag{B-1}$$

A formula of the form of Eq. (B-1) has been used by Frantzich et al<sup>B-1</sup> to represent the time to “critical conditions in the room” from simulations of the CFAST model<sup>B-2</sup>.

It follows from Eq. (B-1) that

$$\ln(X) = \ln(\kappa) + \alpha_1 \ln(A_1) + \alpha_2 \ln(A_2) + \dots + \alpha_n \ln(A_n) \tag{B-2}$$

In other words,  $\ln(X)$  can be expressed as a linear combination of the logarithms of the inputs. It follows from the Generalized Central Limit Theorem (Feller<sup>B-3</sup>) that  $\ln(X)$  tends to have a normal distribution, which implies that  $X$  tends to have a lognormal distribution.

The justification for a lognormal distribution for the time to untenable conditions is supported by results of a large-scale Monte Carlo simulation of a one-zone fire model, carried out at the Centre for Environmental Safety and Risk Engineering (CESARE), Victoria University of Technology (VUT), Melbourne, Australia. The model used was a modified version of a model first developed by researchers at the National Research Council of Canada (NRC) and subsequently modified by researchers at CESARE. It is now known as the NRC/VUT fire growth model. There were sixteen input parameters, which were randomly generated from given distributions.

Figure B-1 is a plot of 1090 ordered values of the time to untenable conditions, in minutes, on a logarithmic scale, against the corresponding quantiles of a standard normal distribution, together with a line of best fit. It can be seen that the assumption of lognormality of the data applies rather well for most of the range. At the

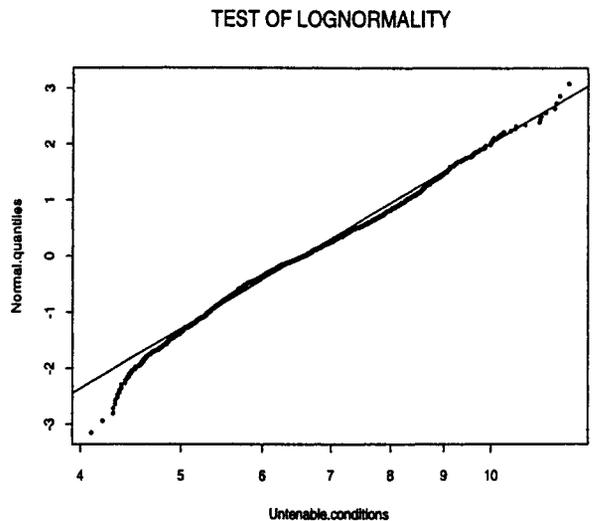


Figure B-1. Lognormality of Time to Untenable Conditions.

lower end of the range, which is the critical part of the distribution for our purposes, the observations tend to be higher than predicted by the lognormal assumption. This means that assuming a lognormal distribution is conservative. The reason for the departure from lognormality at the lower end is that the data appear to have a positive lower end point. This phenomenon is presumably due to the fact that, at the beginning of a fire, the factors which control the subsequent development of the fire, such as fuel mass, room geometry and oxygen supply, do not seriously affect the rate of spread of the fire, so that it takes a fixed minimum amount of time until untenable conditions are reached.

We next need to address the problem that the time to onset of untenable conditions is measured from the point of time of the alerting cue, not from the time of ignition. To see why the change of origin will not significantly affect the shape of the distribution of  $X$ , we show, in Fig. B-2, a scattergram of the time to untenable conditions from ignition against the time to a particular alerting cue, namely heavy smoke (both measured in minutes). It turns out that there is very nearly a linear relation between the two times. All the other times to alerting cues we have examined have shown the same near linearity relation to the time to untenable conditions. It is then clear that the time to untenable conditions, measured from the time of the alerting clue, will also have a lognormal distribution. Indeed, let  $X_1$  be the time to untenable conditions from ignition

Scattergram of Heavy smoke against Untenable conditions

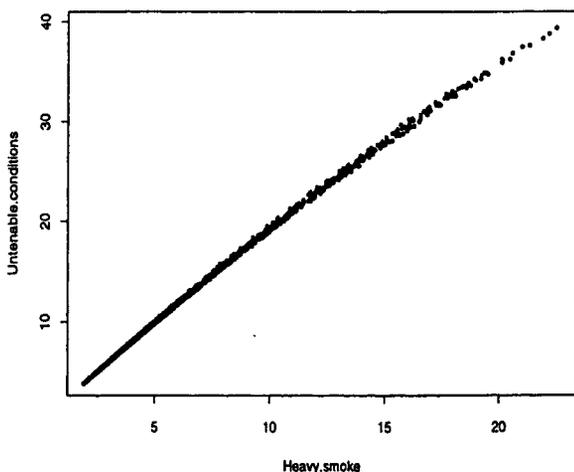


Figure B-2. Scattergram of Untenable Conditions v/s Heavy Smoke.

TEST OF LOGNORMALITY

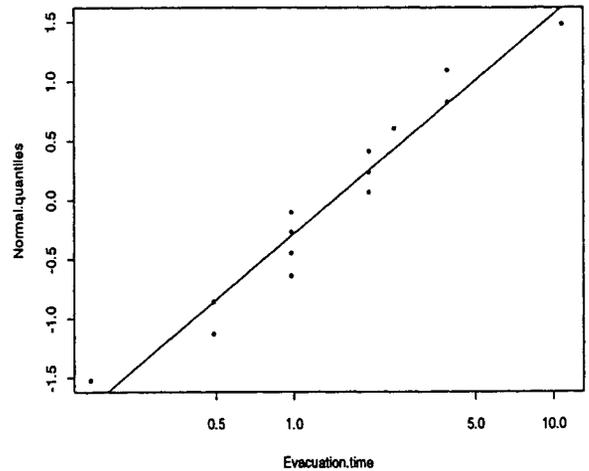


Figure B-3. Lognormality of Time to Evacuation.

and  $X_2$  the time to the alerting cue from ignition. Then  $X = X_1 - X_2$ . But, since we have approximately  $X_2 = \gamma X_1$  for some positive  $\gamma < 1$ ,

$$X = (1 - \gamma)X_1 \tag{B-3}$$

and  $\ln(X) = \ln(1 - \lambda) + \ln(X_1)$ . And since  $\ln(X_1)$  is approximately normally distributed, so is  $\ln(X)$ , with just a change of mean.

We now consider the distribution of the time to evacuation. There are, of course, no computer programs that can perform a Monte Carlo simulation of that variable. We do, however, have the results of research carried out at CESARE on the topic. For details see Brennan.<sup>B-4</sup> Figure B-3 is a plot of 15 ordered values of the time from alerting cue to evacuation, in minutes, on a logarithmic scale, against the corresponding quantiles of a standard normal distribution, together with a line of best fit. Here again there is no evidence to reject the lognormal assumption. In fact, a Kolmogorov-Smirnov test of normality accepts the normality assumption for  $\ln(Y)$ . It must also be remembered that the critical range of the distribution here is the upper range, where the observations tend to be sparse.

## REFERENCES

B-1. Frantzitch, H., Magnusson, S.E., Holmquist, B. and Rydén, J., "Derivation of Partial Safety Factors for Fire Safety Evaluation Using the Reliability Index  $\beta$  Method,"

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- B-3. Feller, W., “An Introduction to Probability Theory and its Applications,” Vol. 11. Wiley, New York, 1966.
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### Appendix C: Code calibration

The following is a short presentation of code calibration procedure, following Melchers.<sup>C-1</sup> The purpose of code calibration is to adjust the reliability index,  $\beta$  for a new proposed code so as to bring its safety level in line with the existing code.

*Step 1: Define Scope.* First of all, the range of designs to be considered must be defined.

*Step 2: Select calibration points.* A number of representative designs are selected for calibration.

*Step 3: Existing design code.* The existing design code is used to carry out the chosen designs.

*Step 4: Define limit states.* A design space, consisting of all basic variables, is chosen. In that space, limit state functions are defined.

*Step 5: Determine statistical properties.* Appropriate statistical properties for each of the basic variables required for calculating the reliability index are derived from available data.

*Step 6: Reliability analysis.* Using an appropriate method of reliability analysis, together with the limit state functions defined in Step 4, each of the designs obtained in Step 3 is analyzed to determine the set of values,  $\beta_C$  for each calibration point.

*Step 7: Select target ( $\beta_T$ ) values.* Designs are categorized according to the  $\beta_C$  values determined. Target values,  $\beta_T$  are chosen for each category of design. Allowance can be made for the consequences of failure, with higher  $\beta_T$  values for high consequence failures. Also, selected  $\beta_T$  values must reflect a trade-off between cost of initial construction and cost of failure consequences. It is recommended to recognize the complexity of the issue and to select the  $\beta_T$  values on a semi-intuitive basis.

### REFERENCES

- C-1. Melchers, R.E., “Structural Reliability: Analysis and Prediction,” Wiley, New York, 1987.