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A THEORETICAL INVESTIGATION INTO THE RESIDUAL DEFORMATION OF STEEL BEAMS AFTER A FIRE

Y. C. Wang
Building Research Establishment
Garston, Watford WD2 7JR
United Kingdom

SUMMARY

A finite element computer program is used to determine the temperature-deflection curve of a loadbearing steel beam throughout the entire heating-cooling duration. From this analysis, the residual deformation of the steel beam is determined. A simple method to calculate this residual deformation is then proposed. Results from this simple method are shown to be in broad agreement with predictions from the more rigorous finite element computer program.

The residual deformation of a steel beam may then be used to assess the potential damage caused by fire and to help establish an approach to calculate the structural fire protection for minimum post-fire repair. In this approach, an appropriate deflection limit is selected at which the steel beam is regarded as serviceable, thus requires no major repair. Fire protection is provided to the steel beam so that its residual deflection after a fire does not exceed this limit. An example is given to show how this simple method is applied.

OBJECTIVE OF RESEARCH

Traditionally, the specification for fire protection for structural steel members is based on results of standard fire resistance tests. In these tests, a structural steel member is subjected to a standard heating regime in a furnace described by the time-temperature curve adopted in various fire resistance test standards such as ASTM E 119[1]. In this context, the fire resistance is the elapsed time of fire exposure to the structural steel member in the furnace before it has reached a set of prescribed failure criteria.

While this traditional approach is simple to use and has been confirmed by the adequate performance of steel structures in fire, there are shortcomings associated with this method.

These shortcomings are that:

(1) Only a very small fraction of structures designed and built can be tested;

(2) The standard heating regime does not simulate real fires;

(3) Fire resistance testing considers the performance of a structural member during the standard fire test only, without concern for its post-test serviceability.

Although there are a large number of research studies on the behavior of real fires and realistic performance of structural steel members under real fire conditions in the last thirty years or so[2], there is very little work on the post-fire behavior of a structure. Particularly, there is no consideration of post-fire serviceability of structural members at the design stage. In this context, the serviceability of a structural member after a fire may be regarded as requiring no replacement. For a steel member, since the criterion for structural replacement is its excessive residual deflection after the fire, the objective of this paper is to present a simple method to calculate this residual deflection so that the post-fire serviceability of a steel member may be addressed at the design stage.

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CURRENT PRACTICE

Although the traditional prescriptive approach[3] based on the results of standard fire resistance tests is still followed in the fire protection industry, there is a trend towards performance-based design for fire resistance. In this performance-based approach, the fire protection to a loadbearing structural member is determined so that this structural member achieves its desired performance under fire conditions.

There are three essential steps in this new approach. Firstly, the fire environment to which the structural member is exposed is determined. Many fire models[4] are available to enable the fire time-temperature relationship to be predicted. For structural applications, simple models based on the assumptions of a post-flashover fire and uniform fire temperature are often used[5]. For practical design, the concept of time equivalency[6] may also be used.

Having established the fire time-temperature relationship, the second step is to calculate the temperature rise in the structural member. For general applications, numerical approaches based on finite element or finite difference method would be more suitable. For design use on structural steel members, simple yet reasonably accurate methods such as that recommended in the European code[7] may be adopted.

Finally, the structural performance is assessed. Since the yield stress and Young's modulus of steel reduce at elevated temperatures, both the strength and stiffness of a structural steel member reduce at high temperatures. This weakened structural performance may not be adequate, hence structural steelwork often requires fire protection.

Due to the importance of safety, structural strength under fire conditions has been the only criterion to judge the adequacy of a structural steel member's performance. For example, in the more advanced steel fire resistant design codes such as the British Standard[8] and the European Standard[7], this structural performance criterion is translated into a set of limiting temperature load ratio relationships for various types of structural steel members.

The load ratio is defined as the ratio of applied load on the steel member under the fire condition to its load carrying capacity at cold condition. The limiting temperature is the steel temperature at which the load carrying capacity of the steel member reduces to the applied load level under the design fire condition. This means that the current design consideration is strength only. The limiting temperature is measured at the most critical location of the steel section. For a steel beam, this is the lower flange of section if a fire is heating up the beam from underneath. For example, Table 1 gives the limiting temperature-load ratio relationship for a steel beam with concrete slabs on top and a fire exposure from below.

Table 1: Limiting Temperatures (°C) for Beams at Different Load Ratios

Load ratio	0.2	0.3	0.4	0.5	0.6	0.7
Limiting temperature (°C)	780	725	680	650	620	590

Whilst this is necessary, it may not be sufficient. A steel building designed on such a concept will be safe under fire conditions. However, the deformation of its structural steel members may be so excessive that they are not reusable after a fire. The implication is that post-fire replacement of major structural members is necessary even though these structural members are safe during the fire. Although the structural repair itself may not be expensive, the cost incurred due to business interruption may be large. The best strategy may be to avoid such major post-fire repairs by specifying a heavier fire protection to the major structural steel members at the design stage so that there is no danger of excessive residual deformation. Obviously, decisions should be made to select the structural steel members that should be fire protected to such a level, it is nevertheless necessary to establish a methodology to evaluate the post-fire residual deformation of these steel members to enable the fire protection strategy to be implemented.

Brief Description of the Finite Element Program

The behavior of steel structures under fire conditions may be analyzed using a number of methods. Amongst which, the finite element method[9] is the most general and versatile. Although many types of elements may be used to divide a structure steel member, the two noded line element[10] is most efficient for frame analysis. Various researchers have used the finite element method to predict the response of steel frames under fire conditions[11,12,13,14]. In the author's development of a finite element program[15], the following main assumptions are made:

(1) The principle of virtual work is adopted.

(2) A structural steel member is modelled by a number of beam elements. Each element has two nodes and each node 6 degrees of freedom, representing the actions of axial deformation, lateral deflection and rotation about both axes of the element cross-section and twist.

(3) Only the effect of normal stress in the longitudinal direction of a structural steel member is considered.

(4) Plane sections before deformation remain plane after deformation.

(5) The cross-section of an element may be consisted of a number of rectangular blocks. Gaussian integration method is used to obtain the contribution of each block to internal stress resultants and the stiffness matrix.

(6) The main difference between the analysis of steel frames at cold condition and at elevated temperatures is that temperature related steel properties have to be included in the latter. The temperature related material properties include thermal strains and the temperature dependent stress-strain relationship.

For steel, its thermal strains include the thermal expansion and creep. However, creep is normally very small. Steel stress-strain curves at elevated temperature may be described by many models. The following three are implemented in the program: the tabulated data from the British Steel test results,[16] the Ramberg-Osgood equation[17] fitted to this test data and the mathematical equation proposed in the Eurocode 3[7]. The stress-strain relationship of steel at various high temperatures, obtained from British Steel tests[16] are shown in Figure 1.

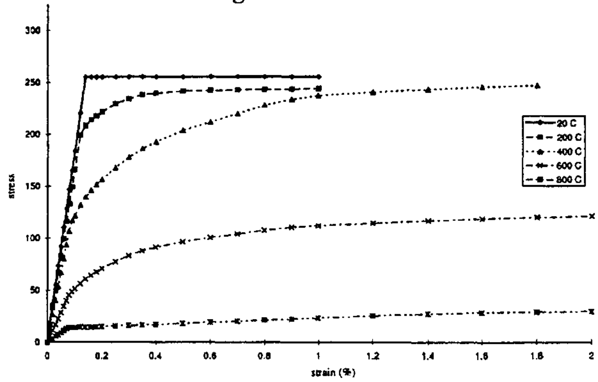

Figure 1. Steel Stress-Strain Curves at High Temperatures, Yield Stress = 255 MPa.

(7) Non-uniform temperature can be incorporated in the analysis. The temperature distribution in the cross-section is input by the user, and the program automatically calculates the temperature values at the Gaussian integration points by linear interpolation.

(8) The program can be used to predict the behavior and fire resistance of an individual steel member and its interaction with other structural members in a steel frame before its failure. The program is not sufficiently developed to trace its post-failure interaction with other structural members because of the numerical procedure adopted in

the program.

The computer program can be used to evaluate the fire resistance of various types of construction such as steel beams and columns, concrete beams and columns, composite beams and columns and steel frames. It can also be used to predict the response of these types of construction under realistic fire conditions. A comprehensive exercise has been conducted to validate the ability of the computer program[15]. In this paper, a few examples are presented to illustrate the accuracy of the program.

Figure 2 compares the predicted maximum deflection history against test results of a simply supported steel beam at elevated temperature in a standard fire resistance test. Test results are extracted from a British Steel compendium[16]. The finite element prediction is very accurate. This figure also indicates that if a steel beam is loaded to its maximum allowable stress, a fire resistance of 30 min cannot be achieved.

Figure 3 compares the predicted response and test result for a steel column in a standard fire resistance test. Prediction using the finite element method follows the test result very closely. Approaching failure, the column starts to contract due to reduced column axial stiffness at elevated temperature.

Figure 4 presents the predicted behavior and test result for a steel portal frame under a simulated real fire[18]. The predicted beam deflection assuming the realistic boundary condition of rigid joints is close to the test result. The behavior of the beam with pin joints is also predicted and is shown in Figure 4 for comparison. Obviously, the behavior of the beam is different if its boundary condition changes. However, according to the current practice for structural fire resistance, the fire resistance of the beam with pin joints would have been used to represent that of the beam with the more realistic boundary condition of rigid joints.

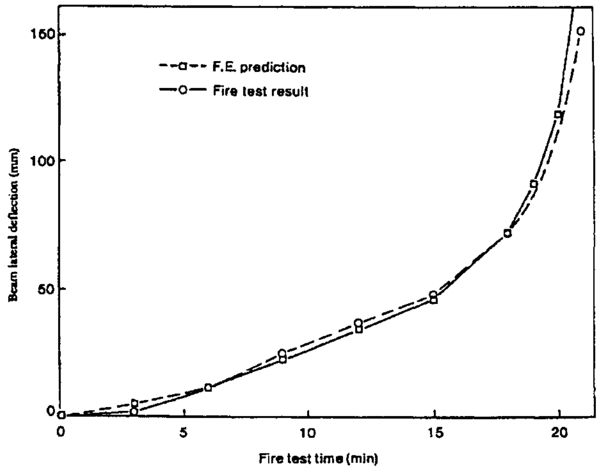

Figure 2. Behavior of a Simply Supported Steel Beam under Fire Condition.

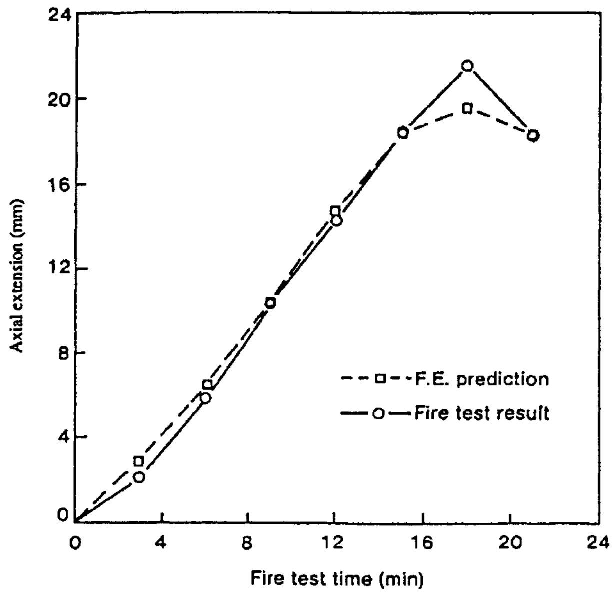

Figure 3. Behavior of a Steel Column under Fire Condition.

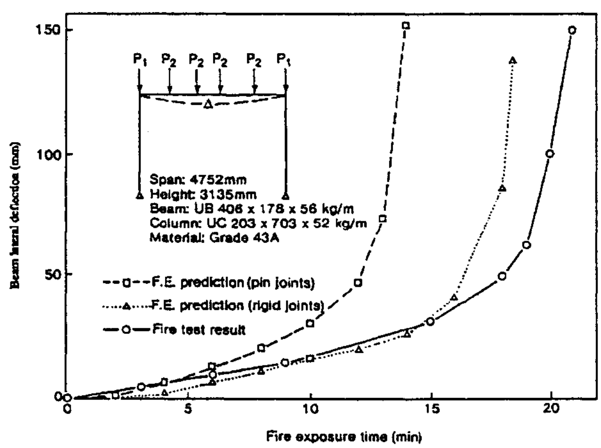

Figure 4. Behavior of a Steel Portal Frame under Fire Condition.

The ability of the finite element program to accurately predict the deflection history of a steel beam at elevated temperature makes it a powerful tool to calculate the post-fire residual deflection of a steel beam. However, such a complicated tool may not be used in a design office. It is therefore important to develop simple methods for design use. The finite element program may be used to validate these simple methods.

Development of a Simple Method

At elevated temperature, the deflection of a structural member is composed of two components: the increasing mechanical deflection from applied load and the thermally induced deflection due to differential thermal expansion. For a steel beam, the thermally induced deflection comes from the temperature difference in its cross-section. The mechanical deflection increases because of the reduced stiffness of steel at high temperatures as illustrated by the high temperature steel stress-strain relationship of Figure 1.

The mechanical deflection of the beam can be further divided into elastic deflection and plastic deflection. Since the stress-strain relationship of steel at elevated temperature is highly non-linear, it is reasonable to assume that the total steel strain is predominantly plastic strain and consequently the total steel beam deflection plastic deflection.

After the steel beam has cooled down, the thermally induced deflection is recovered. Furthermore, the elastic deflection at high temperature will be reduced to that at cool condition because it is reversible. However, the elastic deflection is only a small part of the total deflection such that the post-fire residual deflection of a steel beam is only slightly less than its mechanical deflection at the maximum temperature. The calculation of the steel beam residual deflection may then be based on the stress-strain relationship of steel at the maximum steel temperature.

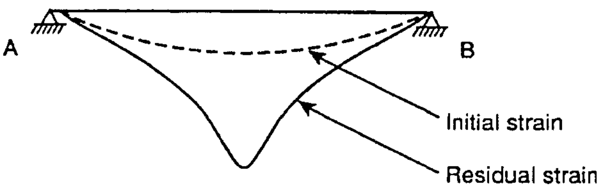

Figure 5. Distribution of Strain in Beam.

The deflection of a beam is obtained from double integration of its curvature. If the post-fire residual curvature distribution in the steel beam is known, its residual deflection is easily calculated. Figure 5 shows qualitatively the initial strain distribution and the corresponding residual strain distribution in a steel beam.

Because of the larger increase in steel strain at higher applied stress in steel at elevated temperature, as indicated in Figure 1, there are sharper increases in curvatures at beam locations where initial stresses are higher. Furthermore, the residual curvatures of these highly stressed locations contribute much more to the total residual deflection of the beam than the less stressed locations. Therefore, for the calculation of residual steel beam deflection, it is reasonable and safe to assume a uniform ratio of residual curvature to the initial curvature as that of the most highly stressed location of the beam. Under this assumption, the ratio of the residual deflection to the initial deflection of the steel beam is equal to this uniform curvature ratio.

The maximum ratio of residual curvature to initial curvature occurs at the location of maximum bending moment, following the reasoning above. To evaluate this maximum ratio, it is necessary to know the stresses at both the top and bottom of the steel section at the location of maximum bending moment. In analogy to plastic design of steel beams at cold condition, both the tensile stress at the bottom and compressive stress at the top of the steel section in the hogging region are calculated as the steel yield stress multiplied by the load ratio. This assumed stress distribution is

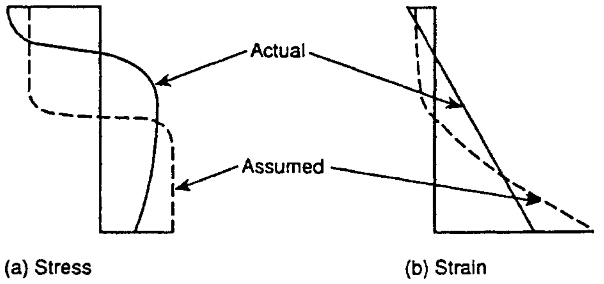

Figure 6. Distribution of Stress and Strain in Steel.

illustrated in Figure 6(a), and its corresponding strain distribution at high temperature is shown in Figure 6(b). The real stress and strain distributions will be like those depicted by the solid lines of Figure 6. According to Figure 6, assuming an equal tensile and compressive stress will overestimate the strain at the lower flange and underestimate the strain at the upper flange.

Since a steel beam is exposed to fire from underneath, its lower flange temperature is much higher than its upper flange temperature. Therefore, the overestimation in the lower flange strain is much higher than the underestimation in the upper flange strain as Figure 1 shows, the steel stress-strain curve becomes very flat at higher temperature and high stress. This implies that using the assumed stress distribution of Figure 6(a) and the associated strain distribution of Figure 6(b) is to overestimate the post-fire residual deflection of the steel beam. This is acceptable because the final results will again be on the safe side.

To summarize, the residual deflection of a steel beam after a fire can be calculated according to the following steps:

(1) Calculate the maximum deflection and maximum curvature of the steel beam at cold condition. The relationship between the maximum curvature and the maximum applied bending moment is expressed as:

$$EI\phi_{max,0} = M_{max} \tag{1}$$

where EI is the stiffness of the steel beam, M_{max} the maximum bending moment and $\phi_{max,0}$ the maximum curvature.

(2) Adopting the assumed stress distribution of Figure 6(a), calculate the strains at the upper and lower flanges of the steel section of Figure 6(b) according to the high temperature stress-strain relationship of steel. The maximum residual curvature of the steel beam is calculated from these two strains.

(3) The ratio of the maximum residual curvature of step (2) to the maximum initial curvature of step (1) gives rise to the maximum curvature ratio.

(4) The maximum residual deflection of the steel beam after a fire is obtained as the maximum initial deflection of the beam at cold condition multiplied by the maximum curvature ratio of step (3).

Parametrical Study

A parametrical study is performed to compare the predictions of the simple calculation method with the finite element program. The parameters investigated are:

Load distribution:	two types as shown in Figure 7
Load ratio:	0.4, 0.5 and 0.6
Temperature distribution:	two types as shown in Figure 8
Maximum steel temperature in lower flange:	400 °C, 500 °C and 600 °C
Steel section sizes:	B1: 305x164UB54, B2: 610x305UB149 and B3: 254x102UB22 according to BS4: part 1[19]
Beam length:	3m, 6m and 9m

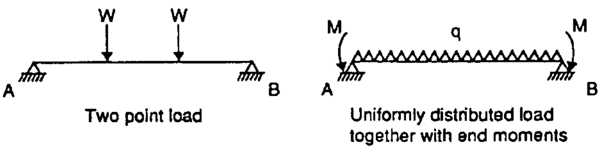

Figure 7. Two Types of Load Distribution.

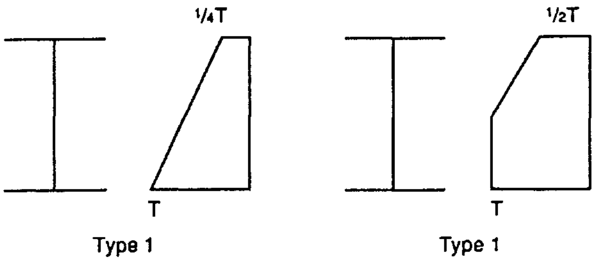

Figure 8. Two Types of Temperature Distribution.

In this study, steel is assumed to have a yield stress of 255 MPa and Young's modulus of 200000 MPa. The two temperature profiles across the steel section may be taken as the two extremes of practical temperature distributions when the beam is exposed to fire underneath and protected by concrete slabs on its top.

Table 2. Results for Load Case 1

Beam type	Temp type	L(m)	δ_i (mm)	δ_r (mm)	δ_r/δ_i	R
B1	T1	3	4.7	15.8	3.4	11.9
		6	18.8	62.3	3.3	11.9
		9	42.3	135.3	3.2	11.9
B1	T2	3	4.7	34.7	7.4	11.9
		6	18.8	135.0	7.2	11.9
		9	42.3	296.1	7.0	11.9
B2	T1	3	2.4	7.9	3.3	11.5
		6	9.6	31.7	3.3	11.5
		9	21.5	71.2	3.3	11.5
B2	T2	3	2.4	17.4	7.3	11.5
		6	9.6	68.7	7.2	11.5
		9	21.5	153.8	7.2	11.5
B3	T1	3	6.1	17.5	2.9	11.5
		6	24.2	67.1	2.8	11.5
		9	54.5	147.4	2.7	11.5
B3	T2	3	6.1	45.0	7.4	11.5
		6	24.2	169.4	7.0	11.5
		9	54.5	376.9	6.9	11.5

(a) Fixed load ratio (0.5), fixed maximum steel temperature (600 °C) variable steel section size.

RESULTS AND DISCUSSION

Tables 2 and 3 present the results of the study. In these tables, the maximum initial deflection of the beam (δ_i), the maximum residual deflection (δ_r), the ratio of these two deflections and the ratio of curvatures predicted using the simple method (R) are reported.

In Tables 2(a) and 3(a), three types of steel section are studied. In other tables, the steel section is B3: 254x102UB22.

Results in these tables indicate that the simple method does not predict any sig-

Max. temp	Temp type	L(m)	δ_i (mm)	δ_r (mm)	δ_r/δ_i	R
400	T1	3	4.7	5.8	1.2	1.3
		6	18.8	23.2	1.2	1.3
		9	42.3	51.3	1.2	1.3
400	T2	3	4.7	7.0	1.5	1.3
		6	18.8	27.9	1.5	1.3
		9	42.3	61.3	1.4	1.3
500	T1	3	4.7	8.3	1.8	2.2
		6	18.8	33.9	1.8	2.2
		9	42.3	73.0	1.7	2.2
500	T2	3	4.7	12.0	2.6	2.3
		6	18.8	48.0	2.6	2.3
		9	42.3	104.2	2.5	2.3

(b) Fixed steel section size, fixed load ratio (0.5), variable maximum steel temperature.

Load ratio	Temp type	L(m)	δ_i (mm)	δ_r (mm)	δ_r/δ_i	R
0.4	T1	3	3.8	5.6	1.5	1.5
		6	15.0	22.4	1.5	1.5
		9	33.8	50.0	1.5	1.5
0.4	T2	3	3.8	8.0	2.1	1.6
		6	15.0	33.2	2.2	1.6
		9	33.8	71.9	2.1	1.6
0.6	T1	3	5.6	11.6	2.1	2.9
		6	22.6	47.1	2.1	2.9
		9	50.7	103.0	2.0	2.9
0.6	T2	3	5.6	18.3	3.3	3.0
		6	22.6	73.8	3.3	3.0
		9	50.7	157.0	3.1	3.0

(c) Fixed steel section size, fixed maximum steel temperature (500 °C) variable load ratio.

Table 3. Results for Load Case 2

Beam type	Temp type	L(m)	δ_i (mm)	δ_r (mm)	δ_r/δ_i	R
B1	T1	3 6 9	3.99 15.95 35.89	11.95 46.65 101.2	3.03 2.92 2.82	11.9 11.9 11.9
B1	T2	3 6 9	3.99 15.95 35.89	24.88 98.22 212.3	6.24 6.16 5.92	11.9 11.9 11.9
B2	T1	3 6 9	2.03 8.12 18.26	6.01 23.82 53.58	2.96 2.93 2.93	11.5 11.5 11.5
B2	T2	3 6 9	2.03 8.12 18.26	12.58 49.85 112.5	6.20 6.14 6.16	11.5 11.5 11.5
B3	T1	3 6 9	5.09 20.37 45.82	13.04 50.24 108.4	2.56 2.47 2.37	11.5 11.5 11.5
B3	T2	3 6 9	5.09 20.37 45.82	31.17 119.7 265.8	6.12 5.88 5.80	11.5 11.5 11.5

(a) Fixed load ratio (0.5), fixed maximum temperature (600 °C) variable steel section size.

Max. temp	Temp type	L(m)	δ_i (mm)	δ_r (mm)	δ_r/δ_i	R
400	T1	3 6 9	3.99 15.95 35.89	4.75 19.00 42.54	1.19 1.19 1.19	1.3 1.3 1.3
400	T2	3 6 9	3.99 15.95 35.89	5.58 22.54 50.59	1.40 1.41 1.41	1.3 1.3 1.3
500	T1	3 6 9	3.99 15.95 35.89	6.51 26.08 56.98	1.63 1.64 1.59	2.2 2.2 2.2
500	T2	3 6 9	3.99 15.95 35.89	9.29 36.97 82.56	2.33 2.32 2.30	2.3 2.3 2.3

(b) Fixed steel section size, fixed load ratio (0.5), variable maximum steel temperature.

Load ratio	Temp type	L(m)	δ_i (mm)	δ_r (mm)	δ_r/δ_i	R
0.4	T1	3 6 9	3.19 12.76 28.71	4.46 18.00 40.14	1.40 1.41 1.40	1.5 1.5 1.5
0.4	T2	3 6 9	3.19 12.76 28.71	6.49 26.64 57.04	2.03 2.09 1.99	1.6 1.6 1.6
0.6	T1	3 6 9	4.79 19.14 43.07	9.26 37.44 80.52	1.93 1.96 1.87	2.9 2.9 2.9
0.6	T2	3 6 9	4.79 19.14 43.07	11.73 54.34 120.0	2.45 2.84 2.79	3.0 3.0 3.0

(c) Fixed steel section size, fixed maximum temperature (500 °C) variable load ratio.

nificant difference in residual deflections of a steel beam for the two different types of non-uniform temperature distribution across the steel section. This is the result of small contribution of elastic deflection to the total deflection. Because of low temperature in the upper flange of steel section, the behavior at this location is almost elastic. The corresponding strain is very small compared to the strain in the lower flange. Hence, using the simple method does not predict any distinctive change in the steel curvature and consequently steel beam residual deflections for the two different types of temperature distribution are almost the same.

However, the finite element program predicts some noticeable difference of residual deflections for the two different types of temperature distribution. The difference is large for beams with high applied load and high maximum temperature, as presented in Tables 2(a) and 3(a) where the maximum steel temperature is 600 °C and the applied load ratio is 0.5.

The difference in results for different types of non-uniform temperature distribution in the steel section can be explained using Figure 6. Because of different assumed temperature distributions in a steel section, its final stress distributions are different. The higher the temperature at the upper flange, the higher its strain to maintain the stress level. This results in higher curvature and higher deflection.

At a higher temperature in the lower flange, the difference is more because of the greater sensitivity of steel strain to a small change in steel temperature as shown in Figure 1.

As both the steel stress and temperature reduce, the difference in residual deflections of a steel beam due to different temperature distributions reduces, as confirmed in Tables 2(b) and 2(c) and 3(b) and 3(c).

Also at lower stress and temperature, the assumed stress distribution and the real stress distribution shown in Figure 6 are closer. Therefore, the predicted residual deflection using the simple method and that predicted using the finite element method are closer.

Other observations are that both the simple method and the finite element method predict that there is very little difference in results between different types of steel section and that the difference caused by different load distributions is very small.

Since the ratios of residual curvature to initial curvature at different locations of the steel beam are different for the two different loading cases (except at the location of the maximum curvature), the fact that there is very little difference in the steel beam residual deflection between these different loading cases may be used to confirm that (1) the curvature of the most highly stressed location of the steel beam is most influential on the residual deflection of the beams and that (2) using the maximum curvature ratio at this location to scale up uniformly the curvature of the whole steel beam is reasonably accurate.

The assumption that the residual deflection of a steel beam is obtained by reducing the thermally induced deflection from its total deflection at the maximum temperature is illustrated in Figure 9. For the theoretical case that the coefficient of thermal expansion of steel is set to zero, the residual deflection is almost equal to the deflection at the highest temperature. In the other curve, the deflection reduction slope as the beam cools down is almost the same as the initial slope of the deflection curve which represents the thermally

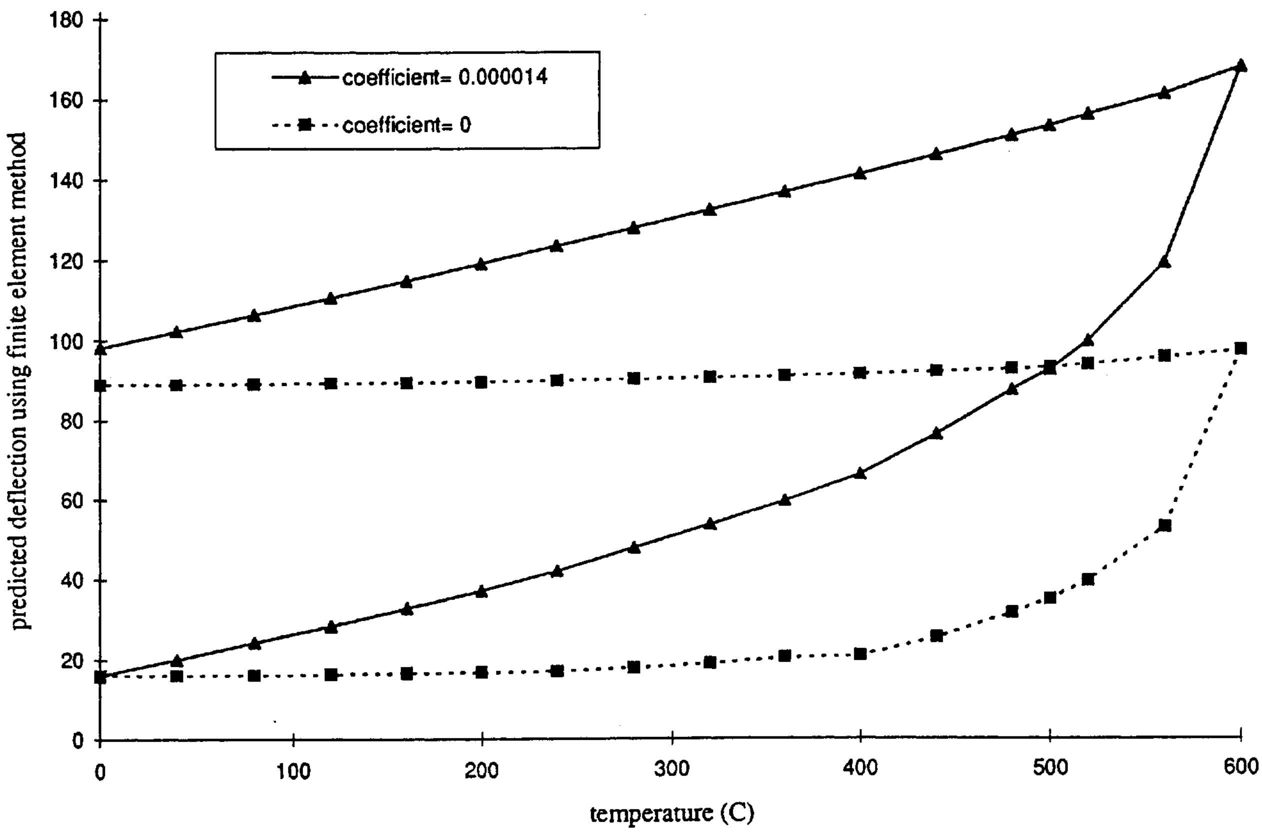

Figure 9. Deflection Curves for Two Values of Coefficient of Thermal Expansion of Steel.

induced deflection rate when there is no change in the steel stress-strain curve hence no change in the steel beam mechanical deflection.

Figure 10 summaries the comparison between the predictions of the simple method and those of the finite element method. It shows at high ratios of residual deflection to initial deflection due to high steel temperature (600 °C), the simple method overpredicts the residual deflection. At the temperature range of interest (400 °C and 500 °C), predictions from the simple method correlate well with those from the finite element method.

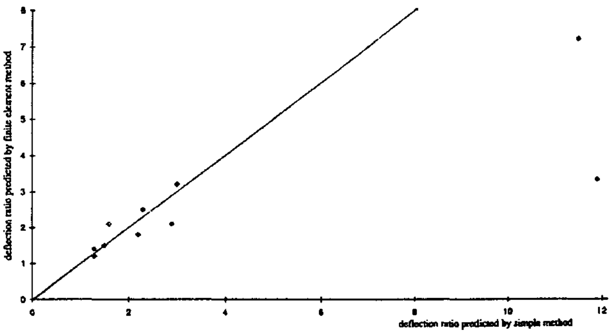

Figure 10. Correlation between Predictions from Finite Element Method and Simple Method.

At a high steel temperature, the steel strain and similarly the residual deflection of the steel beam are very sensitive to the steel temperature, as shown in Figure 1. This gives rise to the difficulty to accurately predict the residual deflection of the steel beam using the simple method at a given high temperature. However, the purpose of design for post-fire service ability is to find a limiting temperature so that after cooling down from this temperature, the residual deflection of the steel beam is less than a specified limit.

Figure 11 compares the predicted residual deflection—maximum temperature relationships using the finite element method and the simple method. The difficulty of accurately predicting the steel beam residual deflection is illustrated. However, at a specified residual deflection limit, the predicted maximum steel temperature using both methods are very close.

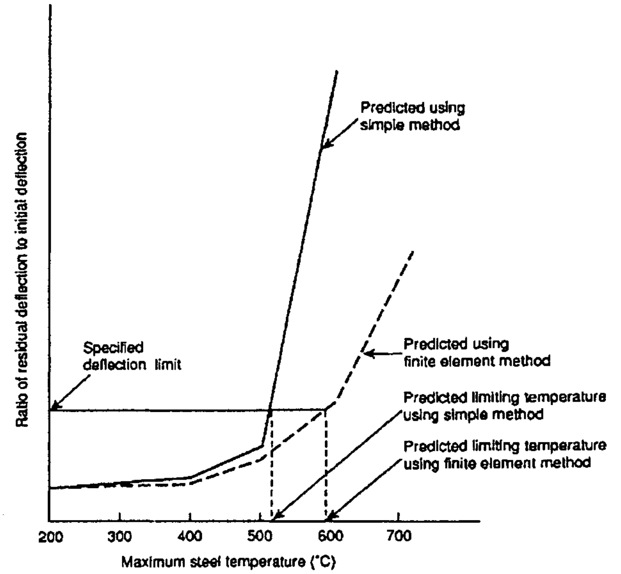

Figure 11. Inaccuracy of Predicted Limiting Temperature Using Simple Method.

Design Example

Having established the method to calculate the residual deflection of a steel beam after a fire, it is useful to indicate the effect on the fire protection of the steel beam caused by changing the fire protection objective from design for steel beam stability under the fire condition to design for post-fire serviceability.

In design for post-fire serviceability, the purpose is to ensure that the calculated residual deflection of a beam after a fire is within an acceptable serviceability limit of deflection. For example, this deflection limit may be specified at L/200. Other basic parameters are:

Beam size: 305x165UB54, second moment of inertia I = 11700 cm^4, plastic modulus = 845 cm^3, h = 310.9 mm, L = 6000 mm.

Steel properties: yield stress = 255 MPa, Young's modulus = 200000 MPa.

Applied load ratio = 0.5, therefore stress =127.5 MPa.

The calculation consists of the following four steps:

(1) Find the maximum deflection at cold condition δ_i and the maximum curvature at ambient temperature $\phi_{max,0}$. According to Table 2(a), δ_i = 18.8 mm. Using Equation 1 gives $\phi_{max,0} = M_{max}/EI = 4.6 \times 10^{-6}$.

(2) Find the allowable residual deflection δ. Assuming a limit of L/200, δ=L/200=30 mm.

(3) Find the allowable maximum curvature at elevated temperature ϕ_{max} according to $\phi_{max}=(\delta/\delta_i)\phi_{max,0} = 7.35 \times 10^{-6}$.

(4) Using iteration method, find a temperature at the lower flange of the beam so that the strains at the upper and lower flanges of the beam at the applied stress of 127.5 MPa give a curvature of 7.35×10^{-6}. From the steel stress-strain curve at elevated temperature obtained by British Steel[16] and shown in Figure 1, the curvature is 9×10^{-6} at 500 °C. At 450 °C, the curvature is about 6.8×10^{-6}. Linear interpolation gives a temperature of about 463 °C. The limiting temperature at the instability of the steel beam is 650 °C according to Table 1, taken from the British Standard.[8]

If a much larger deflection limit, say L/100 is allowed, the limit temperature according to the limited residual deflection would be 528 °C. Comparison between this value with 463 °C for the deflection limit of L/200 suggests that for such a wide range of deflection limits, limiting temperatures may be scattered within a narrow range as indicated in Figure 11.

As an indication, the steel fire protection thicknesses using a nominal spray material are calculated to be 22 mm and 13 mm respectively for limiting steel temperatures of 463 °C and 650 °C[8] assuming a 60 minute fire resistance.

While the extra fire protection cost to reduce the steel temperature rise is probably not insignificant if the design is to ensure post-fire serviceability of the steel beam rather than its stability during a fire, the importance of the steel beam in the building and the cost of replacing the beam after a fire may require the beam to be more heavily protected than the minimum requirement from stability consideration alone.

Conclusions

A simple method is proposed for the calculation of the residual deflection of a steel beam after a fire. This method is based on the assumption that the residual deflection of a steel beam is the same as its deflection at the maximum temperature due to applied load only.

This deflection is calculated by firstly finding out the maximum residual curvature of the steel member at the highest temperature and then scaling up the maximum deflection of the steel beam at cold condition, according to the ratio of the maximum residual curvature at the highest temperature to the maximum initial curvature at cold condition.

Comparisons between predicted results using this method and the more sophisticated finite element method[15] confirm the suitability of using this simple method.

A design example is given to show how this method is applied in design.

The extra fire protection required to limit the post-fire residual deflection of a steel member may be significant. Nevertheless, it may be necessary for some steel members to have such extra fire protection so that their post-fire residual deflections do not exceed the deflection limit for structural replacement in order to minimize the overall repair cost to fire damage.

NOMENCLATURE

E Young's modulus of steel

h steel section height

I steel section second moment of inertia

L beam length

M_{max} maximum bending moment in beam

R ratio of maximum residual deflection to maximum initial deflection, obtained using the simple method

δ maximum allowable beam deflection

δ_i maximum beam deflection at cold condition

δ_r maximum beam residual deflection

ϕ_{max} maximum beam residual curvature

$\phi_{max,o}$ maximum beam curvature at cold condition

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