J. of Fire Prot. Engr., **3** (2), 1991, pp 37-48

NON-DETERMINISTIC MODELLING OF FIRE SPREAD

by

G. Ramachandran
Consultant
Risk Evaluation and Insurance†

SUMMARY

Uncertainties in the pattern of fire development are caused by multiple interactions among physical and chemical processes evolved by a variety of burning materials arranged in different ways. Hence, in a general sense, the spread of fire in a building is non-deterministic and the likely pattern of spread can only be predicted within limits of confidence expressed in probabilistic terms. Non-deterministic models are of two types – probabilistic and stochastic. The object of this paper is to explain, with examples, the distinguishing features of these two types.

In probabilistic models, critical events occurring during fire spread are treated as entirely random and independent. These models deal with the final outcomes, e.g., area damaged, which is sufficient in fire protection and insurance problems not requiring a detailed knowledge of the underlying physical processes. Probabilistic models discussed include probability distributions, logic-probability trees and probabilistic version of a deterministic model.

In stochastic models, critical events occurring sequentially in space and time form a chain and are interconnected by transition probabilities. A state-transition stochastic model is discussed in detail with suggestions for improvement. Other models reviewed briefly include epidemic theory, branching processes, random walk and percolation process.

INTRODUCTION

Apart from changes in environmental conditions, the spread of fire in a building is governed by physical and chemical processes evolved by a variety of burning materials arranged in different ways. Multiple interactions among these processes at different times cause uncertainties in the pattern of fire development. A deterministic model cannot evaluate these uncertainties (errors), although it can simulate different patterns by varying the input values to the parameters of the model. There is a need to determine the relative frequency with which each pattern is likely to occur in a large number of real fires in the type of buildings considered. This frequency, defined as probability, together with its standard deviation (error), can be ascertained by non-deterministic models.

Non-deterministic models (or indeterministic models as defined by Kanury[1]) estimate and predict the extent of fire spread within certain limits of confidence expressed in probabilistic terms. Depending upon the constancy or transience of the lack of certainty (i.e., probability) a non-deterministic model may be either probabilistic or stochastic. The object of this paper is to explain the distinguishing features of these two types of models which complement the deterministic approach.

Probability distributions of damage are discussed in detail with reference to their role in assessing the fire risk and economic value of fire protection measures for a group of buildings. Other models reviewed include probability distributions for evaluating the failure of compartment boundaries, logic-probability trees and statistical treatment of exponential model of fire growth.

The state-transition stochastic model explained in the paper is based on the information available in fire incidence reports. The use of results of fire tests in such a model has been discussed

†*Retired, Head of the Operational Research Section, Fire Research Station, U.K. Address for correspondence:*
93, Wellfield Road, Hatfield, Hertfordshire, AL10 0BY, U.K.

of fire tests in such a model has been discussed by other authors. Attention is drawn to the application of other stochastic models in predicting fire spread; these models include epidemic theory, branching processes, random walk, and percolation processes.

PROBABILISTIC MODELS

In a stochastic model, discussed in the next section, critical events occurring sequentially in space and time form a chain and are connected by probabilities. In a probabilistic model, these events are treated as entirely random, completely independent of the antecedent event(s) and hence, totally blind to any sequentiality of occurrence. The events are merged into a single global event such that the chain is continuous whereas transitions from event to event in the chain are fundamental features of a stochastic model.

Probability models generally deal with the final outcomes which is sufficient in many practical problems not requiring a detailed knowledge of the underlying physical processes. Consider, for example, area damaged d in a fire which is a random variable reaching various levels according to a probability distribution. The probability of damage being less than or equal to d is given by the cumulative distribution function $G(d)$ and the probability of damage exceeding d by $[1 - G(d)]$. Figure 1, on a log scale, is an example based on fire brigade data and shows the relationship between d and $[1 - G(d)]$ for a building with sprinklers and a building without sprinklers.[2] As indicated by this graph and several statistical studies, fire damage has a skewed (non-normal) probability distribution such as log normal. The graph ignores discontinuities due to compartment boundaries and other design features of a building. According to Figure 1, an initial damage of 3 m^2 is likely to occur before the heat generated in a fire is sufficient to activate a sprinkler system.

The probability of damage in a fire exceeding 100 m^2 is about 0.18 if a building has no sprinklers and 0.08 if the building has sprinklers. On the other hand, if a fixed probability level of, say, 0.08 is considered the damage would be 500 m^2 in the absence of sprinklers which would be reduced to 100 m^2 if the building is protected by sprinklers. Based on a log normal distribution fitted to raw data pertaining to Figure 1, the average damage was estimated to be 41.64 m^2 for a sprinklered building and 216.67 m^2 for a non-sprinklered building.

Results based on area damaged can be converted to financial loss figures by using an approximate value for loss per m^2.[2] Probability distribution of financial loss can also be derived directly if observations are available for the entire range possibly covered by this variable. However, loss figures are generally available only for large fires which would necessitate the application of statistical models based on extreme value probability distributions.[3] One of these models developed by Ramachandran was applied by Rogers[4] for assessing the economic value of sprinklers for various industrial and commercial groups of buildings. According to the results of this study, sprinklers reduce significantly the average loss and the probability of loss exceeding any value, particularly in a multi-storey building. As discussed in a recent paper of Ramachandran,[5] probability distributions of area damaged and financial loss are useful in problems concerned with trade-offs between fire protection measures, fire safety codes and fire insurance which

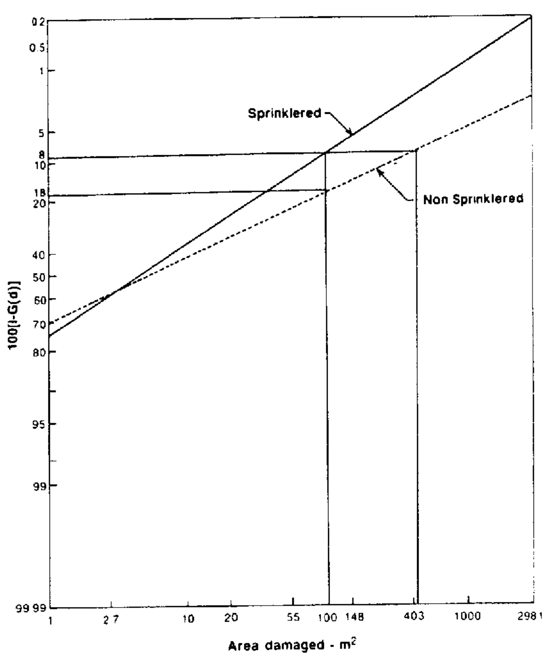

Figure 1. Textile Industry, UK – Probability distribution of area damaged

deal with "collective risk" in a group of buildings rather than risk in individual buildings.

Probability distributions also arise in an assessment of the probability of failure of structural boundaries of a compartment in preventing spread of fire. According to deterministic formulas, severity S expected in a fire depends upon the dimensions of a compartment, thermal inertia of compartment boundaries, ventilation factor and total fire load (weight of combustible materials). The last two of these factors are random variables, and hence severity should be regarded as a random variable. The fire resistance R of a compartment composed of structural elements is also a random variable due to weakness caused by penetrations, doors and other openings in the structural barriers. Assuming appropriate probability distributions for R and S the probability of barrier failure[5] can be estimated for different values of the ratio $(\bar{R}/\bar{S})$, where $\bar{R}$ and $\bar{S}$ are the means of R and S.

Suppose R and S are normally distributed. The probability of barrier failure (or success) in this case is 0.5 if $\bar{R} = \bar{S}$. The probability of failure will be less than 0.5 if $\bar{R} > \bar{S}$ and greater than 0.5 if $\bar{R} < \bar{S}$ For any level prescribed for the probability of failure, the fire resistance required to meet this target can be determined with the aid of probability tables of the normal distribution. For example, for a barrier failure probability of 0.0014 (with a success probability of 0.9986), with a coefficient of variation of 0.15 for both R and S, the mean fire resistance $\bar{R}$ should be set equal to $2\bar{S}$. The value of $\bar{S}$ can be based on deterministic formulas or on the probability distribution of severity as indicated by fire damage sustained in real fires.[5] The ability of structural elements to meet the fire resistance requirement specified by the target value $\bar{R}$ should, of course, be judged by carrying out appropriate laboratory tests with specimens of the elements.

The method mentioned above which is "approximately probabilistic" provides a better quantitative evaluation than the approach based on partial safety factors[6] usually adopted by fire protection engineers. It is possible to develop a "fully probabilistic" model which is currently being attempted by Ramachandran and involves particularly the extreme value distributions related to maximum fire severity and minimum fire resistance.[7]

Logic Trees[8] constitute another type of probability models in which various factors governing the occurrence of an event are placed in their correct sequential order to allow the probabilities associated with the factors to be ascertained. A probability tree is usually devised for estimating a specific goal of fire safety

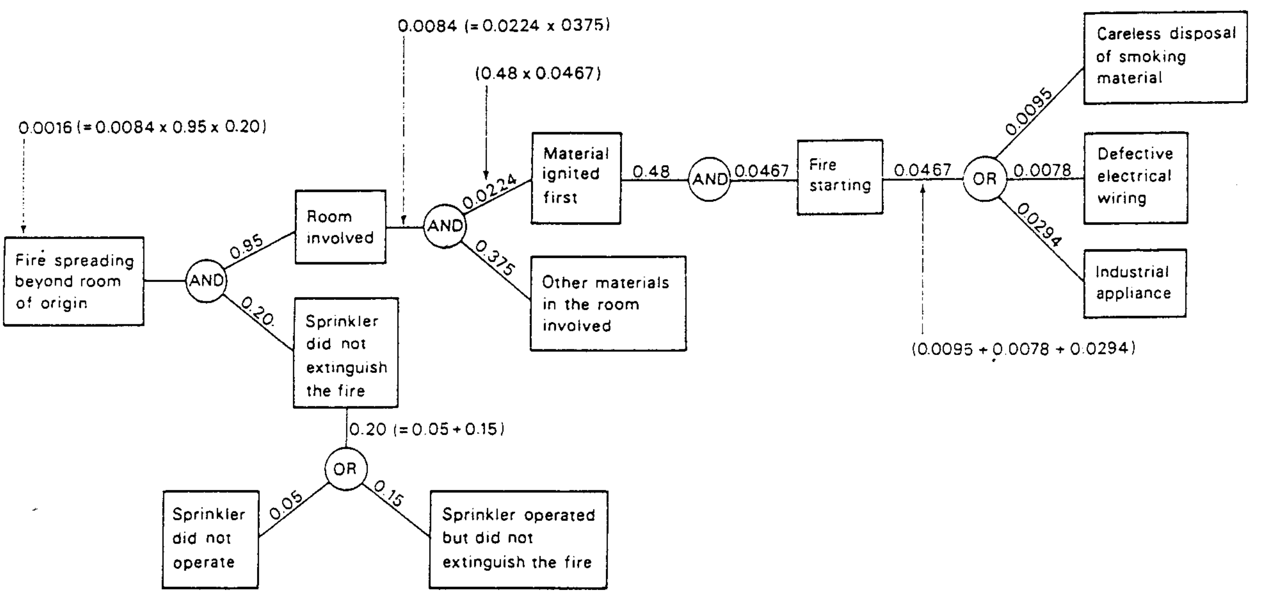

Figure 2. Fault tree analysis for a fire spreading beyond room of origin.

expressed quantitatively. If the desired goal is to avoid a certain level of serious fire, the usual approach is to use an "event tree" in the form of either a "fault tree" or a "success tree"; these two are inverses of each other according to how the goal is expressed.

Figure 2 is a simple example of a fault tree describing only the situation of ignition and spread given that a room has been involved in a fire. The undesirable final "top event" is fire spreading beyond a room protected by sprinklers. A fire confined to the room due to the action of sprinklers is part of the success tree and hence does not appear in Figure 2.

The events in the tree are connected by AND and OR gates. The AND gate pertaining to a top event connects all the constituent sub-events which have to be present to cause the occurrence of the top event. The OR gate connects sub-events, any one (not all) of which could cause the top event. Addition principle of probability theory is used for combining the probabilities associated with the branches of an OR gate and the multiplication principle for the AND gate. It may be noted that the probabilities for the three sources of ignition connected by the OR gate at the right end do not add up to unity. This is due to the assumption that, for the particular room considered, other sources of ignition are either absent or have been eliminated by effective fire prevention measures. Fault trees had their origin in reliability theory and have been applied to a number of accident problems in aerospace, chemical and nuclear industries and more recently to a few fire protection problems.

A deterministic model amenable to probabilistic treatment is the exponential model of fire growth.[9] According to this model

$$q = q_o \exp(\Theta t) \qquad (1)$$

where

q = heat output of fire at time t since the commencement of ignition
q_o = initial heat output at time "zero"
Θ = growth parameter for heat output

The "doubling time" estimated by

$$d = (1/\Theta) \log_e 2 \qquad (2)$$

is the parameter generally used for characterizing different materials in regard to fire development, slow or fast growing.

Equation 1 is approximately valid for a particular material burning under a specific set of environmental and other conditions producing a smouldering or spreading fire. The value of Θ would vary with different sets of conditions. Also, a compartment contains several materials or objects arranged in a certain manner. Hence, the pattern of fire development would depend on the values of Θ and q_o and the degree of overcrowding of objects. Probability of spread would decrease with increasing distances between objects. For predicting the fire development it is, therefore, necessary to use average values of Θ and q_o for all the materials in a compartment rather than the Θ and q_o of a particular material.

Even if results of experiments performed in different countries are pooled, estimates of Θ and q_o are unlikely to be available for several materials. For this reason, Ramachandran[10] suggested the use of information on area damaged and time recorded in fire incidence reports since such data include several materials involved in a fire in a room. An exponential model similar to Equation 1 was used in this investigation which was turned into a linear regression by using the logarithm of area damaged as the dependent variable and time as the independent variable. Estimates were obtained for average values of the growth parameter and the logarithm of area initially ignited. Normal distribution was assumed for calculating the upper limit (0.975 probability point) for the growth parameter to denote the "worst" condition; this corresponds to the lower limit (0.025) for doubling time. The probability of rate of growth in a fire exceeding the upper limit would be 0.025. Correspondingly, the probability of doubling time in a fire being less than its lower limit would be 0.025. The exponential model can be used for assessing the reduction in damage due to early detection of fires.[11,12] Figure 3 is an example.

The time of operation of an extinguishing system such as sprinklers depends on the reliability of the system, the location and heat output rates of materials in a compartment and environmental conditions such as ventilation and humidity. Time of commencement of fire brigade attack on a fire would depend on the availability of fire fighting appliances, traffic congestion on the route and other factors affecting the brigade capability. Hence, varying values should be considered for the time parameter t in Equation 1 although this was not done in the studies mentioned above.

The value of parameter t corresponding to any specified level of heat output or damage would have an element of uncertainty measured by its standard deviation (or coefficient of variation). In fire science literature, averaging is a normal practice to provide quantities such as time-mean and coefficient of variation is referred to as "intensity of fluctuation" — see Cox,[13] for example. By considering the probability distributions of q_0, Θ and t, the "worst" case according to a specified probability level can be identified. However, this problem is beyond the scope of this paper.

Stochastic Models

State Transition Stochastic Models

A fire in a compartment usually starts by igniting a single material or object. As it spreads to other combustible objects, there is a chain of ignitions which could lead to flashover and fully developed conditions. Depending upon the fire resistance of the compartment boundaries, the fire could spread beyond the compartment, compartment to compartment, floor to floor and finally beyond the building. There is, however, a chance (probability) that this chain could break at some stage due to fire fighting and other reasons with the fire burning out or getting extinguished before involving the entire compartment or building of origin.

It is, therefore, apparent that the fire chain contains different states and critical events occurring sequentially in space and time and connected by probabilities. The probabilities represent noise terms superimposed over a deterministic trend in fire growth over space and time. Although a fire experiences deterministic (physiochemical and thermodynamic) processes in its development over time, these processes stochastically undergo branching at random intervals. The fire stays in each state governed by a "temporal" probability distribution and moves from state to state according to a "transition probability". Extinguishment or burning out of fire is an "absorbing" state which fire process cannot leave after entering it.

The scheme mentioned above is a general description of a *state transition stochastic model*. In the limit of zero noise, a stochastic model becomes a deterministic one. If, in the other limit, the trend is ignored (zero trend) and only the overall random noise prevails, the model becomes probabilistic as discussed in the previous section. Table 1 is an example based on fire incidence statistics with the exponential model discussed earlier providing estimates of time since the occurrence of "established burning" at the end of the first state. The time variable depicts the deterministic trend in fire growth while the percentage figures provide noise terms (probabilities) for constructing a

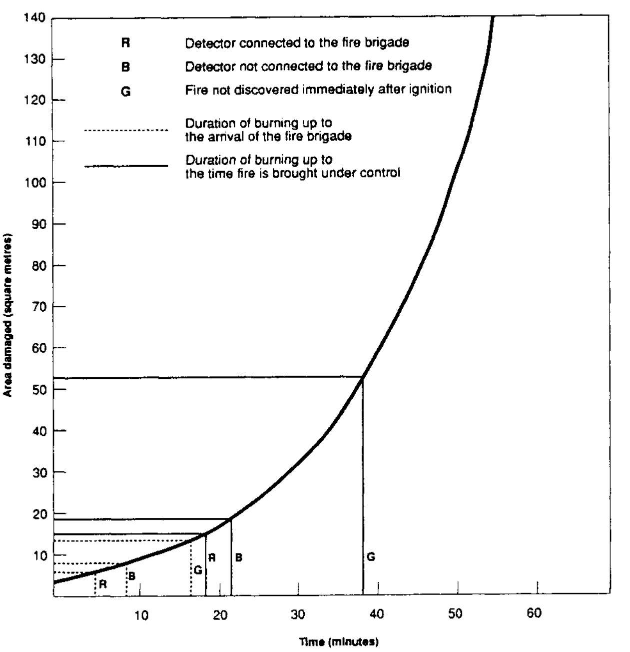

Figure 3. Average time and area damaged.

Table 1

Textile Industry, U.K.
Extent of Fire Spread and Average Area Damaged

Extent of spread	Sprinklered* Average area damaged (m^2)	Sprinklered* % of fires	Sprinklered* Time (min)	Non-sprinklered Average area damaged (m^2)	Non-sprinklered % of fires	Non-sprinklered Time (min)
Confined to item first ignited	4.43	72	0	4.43	49	0
Spread beyond item but confined to room of fire origin						
i) contents only	11.82	19	8.4	15.04	23	6.2
ii) structure involved	75.07	7	24.2	197.41	21	19.4
Spread beyond room	1000.00	2		2000.00	7	
Average	**30.69**	**100**		**187.08**	**100**	

* System operated

probability tree as in Figure 4 to describe the development of a fire through four states. The parameter μ_i (i = 2,3,4) is the conditional probability of confinement or extinguishment in the i^{th} state given that the fire has spread beyond the $(i-1)^{th}$ state with the conditional probability $\lambda_{(i-1)} = [1 - \mu_{(i-1)}]$ Fire spread beyond the building of origin is not considered in this model, hence $\mu_4 = 1$. The product $(\lambda_1 \cdot \lambda_2)$ may be regarded as the probability of flashover while λ_3 is the probability of barrier failure. The overall product $(\lambda_1 \cdot \lambda_2 \cdot \lambda_3)$ is the probability of fire spreading beyond the room of origin.

The probability tree (Figure 4) divides the increasing intensity of a fire into states or realms demarcated by critical events which are functions of the fuel, geometry of the compartment, ventilation opening and fire resistance of the structural barriers. Such a division enables the specification of time and state when different fire protection systems are pressed into action. Sprinklers are activated in the first or second realm while fire resistance comes into operation in the third. It is apparent that sprinklers reduce the probabilities λ_1 and λ_2 and hence $(\lambda_1 \cdot \lambda_2)$ of flashover; consequently, the probability of spreading beyond room is reduced.

Markov Models

In the simple *Markov model* described in Figure 4, the transition probabilities $\lambda_i (i = 1,2,3)$ and hence $\mu_i (= \lambda_i)$ have been regarded as independent of each other which may be a reasonable assumption.[2] Although a constant value independent of time has been assigned for the sake of simplicity, each of the probabilities has a probability distribution involving time spent by fire in the corresponding state. The length of time a fire burns in a given state affects future fire spread. For example, the probability of a wall burn-through increases with fire severity which is a function of time. Also the time spent by fire in a particular state may depend on how that state was reached, i.e., whether the fire is growing or receding.[2] Some fires may grow quickly, and some grow slowly depending on high or low heat release.

Berlin[14] used uniform, normal and log normal distributions to describe "temporal" probability distributions for different states. He considered six realms (states) for residential occupancies — the non-fire state, sustained burning, vigorous burning, interactive burning, remote burning and full room involvement. These realms were based on measurable criteria such as heat release rate and air temperature. Berlin described the variability of several fire effects such as those expressed in terms of maximum extent of flame spread, the probability of self-termination and the distribution of fire intensity. His state transition model incorporated fire test data instead of professional judgement as the primary source of information while fire incidence data was suggested for partial validation of the results.

Ramachandran[15] proposed a state transition model similar to Figure 4 in the overall framework but expanded to include sub realms (periods) each of fixed duration of 5 minutes for evaluating transition probabilities as functions of time. The states defined by Aoki[16] were based on the physical extent of fire spread and his analysis was somewhat similar to that of Ramachandran; Morishita *et al*[17] divided the extent of spread into seven phases. Williamson[18] introduced a state transition model for analysing and reporting the results of experiments performed under conditions resembling actual fire conditions.

Network Theory

Network theory has been considered by some authors, transforming a building into a network with rooms as nodes; the links between nodes represent possible paths for fire spread. Ling and Williamson[19] include the element of time and probability for each link as well as the confinement of fire by fire resistive building elements. Beck[20] has discussed a series of state transition models and interrelated deterministic models to represent the interaction between human behaviour and fire growth.

A few other models considered in fire science literature are worth reviewing. Albini and Rand[21] envisage a large urban area with fires in "locales" which may be single buildings or blocks of buildings. A number of these are presumed to be alight initially and randomly distributed and to stay alight for a time t in the absence of fire fighting. At time t this "generation" of fires can spread fire and then die out leaving a second "generation" to burn for a second period t. Fire spread is assumed to take place only at the end of each fire interval. For the $(n + 1)^{th}$ interval, the "a priori" probability that any locale is burning is P_n and that it has not yet been burnt is A_n. It follows that

$$A_n = (1 - P_o)(1 - P_1) \dots (1 - P_n)$$

$$P_{(n+1)} = A_n \cdot B_n$$

where B_n is the probability that during the $(n + 1)^{th}$ interval fires spread into the "locale" previously unburnt. Albini and Rand have discussed a method for estimating approximately B_n and its upper and lower bounds.

An extended version of the model described above includes fire fighting assuming that fire fighting effort is constant. This constant denotes the fraction of burning locales which all fire fighters in a city could extinguish during a given time interval out of all possible burning locales. If a fire is not extinguished it may or may not spread the fire; if extinguished it cannot spread. Albini and Rand considered directional spread of fire assuming that from an isolated locale the probability of spread forwards and backwards was the same and the directional element in the spread arose only from the initial condition. Spatial variation was

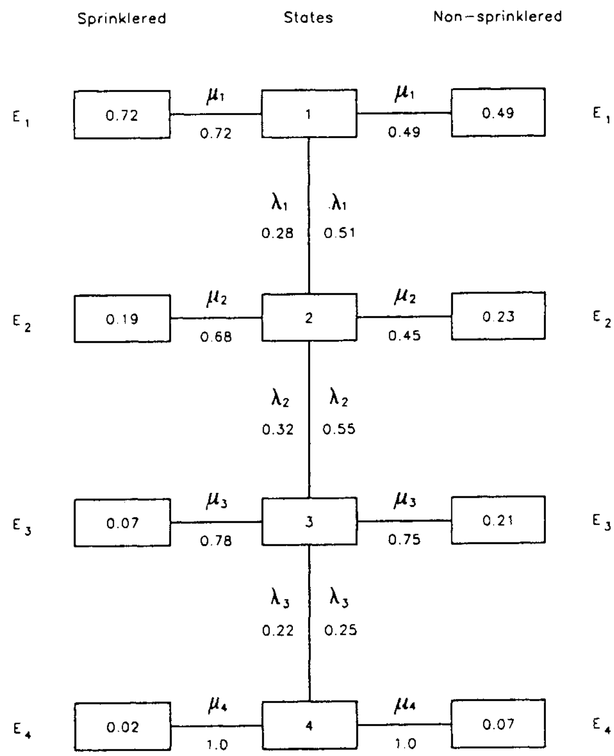

Figure 4. Probability tree (Textile Industry).

included in the model by connecting the probability of spread to the probability that any building was itself burning and separated from any of its adjacent buildings not yet burning by less than the appropriate "safe" distance for radiation or brand transfer.

Epidemic Theory and Branching Process

Albini and Rand model has some similarity with chain-binomial models of Reed and Frost[22] for the spread of an epidemic. Thomas[23] drew attention to the possible relevance of *epidemic theory* to fire spread in a building and compared the model of Albini and Rand with a deterministic epidemic model based on a continuous propensity to spread fire. He found the results of both the models to be in reasonable agreement as to their basic features but concluded that neither would be appropriate for dealing with spread in a single building where the number of "locales" is not large. For such a situation a stochastic treatment is necessary; this will allow for the finite chance that the initiating fires can burn out before spreading, a chance which is negligible when the number of initial fires is large.

The studies mentioned above suggest the relevance of *branching processes*[24] for fire spread in a building in which case a material (first generation) ignited first ignites one or more other materials (second generation) which ignite other materials (third generation) and so on, leading to the spread of fire throughout the building. The number of offsprings (burning materials) would vary randomly from one generation to another depending on the distances between ignited and unignited materials, ventilation and other factors affecting fire spread. The first two generations will relate to materials in the room of fire origin as in Figure 4 while the subsequent generations will involve materials in other rooms.

Random Walk

In a simple stochastic representation, the fire process involving any generation of burning materials or all generations considered together can be regarded as a *random walk*. The fire takes a random step every short period either to spread with a probability λ or to get extinguished (or burn out) with a probability μ (= 1 - λ). The parameter λ denotes the success probability of the fire in destroying, say, a unit of the floor area. The parameter μ denotes the success probability of an extinguishing agent. The problem is similar to two gamblers, A (fire) and B (extinguishing agent), playing a sequence of games, the probability of A winning any particular game being λ. If he wins a game he acquires a unit stake from B and if he loses the game, he does not gain any stake. In the latter case he does not lose his own unit stake to B; an already burnt out area is a loss that cannot be regained. Extinguishment can also be considered as an "absorbing boundary" to the random walk just as an "absorbing state" in a state transition model.[15]

A random walk as described above will lead to an exponential model[15] according to which the (cumulative) probability distribution function of duration of burning (until extinguishment) is given by

$$F(t) = 1 - \exp(-\mu \cdot t) \tag{3}$$

Equation (3) denotes the probability that the duration of burning is less than or equal to t while

$$\begin{aligned} Q(t) &= 1 - F(t) \\ &= \exp(-\mu \cdot t) \end{aligned} \tag{4}$$

denotes the probability that the duration of burning is greater than t. If $c = \mu - \lambda$, since $\mu + \lambda = 1$, $\mu = (1 + c)/2$ in which case

$$Q(t) = \exp[-(1 + c)t/2] \tag{5}$$

The fire fighting effort is adequate if c is positive with μ greater than λ and hence greater than 1/2; it is inadequate if c is negative with μ less than λ and hence less than 1/2. If $c = 0$ such that $\mu = \lambda = 1/2$, there is an equal balance between fire fighting efforts and the propensity of fire to spread.

Associated with t, there is a damage x which may be expressed in terms of area or financial value destroyed. If it is assumed that x is approximately proportional to heat output, from Equation 1, logarithm of x is proportional to t:

$$\log_e x = k \cdot t \qquad (6)$$

It may be deduced from Equation 4 that the probability of damage exceeding the value x is given by

$$\Phi(x) = x^{-w}, x > 1 \qquad (7)$$

where $w = \mu/k$.

Equation 7 is known as Pareto distribution which is used in economic theories concerned with, for example, income distribution to describe the fact that there are a large number of people with low incomes and a small number of people with high incomes. The damage is small in most of the fires; high levels of damage occur only in a small number of fires. The use of Pareto distribution for fire damage originally proposed by Benckert and Sternberg[25] was later supported by Mandelbrot[26] who derived this distribution following a *random walk process*. For all classes of Swedish houses outside Stockholm the value of the exponent w was found to vary between 0.45 and 0.56. A value of $w = 0.5$ with

$$\Phi(x) = x^{-0.5} \qquad (8)$$

would generally indicate, as discussed with reference to Equation 5, an equal balance between fire fighting efforts and the propensity of fire to spread and cause damage. It may also be seen that with $c = 0$ or $\mu = 1/2$ and $k = 1$ in Equation 6, Equation 4 reduces to Equation 8.

The parameter μ in Equations 3 and 4 is known as the "hazard" or "failure rate" given by

$$f(t)/Q(t) \qquad (9)$$

where $f(t)$ is the probability density function obtained as the derivative of $F(t)$. A constant value for this parameter would denote "random failure rate" which is somewhat unrealistic particularly for a fire which is fought. As discussed by Ramachandran[27] the value of μ, as a function of time (t), would eventually increase ("wear out failure") and exceed λ since fire extinguishing efforts would succeed ultimately. The failure rate would be decreasing in the early stages of fire development denoting a success for fire in spreading, constant for sometime and increasing later. Hence, the failure rate as a function of time would resemble a "bath tub".

Percolation Process

In a *random walk* the randomness is a property of the moving object whereas, in a *percolation process*,[28] randomness is a property of the space in which the object moves. In the latter approach, the walk can take place on a graph consisting of a number of sites, connected by directed "bonds," passage being possible only along such a "bond." Buildings in a city or compartments and other areas in a building are also connected by directed bonds with flow (spread of fire) being possible only along the bonds. Each bond has an independent probability of blocking or preventing fire spread.

Apparently for the reason mentioned above, Hori[29] considered percolation theory to modelling of fire spread from building to building in an urban area. Sasaki and Jin[30] were concerned with the actual application of this model and estimation of probabilities of fire spread. By using the data contained in the fire incidence reports for Tokyo, urban fires were simulated and the average number of burnt buildings per fire estimated. Apart from distances between buildings and wind velocity, the following factors were also regarded as having some effect on the probability of fire spread — building construction, building size and shape, window area, number of windows, indoor construction material, furniture, wall, fence, garden and tree.

For predicting the damage to buildings and other properties resulting from incendiary or nuclear attacks, Phung and Willoughby[31] considered two types of stochastic models. In the first model, the entire fire front was regarded as a random walker moving along a linear row of cells or small square areas. In each short time interval the front may be in one of three states — die or stop permanently, spread or move one cell forward, pause or stay where it is. Simple probability considerations provided an estimate of the probability P_n that at time t the fire will be n cell units long after an initial condition of being lit at time zero:

$$P_n = \exp[-(\lambda + \mu)t]\,\lambda^n t^n$$

The parameters λ and μ are respectively the probabilities for forward spread and for burn out during a short time interval. The fire will stay where it is with probability $[1 - (\mu + \lambda)]$.

The second stochastic model of Phung and Willoughby was called fuel-state model because it dealt explicitly with the state of the fuel in each cell. In the burning process the fuel changes from the unignited to the burn out state passing through the flaming state. A cell will be in one of these three states at any time with probabilities U, F and B for unignited, flaming and burn out states. In a two-dimensional array of cells, the cell dimension can be so chosen that a burning cell can ignite the immediate neighbour cells but not those which are further away. Under this assumption an unignited cell can be ignited by one or more of its 8 immediate burning neighbours with probability

$$P = 1 - (1 - P_1)(1 - P_2) \ldots (1 - P_8)$$

where P_1, P_2 ... P_8 denote the chances of ignition by the neighbours. These eight spread probabilities are not necessarily symmetrical due to factors such as wind and topography. Using the formulation described above, differential equations are derived for U, F and B for each cell with $(U + F + B) = 1$, solutions of which can be obtained by numerical calculations using computers, if necessary.

CONCLUSION

The historical development of deterministic modelling has been in the prediction of patterns of fire growth in a building as though the implied relationships based on scientific theories and experimental data were exactly fulfilled in real fire situations. The uncertainties (errors) in the predicted patterns caused by several factors are not quantitatively evaluated and specified in the deterministic approach. Such is not the case with non-deterministic models which explicitly introduce random perturbations and lead only to probabilistic conclusions.

Non-deterministic models are of two types: probabilistic and stochastic. The former type of models deal with probable final outcomes such as extent of spread, area damaged and financial loss and do not provide any picture of fire development in a building over a period of time. These models may be sufficient for fire protection and insurance problems concerned with "collective risk" in a group of buildings rather than risk in individual buildings. Stochastic models, on the other hand, can predict the probable growth of fire over space and time in a particular building with specific design features and fire protection measures. Most of these models, except the simple version in Figure 4, involve computations more complex than probabilistic models.

Among probabilistic models, logic trees, particularly, fault trees enable one to trace the sub-events leading to an undesirable top (final) event. The risk, probability of occurrence, associated with the top event can thus be reduced by identifying and eliminating (or reducing) the risk related to one or more of the sub-events. Logic trees are also useful in assessing the reliability of active fire protection systems such as detectors and sprinklers. However, the discrete probabilities provided by these methods may not be sufficient for any problem requiring probabilities of damage levels varying randomly and continuously around an average value with a standard deviation. Such problems include cost-benefit studies of passive and active fire protection measures (including trade-offs between measures), reliability of structural (compartment) fire resistance and calculation of risk premiums for fire insurance. Probability distributions provide better tools for these problems, but require a good deal of statistical data for estimating the parameters.

As discussed in the paper, a deterministic model amenable to probabilistic treatment is the exponential model of fire growth, estimation of whose parameters can be based on statistical data on area damaged or experimental data on heat output or release rates. The model can also be applied for estimating the rate of growth of smoke if sufficient data are available for the area damaged by this combustion product. Smoke travels, perhaps, four or more times faster than fire with a "doubling time" one-fourth or a smaller fraction of the "doubling time" for fire. As shown in Table 1, an exponential model depicts the deterministic fire growth as a function of time over which probabilities can be superimposed through a probability tree (Figure 4) providing parameters for a state transition model. Deterministic and stochastic models can thus be coupled for attaching probabilities to patterns of fire spread in a compartment simulated by a deterministic model.

Complementing the deterministic approach, stochastic models provide powerful tools for assessing property and life risk from fires in a particular building. Among these, state transition models have the potential for utilizing both experimental and statistical data for predicting fire spread in a compartment. Probabilities provided by these models can be used as inputs for a network model but this problem requires some further research. Network theory appears to be the best approach for predicting fire and smoke spread from compartment to compartment including corridors and other spaces in a building but an application of this technique is contingent upon the availability of data which, at this time, is still mostly lacking.[32] For fire or smoke spread throughout a building percolation process (which is somewhat similar to flow through a network) has some practical application, while a branching process in stochastic environment, although theoretically very sound, would be a complex model to develop and apply.

REFERENCES

1. Kanury, A.M., "On the Craft of Modelling in Engineering and Science," *Fire Safety Journal*, **12**, 65-74, 1987.
2. Ramachandran, G., "Probabilistic Approach to Fire Risk Evaluation," *Fire Technology*, **24**, 3, 204-226, 1988.
3. Ramachandran, G., "Properties of Extreme Order Statistics and Their Application to Fire Protection and Insurance Problems," *Fire Safety Journal*, **5**, 59-76, 1982.
4. Rogers, F.E., *Fire Losses and the Effect of Sprinkler Protection of Buildings in a Variety of Industries and Trades*, Current Paper CP9/77, Building Research Establishment, Fire Research Station, Borehamwood, Herts., U.K., 1977.
5. Ramachandran, G., "Probability-Based Fire Safety Code," *Journal of Fire Protection Engineering*, **2** (3), 75-91, 1990.
6. "A Conceptual Approach Towards a Probability Based Design Guide for Structural Fire Safety," CIB W14 Workshop Report, *Fire Safety Journal*, **6**, 1, 1983.
7. Ramachandran, G., "Extreme Value Theory," *SFPE Handbook of Fire Protection Engineering*, DiNenno, P.J. (Ed.) *et al.*, National Fire Protection Association, Quincy, MA, USA, Chapter 4-4, 28-33, 1988.
8. Berlin, G.N., "Probability Models in Fire Protection Engineering," *SFPE Handbook of Fire Protection Engineering*, National Fire Protection Association, Quincy, MA, USA, Chapter 4-6, 43-52, 1988.
9. Thomas, P.H., *Fires in Model Rooms*, Current Paper CP32/74. Building Research Establishment, Fire Research Station, Borehamwood, Herts., U.K., 1974.
10. Ramachandran, G., "Exponential Model of Fire Growth," *Fire Safety Science: Proceedings of the First International Symposium* (Eds.) C.E. Grant and P.J. Pagni, Hemisphere Publishing Corporation, New York, 657-666, 1986.
11. Ramachandran, G., *Economic Value of Automatic Fire Detectors*, Information Paper IP27/80. Building Research Establishment, Fire Research Station, Borehamwood, Herts., U.K., 1980.
12. Ramachandran, G., and Chandler, S.E., "The Economic Value of Fire Detectors," *Fire Surveyor*, **13** (2), 8-14, 1984.
13. Cox, G., "On Radiant Heat Transfer from Turbulent Flames," *Combustion Science and Technology*, **17**, 75-78, 1977.
14. Berlin, G.N., "Managing the Variability of Fire Behaviour," *Fire Technology*, **16**, 287-302, 1980.
15. Ramachandran, G., "Stochastic Modelling of Fire Growth," *Fire Safety: Science and Engineering*, ASTM STP 882, (Ed) T.Z. Harmathy. American Society for Testing and Materials, Philadelphia, 122-144, 1985.
16. Aoki, Y. *Studies on Probabilistic Spread of Fire*, Research Paper No. 80, Building Research Institute, Tokyo, Japan, 1978.
17. Morishita, Y., *Establishment of Evaluating Method for Fire Safety Performance*, Report, Research Project on Total Evaluating System on Housing Performances, Building Research Institute, Tokyo, Japan, 1977.

18. Williamson, R.B., "Fire Performance Under Full-Scale Test Conditions - A State Transition Model," *Sixteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1357-1371, 1976.

19. Ling, W.T.C., and Williamson, R.B., "The Modelling of Fire Spread Through Probabilistic Networks," *Fire Safety Journal*, **9**, 1986.

20. Beck, V.R., "A Cost-Effective Decision-Making Model for Building Fire Safety and Protection," *Fire Safety Journal*, **12**, 121-138, 1987.

21. Albini, F.A., and Rand, S., *Statistical Considerations on the Spread of Fire*, IDA Research and Engineering Support Division, Washington D.C., 1964.

22. Bailey, N.J.T., "Reed and Frost Model" in *The Elements of Stochastic Processes*, (Chapter 12, Section 5), John Wiley, New York, 1964.

23. Thomas, P.H., *Some Possible Applications of the Theory of Stochastic Processes to the Spread of Fire*, Internal Note No. 223, Fire Research Station, Borehamwood, Herts., U.K., 1965.

24. Harris, T.E., *The Theory of Branching Processes*, Springer-Verlag, Berlin, 1963.

25. Benkert, L.G., and Sternberg, I., "An Attempt to Find an Expression for the Distribution of Fire Damage Amount," *Transactions of the Fifteenth International Congress of Actuaries*, **II**, 288-294, 1957.

26. Mandelbrot, B., "Random Walks, Fire Damage Amount and Other Paretian Risk Phenomena," *Operations Research*, **12**, 582-585, 1964.

27. Ramachandran, G., "The Poisson Process and Fire Loss Distribution," *Bulletin of the International Statistical Institute*, **43**, 2, 234-236, 1969.

28. Broadbent, S.R., and Hammersley, J.M., "Percolation Processes, 1, Crystals and Mazes," *Proceedings of the Cambridge Philosophical Society*, **53**, 629-41, 1957.

29. Hori, M., "Theory of Percolation and its Applications," *Nippon Tokeigakkai-shi* **3**, 19, 1972.

30. Sasaki, H. and Jin, T., *Probability of Fire Spread in Urban Fires and Their Simulations*, Report No. 47, Fire Research Institute, Tokyo, Japan, 1979.

31. Phung, P.D. and Willoughby, A.B., *Prediction Models for Fire Spread Following Nuclear Attacks*, Report No. URS 641-6, URS Corporation, Burlingame, California, 1965.

32. Ling, W.C.T. and Williamson, R.B., "The Use of Probabilistic Networks for Analysis of Smoke Spread and the Egress of People in Buildings," *Fire Safety Science: Proceedings of the First International Symposium*. (Eds.) C.E. Grant and P.J. Pagni. Hemisphere Publishing Corporation, New York, 953-962, 1986.