Appendix A

For the dichotomous item, the KL divergence in the first component of (11) is obtained as

$$K_1 = P(\theta_0)\log\frac{P(\theta_0)}{\lambda P(\theta_0)+(1-\lambda)P(\theta)} + [1-P(\theta_0)]\log\frac{1-P(\theta_0)}{\lambda[1-P(\theta_0)]+(1-\lambda)[1-P(\theta)]}.$$

The second order derivative of the K_1 evaluated at θ_0 can be found as

$$\left.\frac{\partial^2 K_1}{\partial\theta^2}\right|_{\theta=\theta_0} = \frac{(1-\lambda)^2\left[P'(\theta_0)\right]^2}{P(\theta_0)\left[1-P(\theta_0)\right]}.$$

Likewise, the KL divergence in the second component in (11) is obtained as

$$K_2 = P(\theta)\log\frac{P(\theta)}{\lambda P(\theta_0)+(1-\lambda)P(\theta)} + [1-P(\theta)]\log\frac{1-P(\theta)}{\lambda[1-P(\theta_0)]+(1-\lambda)[1-P(\theta)]},$$

which gives the second derivative at θ_0 as

$$\left.\frac{\partial^2 K_2}{\partial\theta^2}\right|_{\theta=\theta_0} = \frac{\lambda^2\left[P'(\theta_0)\right]^2}{P(\theta_0)\left[1-P(\theta_0)\right]}.$$

Plugging in the corresponding values, the second derivative of the JS divergence at θ_0 is found to be

$$\begin{aligned}\left.\frac{\partial^2 JS(\theta_0\,||\,\theta)}{\partial\theta^2}\right|_{\theta=\theta_0} &= \lambda\left.\frac{\partial^2 K_1}{\partial\theta^2}\right|_{\theta=\theta_0} + (1-\lambda)\left.\frac{\partial^2 K_2}{\partial\theta^2}\right|_{\theta=\theta_0} = \frac{\lambda(1-\lambda)\left[P'(\theta_0)\right]^2}{P(\theta_0)\left[1-P(\theta_0)\right]}\\ &= \lambda(1-\lambda)I(\theta_0).\end{aligned}$$

Appendix B

Reported in Table A.1 are summary statistics of the item parameter estimates used in CD-CAT simulations. The first column shows the type of item parameters in sequence: guessing (denoted as g), slipping (denoted as s), item discrimination index (IDI; obtained as $1-g-s$), discrimination (denoted as a), and difficulty (denoted as b). The g and s parameters in the Reduced RUM were obtained through the reparameterization within the NIDA model (Junker & Sijtsma, 2001; Rupp, Templin, & Henson, 2010, p.124). In general, the parameter estimates had small standard errors across all items. The item pools were relatively highly discriminating in terms of both the CDM and the IRM.

A.1 Descriptive Statistics of Item Parameter Estimates

	DINA			DINO			Reduced RUM		
	Mean	SD	SE	Mean	SD	SE	Mean	SD	SE
g	.155	.060	.011	.155	.063	.019	.053	.036	.015
s	.151	.061	.017	.150	.058	.014	.109	.054	.016
IDI	.693	.087	-	.695	.087	-	.838	.064	-
a	.977	.227	.062	.983	.235	.062	1.374	.260	.083
b	.268	.222	.053	-.289	.205	.053	.189	.183	.040

Note. SD=standard deviation of parameter estimates. SE=standard error of estimates. g=guessing. s=slipping. IDI=item discrimination index. a=discrimination. b=difficulty.

To further evaluate the interrelationship between the item parameter estimates, we report correlation matrices for each item pool in Table A.2. The results indicate that g values were negatively correlated with IDI, a, and b. A similar pattern was observed for s with IDI and a estimates. The s values were moderately positively correlated with b, indicating that the higher the slipping, the harder the item was. One pronounced pattern in the correlation matrices across all CDMs was that the IDI and a values were very strongly correlated. This suggests that item selection weighted on a particular parameter would also retain much information for the other.

In Table A.3 summary statistics of the item parameter estimates obtained from the

A.2 Correlation between Item Parameter Estimates

	DINA				DINO				Reduced RUM			
	g	s	IDI	a	g	s	IDI	a	g	s	IDI	a
s	.049				.047				-.059			
IDI	-.721	-.728			-.753	-.693			-.522	-.821		
a	-.713	-.619	.919		-.652	-.678	.918		-.631	-.622	.892	
b	-.538	.571	-.027	.120	-.589	.468	.116	-.051	-.805	.561	-.019	.214

Note. g=guessing. s=slipping. IDI=item discrimination index. a=discrimination. b=difficulty.

A.3 Descriptive Statistics of Item Parameter Estimates in the Empirical Item Pool

	g	s	IDI	a	b	c
Mean	.401	.178	.421	1.011	-.194	.238
SD	.170	.158	.151	.444	.870	.120
SE	.020	.041	-	.129	.162	.054

Note. g=guessing. s=slipping. IDI=item discrimination index. a=discrimination. b=difficulty. c=lower-asymptote. SD=standard deviation of parameter estimates. SE=standard error of estimates.

empirical data are presented. The items from the empirical data generally had high discriminating power and tended to measure examinees with slightly below the mean ability levels. We also report the correlational structure of the parameter estimates to get a detailed description of the item pool characteristics. The correlation matrix between the item parameter estimates was observed as

$$\Sigma = \begin{array}{c} \\ s \\ \text{IDI} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccc} g & s & \text{IDI} & a & b \end{array} \\ \begin{pmatrix} -.579 & & & & \\ -.523 & -.393 & & & \\ -.365 & -.140 & .558 & & \\ -.576 & .872 & -.262 & .008 & \\ .560 & -.070 & -.558 & -.260 & .116 \end{pmatrix} \end{array}.$$