**Appendix**

In the derivation of item information functions of the GGUM-RANK model, Equation 14 involves first-order and second-order partial derivatives with respect to . We will demonstrate the derivatives of the GGUM-RANK triplet model but tetrad model can be extended.

Let us define the single-statement response probabilities and their first- and second-order derivatives as follows.

(A.1a)

(A.1b)

(A.1c)

where represents a vector of single-statement X response probability parameters (i.e., GGUM parameters; , and ) and is latent trait for a dimension associated with a statement X. Also, we additionally define

, (A.2a)

, (A.2b)

. (A.2c)

Because of space limitation, we will only demonstrate the derivatives of , which is illustrated in Equation 9 in the manuscript. The rest of the derivative procedures can be obtained from the first author. The first partial derivatives with respect to are

The same partial derivative rule (i.e., product rule) can be applied for other possible ranks (i.e., A>C>B, B>A>C, B>C>A, C>A>B or C>B>A). The second derivatives in Equation 14 can be similarly obtained applying product rule to Equations A.3a – A.3c. The second-order partial derivatives will not show in this paper due to space limitations, but they can be obtained by requesting from the corresponding author. Finally, note that the first- and second-order partial derivatives of the general GGUM probability function with respect to in Equations A.1a – A.lc and A.2a – A.2c are derived in Appendix B in Roberts et al. (2000).