**Appendix**

 In the derivation of item information functions of the GGUM-RANK model, Equation 14 involves first-order and second-order partial derivatives with respect to $θ$. We will demonstrate the derivatives of the GGUM-RANK triplet model but tetrad model can be extended.

Let us define the single-statement response probabilities and their first- and second-order derivatives as follows.

$A=P\_{A}\left(Z=1|θ\_{d\_{A}},β\_{A}\right) , A'=P\_{A}'\left(Z=1|θ\_{d\_{A}},β\_{A}\right), and A''= P\_{A}''\left(Z=1|θ\_{d\_{A}},β\_{A}\right). $ (A.1a)

$B=P\_{B}\left(Z=1|θ\_{d\_{B}},β\_{B}\right) , B'=P\_{B}'\left(Z=1|θ\_{d\_{B}},β\_{B}\right), and B''= P\_{B}''\left(Z=1|θ\_{d\_{B}},β\_{B}\right).$ (A.1b)

$C=P\_{C}\left(Z=1|θ\_{d\_{C}},β\_{C}\right) , C'=P\_{C}'\left(Z=1|θ\_{d\_{C}},β\_{C}\right), and C''= P\_{C}''\left(Z=1|θ\_{d\_{C}},β\_{C}\right).$ (A.1c)

where $β\_{X}$represents a vector of single-statement X response probability parameters (i.e., GGUM parameters; $α$, $δ$ and $τ$) and $θ\_{d\_{X}}$ is latent trait for a dimension associated with a statement X. Also, we additionally define

$D=1-P\_{A}\left(Z=1|θ\_{d\_{A}},β\_{A}\right)=1-A$, (A.2a)

$E=1-P\_{B}\left(Z=1|θ\_{d\_{B}},β\_{B}\right)=1-B$, (A.2b)

$F=1-P\_{C}\left(Z=1|θ\_{d\_{C}},β\_{C}\right)=1-C$. (A.2c)

Because of space limitation, we will only demonstrate the derivatives of $P\_{A>B>C}(θ\_{d\_{A}},θ\_{d\_{B}},θ\_{d\_{C}})$, which is illustrated in Equation 9 in the manuscript. The rest of the derivative procedures can be obtained from the first author. The first partial derivatives with respect to $θ\_{d\_{A}},θ\_{d\_{B}},θ\_{d\_{C}}$ are

$$\frac{∂P\_{A}\left(θ\right)}{∂θ\_{d\_{A}}}=\frac{∂}{∂θ\_{d\_{A}}}\left(\frac{AEF}{AEF+DBF+DEC}\right)\left(\frac{BF}{BF+EC}\right) $$

$$=\left(\frac{\left(A^{'}EF\right)\left(AEF+DBF+DEC\right)-(AEF)(A^{'}EF+D^{'}BF+D^{'}EC)}{\left(AEF+DBF+DEC\right)^{2}}\right)\left(\frac{BF}{BF+EC}\right). (A.3a)$$

$$\frac{∂P\_{B}\left(θ\right)}{∂θ\_{d\_{B}}}=\frac{∂}{∂θ\_{d\_{B}}}\left(\frac{AEF}{AEF+DBF+DEC}\right)\left(\frac{BF}{BF+EC}\right) $$

$$=\left(\frac{\left(AE^{'}F\right)\left(AEF+DBF+DEC\right)-\left(AEF\right)\left(AE^{'}F+DB^{'}F+DE^{'}C\right)}{\left(AEF+DBF+DEC\right)^{2}}\right)\left(\frac{BF}{BF+EC}\right)+ $$

$$\left(\frac{AEF}{AEF+DBF+DEC}\right)\left(\frac{\left(B^{'}F\right)\left(BF+EC\right)-(BF)(B^{'}F+E^{'}C)}{\left(BF+EC\right)^{2}}\right). (A.3b)$$

$$\frac{∂P\_{C}\left(θ\right)}{∂θ\_{d\_{C}}}=\frac{∂}{∂θ\_{d\_{C}}}\left(\frac{AEF}{AEF+DBF+DEC}\right)\left(\frac{BF}{BF+EC}\right) $$

$$=\left(\frac{\left(AEF^{'}\right)\left(AEF+DBF+DEC\right)-\left(AEF\right)\left(AEF^{'}+DBF^{'}+DEC^{'}\right)}{\left(AEF+DBF+DEC\right)^{2}}\right)\left(\frac{BF}{BF+EC}\right)+ $$

$$\left(\frac{AEF}{AEF+DBF+DEC}\right)\left(\frac{\left(BF^{'}\right)\left(BF+EC\right)-(BF)(BF^{'}+EC^{'})}{\left(BF+EC\right)^{2}}\right). (A.3c)$$

The same partial derivative rule (i.e., product rule) can be applied for other possible ranks (i.e., A>C>B, B>A>C, B>C>A, C>A>B or C>B>A). The second derivatives in Equation 14 can be similarly obtained applying product rule to Equations A.3a – A.3c. The second-order partial derivatives will not show in this paper due to space limitations, but they can be obtained by requesting from the corresponding author. Finally, note that the first- and second-order partial derivatives of the general GGUM probability function with respect to $θ$ in Equations A.1a – A.lc and A.2a – A.2c are derived in Appendix B in Roberts et al. (2000).