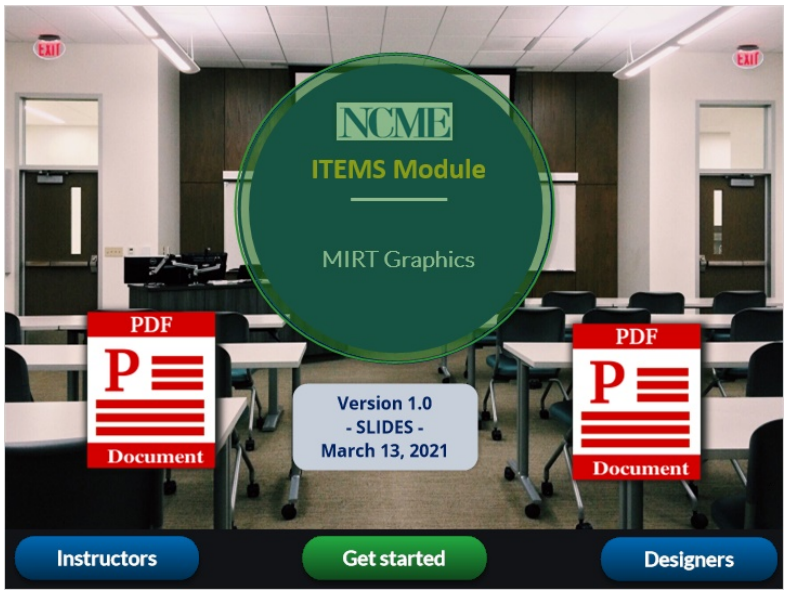
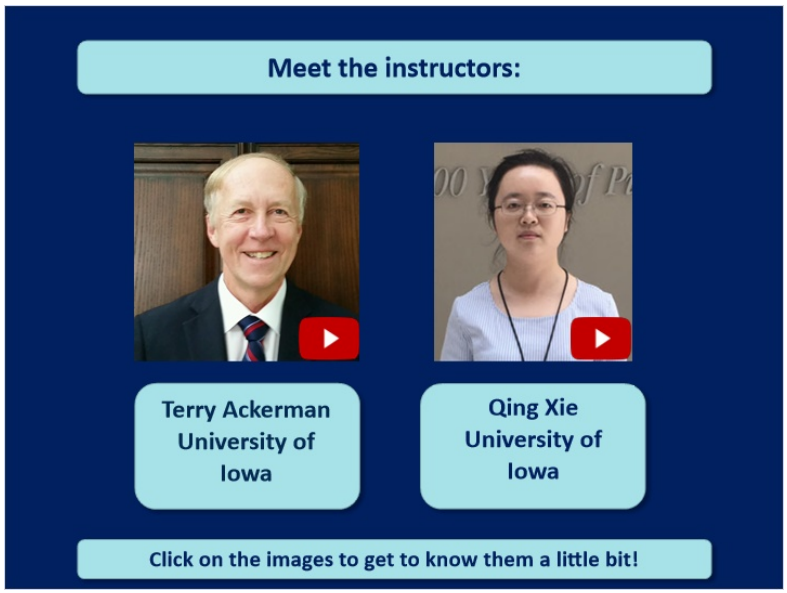
ITEMS: MIRT Graphics

1. Module Overview

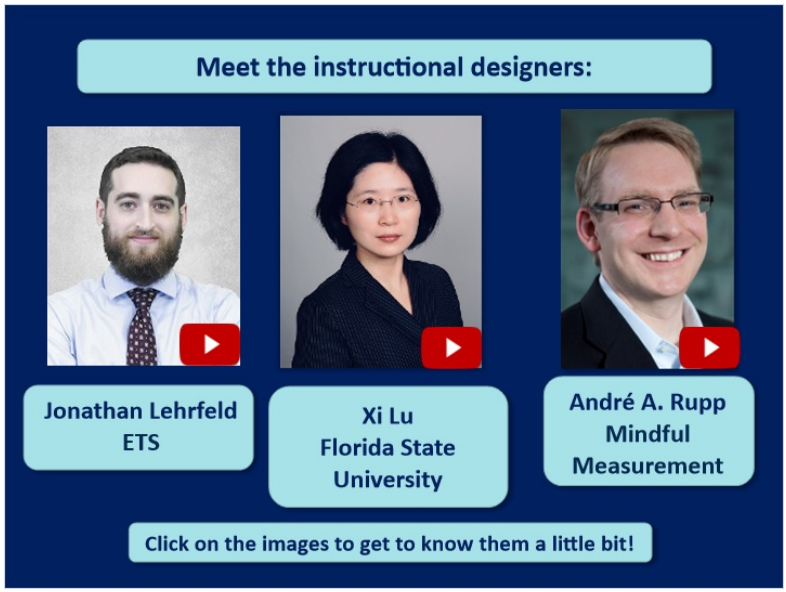
1.1 Module Cover



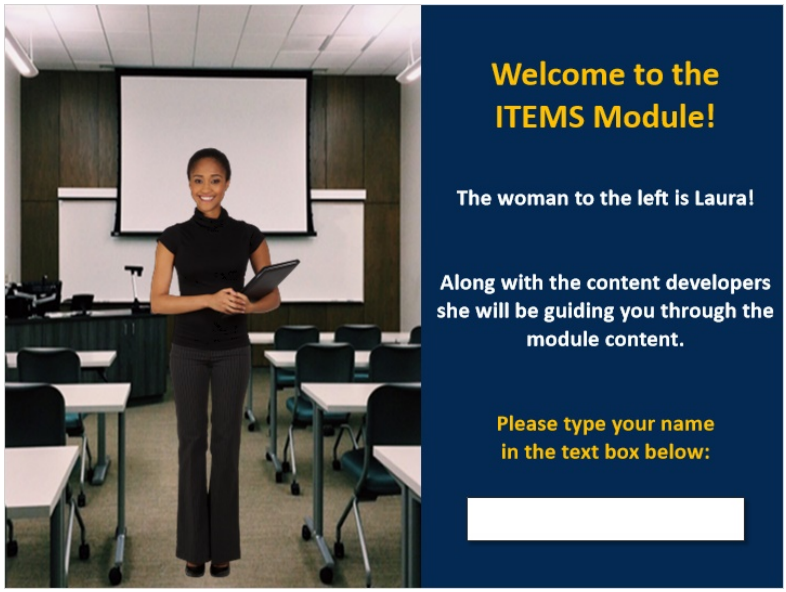
1.2 Instructors



1.3 Designers



1.4 Welcome



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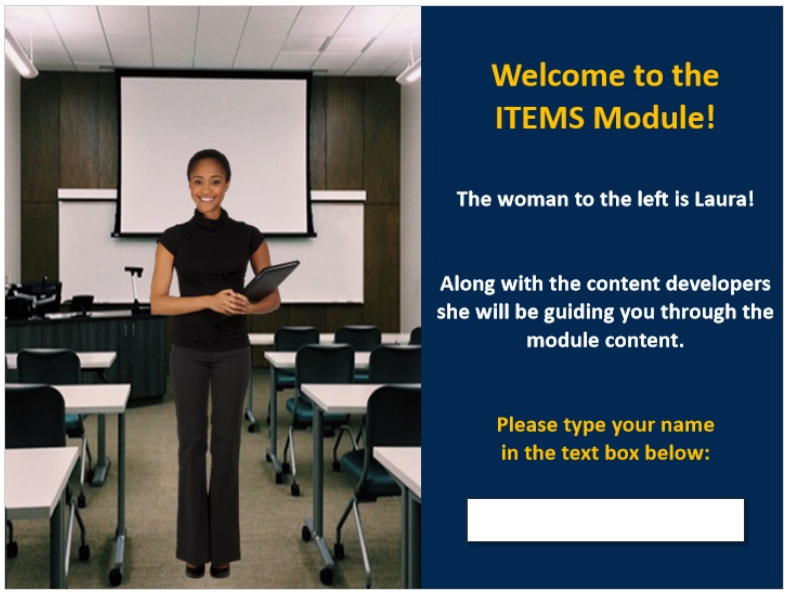
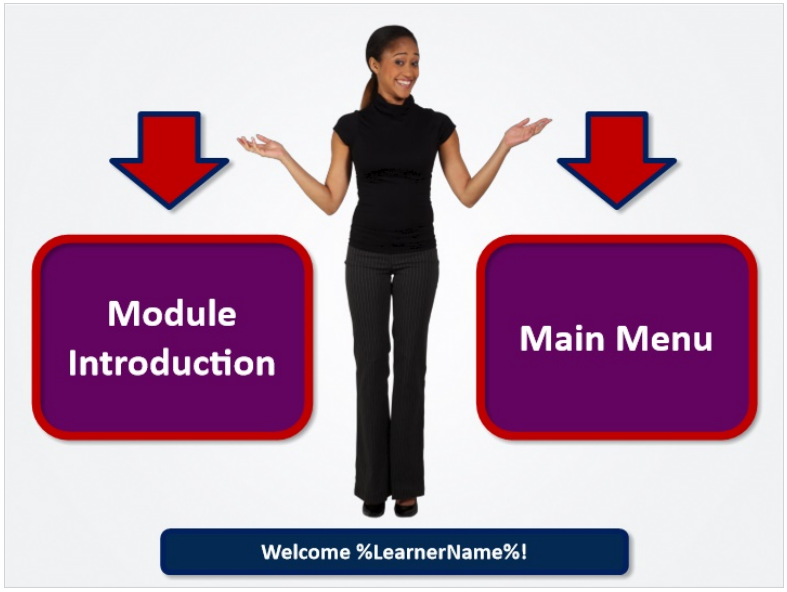
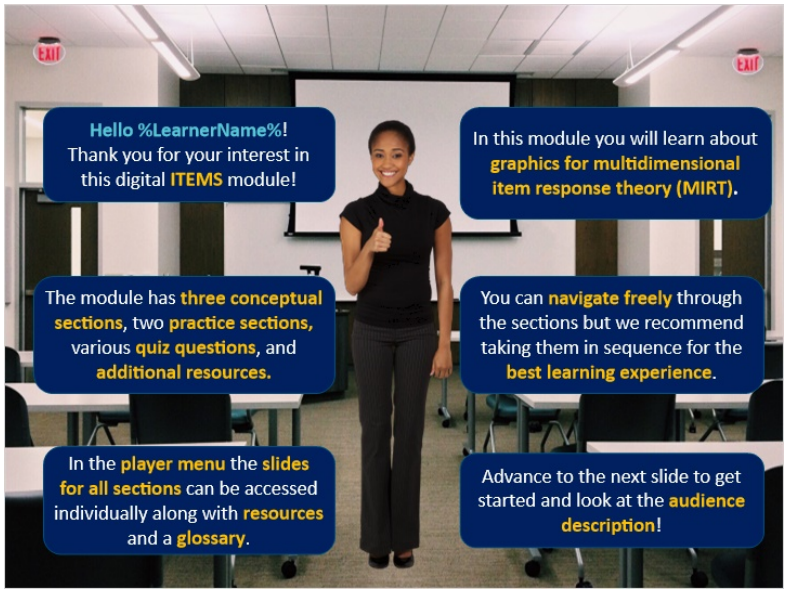


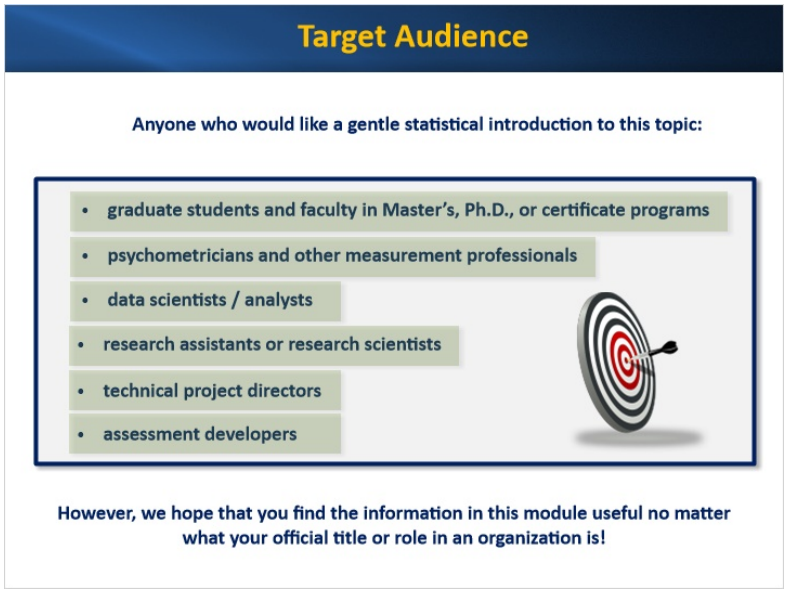
1.5 Path Choice



1.6 Overview

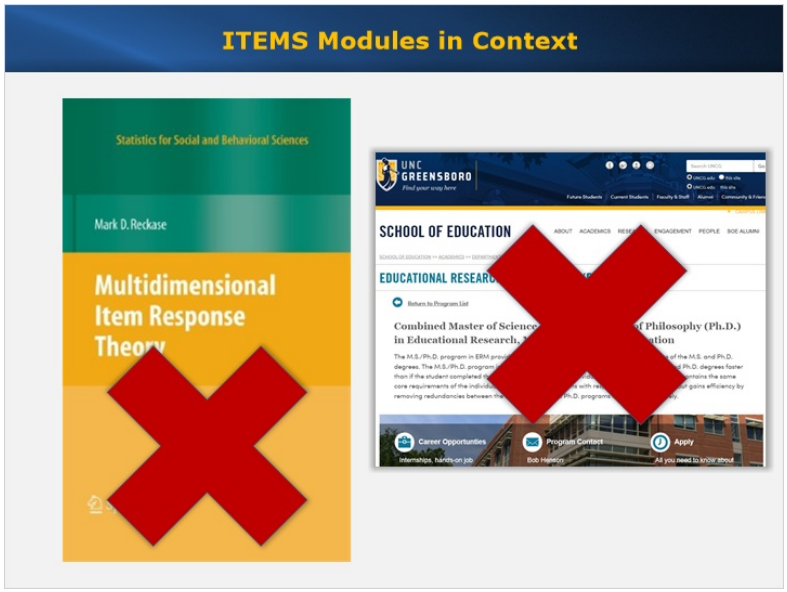


1.7 Target Audience

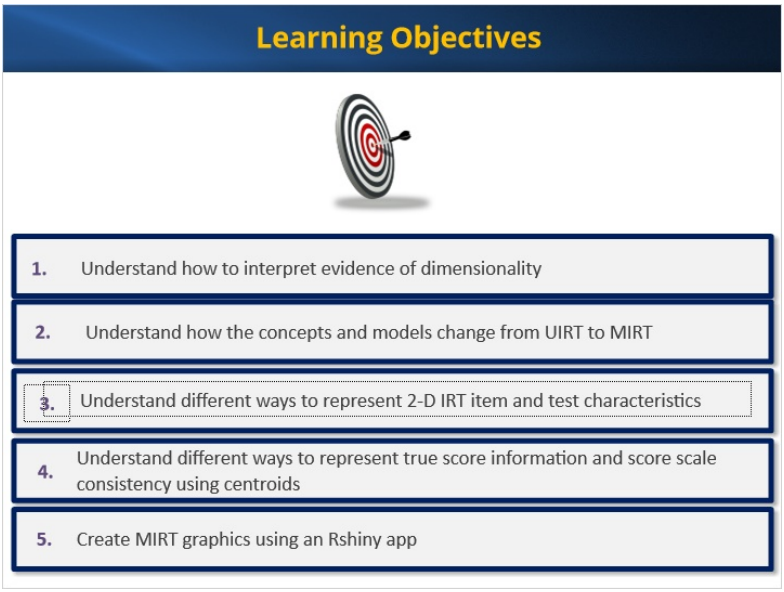


1.8 Expecations (I)

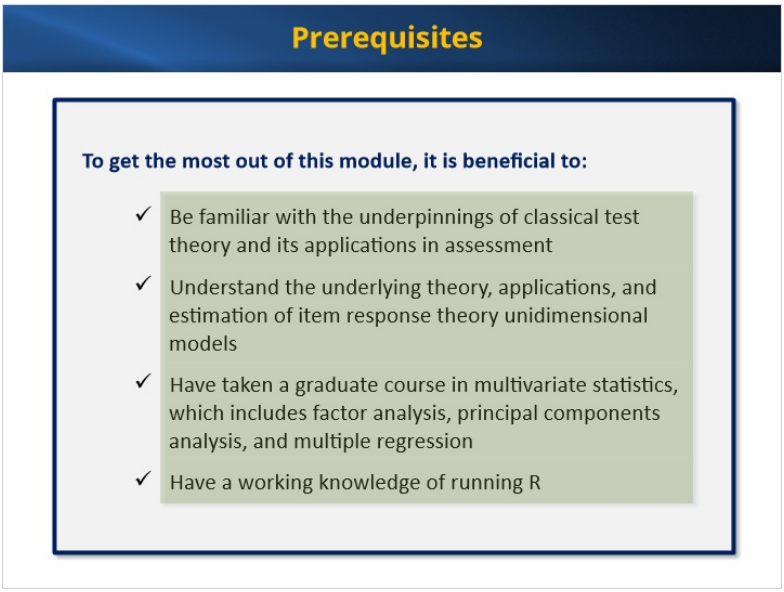
1.9 Expectations (II)



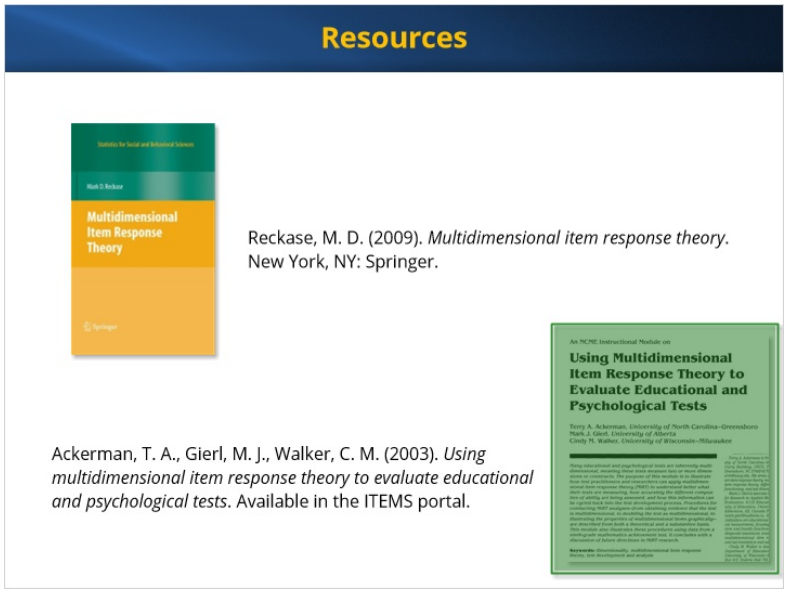
1.10 Learning Objectives



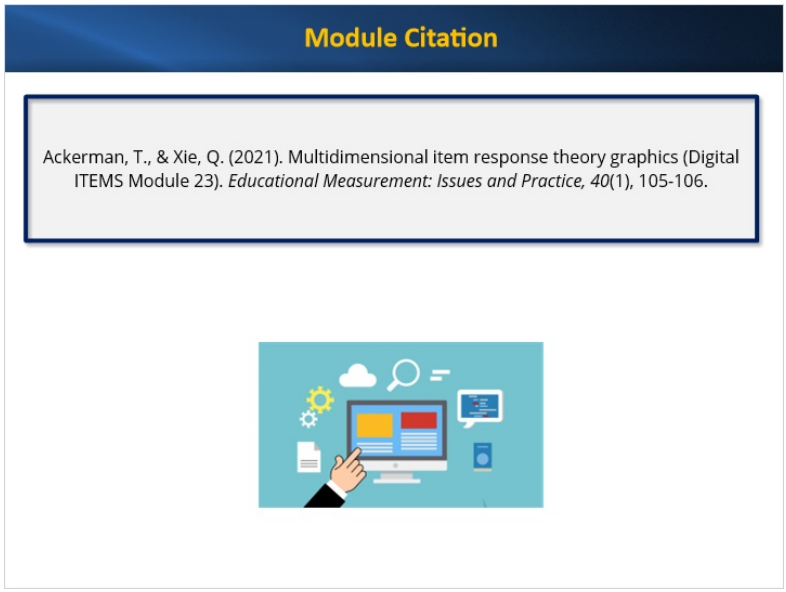
1.11 Prerequisites



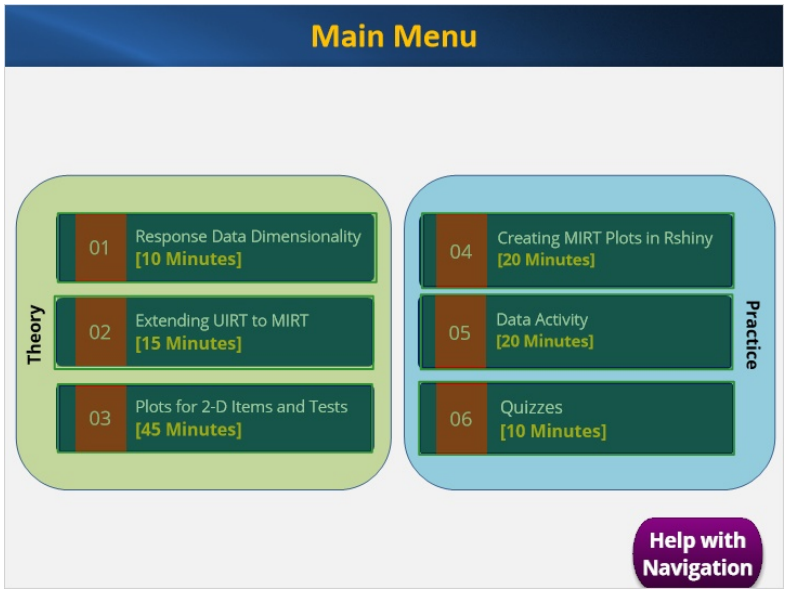
1.12 Resources



1.13 Module Citation



1.14 Main Menu

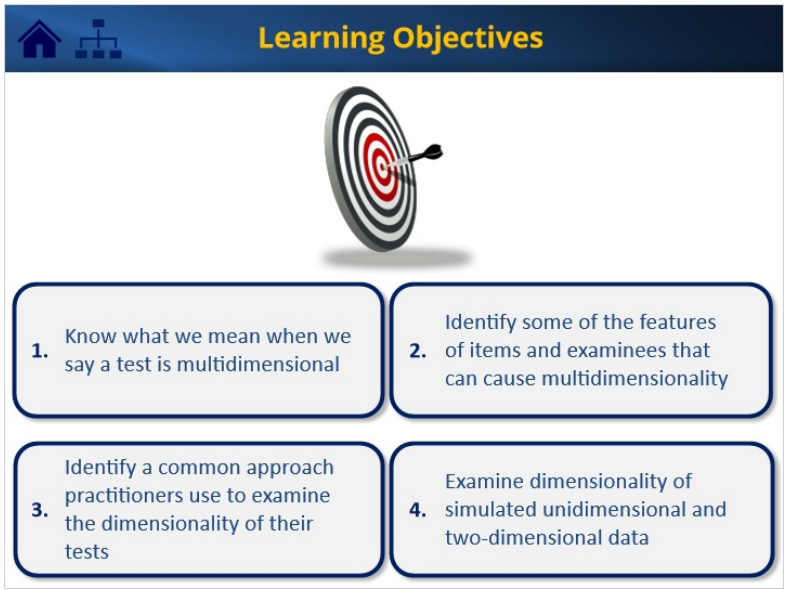


Navigation (Slide Layer)

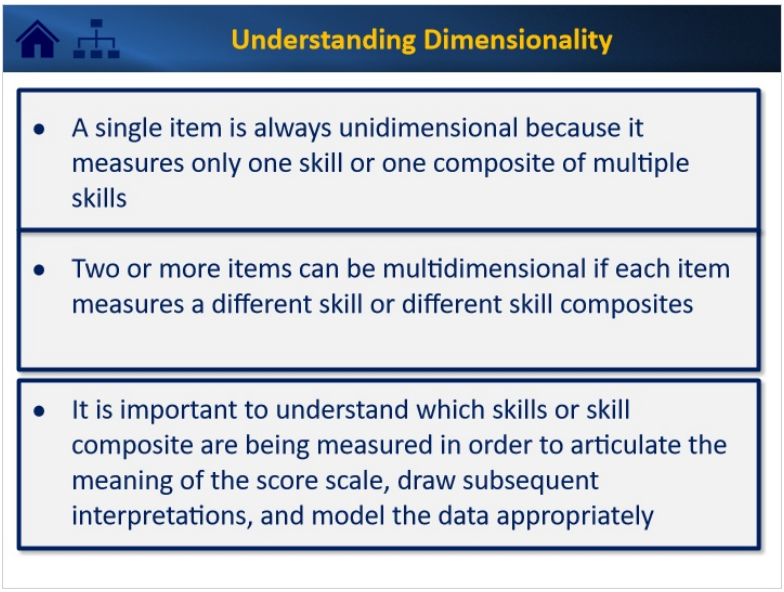
2. Response Data Dimensionality

2.1 Cover: Response Data Dimensionality

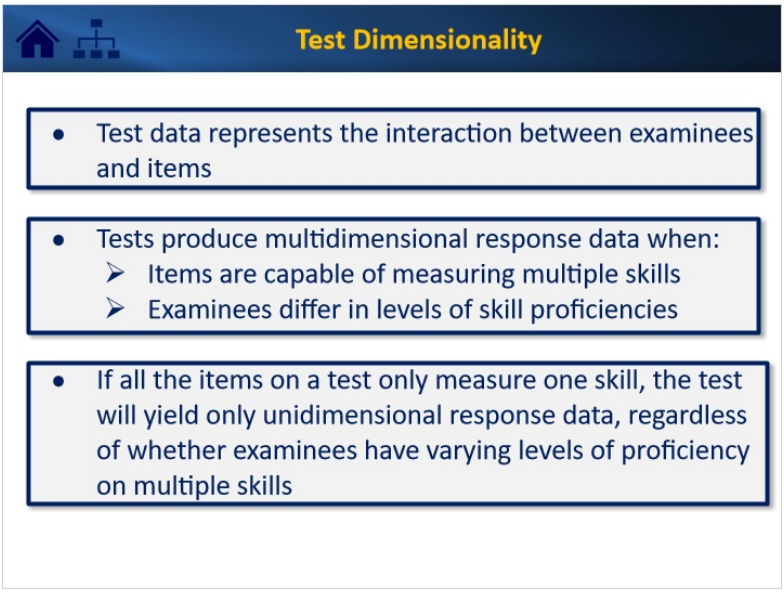
2.2 Learning Objectives: Response Data Dimensionality



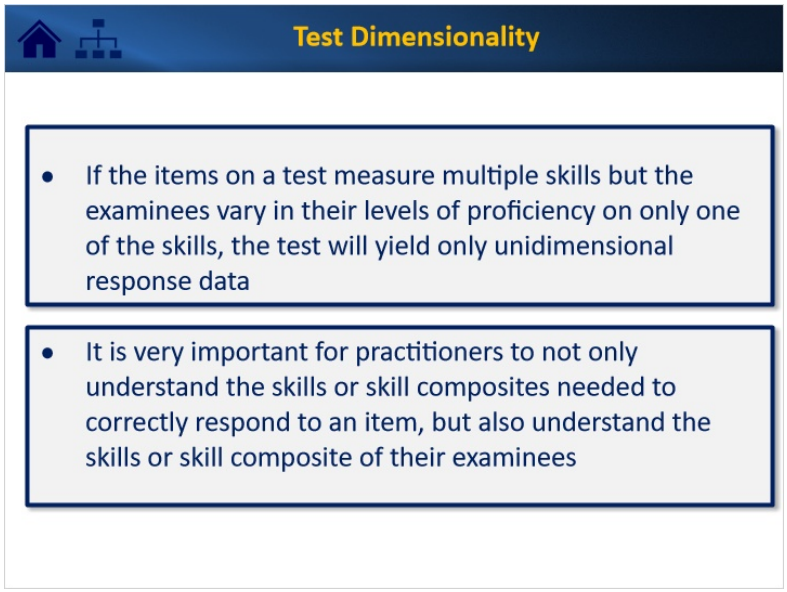
2.3 Understanding Dimensionality



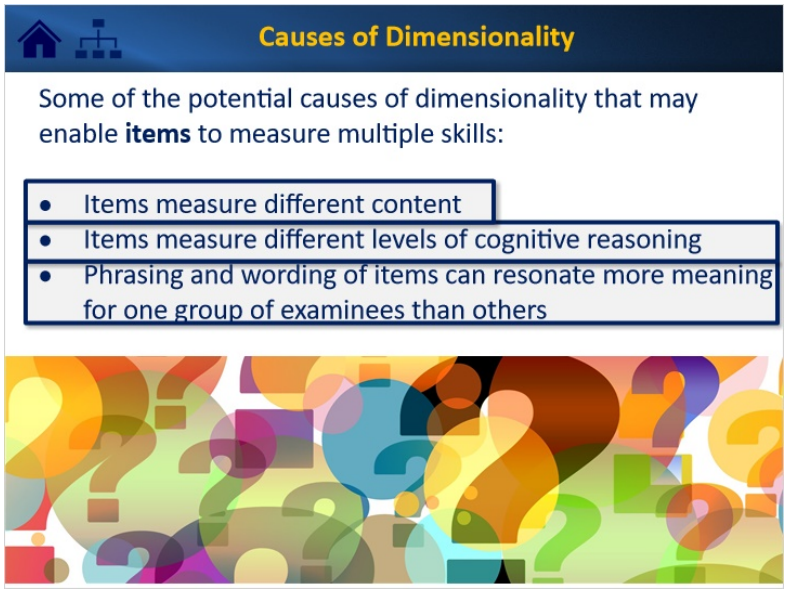
2.4 Test Dimensionality (I)



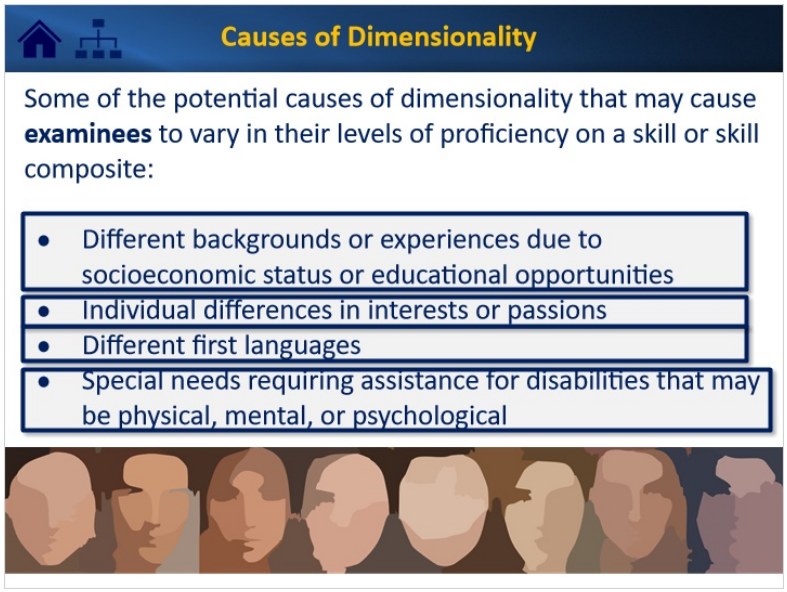
2.5 Dimensionality (II)



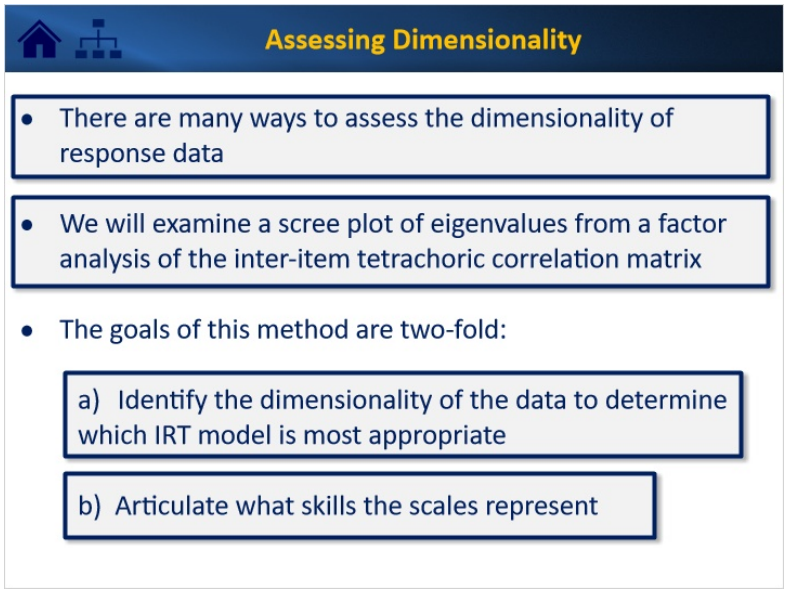
2.6 Causes of Dimensionality (I)



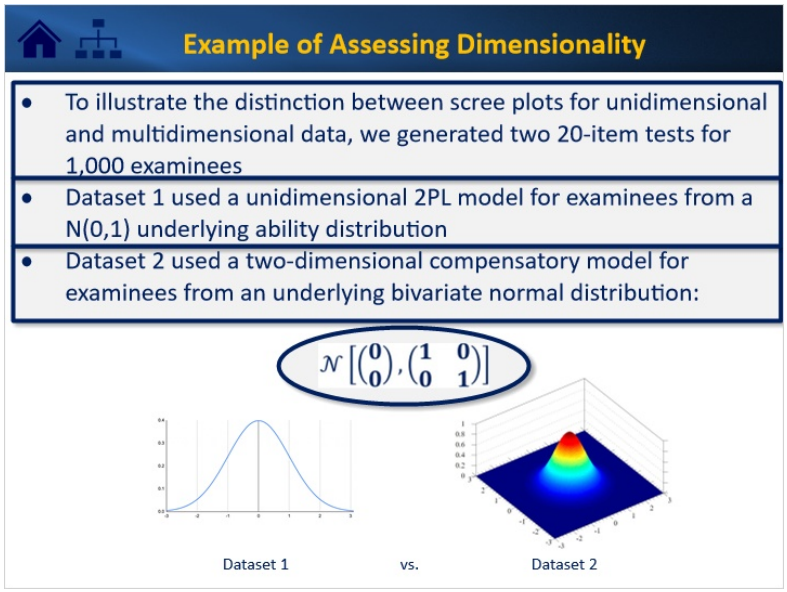
2.7 Causes of Dimensionality (II)



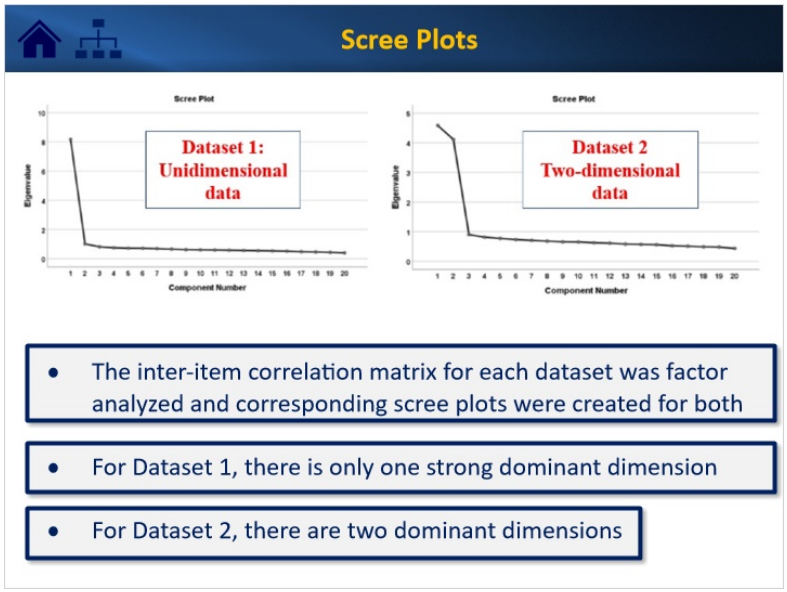
2.8 Assessing Dimensionality



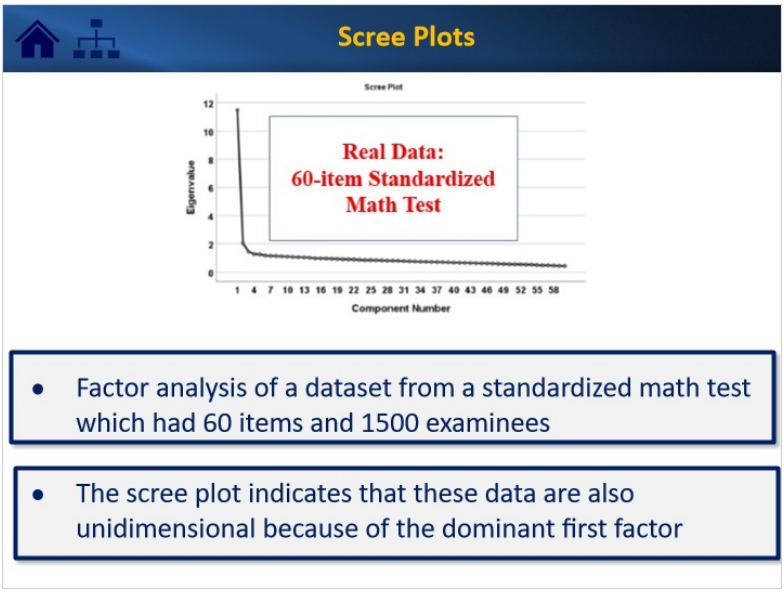
2.9 Example of Assessing Dimensionality



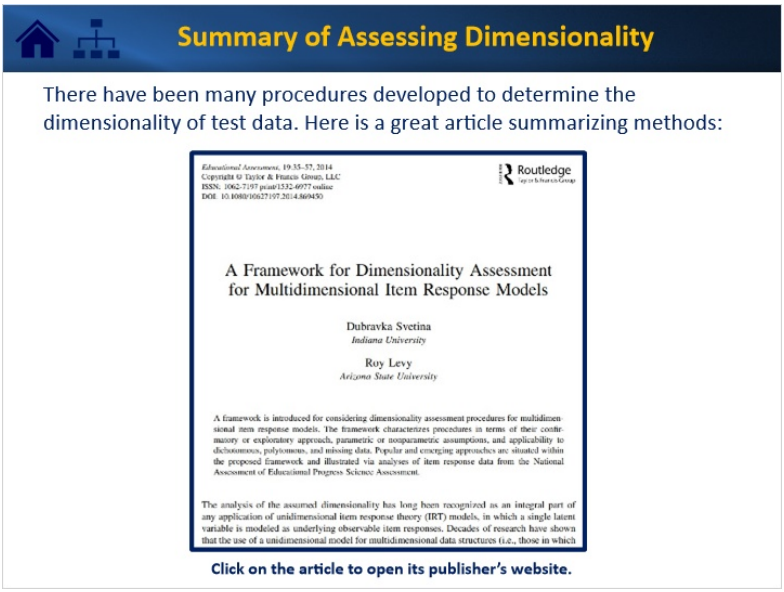
2.10 Scree Plots (I)



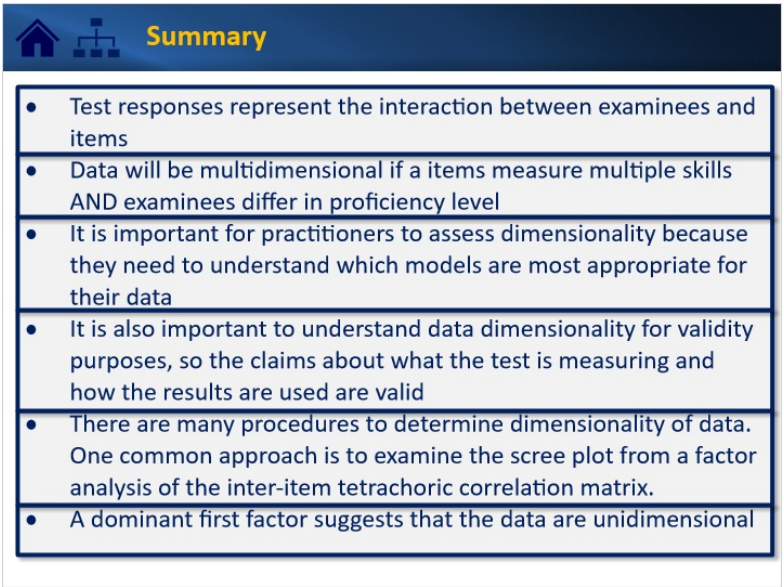
2.11 Scree Plots (II)



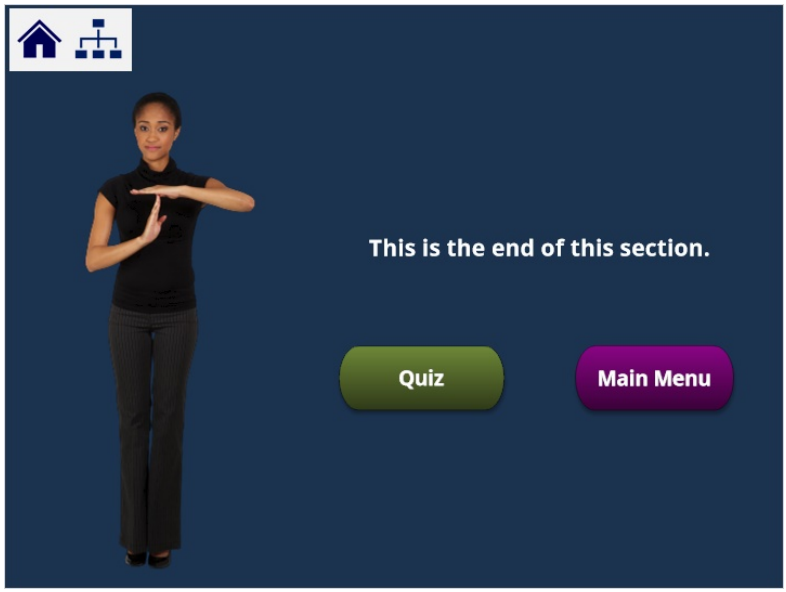
2.12 Summary of Assessing Dimensionality



2.13 Summary: Response Data Dimensionality

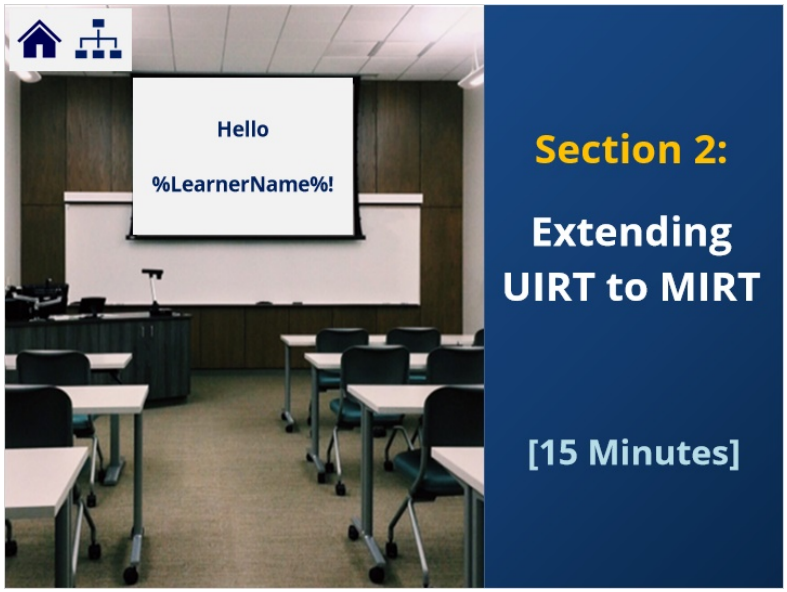


2.14 Bookend: Response Data Dimensionality

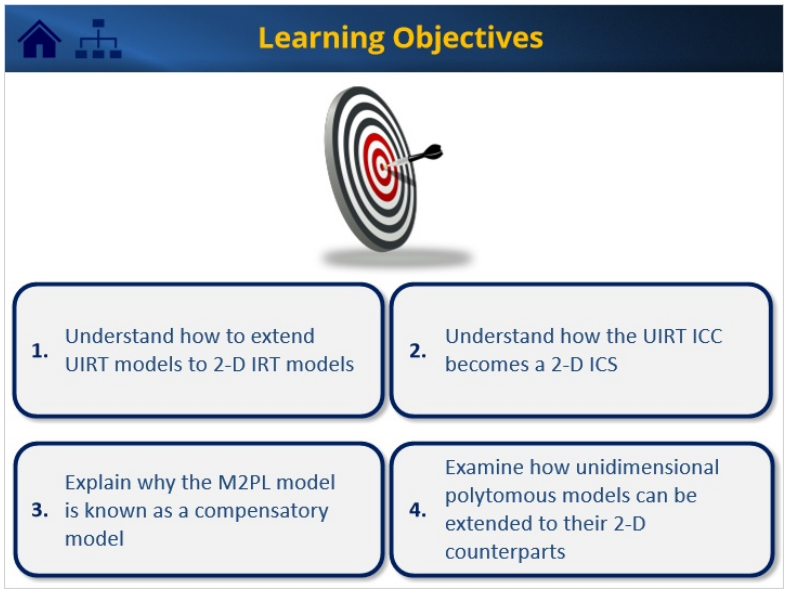


3. Extending UIRT to MIRT

3.1 Cover: Extending UIRT to MIRT



3.2 Learning Objectives: Extending UIRT to MIRT



3.3 The 2PL IRT Model

The 2-PL IRT Model

In the unidimensional 2PL model, the probability of examinee *j* responding correctly to item *i* can be expressed as:

$$P_{ij} = \frac{1}{1 + e^{-1.7a_i(\theta_j - b_i)}}$$

Discrimination parameter

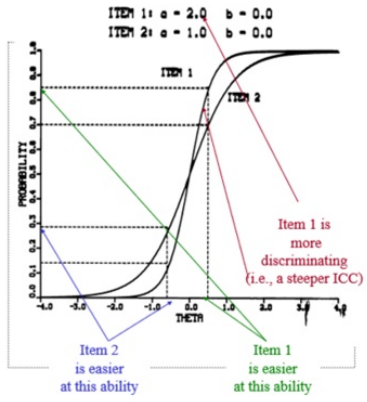
Latent ability parameter

Difficulty parameter

3.4 Item Characteristic Curves for UIRT 2PL

Item Characteristic Curve (UIRT-2PL)

- Graphically, the 2PL model for an individual item can be illustrated as an item characteristic curve (ICC)
- In this model, the steepness of the ICC relates directly to the a parameter (discrimination) and the location relates to the b parameter (difficulty)



3.5 The 2-Dimensional Compensatory Model

The Two-dimensional Compensatory Model

- In the multidimensional 2PL model (M2PL), the probability of examinee *j* responding correctly to item *i* can be expressed as:

$$P_{ij} = \frac{1}{1 + e^{-1.7\left(a_{1i}\theta_{1j} + a_{2i}\theta_{2j} + d_i\right)}}$$

Discrimination parameters

Latent ability parameters

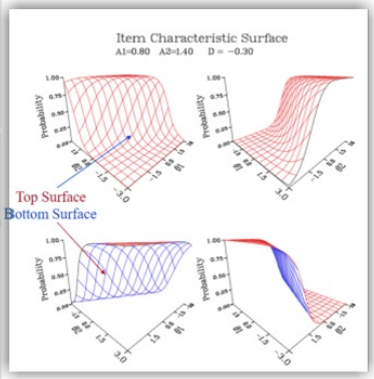
Difficulty parameter

- There are discrimination parameters a_1 and a_2 and ability parameters θ_1 and θ_2
- Regardless of the number of dimensions, there is only ever one difficulty parameter, *d*

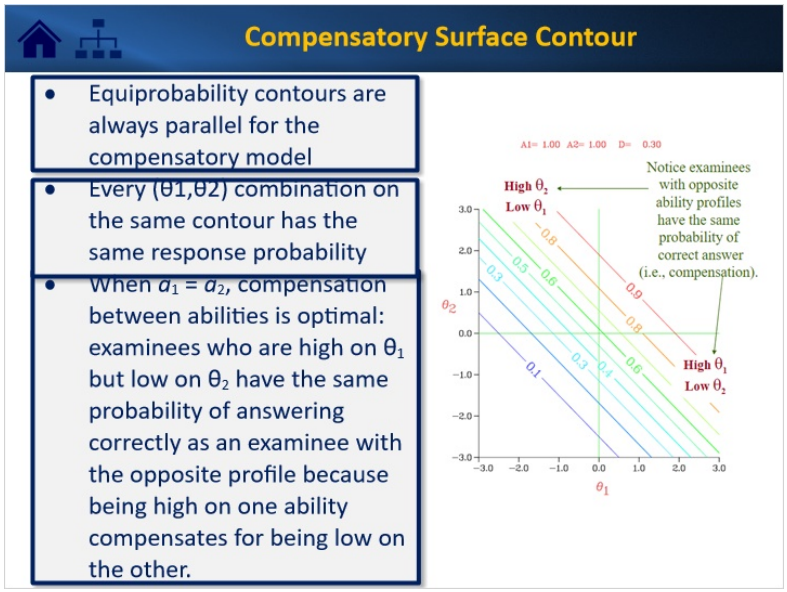
3.6 2-D Compensatory Item Surface

2-D Compensatory Item Surface

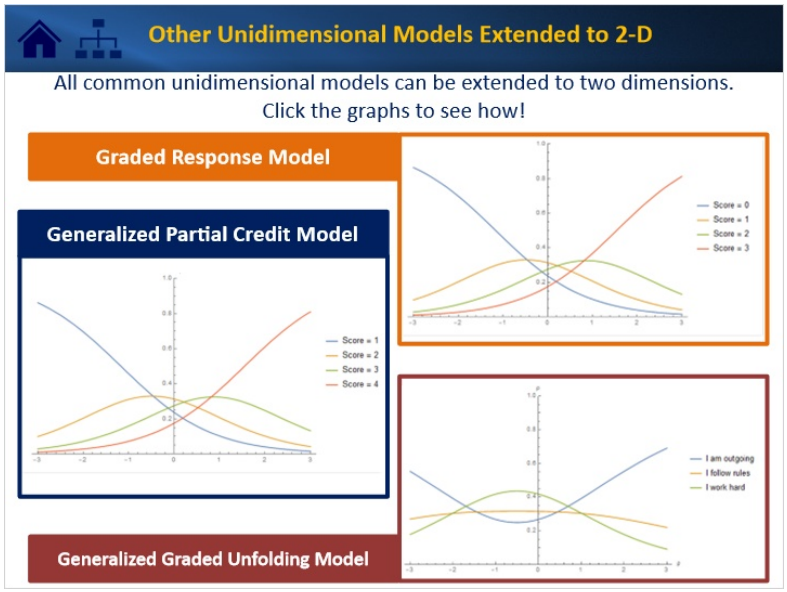
- Under the 2-D compensatory model, items can be illustrated with item characteristic surfaces (ICSs)
- The steepness of the ICS relates directly to the a parameters
- The relative sizes of the *a*s indicate which composite of θ_1 and θ_2 is being best measured. If $a_1 = a_2$, θ_1 and θ_2 are being measured equally well. If $a_1 > a_2$, θ_1 is being given greater emphasis, and if $a_1 < a_2$, θ_2 is being given greater emphasis.



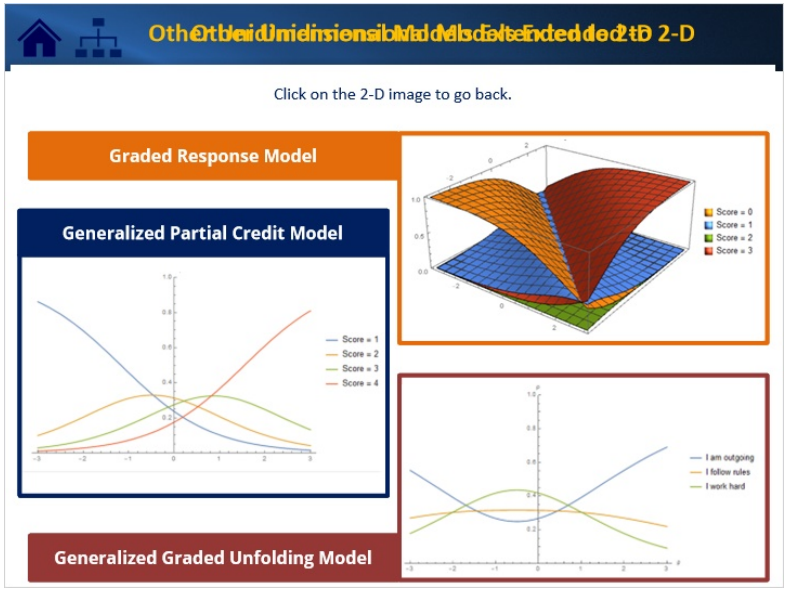
3.7 Compensatory Surface Contour



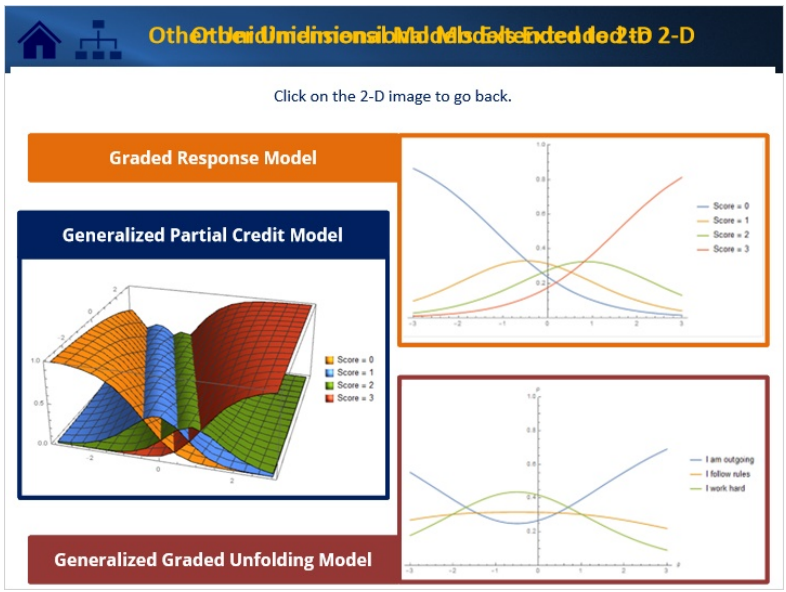
3.8 Other Unidimensional Models Extended to 2D



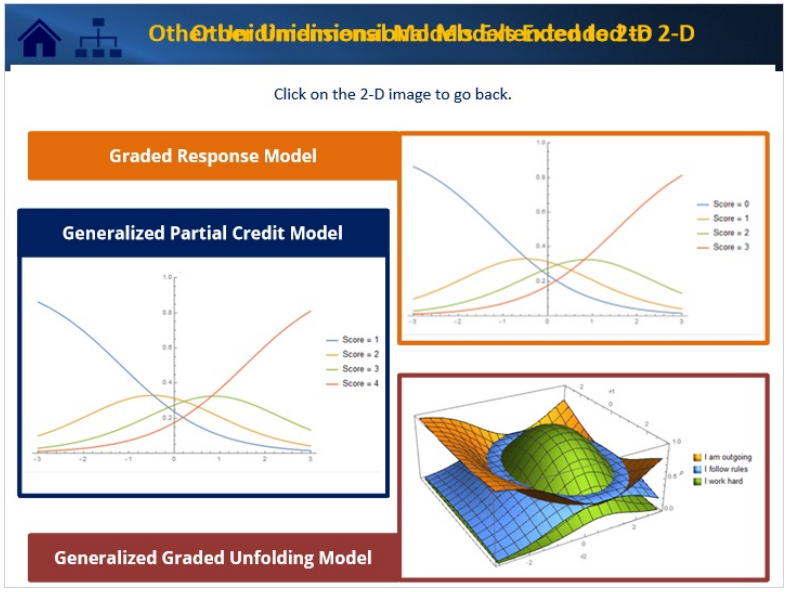
2DGRM (Slide Layer)



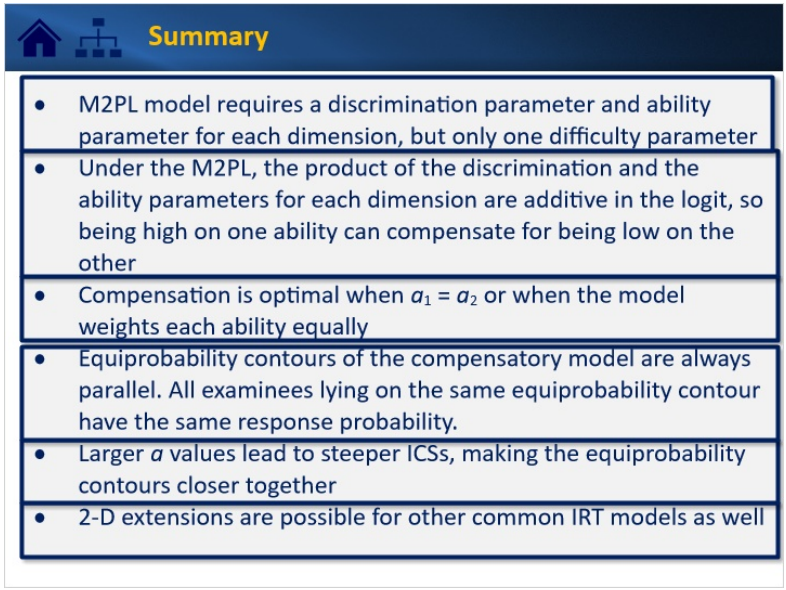
2DGPCM (Slide Layer)



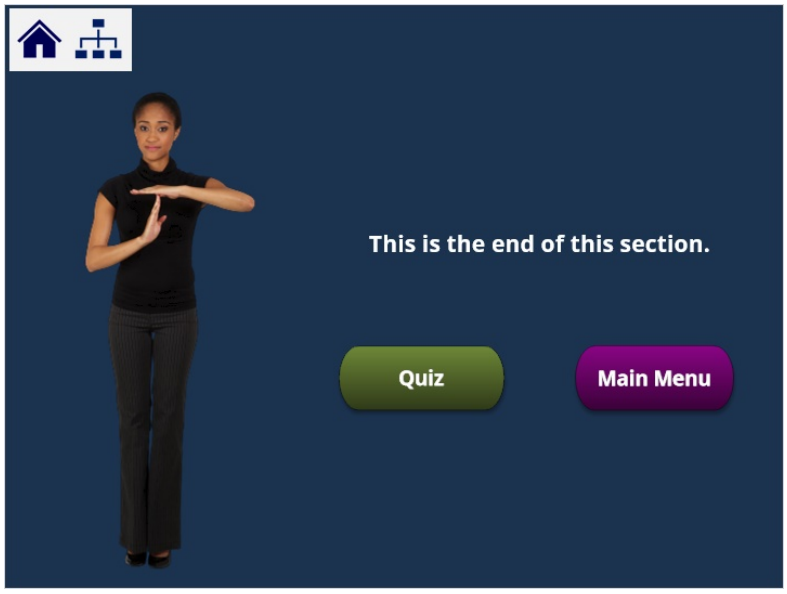
2DGGUM (Slide Layer)



3.9 Summary: UIRT and MIRT Concepts

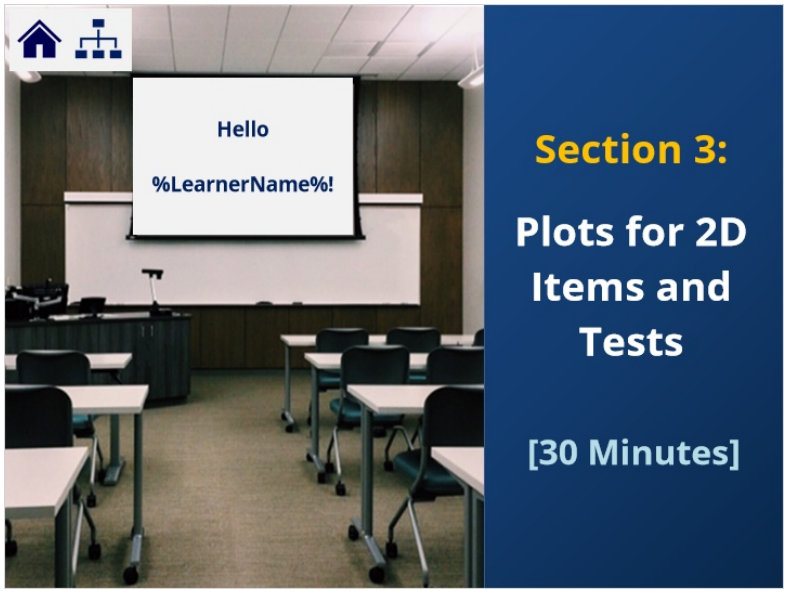


3.10 Bookend: Extending UIRT to MIRT

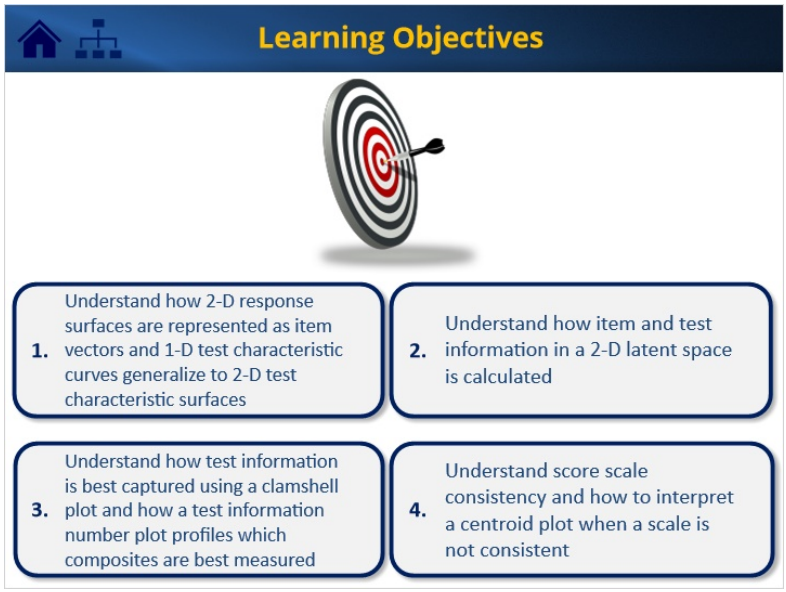


4. Plots for 2D Items and Tests

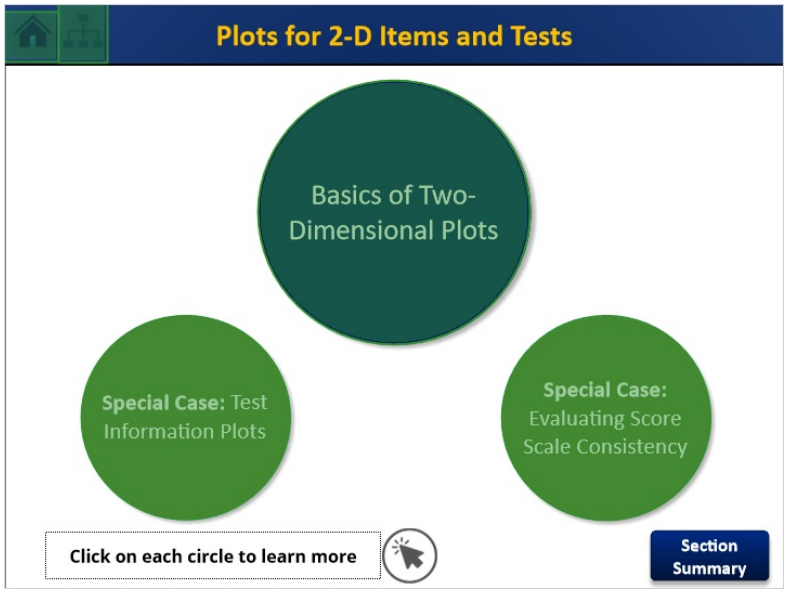
4.1 Cover: Plots for 2D Items and Tests



4.2 Learning Objectives: Plots for 2D Items and Tests

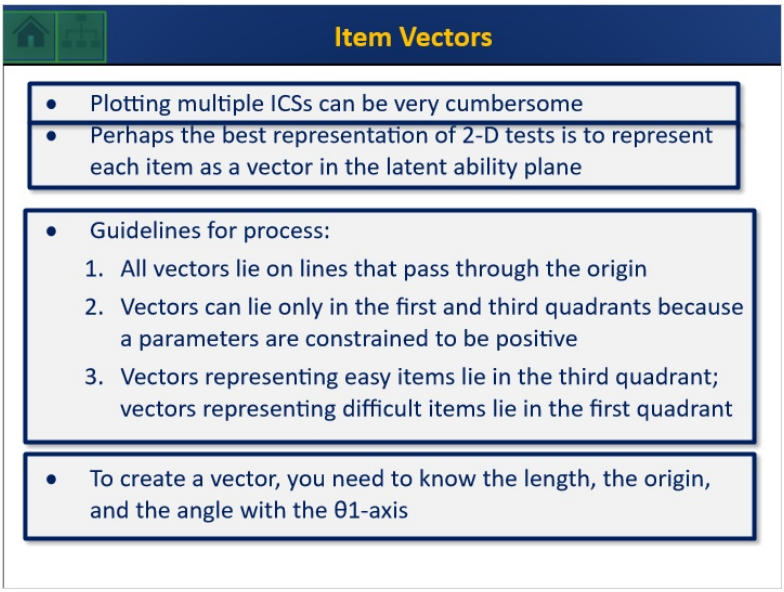


4.3 Topic Selection



4.4 Bookmark: Basics of 2-D Plots

4.5 Item Vectors (I)



4.6 Item Vectors (II)

Item Vectors

- The length of the vector indicates how discriminating the item is. This value is called MDISC.

$$\text{MDISC} = \sqrt{(a_1^2 + a_2^2)}$$

- The tail of the vector lies on the p = .50 equiprobability contour. The signed distance from the origin to this contour is denoted as MDIFF

$$\text{MDIFF} = \frac{-d}{\text{MDISC}}$$

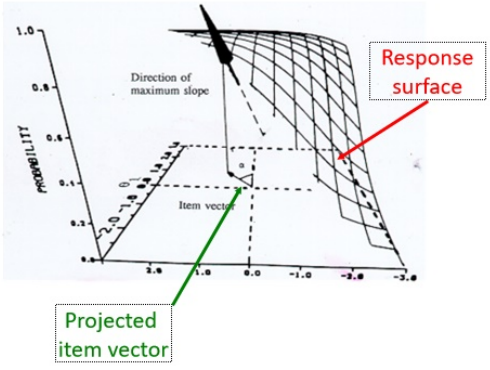
- The angular direction indicates the composite of ability that the item is best measuring

$$\alpha = \cos^{-1}\left(\frac{a_1}{\text{MDISC}}\right)$$

4.7 Item Vectors (III)

Item Vectors

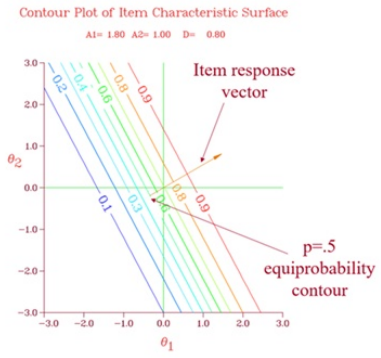
Vectors are actually projections of the direction of maximum discrimination or slope onto the latent ability plane



4.8 Item Vectors (III)

Item Vectors

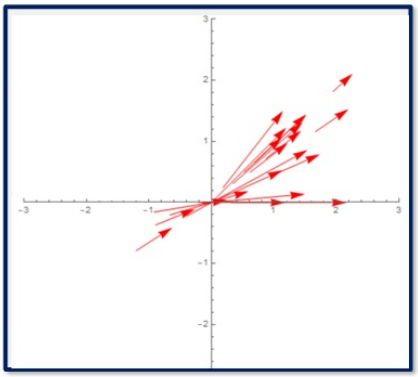
In the figure below, an item vector is plotted onto a contour plot of the item's surface. Note that the vector is orthogonal to the p = .5 contour. The closer together the contours, the larger the MDISC value and the longer the vector.



4.9 Item Vectors (IV)

Item Vectors

- Here is a vector plot of 2-D estimated parameters from a 20-item math quiz
- If a single score is reported then it is paramount that the vectors lie in a narrow sector (i.e., items are measuring similar composites)
- This is referred to as the Validity Sector



4.10 Item Vectors (V)

Item Vectors

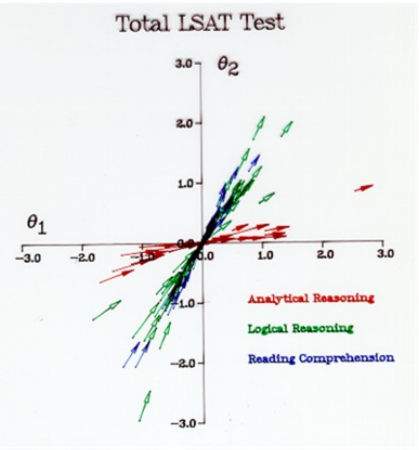
By color coding the vectors to match different content areas we can determine

- Are items from a certain content area more discriminating or more difficult?
- Do different items from different content areas measure different ability composites?
- How similar are the vector profiles for different yet "parallel" forms?

4.11 Example of Item Vectors for the 101-Item LSAT

Example of Item Vectors for the 101-item LSAT

- Here is an example of color-coding vectors from a 101-item LSAT test with three categories of items
- Note how each category lies in a narrow sector
- By color-coding we can also get a better sense of how to define our latent abilities from our table of specifications for each content area



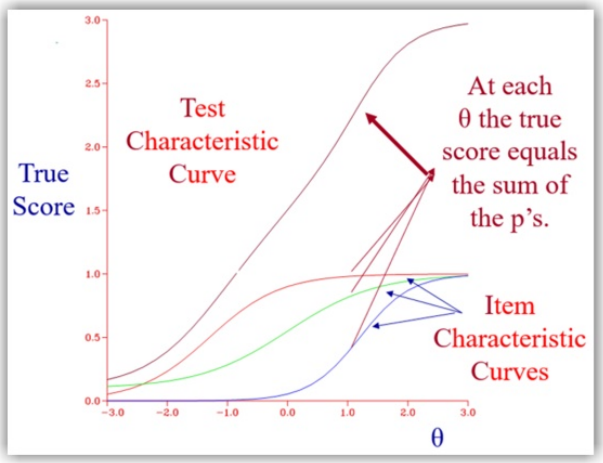
4.12 Test Characteristic Curves

Test Characteristic Curves

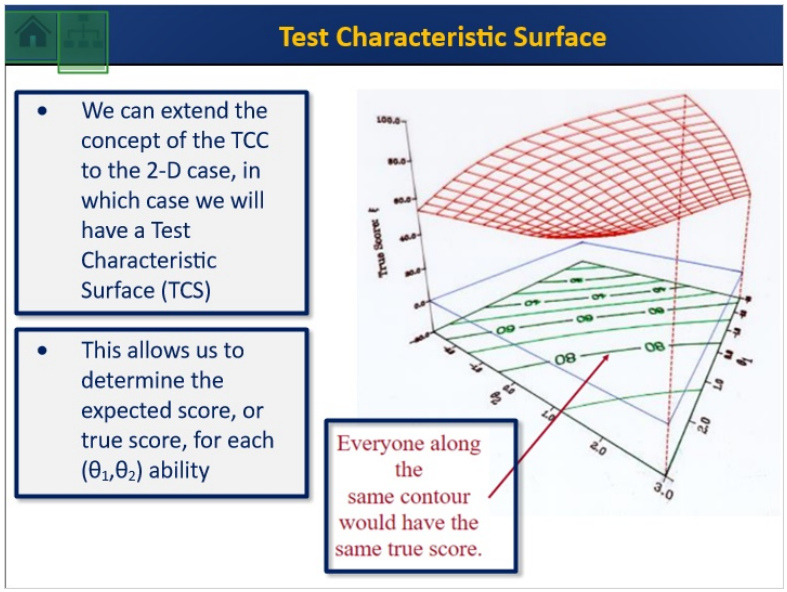
- In unidimensional IRT models, we can "sum up" the probabilities at each theta from ICCs to obtain a test characteristic curve (TCC)
- This curve enables us to determine the expected score (e.g., true score) of each ability on the θ-axis
- This curve has many functions, including mapping the θ scale onto the true score scale and equating across test forms.

4.13 Example of a Test Characteristic Curve

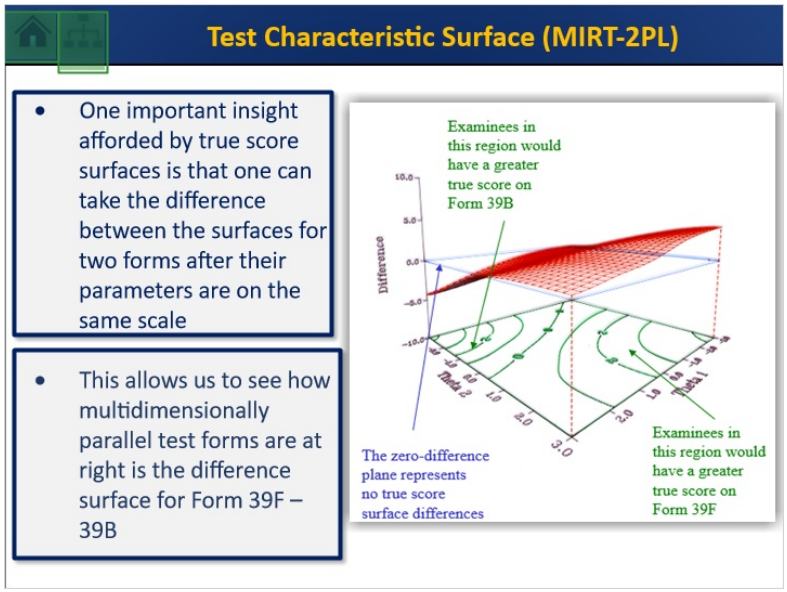
Example of a Test Characteristic Curve



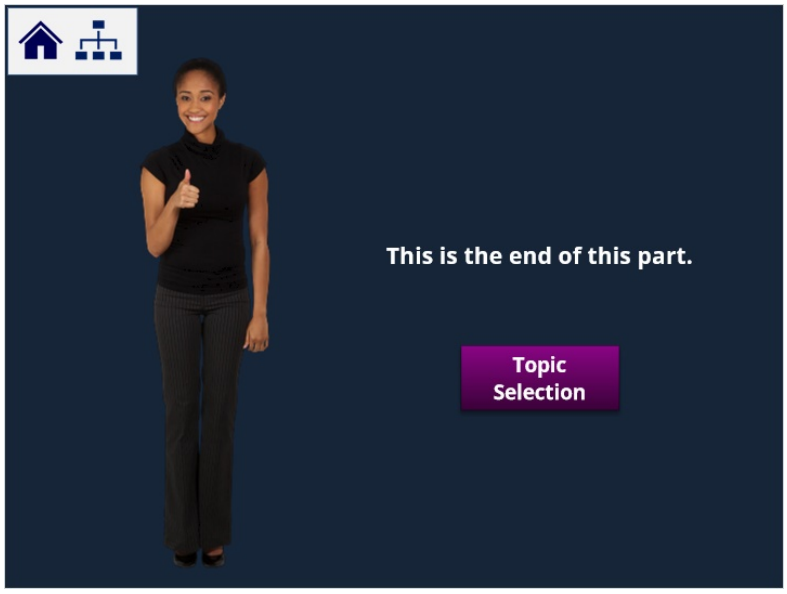
4.14 Test Characteristic Surface



4.15 Test Characteristic Surface (II)

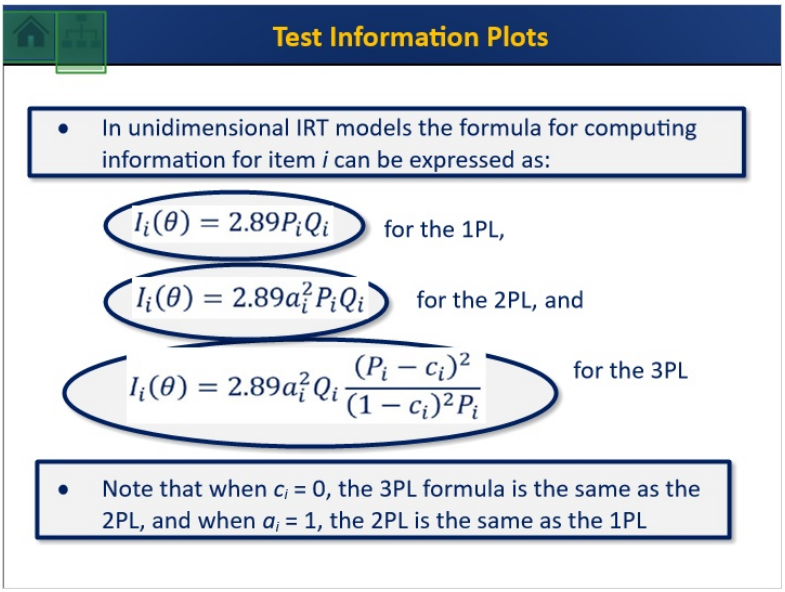


4.16 Bookend: Basics of 2-D Plots

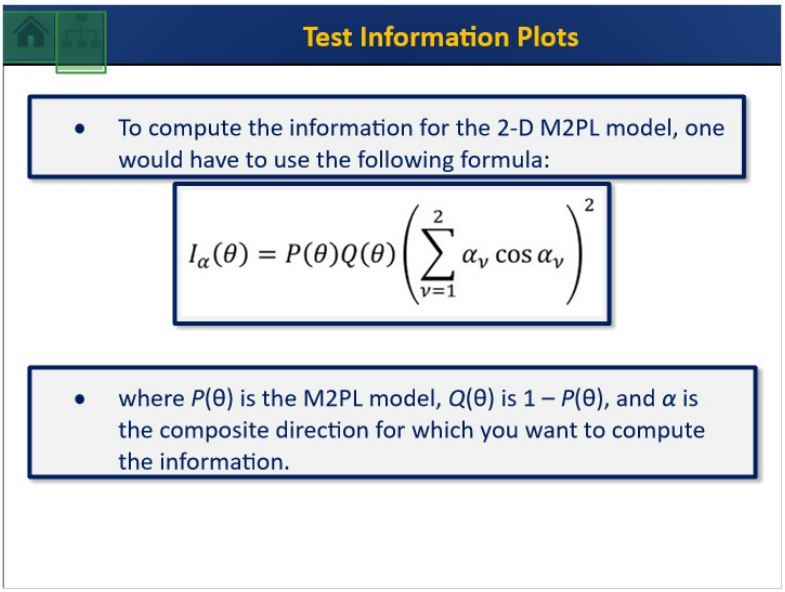


4.17 Bookmark: Test Information Plots

4.18 Test Information Plots (I)



4.19 Test Information Plots (II)



4.20 Test Information Plots (III)

Test Information Plots

- For example, assume $a_1 = 1$, $a_2 = 1$ and $d = 0$. To calculate the information in a 45° angle at the point (0,0), first calculate P(0,0) and Q(0,0).
- Under the M2PL , P(0,0) = .5 and Q(0,0) = .5, so:

$$I_{45}(1,1) = P(\boldsymbol{\theta})Q(\boldsymbol{\theta})(\sum_{v=1}^{m} \alpha_v cos\alpha_v)^2$$
$$I_{45}(1,1) = (.5)(.5)((1.0)(\cos 45) + (1.0)(\cos (45))^2$$
$$I_{45}(1,1) = (.5)(.5)[(1.0)(.707) + (1.0)(.707)]^2$$
$$I_{45}(1,1) = .499$$

- Continuing to move away from the origin in the same direction, P(1,1) = .967 and Q(1,1) = .033, yielding $I_{45}(1,1) = .064$. If we move out further to (2,2), $I_{45}(2,2) = .002$. Information decreases because the ICS flattens out in this direction.

4.21 Test Information Plots (IV)

Test Information Plots

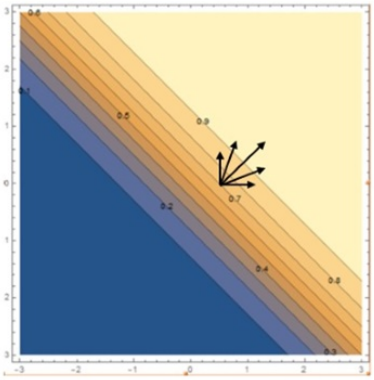
- Assume we are still working with the same item but now want to calculate $I_{30}(0,0)$:

$$I_{30}(0,0) = P(\boldsymbol{\theta})Q(\boldsymbol{\theta})(\sum_{v=1}^{m} \alpha_v cos\alpha_v)^2$$
$$I_{30}(0,0) = (.5)(.5)((1.0)(\cos 30) + (1.0)(\cos 60))^2$$
$$I_{30}(0,0) = (.5)(.5)[(1.0)(.866) + (1.0)(.5)]^2$$
$$I_{30}(0,0) = .466$$

- Further, $I_0(0,0) = .25$. The angle of maximum information when $a_1 = a_2$ is at 45°, or orthogonal to the equiprobability contours. As we move away from this direction, the surface is not as steep, the item is not as discriminating, and the information decreases.

4.22 Test Information Plots

Test Information Plots



The previous calculations are illustrated with information vectors at 0°, 30°, 45°, 60°, and 90°. The lengths of the vectors indicate the amount of information. The longest vector is at 45° and is orthogonal to the equiprobability contours.

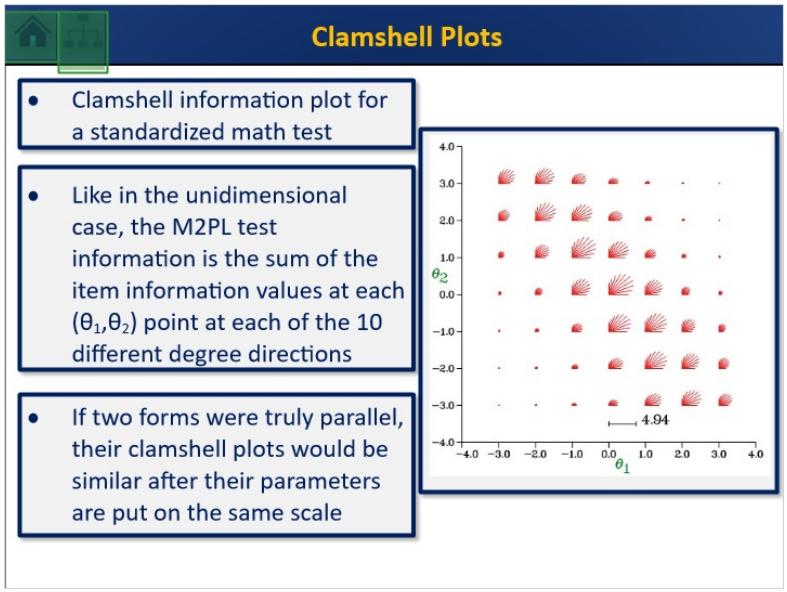
4.23 Clamshell Plots (I)

Clamshell Plots

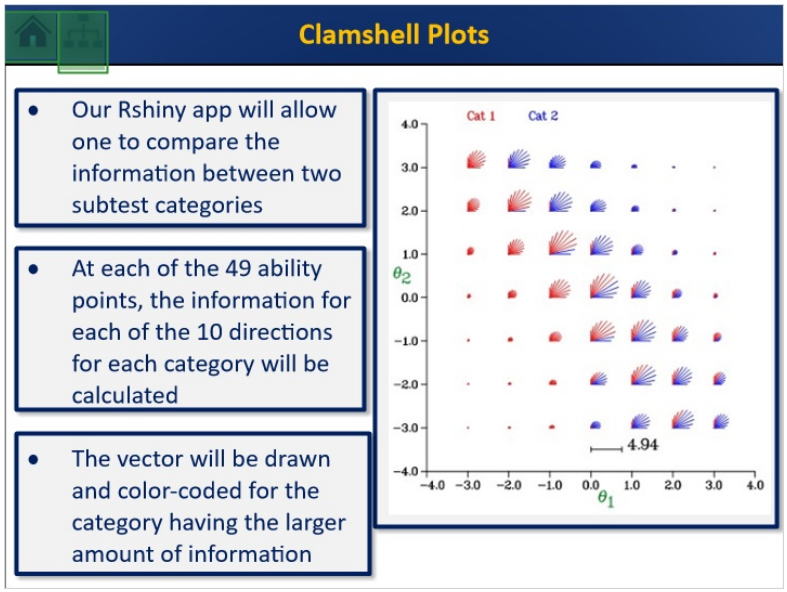
Reckase and McKinley (1991) show a way to illustrate the amount of information for the M2PL model using a "clamshell" plot. To create this plot:

1. The amount of information is computed as 49 uniformly spaced points on a 7x7 grid on the (θ_1,θ_2) plane
2. At each point, the amount of information the test provides is computed for ten different directions (or ability composites) from 0 to 90 degrees in 10-degree increments
3. The resulting ten vectors at each ability point resemble a clamshell

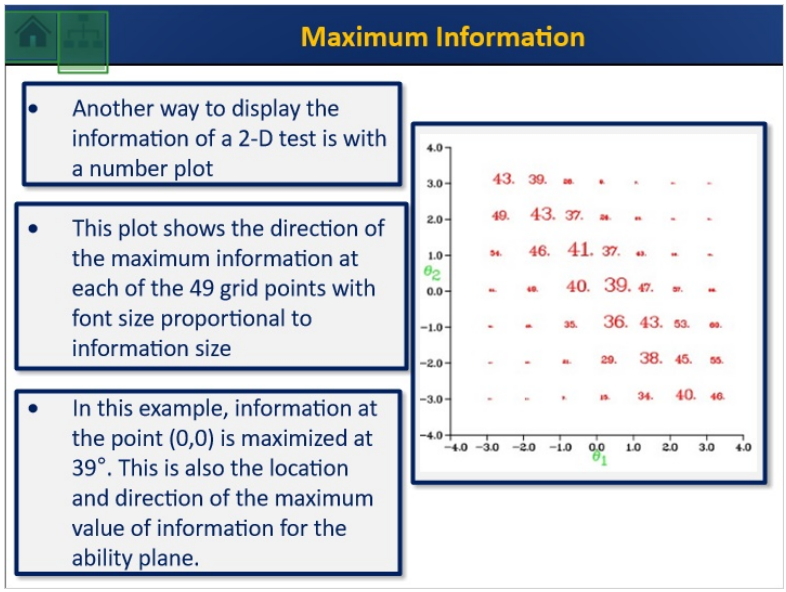
4.24 Clamshell Plots (II)



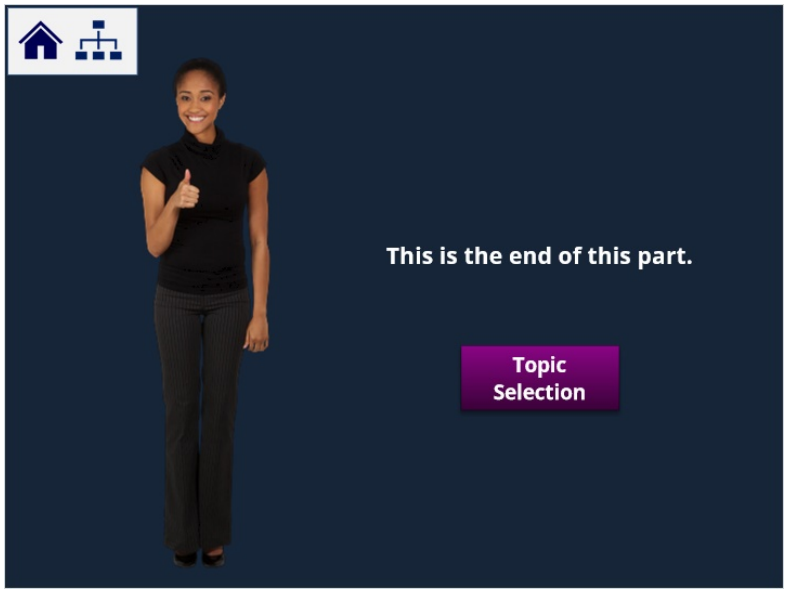
4.25 Clamshell Plots (III)



4.26 Maximum Information

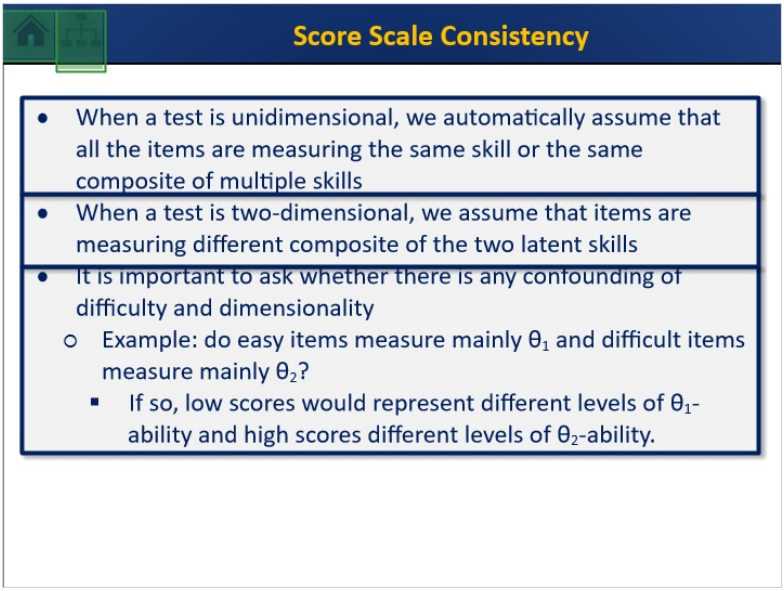


4.27 Bookend: Examining Score Scale Consistency

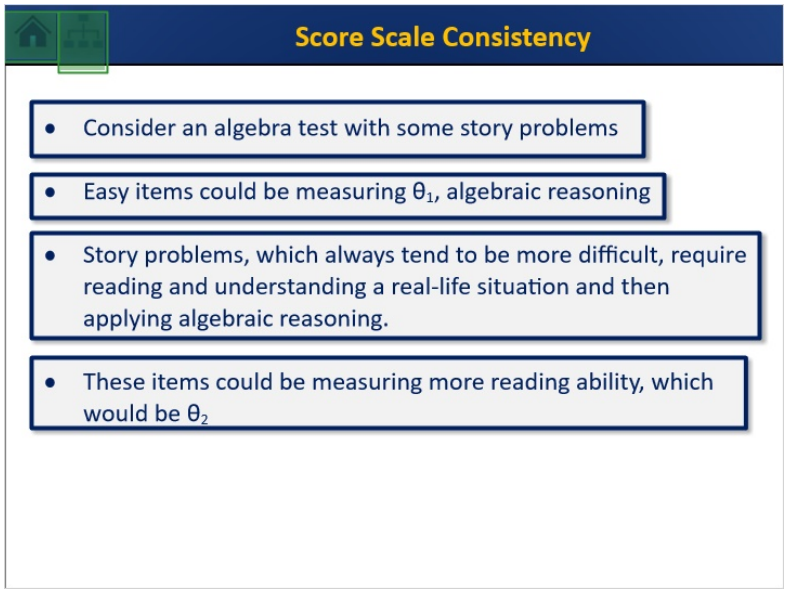


4.28 Bookmark: Examining Score Scale Consistency

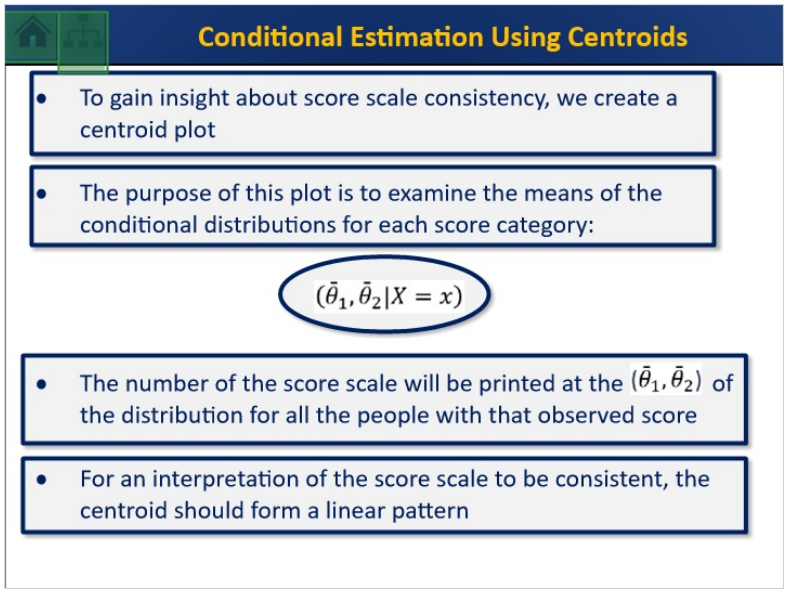
4.29 Score Scale Consistency (I)



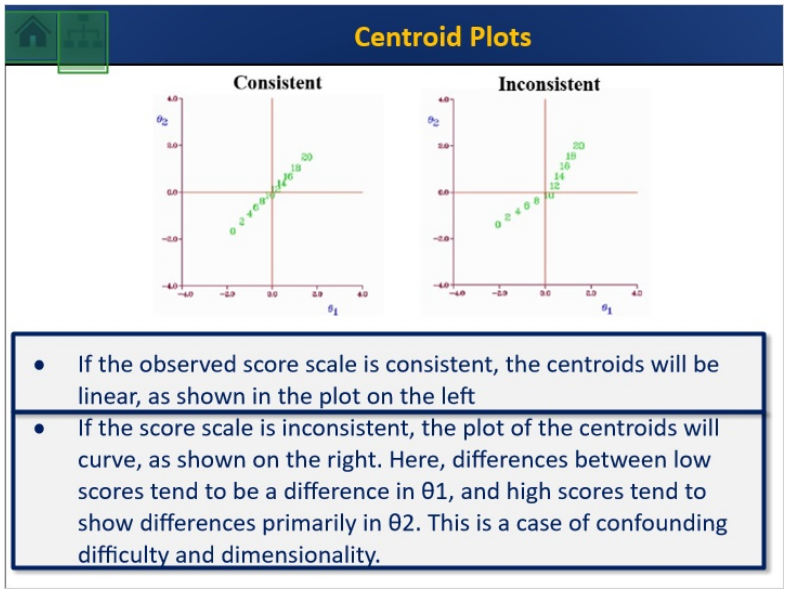
4.30 Score Scale Consistency (II)



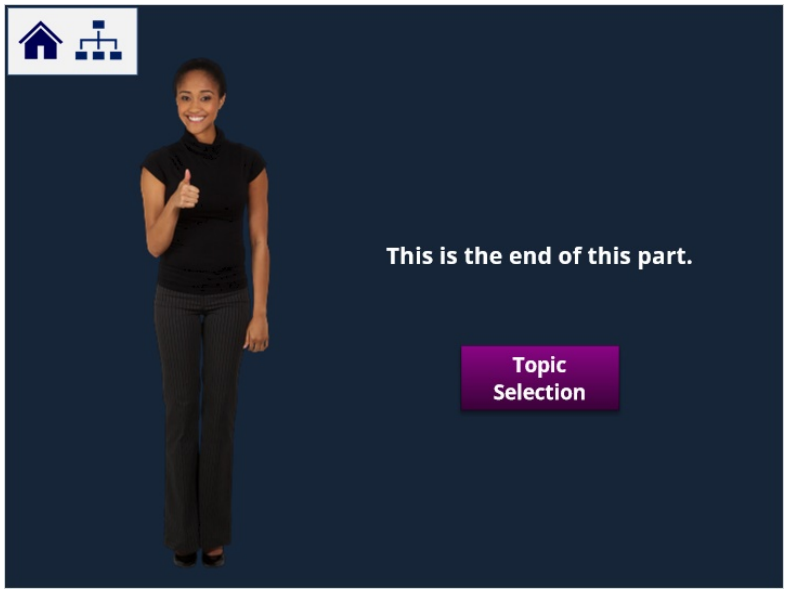
4.31 Conditional Estimation Using Centroids



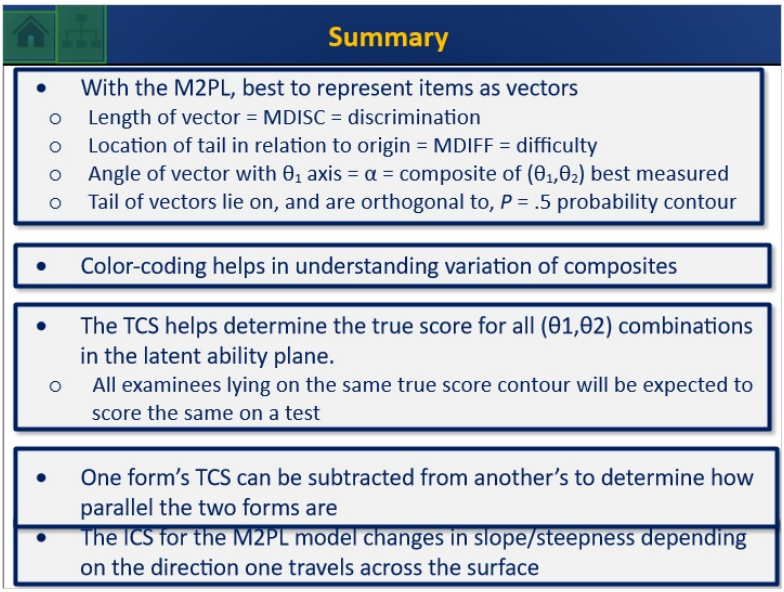
4.32 Centroid Plots



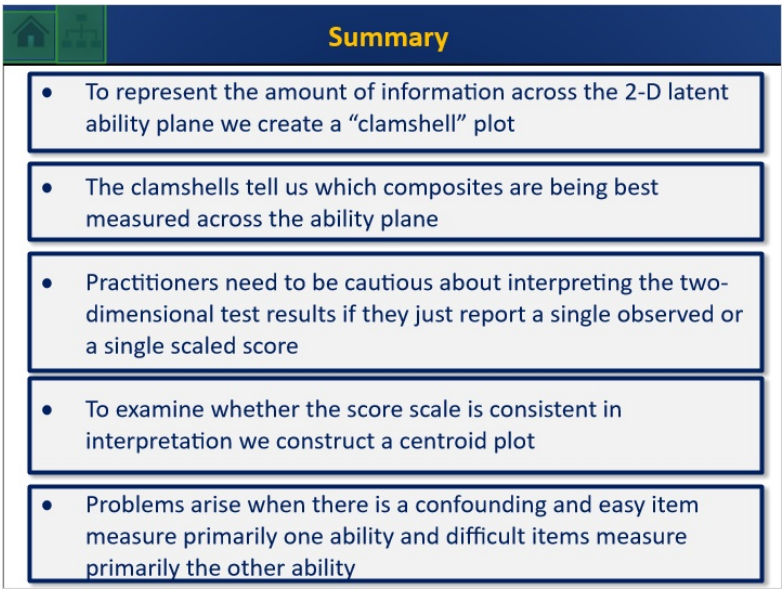
4.33 Bookend: Test Information



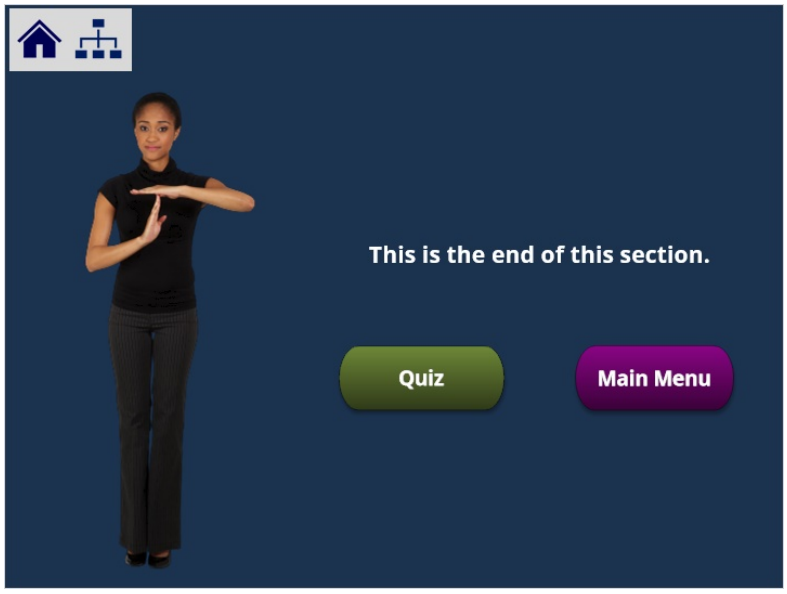
4.34 Summary: Plots for 2D Items and Tests (I)



4.35 Summary: Plots for 2D Items and Tests (II)



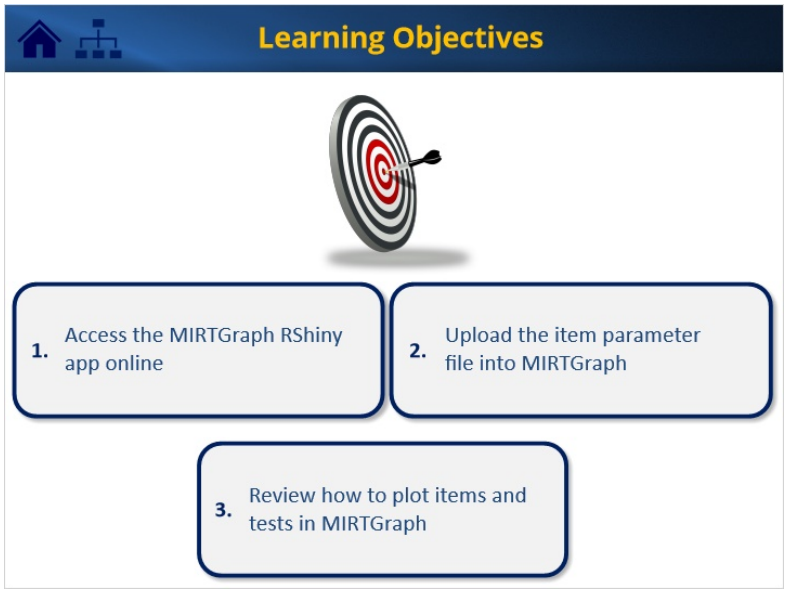
4.36 Bookend: Plots for 2D Items and Tests



5. MIRT Plots in RShiny

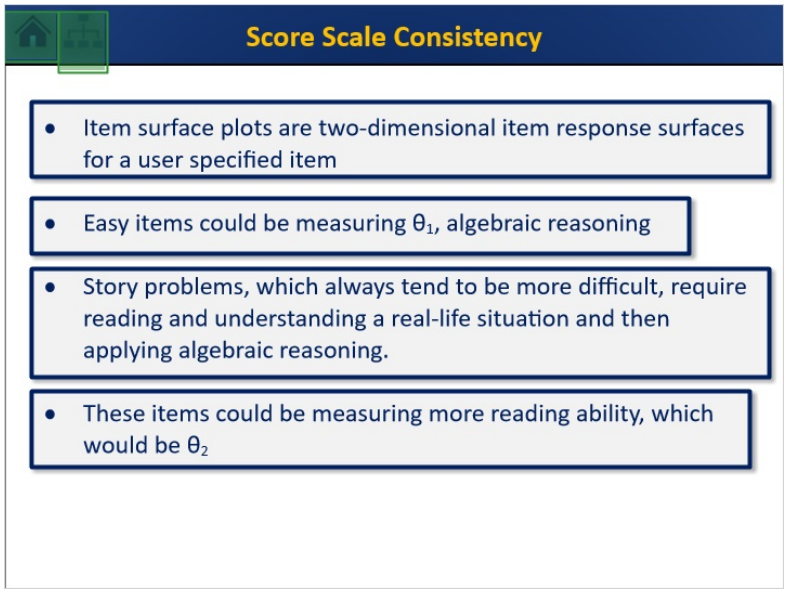
5.1 Cover: MIRT Plots in Rshiny

5.2 Learning Objectives: MIRT Plots in RShiny

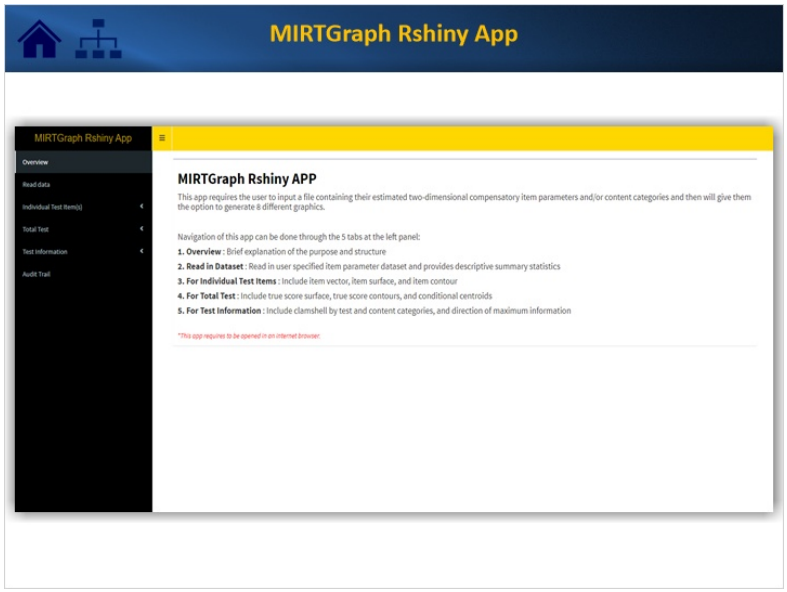


5.3 Access to RShiny

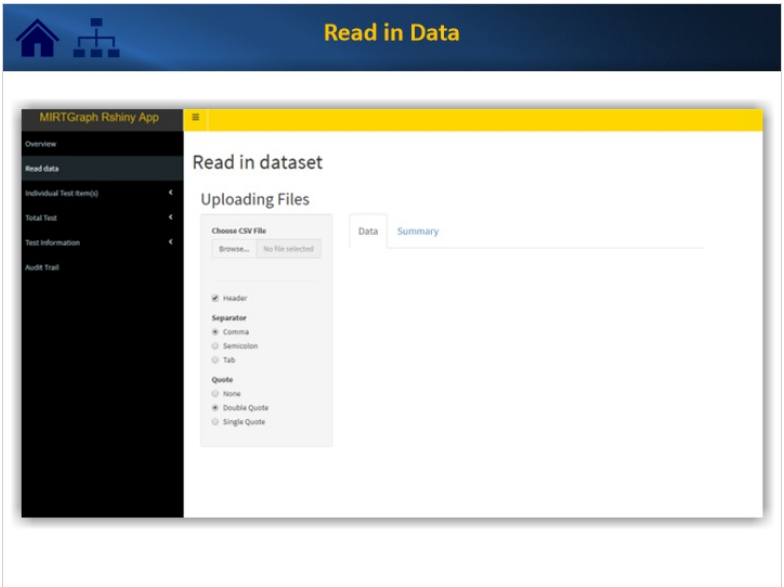
5.4 FORMATTT



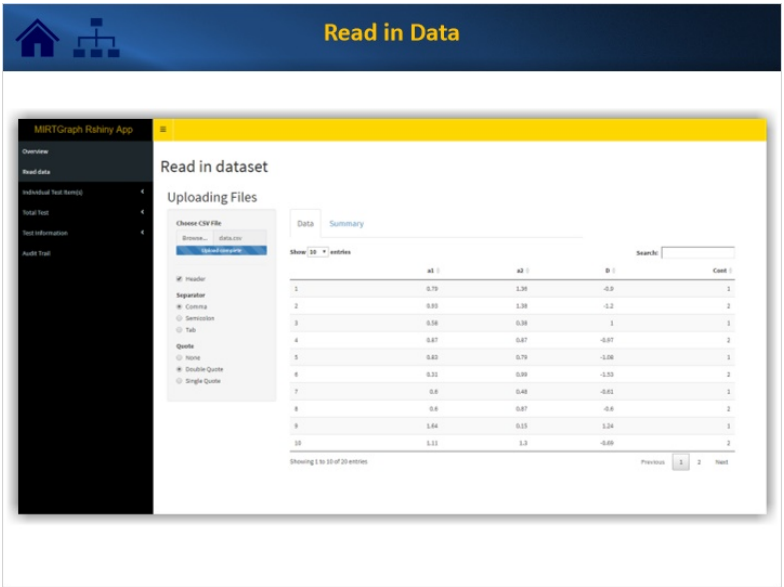
5.5 MIRTGraph RShiny App



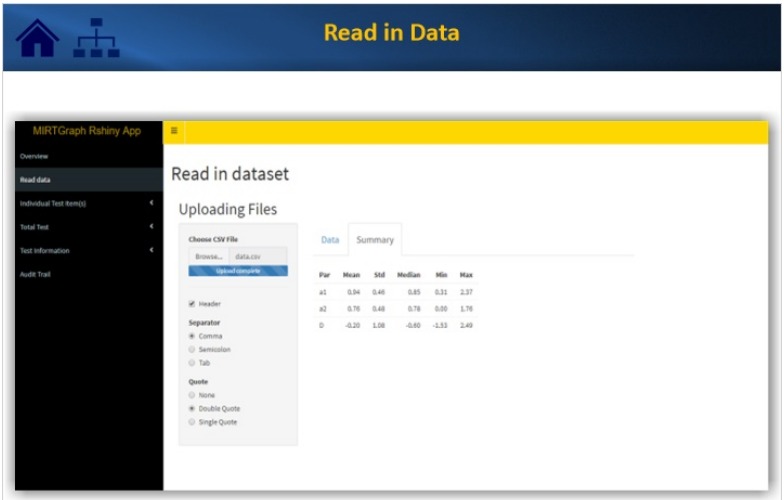
5.6 Read in Data (I)



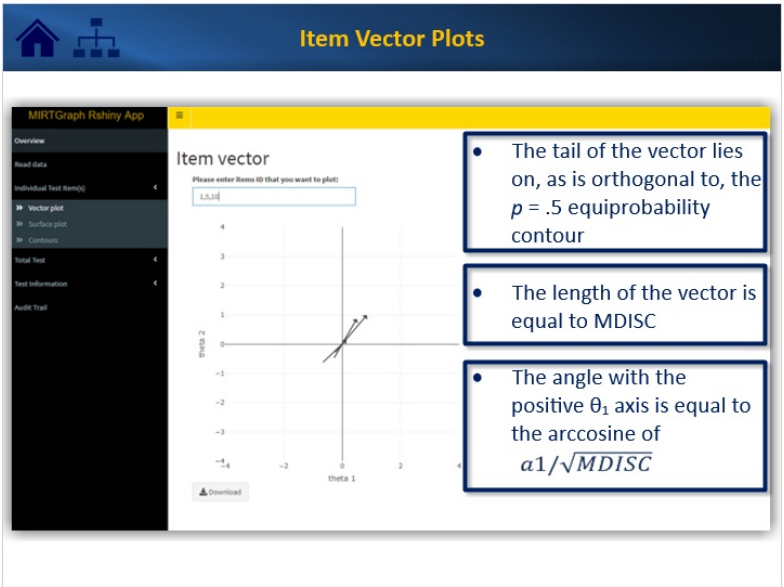
5.7 Read in Data (II)



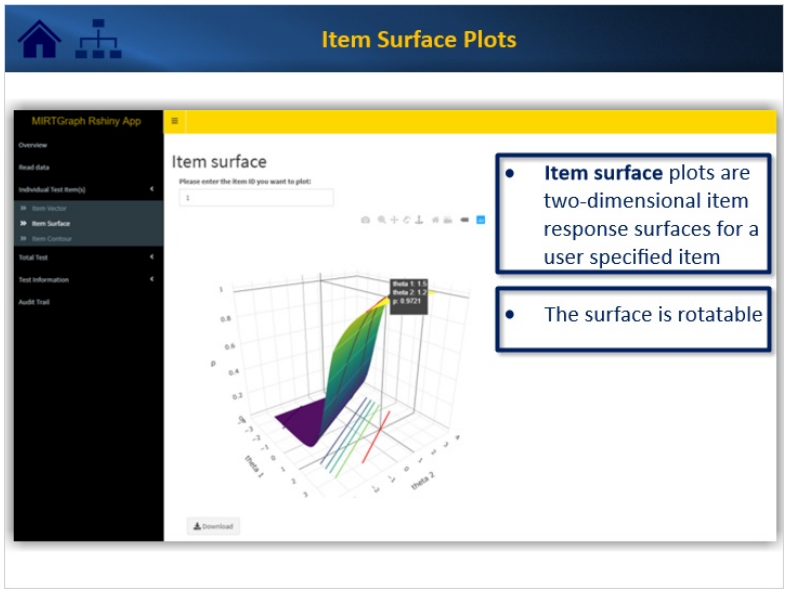
5.8 Read in Data (III)



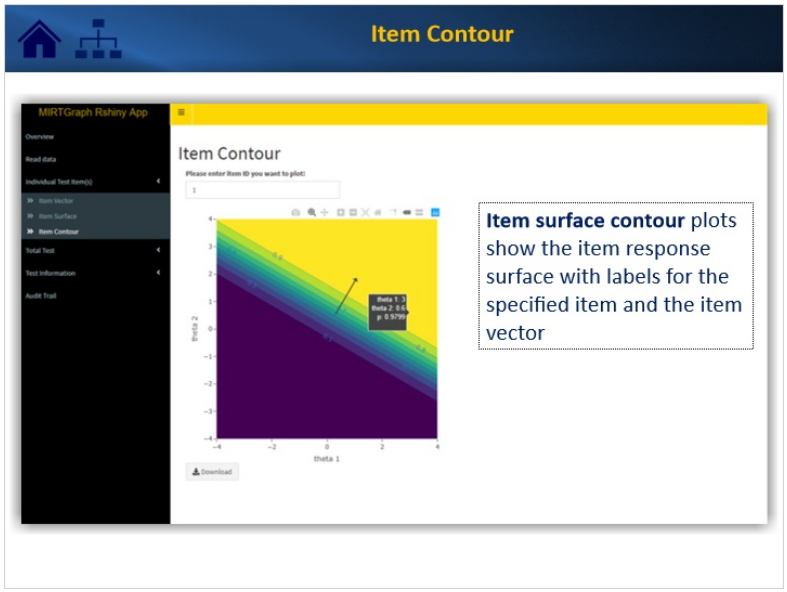
5.9 Item Vector Plots



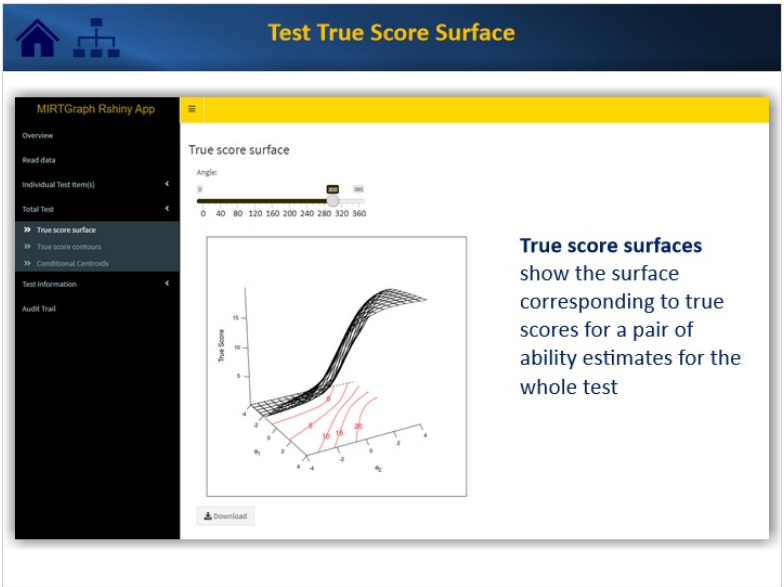
5.10 Item Surface Plots



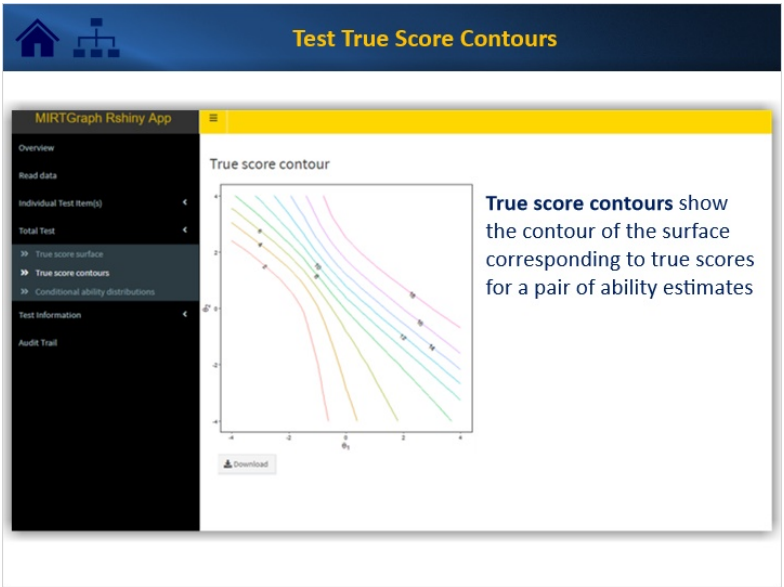
5.11 Item Contour Plots



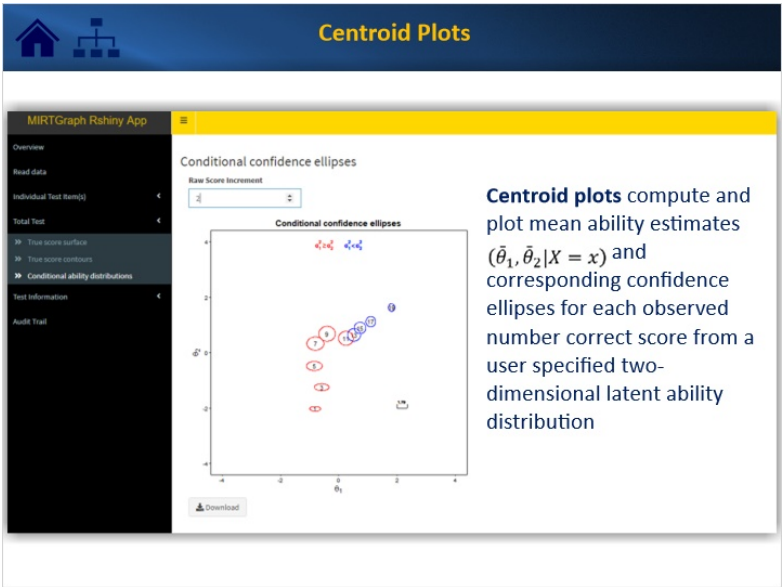
5.12 Test True Score Surfaces



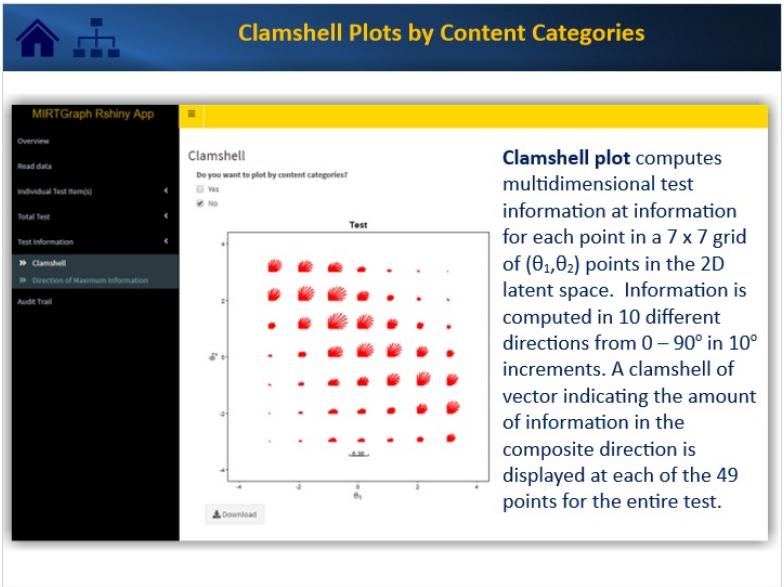
5.13 Test True Score Contours



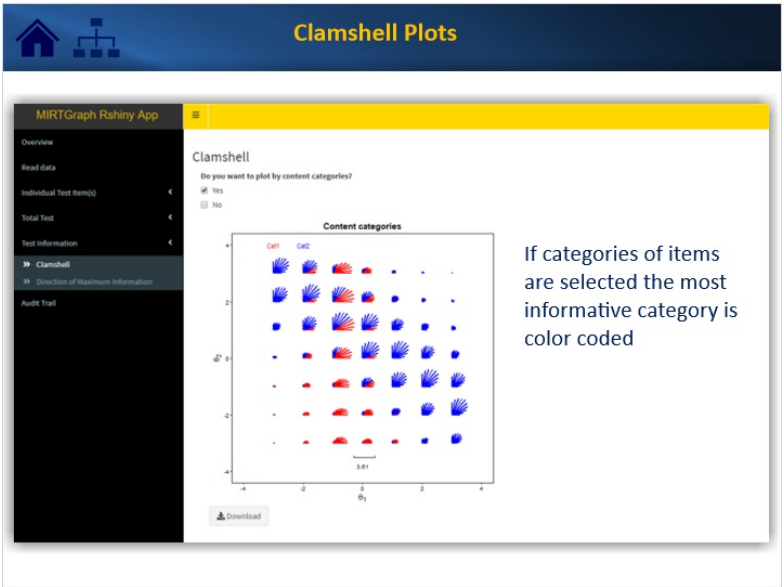
5.14 Centroid Plots



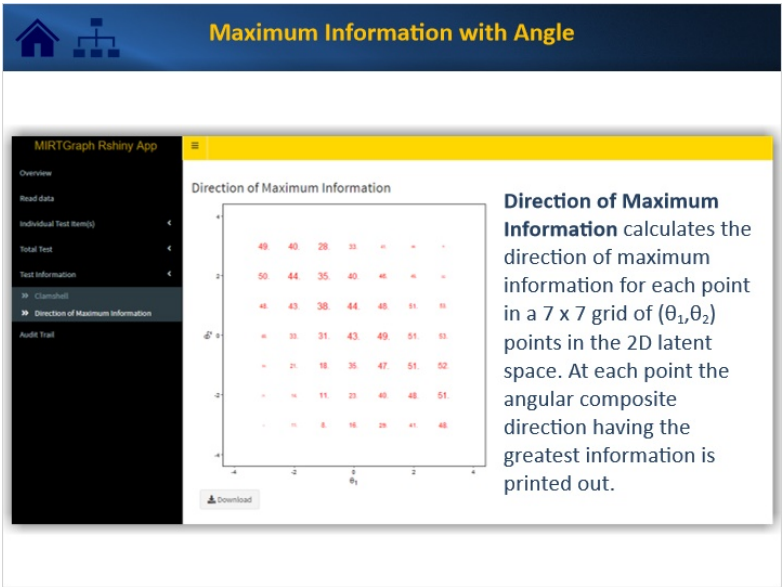
5.15 Clamshell Plots by Content Categories



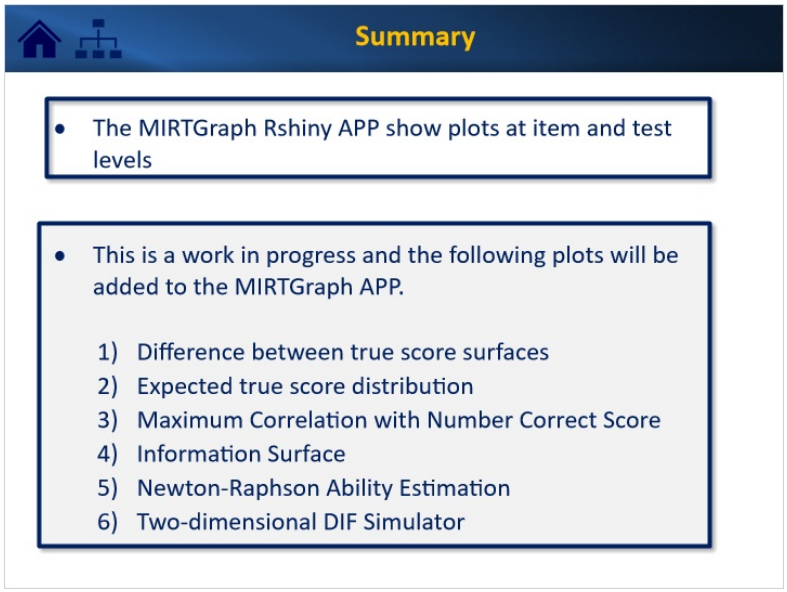
5.16 Clamshell Plots



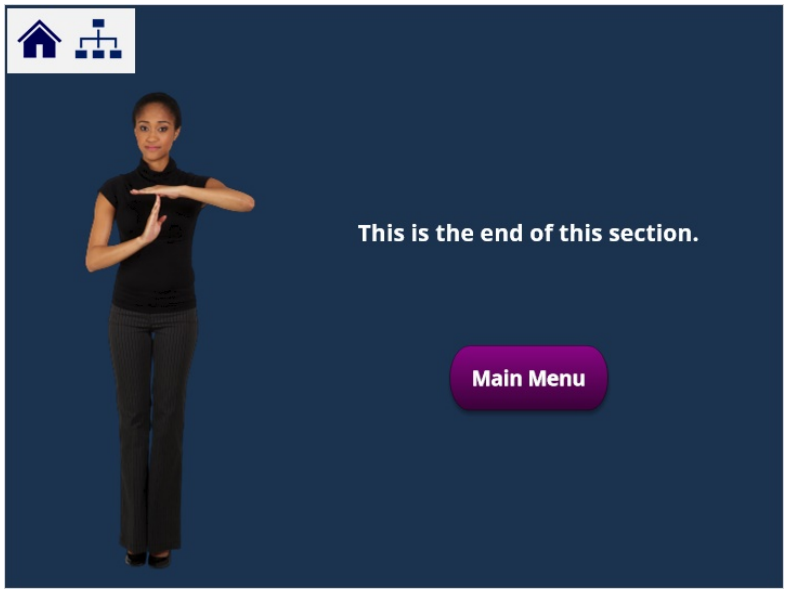
5.17 Maximum Information with Angle



5.18 Summary



5.19 Bookend: Plots for 2D Items and Tests

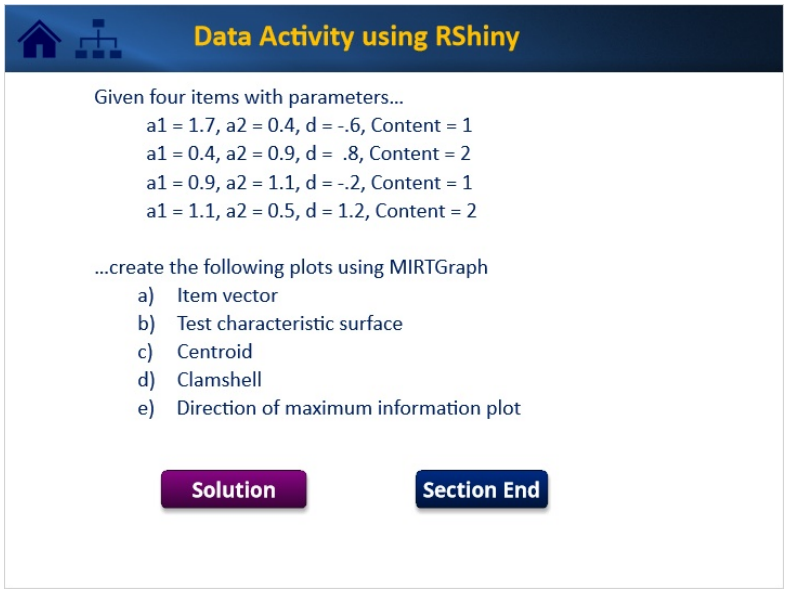


6. Data Activity

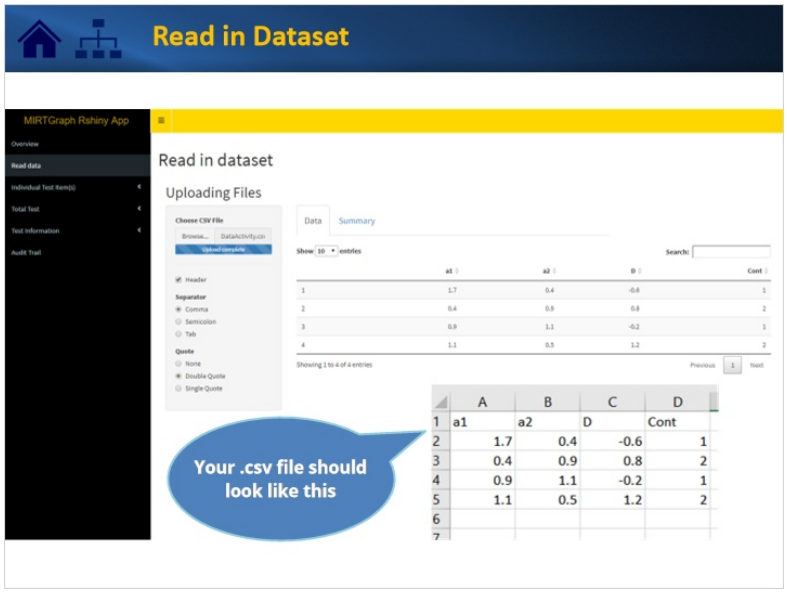
6.1 Cover: Data Activity

6.2 Access to RShiny

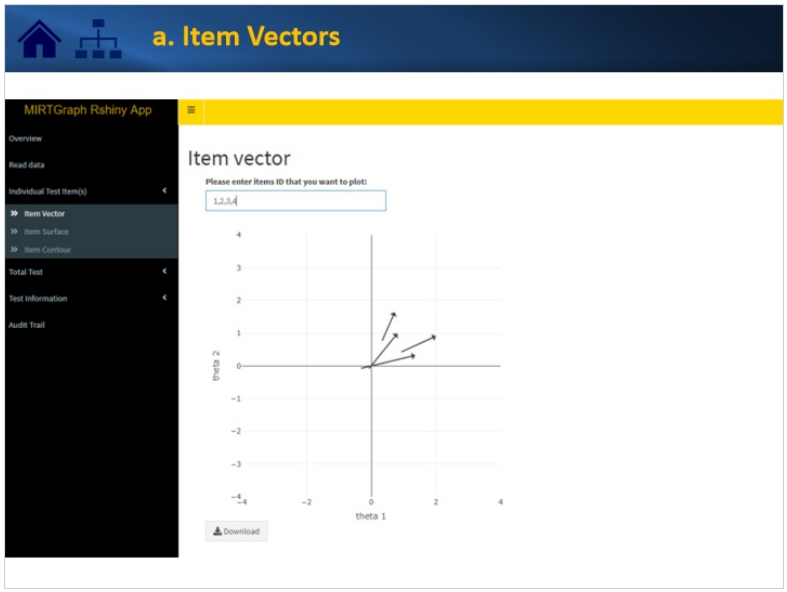
6.3 Data Activity using RShiny



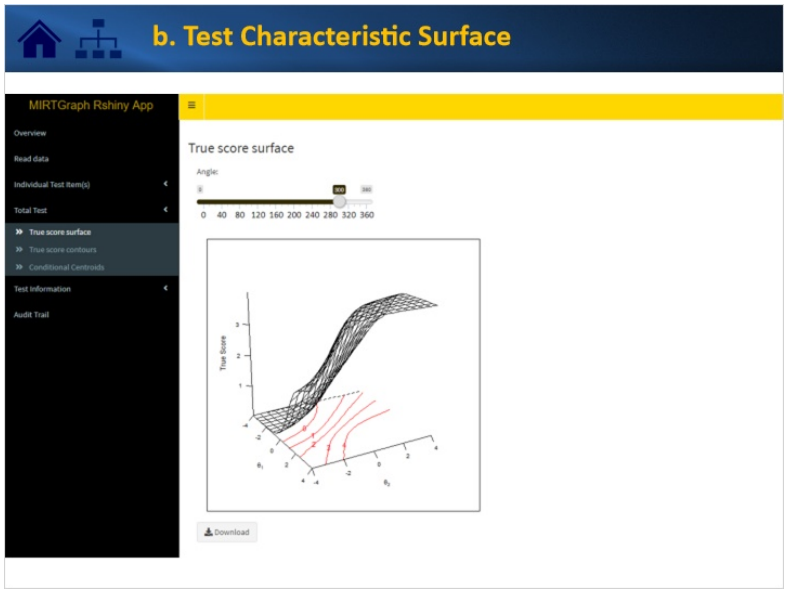
6.4 Read in Dataset



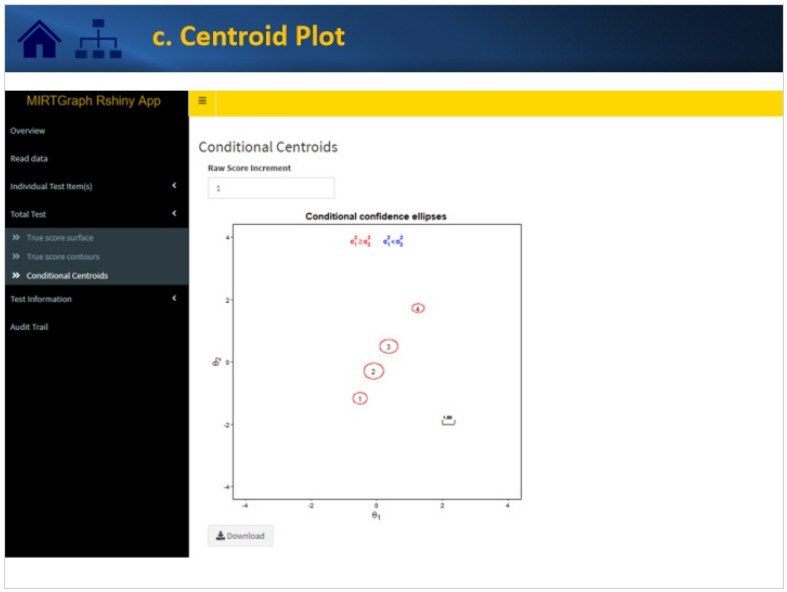
6.5 Item Vectors



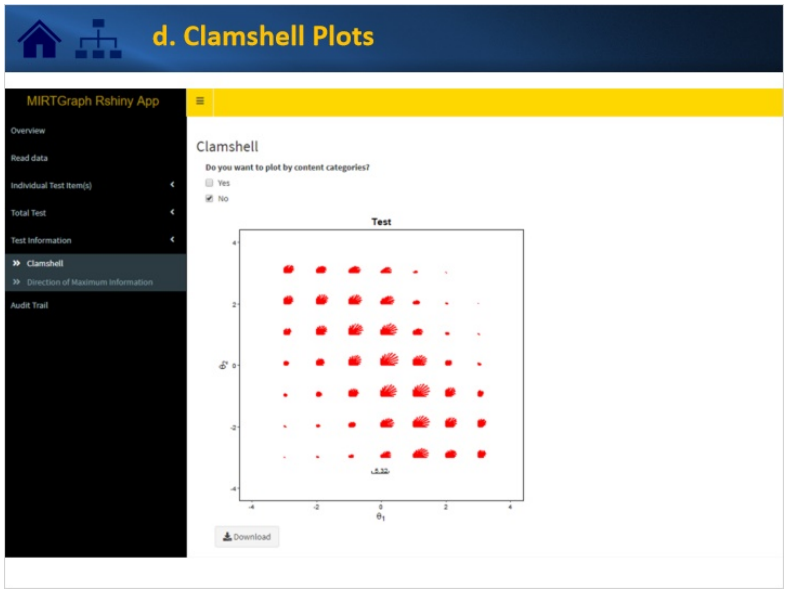
6.6 Test Characteristic Surface



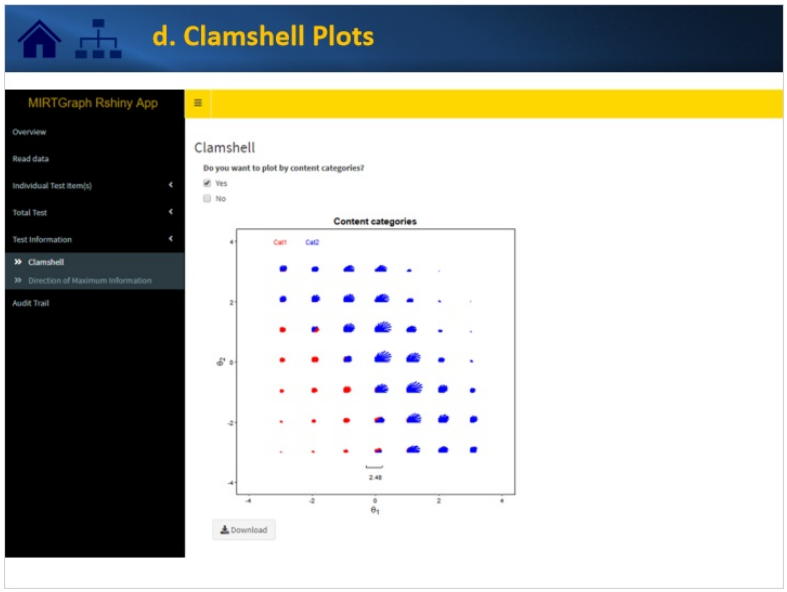
6.7 Centroid Plot



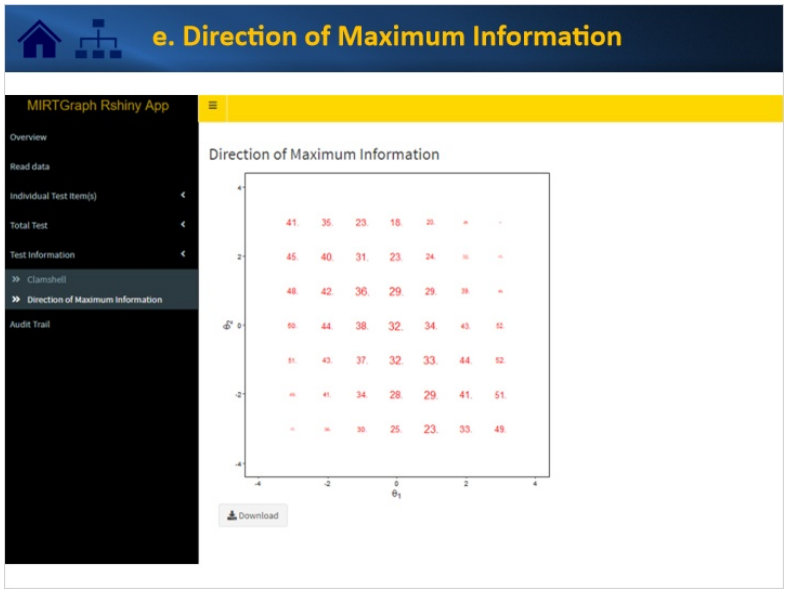
6.8 Clamshell Plots (I)



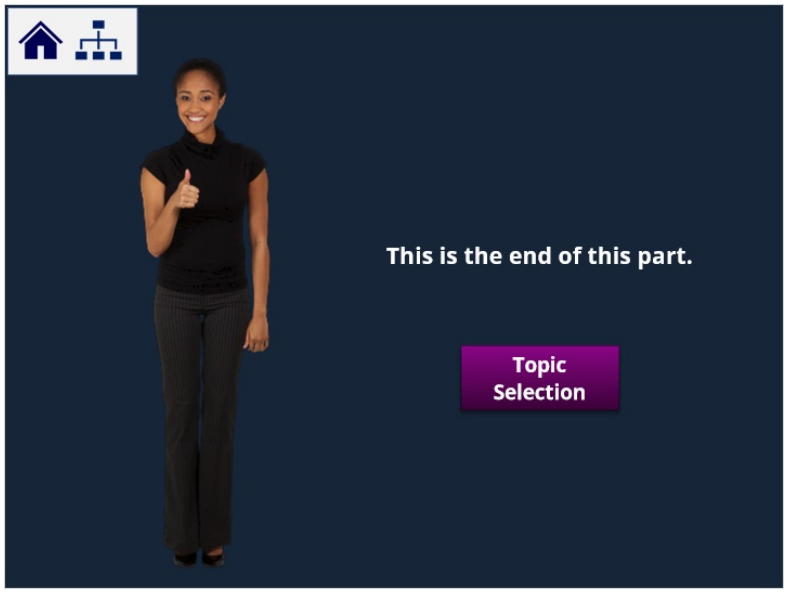
6.9 Clamshell Plots (II)



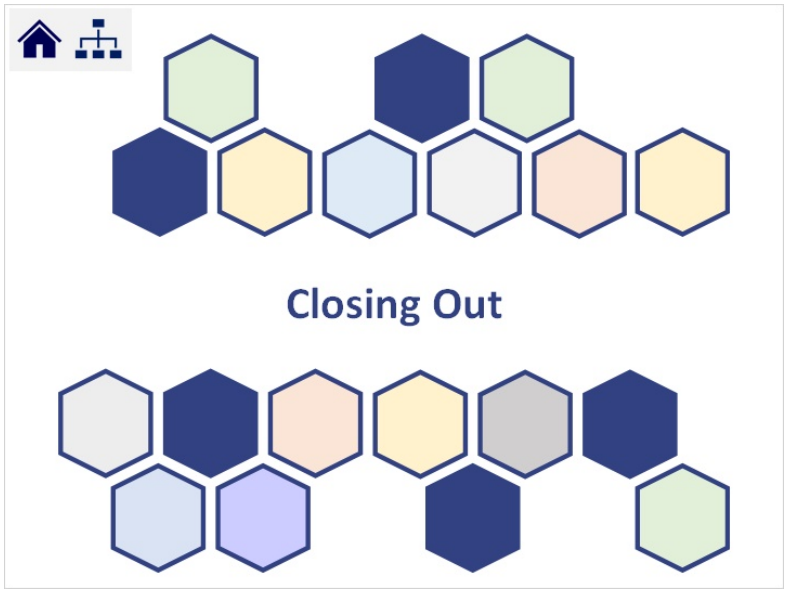
6.10 Direction of Maximum Information



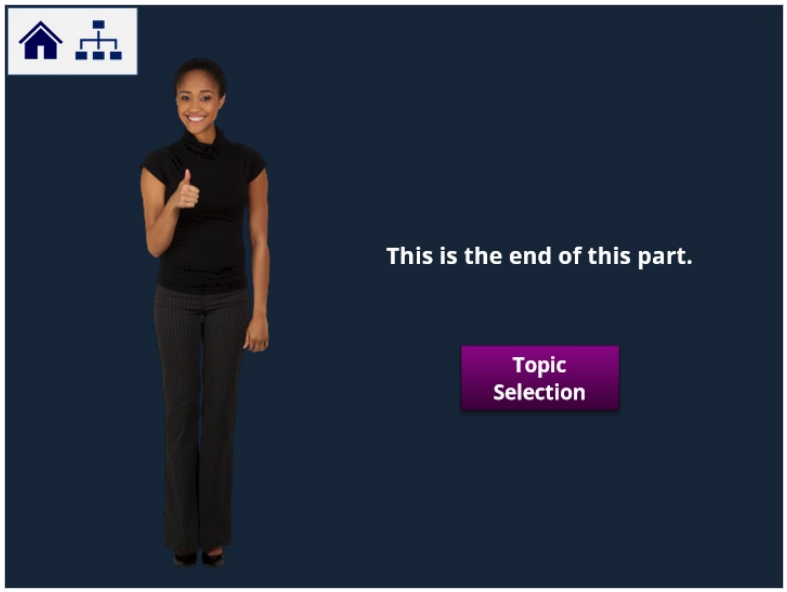
6.11 Bookend: Data Activity Solutions



6.12 End of Section

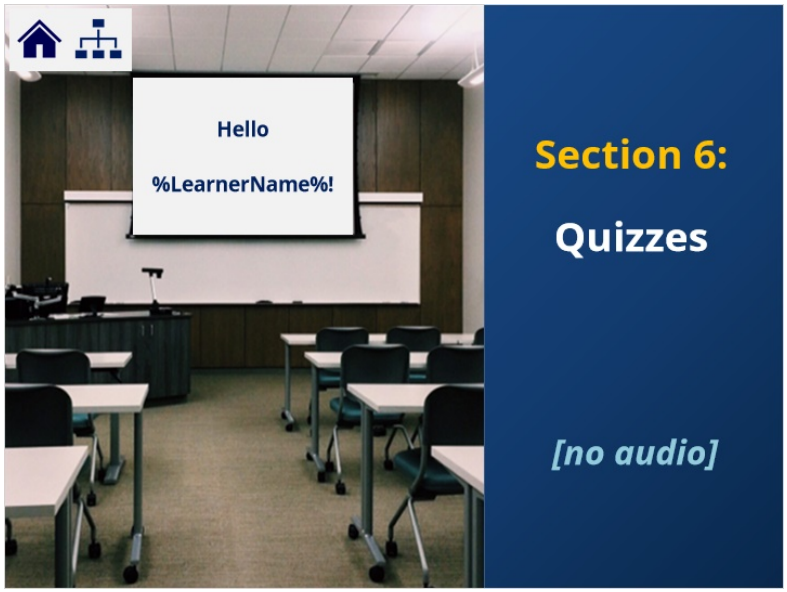


6.13 Bookend: Data Activity



7. Quizzes

7.1 Cover: Quizzes



7.2 Quiz Selection