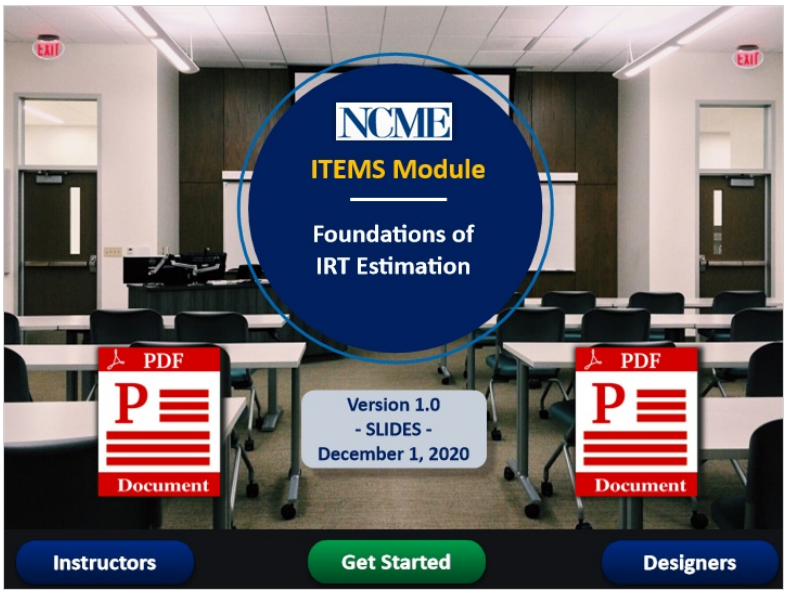
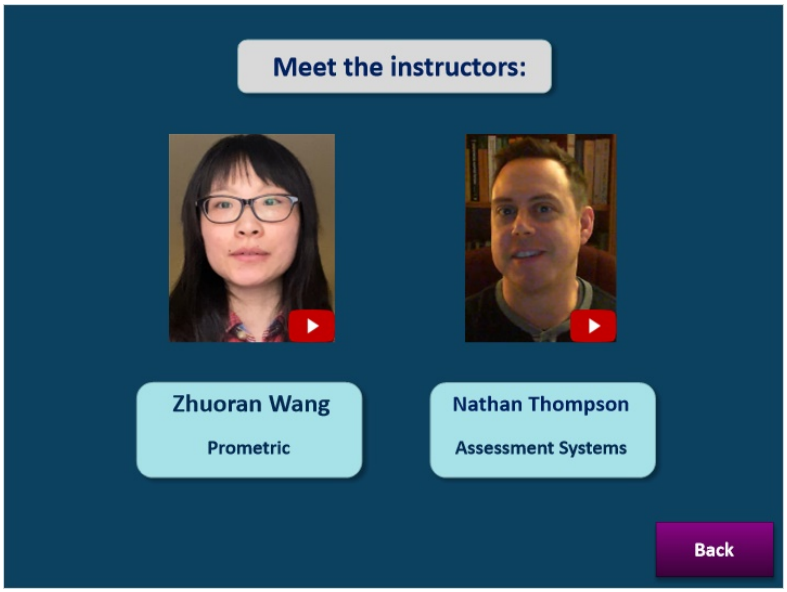
DM19 SLIDES (IRT Estimation, Version 1.0)

1. Module Overview

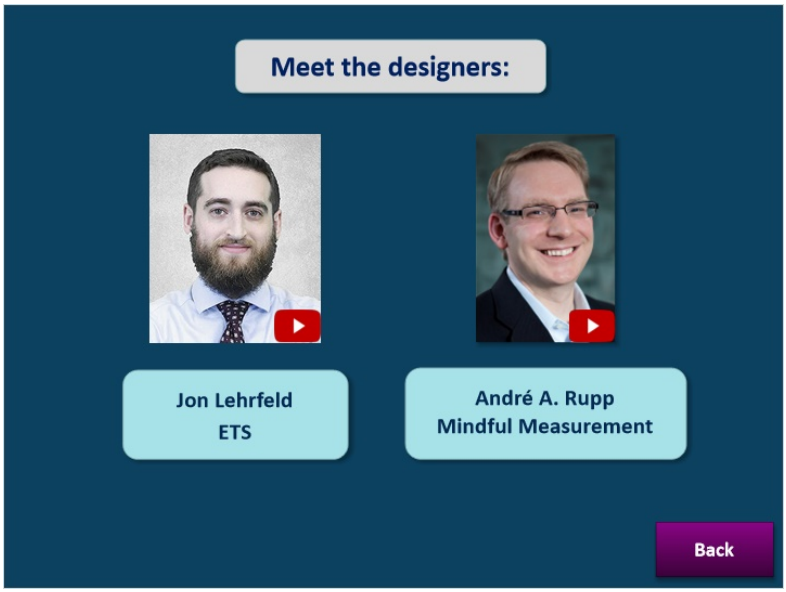
1.1 Module Cover (START)



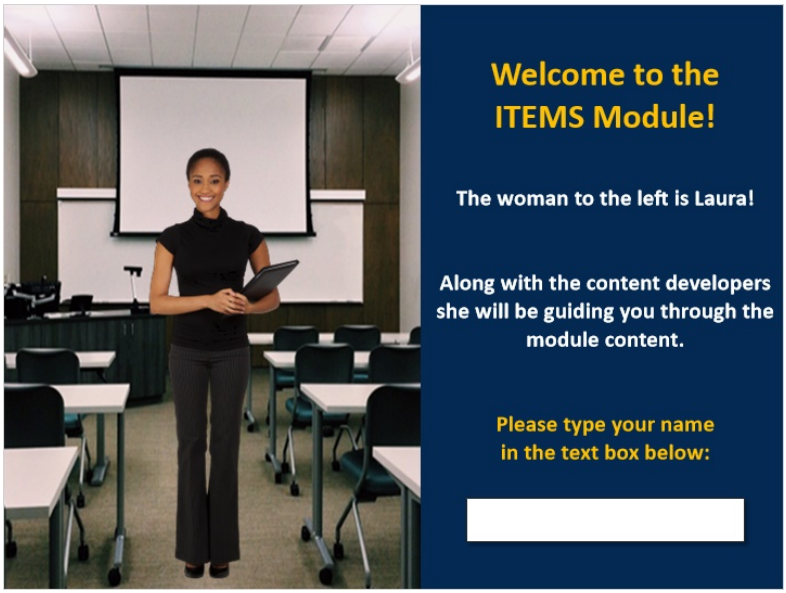
1.2 Instructors



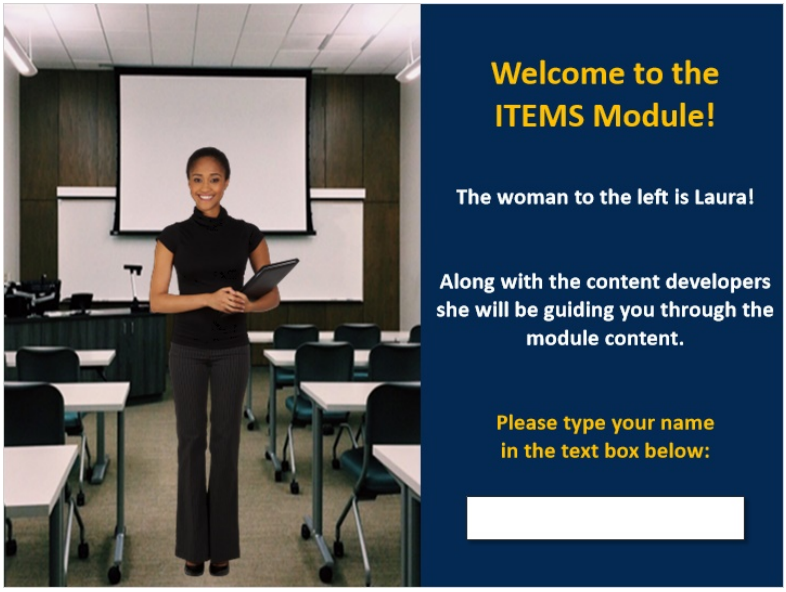
1.3 Designers



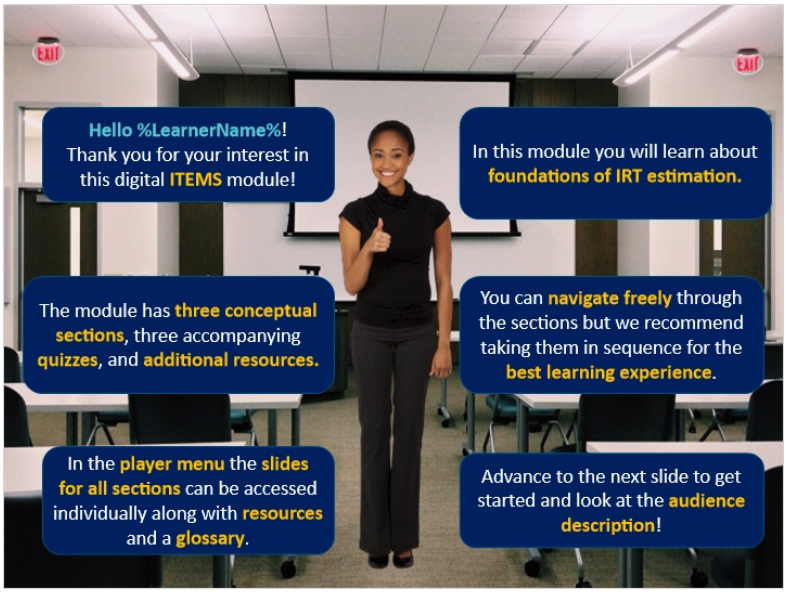
1.4 Welcome



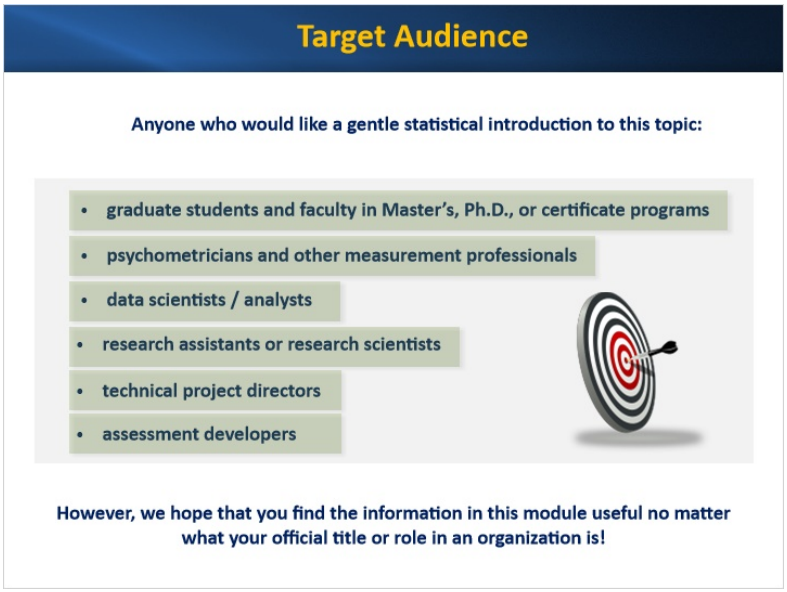
Untitled Layer 1 (Slide Layer)



1.5 Overview

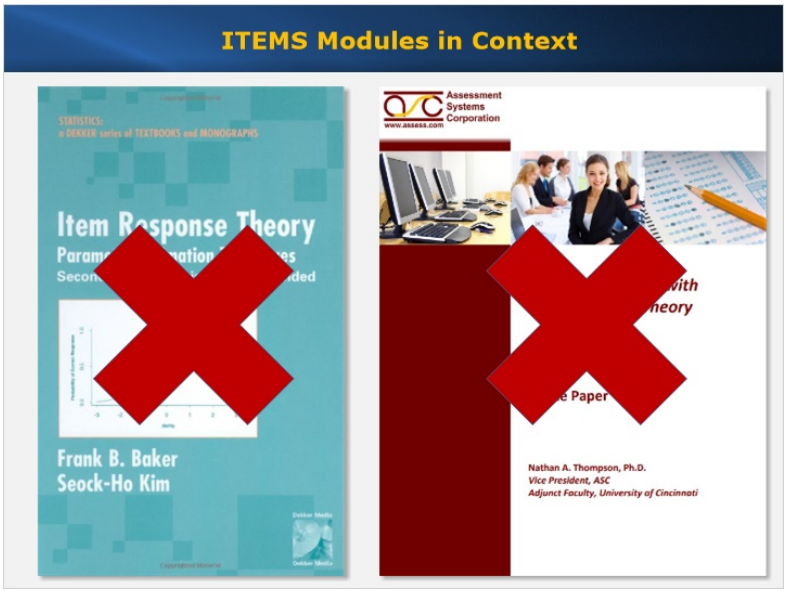


1.6 Target Audience

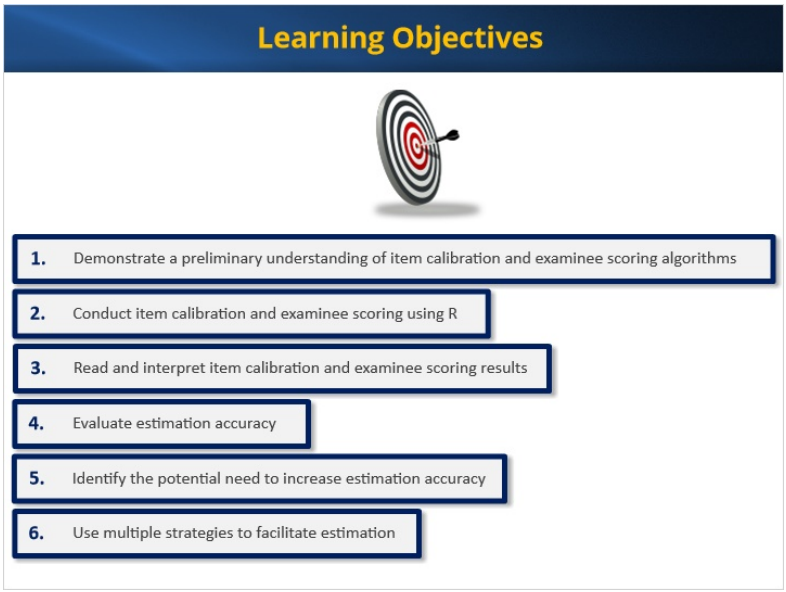


1.7 Expecations (I)

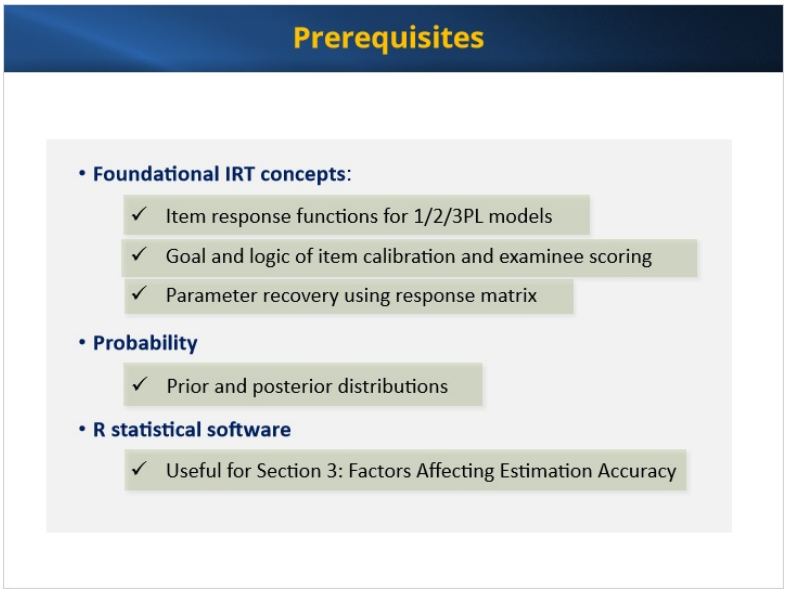
1.8 Expectations (II)



1.9 Learning Objectives



1.10 Prerequisites



1.11 Resources

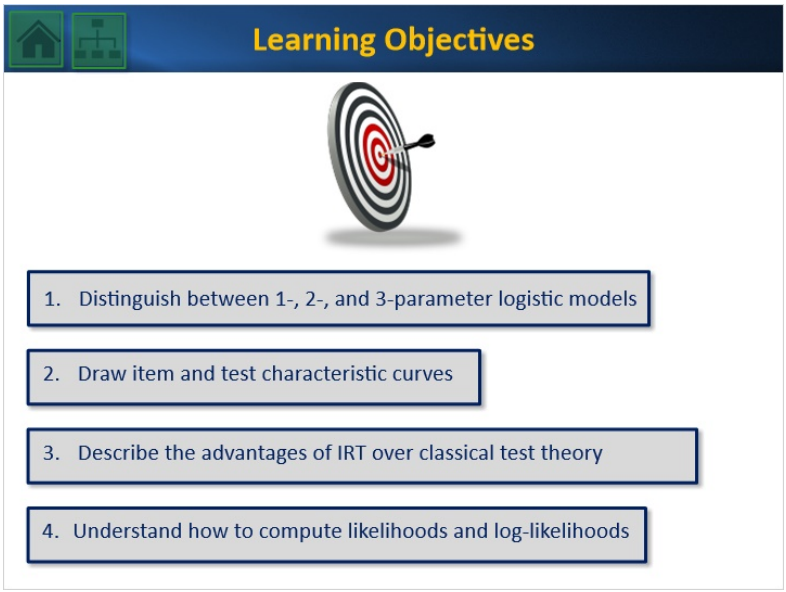
References (Slide Layer)

1.12 Main Menu

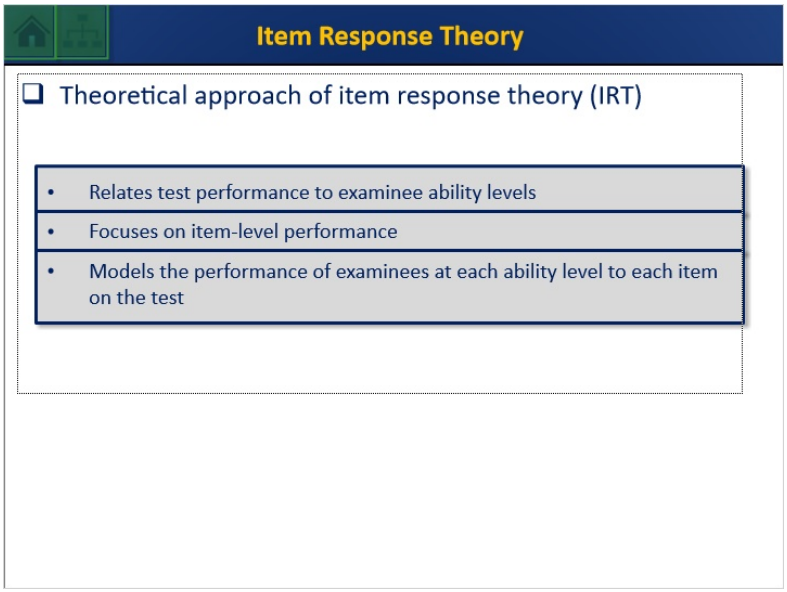
2. Section 1: IRT Basics

2.1 Cover: IRT Basics

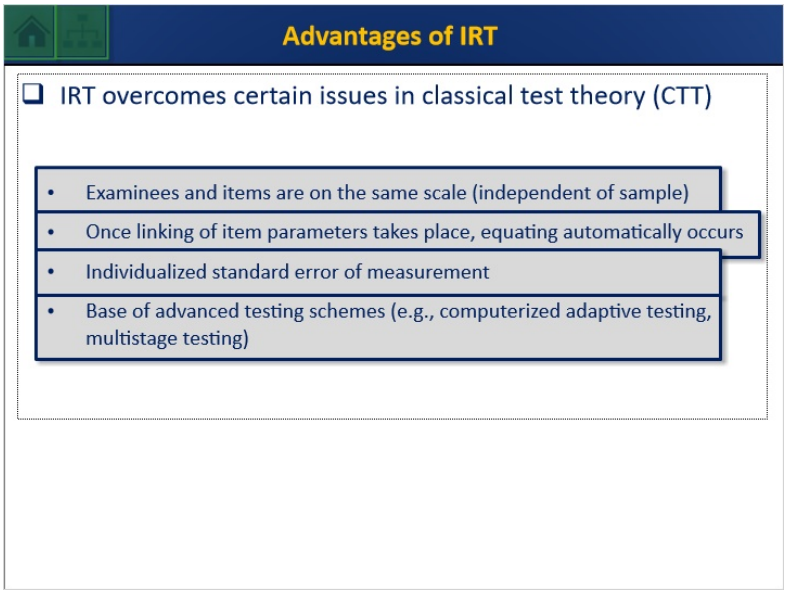
2.2 IRT Basics: Learning Objectives



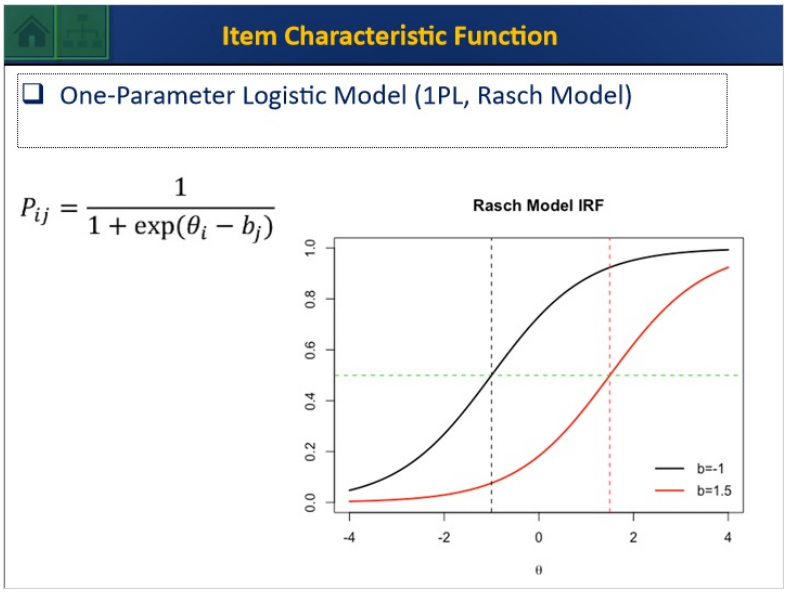
2.3 Item Response Theory



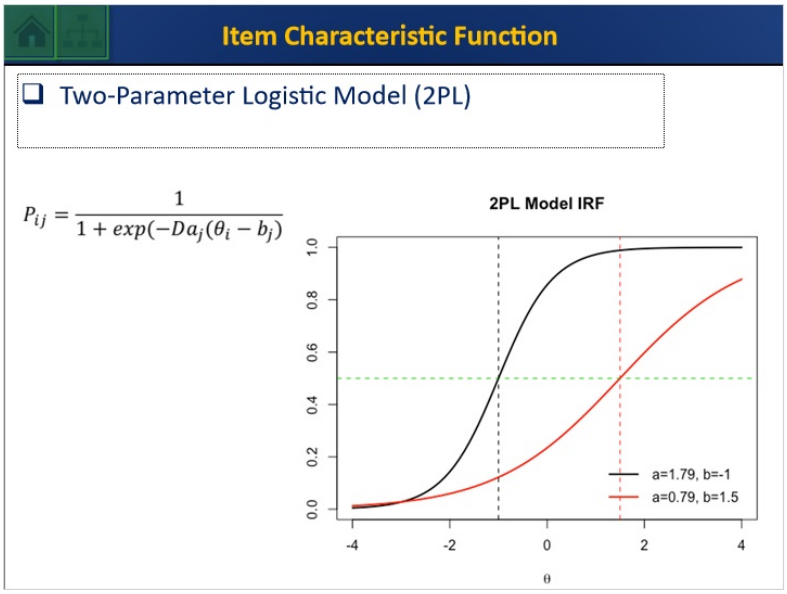
2.4 Advantages of IRT



2.5 Item Characteristic Functions I



2.6 Item Characteristic Functions II

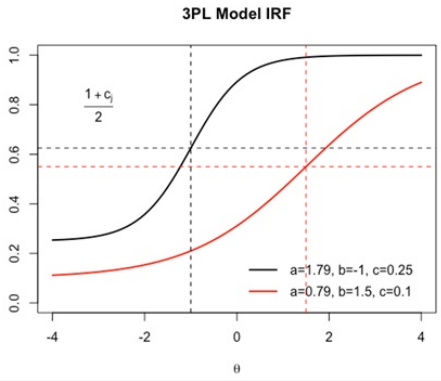


2.7 Item Characteristic Functions III

Item Characteristic Function

- Three-Parameter Logistic Model (3PL)

$$P_{ij} = c + \frac{1-c}{1+\exp(-Da_j(\theta_i - b_j)}$$



2.8 Simulation Example

Simulation Example

- 2PL Example
  - 5 items
  - a~Lognormal(0,1)
  - b~Normal(0,1)
  - θ~Normal(0,1)
  - Response for examinee #1 is (0, 1, 0, 0, 1) ["Mixed" response]

2.9 Item Information

Item Information

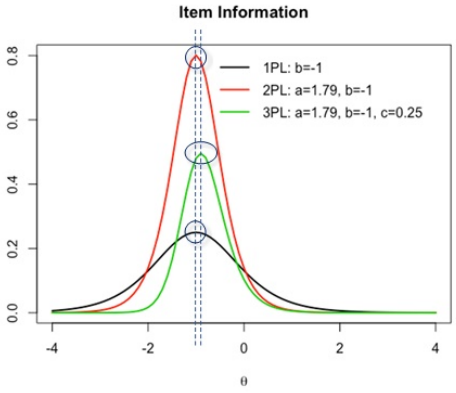
- IRT advances the concept of item and test information to replace reliability.

1PL: $I_{ij} = P_{ij}Q_{ij}$

2PL: $I_{ij} = a_j^2 P_{ij}Q_{ij}$

3PL: $I_{ij} = a_j^2 \frac{(P_{ij}-c_j)^2 Q_{ij}}{(1-c_j)^2 P_{ij}}$

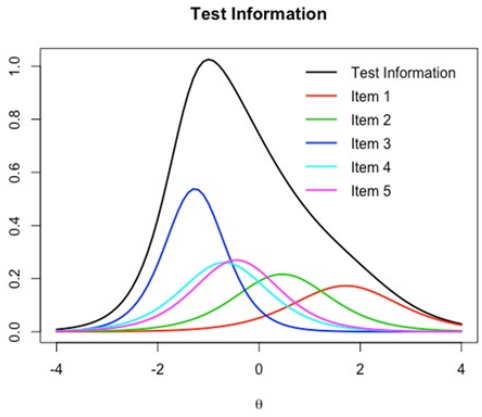
where $Q_{ij} = 1 - P_{ij}$



2.10 Test Information

Test Information

- Test information is the sum of all item information

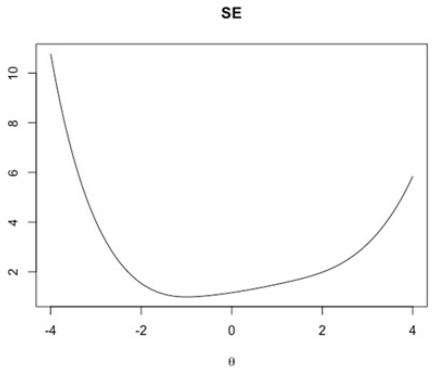


2.11 Standard Error of Measurement

Standard Error of Measurement

- Standard error of measurement (SE) is the reciprocal of the test information at a given ability level.

$$SE_i = \frac{1}{\sqrt{I_i}}$$

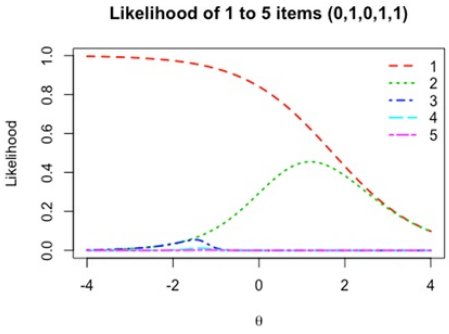


2.12 Likelihood

Likelihood

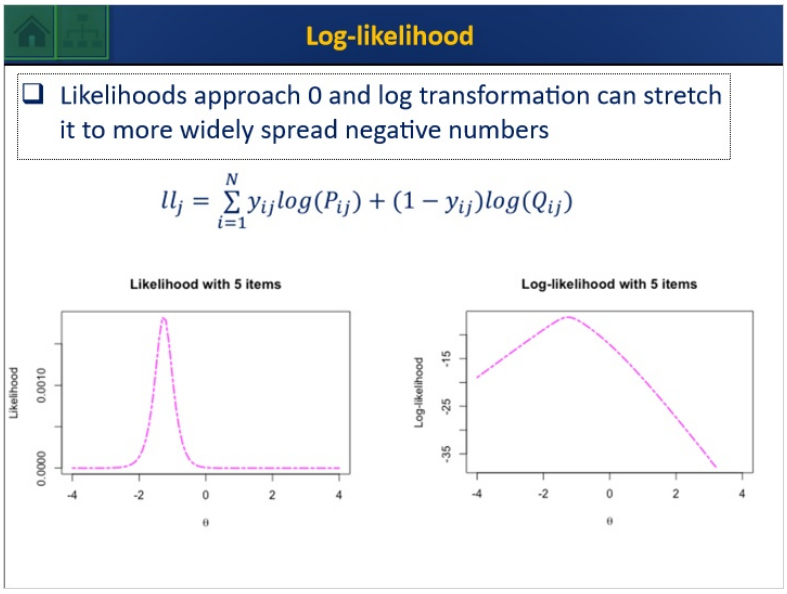
- Likelihood is the plausibility of a response pattern

$$L_i = \prod_{j=1}^{J} P_{ij}^{y_{ij}} Q_{ij}^{1-y_{ij}} = Q_{i1} \times P_{i2} \times Q_{i3} \times P_{i4} \times P_{i5}$$

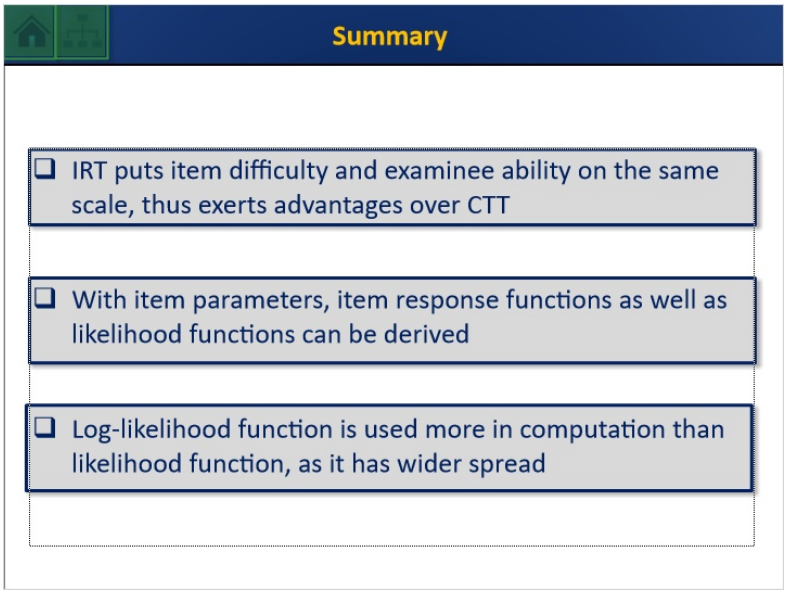


where y_{ij} is examinee i's response on item j

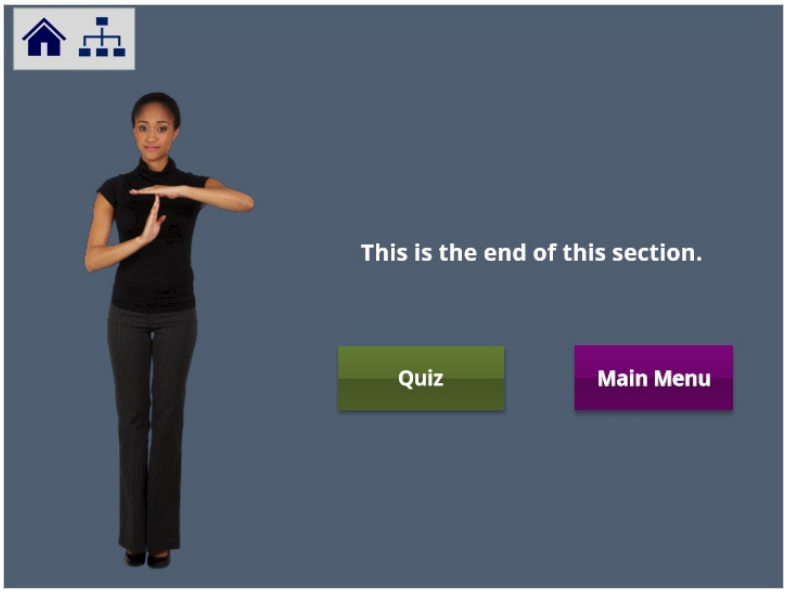
2.13 Log-Likelihood



2.14 Summary



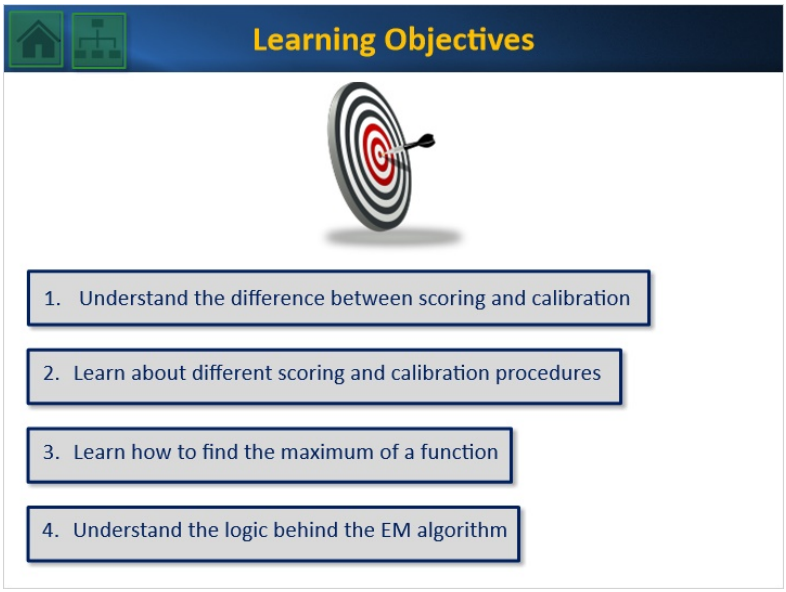
2.15 Bookend: IRT Basics



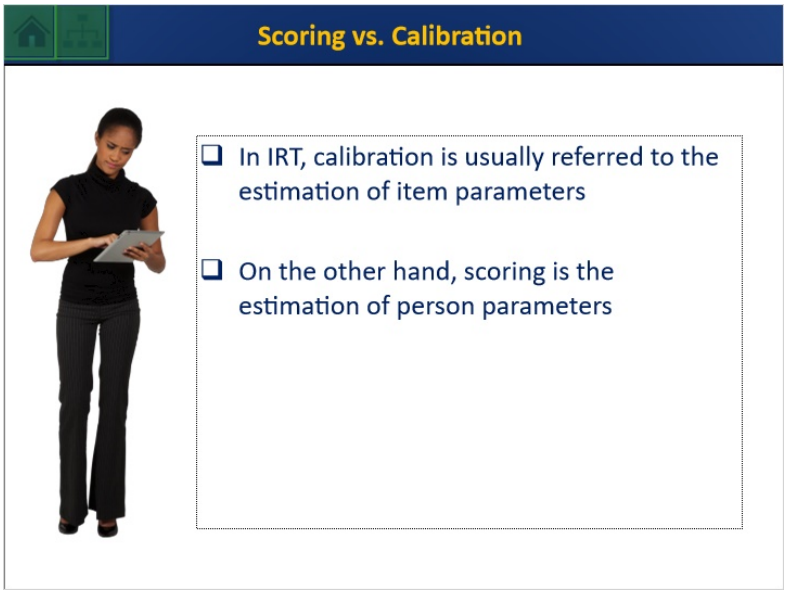
3. Section 2: Calibration and Scoring

3.1 Cover: Calibration and Scoring

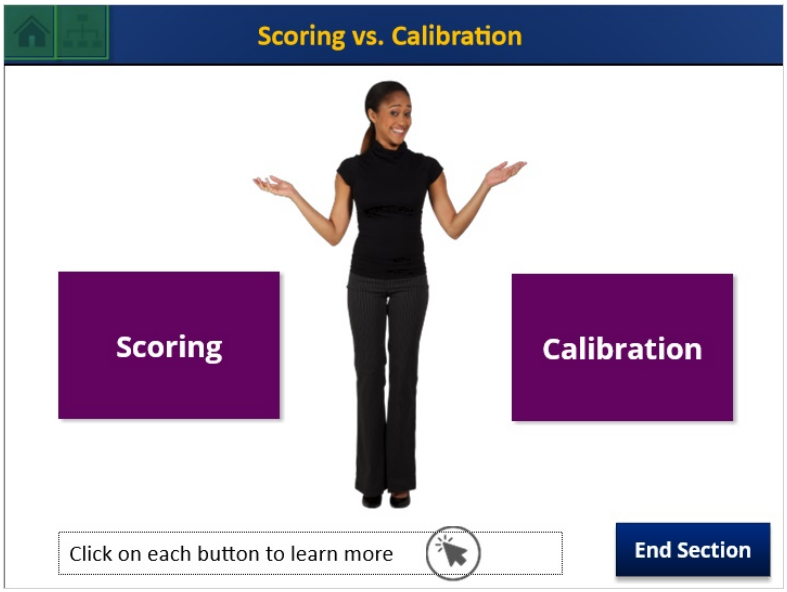
3.2 Estimation Algorithms: Learning Objectives



3.3 Scoring vs. Calibration I

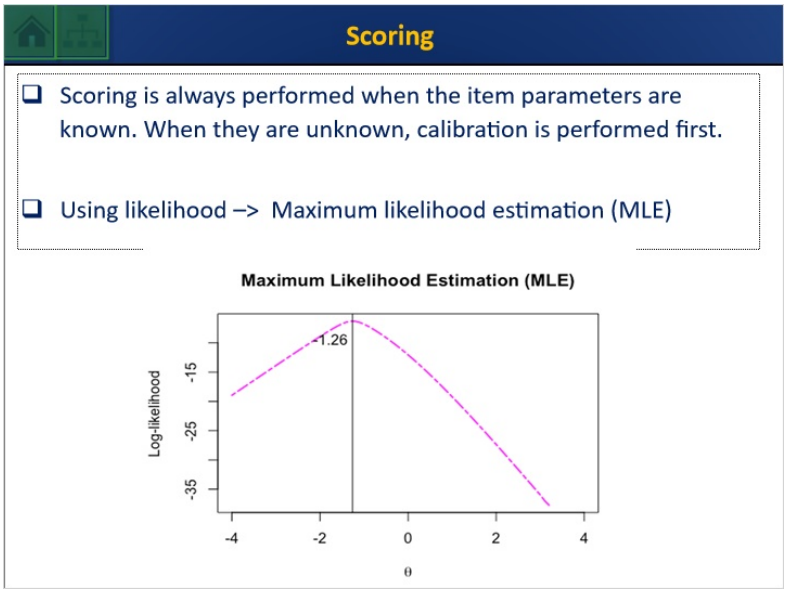


3.4 Topic Selection

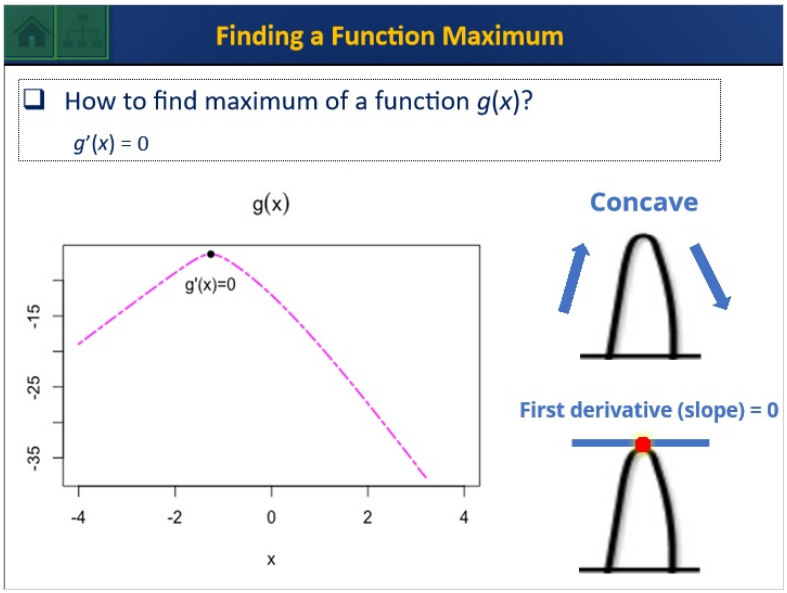


3.5 Bookmark: Scoring

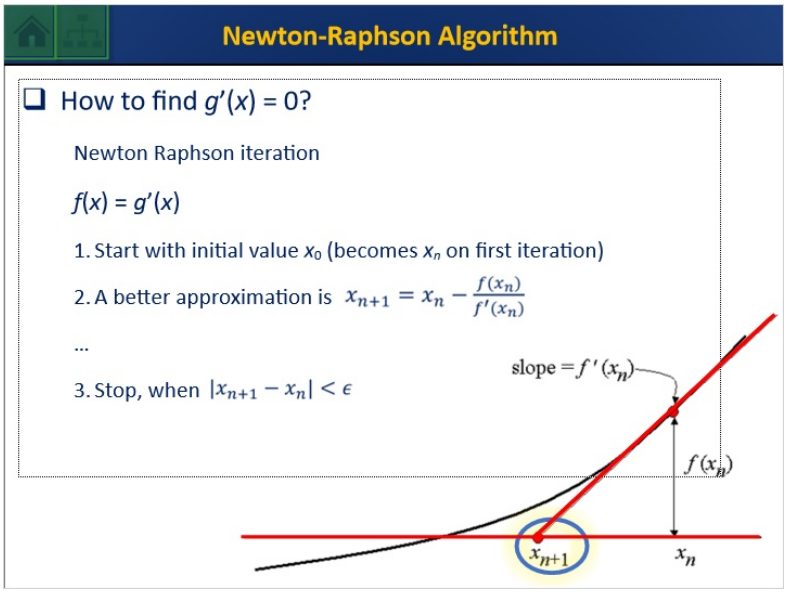
3.6 Scoring: MLE



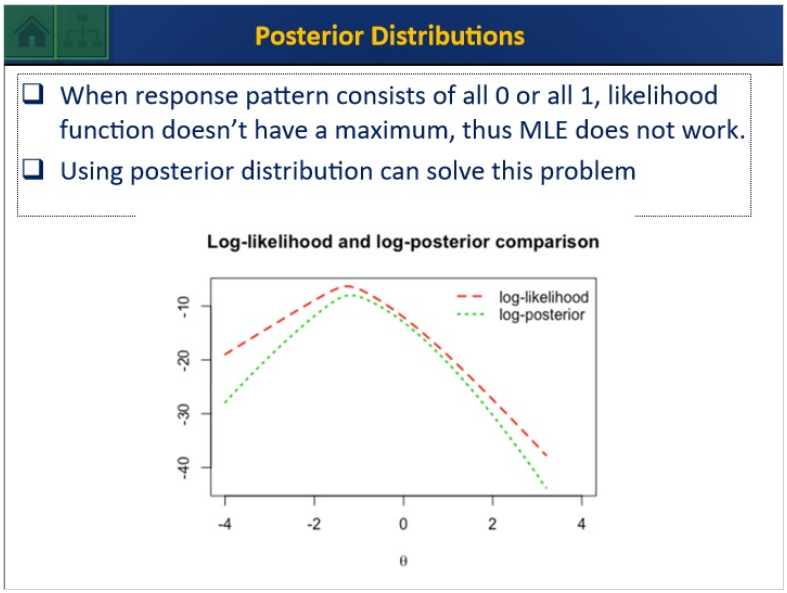
3.7 Finding a Function Maximum



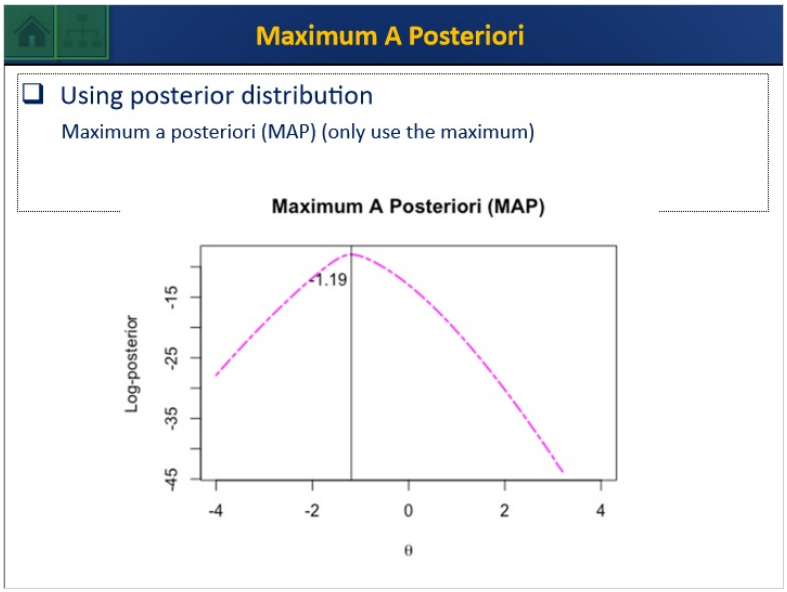
3.8 Newton-Raphson Algorithm



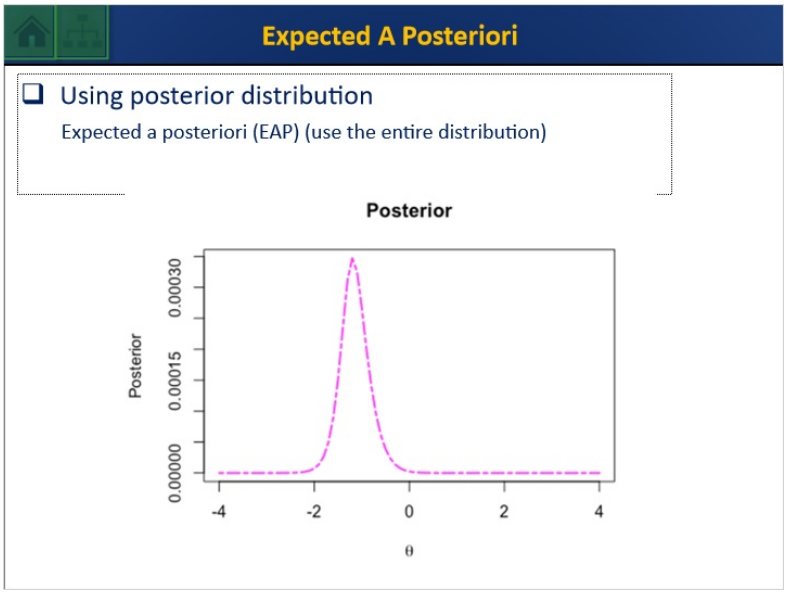
3.9 Posterior Distributions



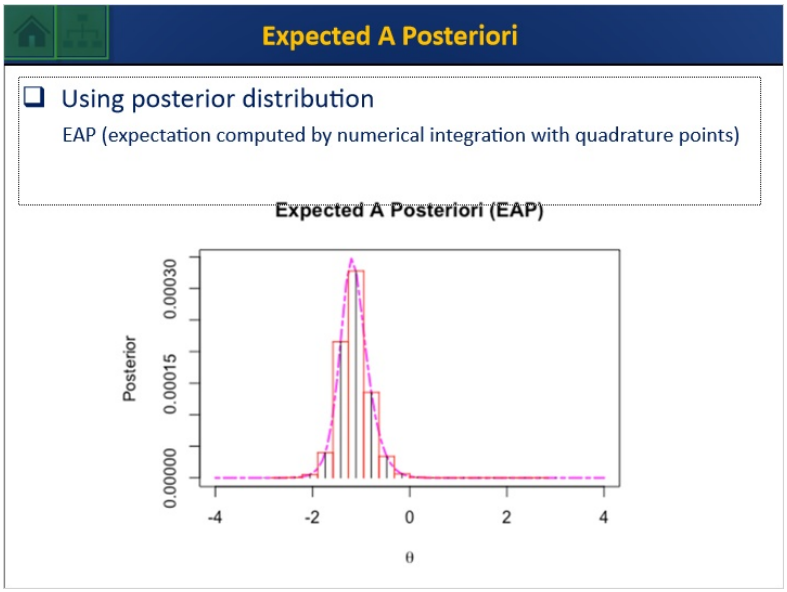
3.10 Maximum A Posteriori



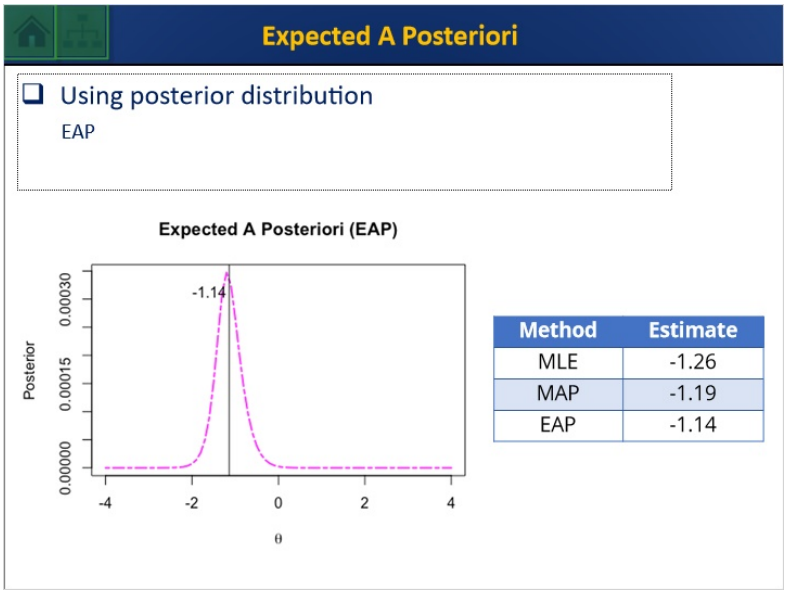
3.11 Expected A Posteriori I



3.12 Expected A Posteriori II

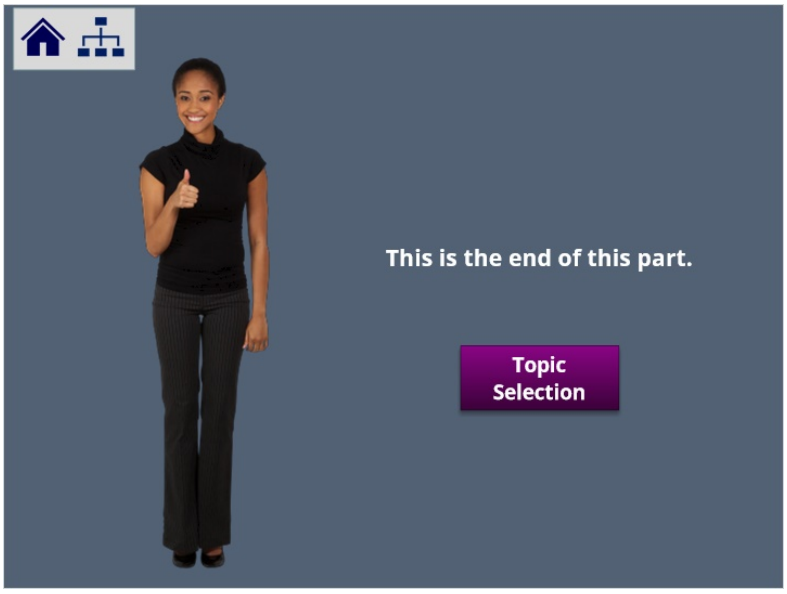


3.13 Expected A Posteriori III



Method	Estimate
MLE	-1.26
MAP	-1.19
EAP	-1.14

3.14 Bookend: Scoring



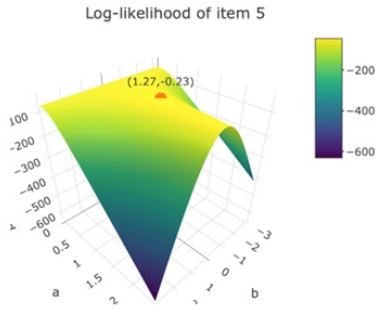
3.15 Bookmark: Calibration

3.16 Calibration with Unknown Item Parameters

Calibration with Known Examinee Parameters

- If examinee parameters are known, MLE is used

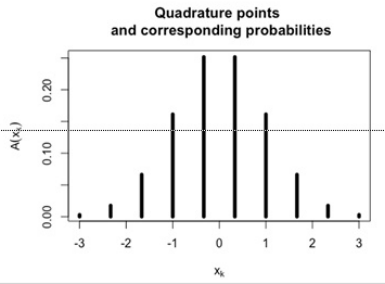
$$ll_j = \sum_{i=1}^{N} y_{ij} log(P_{ij}) + (1 - y_{ij}) log(Q_{ij})$$



3.17 Calibration with Unknown Item and Person Parameters

Calibration with Unknown Item and Person Parameters

- Marginal maximum likelihood estimation (MMLE)
  - Although examinee parameters are unknown, we can assume a known population distribution and replace the person parameters with quadrature points.
  - x_k are quadrature points and $A(x_k)$ are corresponding quadrature weights/ probabilities under the population distribution
  - The quadrature weights sum to 1



3.18 MMLE Log-Likelihood

MMLE Log-Likelihood

- MMLE
  - Replace single response y_{ij} with $\bar{r}_{kj}$ which is number of correct answer in group *k*; replace 1–y_{ij} with $\bar{n}_k - \bar{r}_{kj}$, where $\bar{n}_k$ is the size of group *k*.
  - $\bar{n}_k$ and $\bar{r}_{kj}$ are called artificial data
  - The log-likelihood function for item *j* becomes:

$$ll_j = \sum_{i=1}^{N} \bar{r}_{kj} log(P_{kj}) + (\bar{n}_k - \bar{r}_{kj}) log(Q_{kj})$$

3.19 MMLE Log-Likelihood II

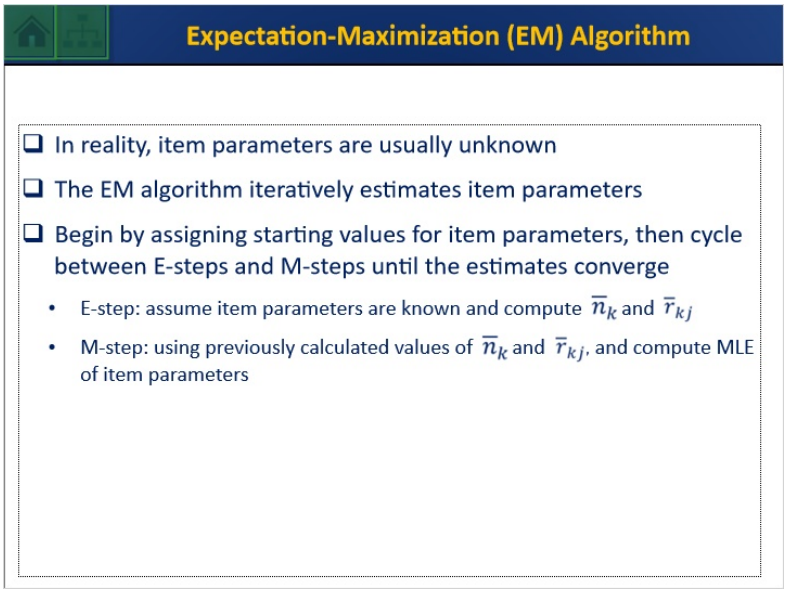
MMLE Log-Likelihood

- MMLE
  - $L_{ik} = \prod_{j=1}^{J} P_{kj}^{y_{ij}} Q_{kj}^{1-y_{ij}}$ is the likelihood of a examinee *i* with ability x_k
  - Compute $\bar{n}_k$ and $\bar{r}_{kj}$ based on known item parameters and responses

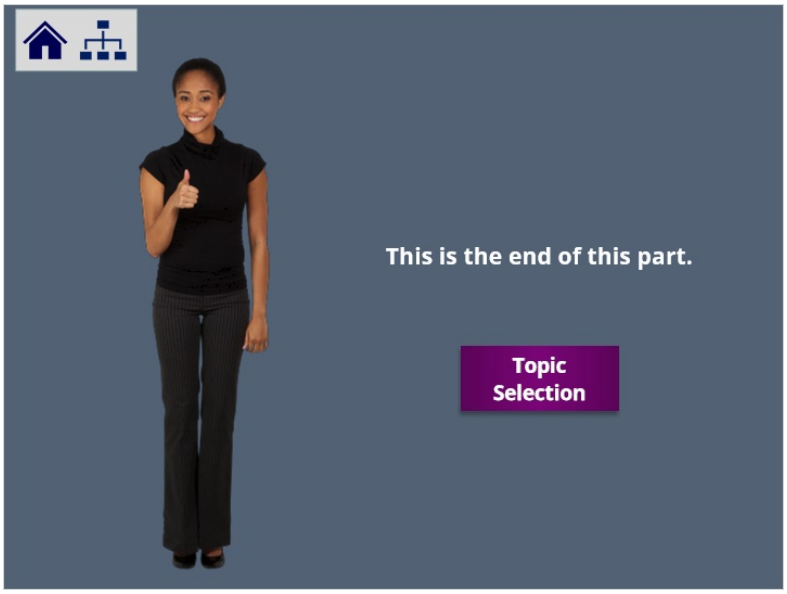
$$\bar{n}_k = \sum_{i=1}^{N} \frac{L_{ik} A(x_k)}{\sum_{k=1}^{K} L_{ik} A(x_k)}$$

$$\bar{r}_{jk} = \sum_{i=1}^{N} \frac{y_{ij} L_{ik} A(x_k)}{\sum_{k=1}^{K} L_{ik} A(x_k)}$$

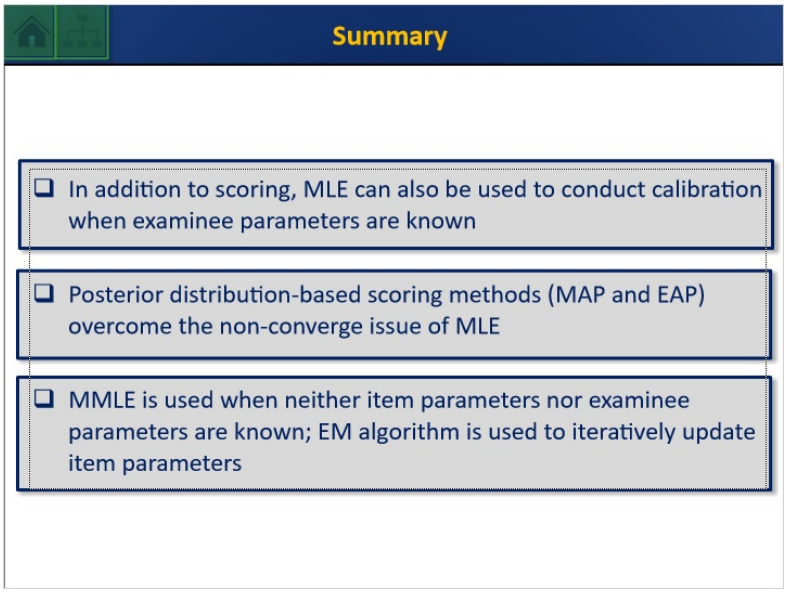
3.20 EM Algorithm



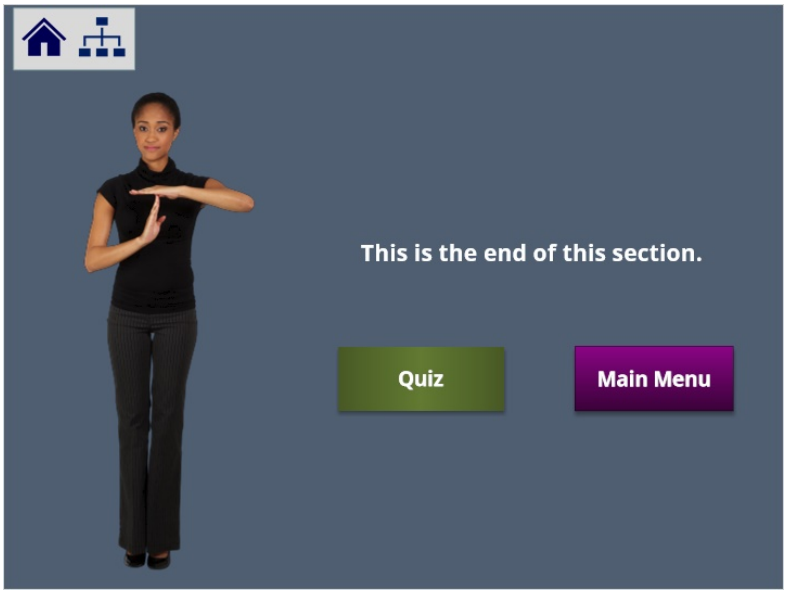
3.21 Bookend: Calibration



3.22 Summary

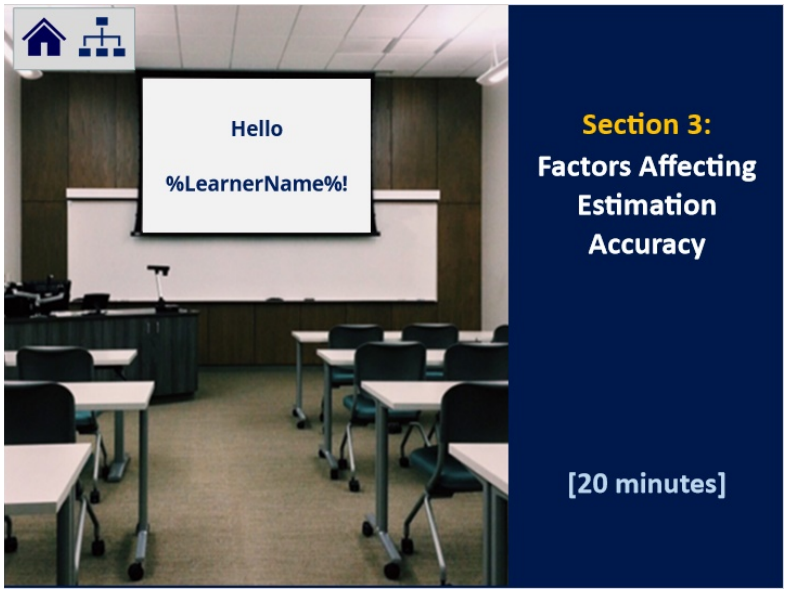


3.23 Bookend: IRT Basics

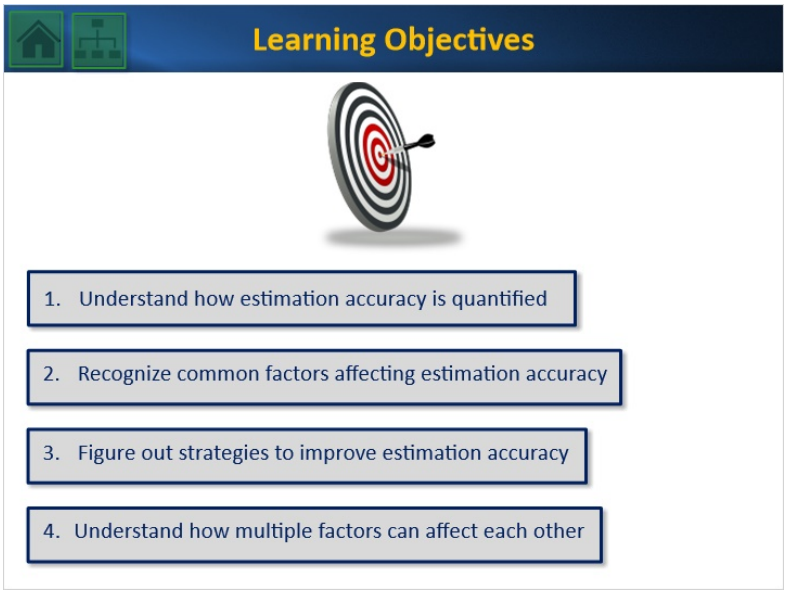


4. Section 3: Factors Affecting Estimation Accuracy

4.1 Cover: Factors Affecting Estimation Accuracy



4.2 Factors Affecting Estimation Accuracy: Learning Objectives



4.3 Estimation Accuracy Evaluation Criteria

Estimation Accuracy Evaluation Criteria

- Bias and root mean square error (RMSE) are used to evaluate the estimation accurac of calibration and scoring
- π is the true value of a parameter and $\hat{\pi}$ is the estimated value of this parameter

Bias:

$$\frac{\sum_{i=1}^{n}(\hat{\pi}_i - \pi_i)}{n}$$

RMSE:

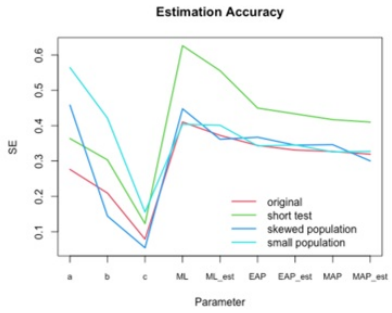
$$\sqrt{\frac{\sum_{i=1}^{n}(\hat{\pi}_i - \pi_i)^2}{n}}$$

- The *proportional* bias and RMSE are computed as the proportion of bias and RMSE of a parameter over the SD of that parameter.

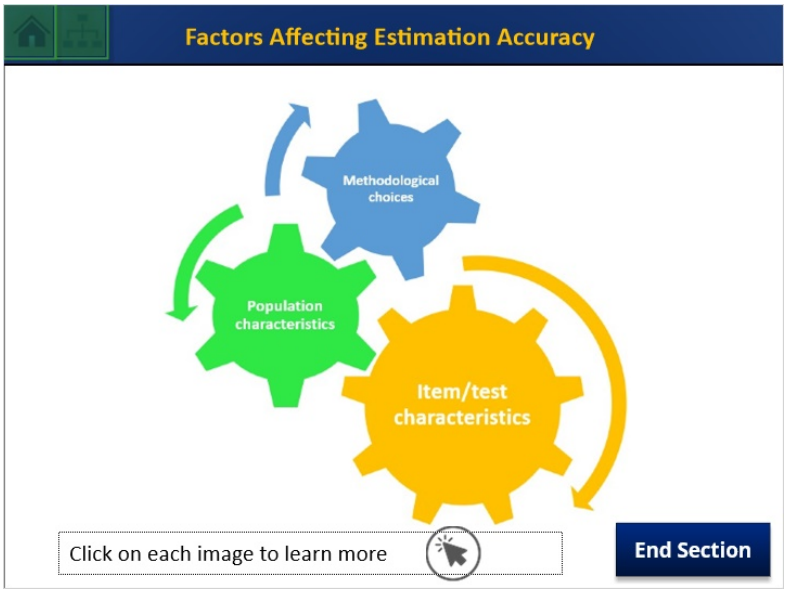
4.4 Identify the Need to Improve Estimation Accuracy

Identify the Need to Improve Estimation Accuracy

- How to identify the need of improving estimation accuracy?
  - Look at population size.
    - 500 for 2PL
    - 1000 for 3PL
  - Look at SE of parameter estimation (especially *a* parameter)

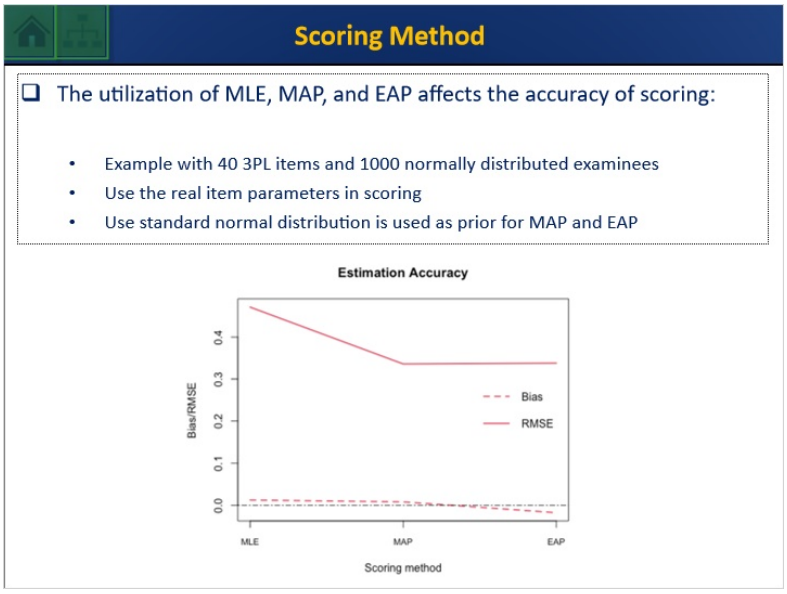


4.5 Topic Selection

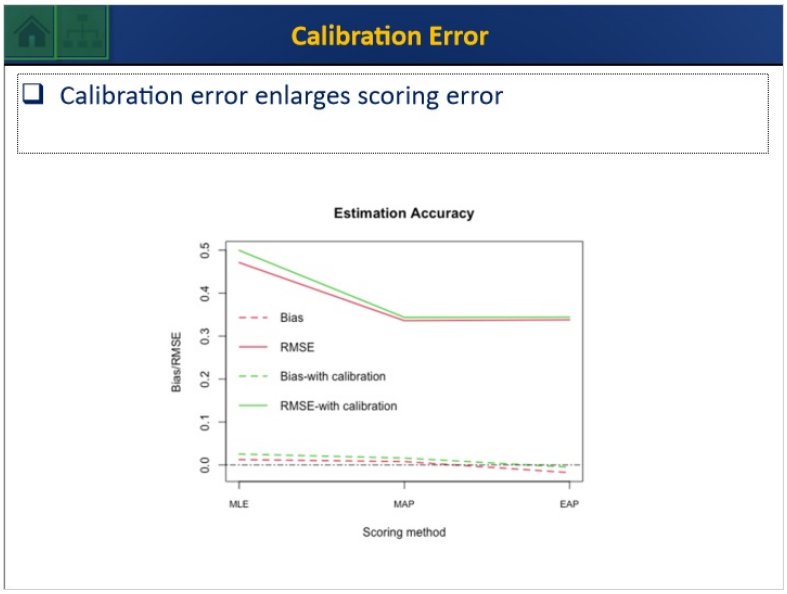


4.6 Bookmark: Methodological Choices

4.7 Scoring Method



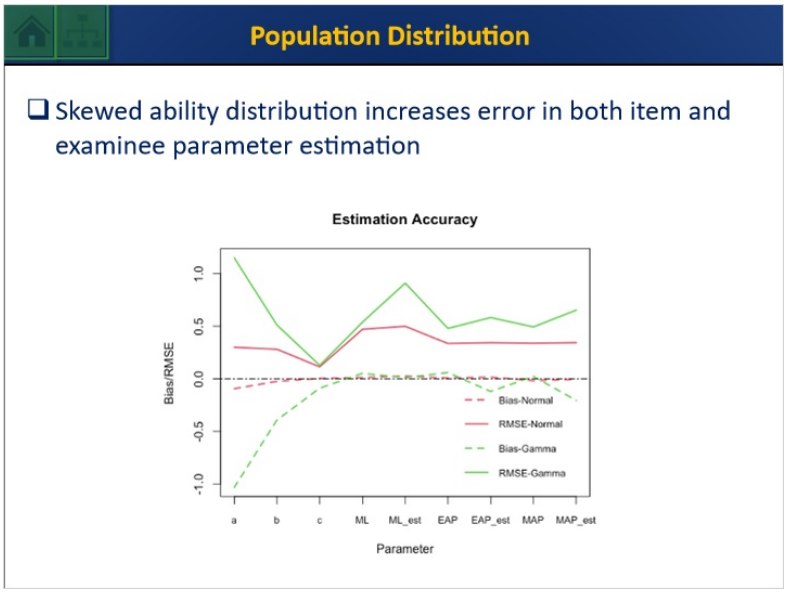
4.8 Calibration Error



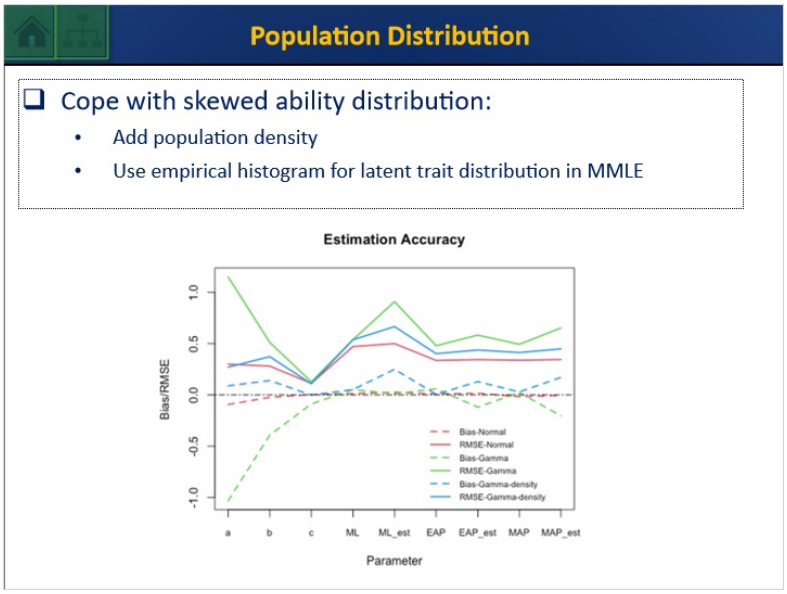
4.9 Bookend: Calibration

4.10 Bookmark: Population Characteristics

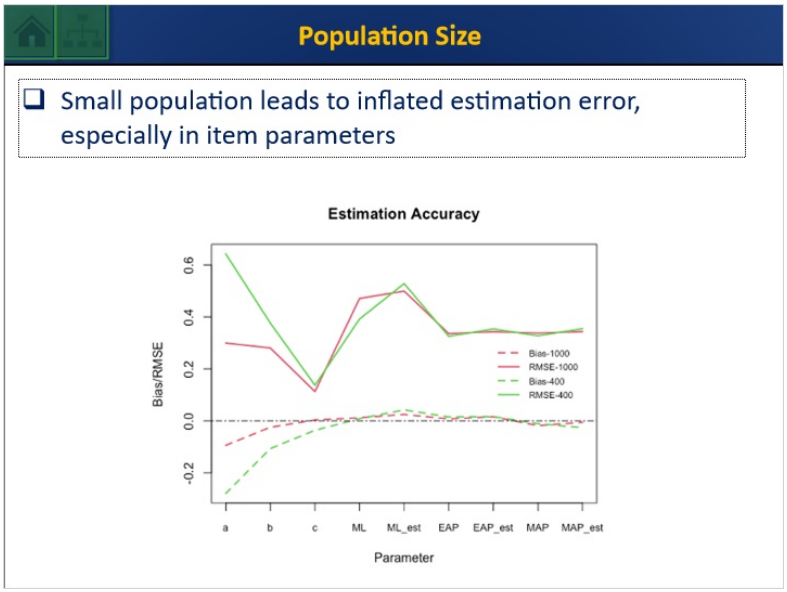
4.11 Population Distribution I



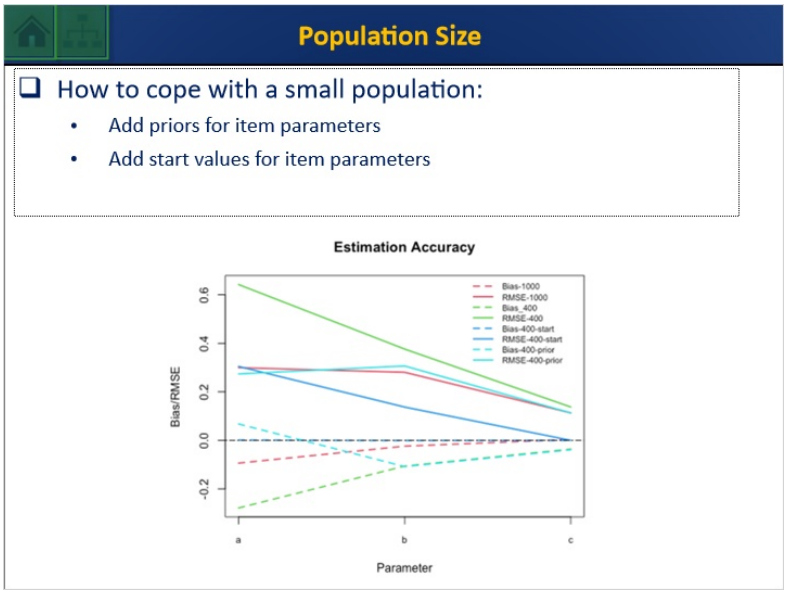
4.12 Population Distribution II



4.13 Population Size I



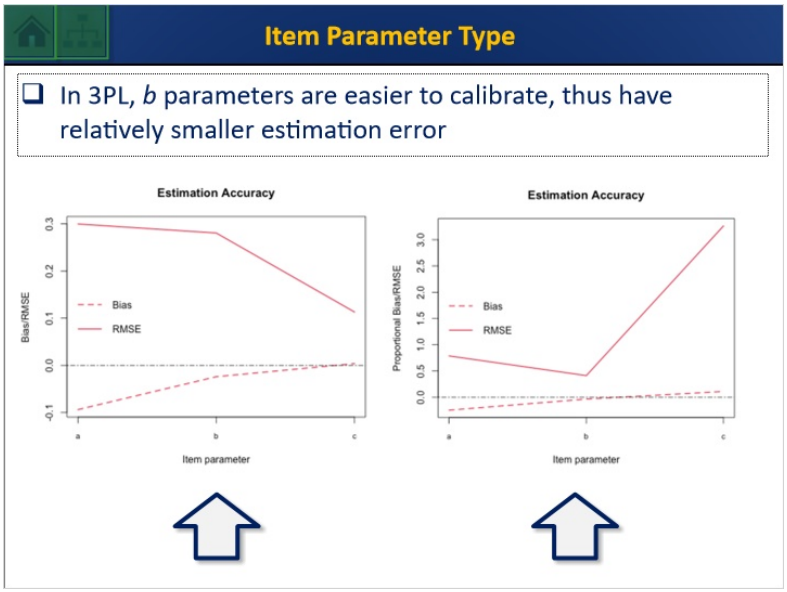
4.14 Population Size II



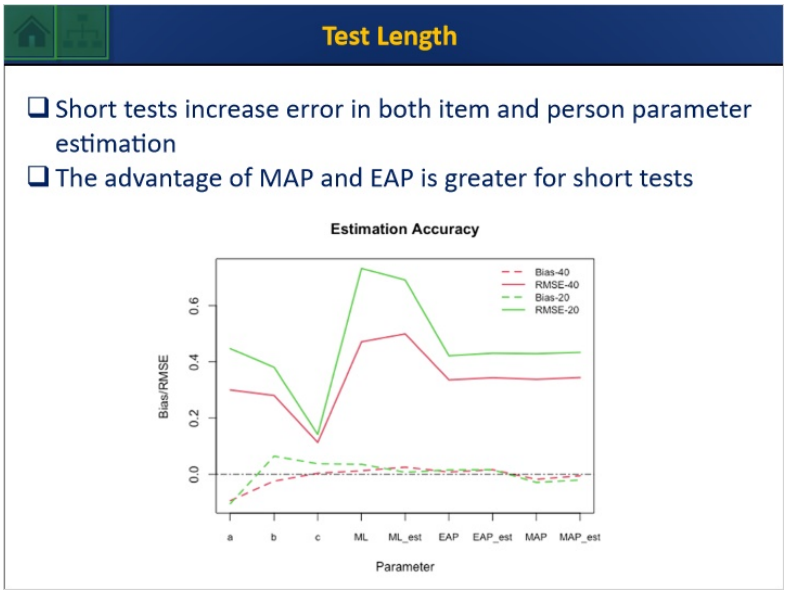
4.15 Bookend: Calibration

4.16 Bookmark: Item and Test Characteristics

4.17 Item Parameter Types

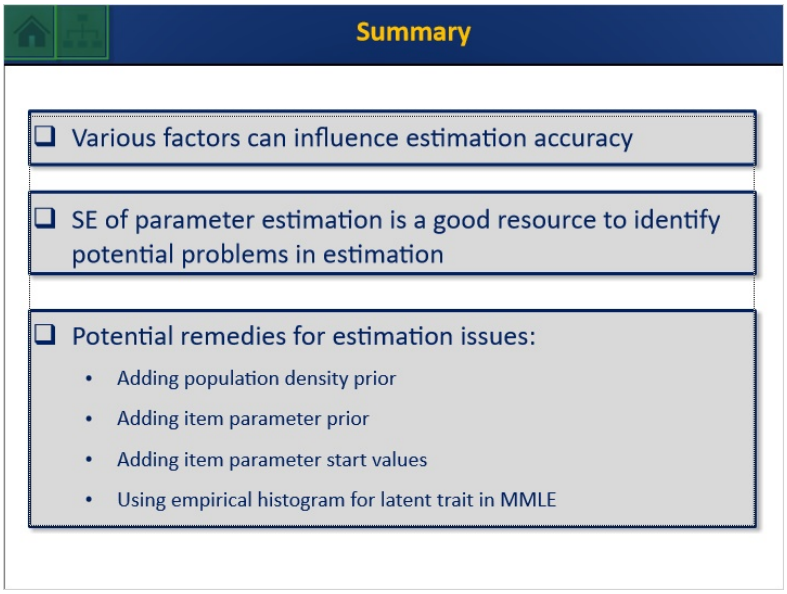


4.18 Test Length

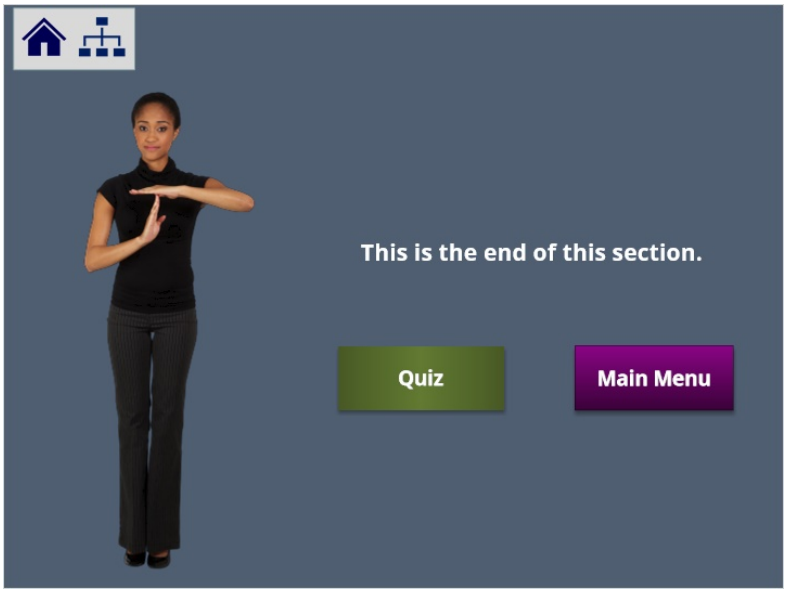


4.19 Bookend: Calibration

4.20 Summary



4.21 Bookend: IRT Basics



4.22 Module Cover (START)