INDUSTRIAL STRENGTH COMPASS FOR OPTIMIZATION VIA SIMULATION

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ABSTRACT

Industrial Strength COMPASS (ISC) is a specific implementation of a general framework for solving optimization via simulation problems over integer-ordered decision variables. ISC can handle problems of realistic size and complexity, while providing well-defined convergence guarantees. We describe ISC and provide a link to the supporting theory and algorithms.

1 INTRODUCTION

The popularity of stochastic simulation for system design and analysis has been driven by a sequence of key advances: Implementation of intuitive process-interaction (network) modeling paradigms; the development of graphical user interfaces for model development; convenient animation of simulation results; interapplication communication between simulation and other software; and integrated toolkits for optimization of simulated system performance (see Nance and Sargent 2002 for a more comprehensive overview of the evolution of discrete-event, stochastic simulation). Only this last advance—which we call *optimization via simulation (OvS)*—is in analysis capability (analysis methods have certainly been incorporated into simulation software, but are probably not responsible for its popularity). Although not every simulation problem requires optimization, it is rare to find an application where the analyst is uninterested in "the best" settings for the simulated system, and in many cases finding a good system design is the reason the simulation was constructed.

2 BACKGROUND

As noted by a number of authors (e.g., Fu 2002), there has been a disconnect between *research* on OvS and the *practice* of OvS, as represented by the commercial OvS products. Simply stated, the impact of published research on commercial software has been limited. Nevertheless, commercial OvS software has been successful because (a) there was and is a substantial market for actually doing OvS in practice, (b) the products are able to handle realistic problems with complex objectives and constraints, while delivering results in a timely manner, and (c) they are integrated into simulation modeling software. The research community, on the other hand, has focused on OvS *correctness*, as quantified by convergence or "correct selection" guarantees. These properties are easiest to prove (which is not to say easy to prove) for simple, elegant algorithms that leave a host of implementation issues unresolved (e.g., incorporating complex constraints and providing stopping rules). Correctness, in this formal sense, is not a feature of the commercial products, but correctness matters because in the presence of stochastic noise it is possible that neither the true quality of the selected solution nor the estimate of how it will actually perform may be acceptable without them.

In the last decade there has been significant research activity aimed at bridging this divide, and we believe it has reached a level of maturity that supports a first step toward developing OvS software that offers correctness guarantees while also being competitive with the features provided by commercial products. This talk will report on one such step, which we call *Industrial Strength COMPASS (ISC)*. The name is derived from the Convergent Optimization via Most Promising Area Stochastic Search algorithm of Hong and Nelson (2006), which is the core of ISC. ISC is a specific instance of a high-level framework for OvS algorithms that consists of three phases: Global, Local and Clean Up. While we are guilty of drawing heavily on our own research in turning this framework into a specific algorithm, our approach is also strongly influenced by the foundational work of Andradóttir (1995, 1999).

3 ISC

We are interested in solving the following problem:

$$\text{Minimize} \quad g(\mathbf{x}) = \mathrm{E}\left[G(\mathbf{x})\right] \tag{1}$$

subject to $\mathbf{x} \in \Theta = \Phi \cap \mathcal{Z}^d$, where $\mathbf{x}$ is a vector of d integer-ordered decision variables in a feasible region $\Phi \subset \Re^d$, possibly defined by a set of constraints. We assume that Φ is compact and convex, and that $|\Theta| < \infty$ (but probably quite large). The random variable $G(\mathbf{x})$ typically has no closed form, but can be observed through simulation experiments at $\mathbf{x}$. We assume that $\mathrm{Var}[G(\mathbf{x})] < \infty$ for all $\mathbf{x} \in \Theta$, and that we can simulate independent and identically distributed replications, $G_1(\mathbf{x}), G_2(\mathbf{x}), \ldots$ at any $\mathbf{x}$. Problem (1) is called a discrete optimization-via-simulation (DOvS) problem, and we refer to any $\mathbf{x}$ as a potential "solution." DOvS problems arise in many areas of operations research and management sciences. For instance, the following problems can all be modeled as DOvS problems: capacity planning, where the capacities of all workstations need to be determined; call center staffing, where the agents are allocated to different departments and different time periods; and supply-chain management, where inventory levels are critical decisions.

The algorithm framework includes three phases: Global, Local and Clean Up. The Global Phase explores the whole feasible region and identifies several good solution seeds; the Local Phase takes one seed at a time and finds a locally optimal solution; and the Clean-Up Phase selects the best from the set of solutions identified in the Local Phase and also estimates its expected performance.

In the Global Phase, we want to quickly identify a number of solution seeds that may lead to competitive locally optimal solutions in the second phase, and also facilitate a quick start for the local search of the second phase. To insure that the algorithm used in this phase has good large-sample properties, we require it to be globally convergent if the simulation effort of this phase goes to infinity.

Although the algorithm of the first phase should be globally convergent, it will transition to the second phase in practice. The transition rules can be effort based or quality based. An effort-based rule transitions from the first phase to the second phase after the simulation budget of the first phase is consumed; a quality-based rule transitions if it is clear that the seeds can lead to high-quality solutions in the second phase.

The Local Phase starts with the best solution seed identified in the first phase and uses efficient local search to find a locally optimal solution; it then goes on to the second-best solution seed and so on until exhausting all solution seeds. To insure good performance of the algorithm used in this stage, we require it to be locally convergent as the simulation effort goes to infinity regardless of the quality of the seeds it is provided. We define a *local minimum* as follows (this definition is also used in Hong and Nelson 2006, 2007):

Definition 1 (Local Minimum) *Let* $\mathcal{N}(\mathbf{x}) = \{\mathbf{y} : \mathbf{y} \in \Theta \text{ and } \|\mathbf{x} - \mathbf{y}\| = 1\}$ *be the local neighborhood of* $\mathbf{x} \in \Theta$*, where* $\|\mathbf{x} - \mathbf{y}\|$ *denotes the Euclidean distance between* $\mathbf{x}$ *and* $\mathbf{y}$*. Then* $\mathbf{x}$ *is a* local minimum *if* $\mathbf{x} \in \Theta$ *and either* $\mathcal{N}(\mathbf{x}) = \emptyset$ *or* $g(\mathbf{x}) \leq g(\mathbf{y})$ *for all* $\mathbf{y} \in \mathcal{N}(\mathbf{x})$*. Let* $\mathcal{M}$ *denote the set of local minimizers of the function* g *in* Θ*.*

Since the quality of the solutions found in the Local Phase is critical to the performance of the algorithm, in practice we require each solution obtained in this phase to be a local minimum with probability at least $1 - \alpha_L$. Therefore, as the PCS $1 - \alpha_L \to 1$, the effort goes to infinity and the algorithm is guaranteed to be locally convergent.

The transition between the second and third phases can be either effort based or quality based. For the effort-based rule, transition happens when the simulation budget of the second phase is exhausted. If there is no effort limit, the algorithm transitions once all solutions seeds are searched. The Local Phase returns a set of solutions $\mathcal{L}$, and is designed to give high confidence that $\mathcal{L} \subset \mathcal{M}$.

In the Clean-Up Phase, we want to select the best solution among the locally optimal solutions $\mathcal{L}$ found in the second phase, and the actual value of the selected solution also needs to be estimated to within $\pm\delta_C$, all with confidence level $\geq 1 - \alpha_C$, where $\delta_C > 0$ is set by the user and is the only parameter that the user must set (sensible defaults are provided for all others). As $1 - \alpha_C \to 1$ the best solution is therefore selected with probability 1.

How can we establish that we are "competitive with the features provided by commercial products?" We do so by comparing ISC to OptQuest (OptTek Systems, Inc.) on DOvS problems. If OptQuest is not the best of the commercial products, it is certainly a good representative of them and it is very widely used. We use OptQuest to establish a competitive benchmark for optimization performance as a function of simulation effort, and we count on OptQuest to deliver good solutions quickly (which in our experience it does). *We do not expect to beat OptQuest in any comprehensive sense.* OptQuest has had years of development, and its algorithms are smart and efficient. *We consider ISC to be a success if it can deliver as good or better solution quality as OptQuest without expending substantially more sim-*

ulation effort. This constitutes "success" because ISC provides convergence guarantees and inference that OptQuest does not. Our only potential advantage comes from how we deal with the stochastic aspect of the problem, which is fundamentally different from any of the commercial products.

We show empirically that ISC achieves our goals: In some cases (noisy problem, multimodal response surface) demonstrating superior performance to OptQuest; in others (low noise, more regular response surface) being beaten by OptQuest in terms of relative effort expended, but still not expending much additional absolute effort; and in yet others having performance almost indistinguishable from OptQuest. Comparison of COMPASS-based approaches with other convergent algorithms can be found in Hong and Nelson (2006).

4 CONCLUSIONS

An overriding objective in the development of ISC was to have an algorithm that could stop on its own with well-defined guarantees. This objective led us to search for locally optimal solutions and use ranking-and-selection procedures to establish and compare them. ISC is not designed to work well under a strict and tightly time-constrained budget—since the user would have to provide effort-based transition rules for each phase—nor does it exploit an essentially unlimited budget. We contend that the time required to develop a detailed simulation model, and the impact of the decision that will be based on it, argue in favor of a liberal, but not infinite, computer budget in most situations.

Papers describing the theory and specific algorithms behind ISC, as well as the detailed empirical evaluation of it, may be found at `www.ISCompass.net`. The web site also provides an implementation of ISC in `C++` that may be downloaded and used for noncommercial purposes.

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