Service Oriented Line Planning and Timetabling for Passenger Trains

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1. Introduction

Planning of railways operations has been practiced for almost two centuries. Over the years, with the development of the railway systems this task has become more and more complex. The planning process consists of several phases, the fundamental ones are: line planning, timetabling, rolling stock circulation, platforming and crew scheduling. For surveys on various railway planning optimization problems, see Bussieck et al. (1997), Cordeau et al. (1998), Caprara et al. (2006) and Törnquist (2006).

This study considers the planning of lines and a timetable in a regional or a metropolitan train system with the goal of minimizing operational costs and user inconvenience. Operational costs are modeled as the total engine time and user inconvenience as the total passenger *journey time*. Journey time includes riding times and waiting times at origin and transfer stations.

The Line Planning Problem is defined as follows: Given railway infrastructure, traveling time over each of its segments and passenger demands for journeys, determine the set of lines and their frequencies. For a recent survey on line planning see Schöbel (2011). Bussieck et al. (1996) and Schöbel (2011) argue that user inconvenience should be measured by the total time passengers spend in the railway system, i.e. journey time. However, since a timetable is required for the calculation of the journey time this cannot be done during the line planning phase.

The Train Timetabling Problem is defined as follows: For a given set of lines and frequencies, determine the arrival and departure time at each block (track section) and station such that a set of safety constraints are satisfied. In regional systems the rail infrastructure is a limited resource on which several lines compete. Consequently, the safety constraints are more complicated in compare to other transportation systems.

A common practice in timetabling of regional rail systems is to schedule the trains in a cyclic manner. In such timetables, each event is repeated every cycle, e.g., every hour. This approach is preferred by both passengers and planners. See for example, Odijk (1996) and Liebchen and Möhring (2007).

Operational costs and user inconvenience are largely affected by decisions made both at the line planning phase and the timetabling phase. Hence simultaneously solving the two problems may be beneficial. Several previous studies have integrated some aspects of the two phases. See, Ceder and Israeli (1992), Gorman (1998), Lindner (2000), Borndörfer et al. (2007) Michaelis and Schöbel (2009).

No previous study on railway planning has integrated line planning, timetabling and routing decisions of the passengers. The contribution of this study is in presenting a model that combines the three problems and an algorithm to solve it.

The solution method presented in this paper is based on the paradigm of the Cross Entropy (CE) metaheuristic, introduced by Rubinstein (1999). See Also: Rubinstein and Kroese (2004) and Margolin (2005). Cross Entropy is an evolutionary metaheuristic that iteratively applies the following two phases:

1. Generation and evaluation of a sample of random solutions according to a specified random mechanism.
2. Updating the parameters of the random mechanism, on the basis of these solutions, in order to produce a "better" sample in the next iteration.

The updated mechanism delivers a better solution with higher probability as compare to the one of the previous iteration.

In the rest of this abstract we present a formal definition of the Service Oriented Line Planning and Timetabling Problem and apply the CE meta-heuristic technique to solve it. The results of a numerical study based on actual data from the Israeli railway system are then reported.

2. Problem Definition

The Service Oriented Line Planning and Timetabling Problem (SOLPTP) is defined as follow: Given a *pool of routes*, *passenger demand for journeys*, *cycle time*, *horizon time*, *safety and operational restrictions* find a *line plan* and a *cyclic timetable* that minimizes the total cost associated with travel time of all passengers and with operation of all trains.

The input consists of information about the system infrastructure, i.e. blocks and stations. A sequence of blocks and stations between two major stations that may be traversed by a train is called a *route*. A set of all possible routes to choose from is given (referred to as the *route pool)*. Lastly, the passenger demand for journeys is given in the form of Origin-Destination (O-D) matrices, one matrix for each period of the planning horizon.

. For a solution to be feasible it must satisfy several safety and operational constraints. Examples for these constraints are: a) only one train is allowed at each block at a time b) a minimal stopping time at a station is required to allow embarking and disembarking of passengers and c) a minimal time interval is required for a passenger to switch trains at station.

The goal is to minimize a weighted sum of the total engine time and the total journey time. The output consists of a set of chosen routes to be used periodically, and the corresponding cyclic timetable.

3. Algorithm

An outline of the Cross Entropy algorithm is given in Figure 1. A set of solutions are generated by an initial arbitrary random mechanism, line planes and cyclic timetables are constructed and evaluated. The random mechanism is updated based on these solutions and the process is repeated until some stopping criteria are met.

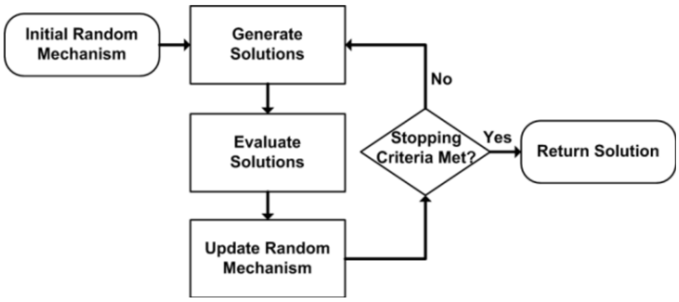

Figure 1: Outline of the CE algorithm

A solution is encoded by a set of *genes* referred as a *chromosome*. Each gene represents a possible train and corresponds to a route in the route pool. A *gene* contains the following information:

- In Use (IU) – A Boolean that indicates whether or not this train is being used in this solution.
- Gate Time (GT) – An integer that indicates the earliest time (in minutes) in which the train can enter its first block.
- Stopping Stations (SS) – Boolean vector with an element for each station along the train's route. A value of 1 in the j^{th} elements of this vector indicates that the train stops at its j^{th} station. Otherwise, the train may pass the j^{th} station without stopping there.

The evaluation of a chromosome consists of four stages. First, a feasible solution is constructed, i.e., a line plan and a cyclic timetable. Second, a graph representation of all feasible passenger itineraries is constructed. Third, the shortest path from each node in this graph to each station is calculated in order to obtain the optimal itineraries for each flow of passengers. Finally, the total journey time of all passengers and total engine time are calculated.

The construction of a line plan and a cyclic timetable is based on a greedy insertion algorithm that was devised especially for the SOLPTP. The description of this algorithm is omitted form this abstract.

Given a timetable, in order to evaluate the total journey time of all passengers, a graphical representation of all possible itineraries is created. The *itineraries graph* is a directed graph consisting of a node for each event in the timetable. A node is characterized by: station, train, type (arrival/departure), and time. Calculating the journey time of a passenger is equivalent to finding the shortest path from the departure node in the origin station following his arrival, to a node in the destination station. This calculation needs to be done for each passenger flow.

There are known algorithms for all-pairs shortest paths, such as the Floyd Warshall algorithm with complexity of $O(N^3)$ and Johnson's algorithm with complexity of $O(N^2 \log N + NA)$, where N is the number of nodes and A is number of arcs. See for example Cormen et al. (2001). However, taking in to account special properties of itineraries graph, we devise an algorithm that allows us to calculate all journey times in $O(NK)$, where K is the number of stations in the system

Summing the journey times over all passenger flows we receive the total journey time. The objective function is a weighted sum of the total engine time and total journey time. The weights reflect the ratio between the cost of engine hour and the cost of "passenger hour" as viewed by the planner.

Recall that each solution is represented by a chromosome that is built of a set of genes. Each gene consists of Boolean variables (IU and SS) and an integer variable (GT). For each of these variables a probability function is created. The Boolean variables are sampled from a Bernoulli distribution and the integer variables are sampled from a general discrete probability function. Based on these distributions a generation of chromosomes is created. After constructing a generation of solutions and evaluating them, an *elite group* is selected, which is the γ lower (best) quantile of the solutions. New distributions are calculated based on the elite group and are exponentially smoothed by a weighted average with the distribution functions of the previous generation

Three stopping criteria are used, namely: stop when the distribution parameters converge, stop when there is no improvement in the objective number for a predefined number of CE iterations or stop when a predefined time limit is exceeded.

4. Numerical Experiments

The algorithm was benchmarked against the actual timetable being in use by Israel Railways. In order to calibrate the CE algorithm and to check the sensitivity of the algorithm to its parameters a full factorial experiment was conducted with the following three algorithm parameters: Smoothing level, Generation Size, and deciding whether to keep the elite group in the following generation, i.e. *Elitism*. An ANOVA analysis showed that the main factors had a significant effect but rather small in terms of the objective function value. In addition, the gap between the best and worse solutions obtained was less than 2.5%. This may suggest that the algorithm is not extremely sensitive to changes in the settings of its parameters, which is good news.

To compare the solutions obtained by the CE algorithm with the existing timetable, the ratio between operational costs and user inconvenience must be specified. Since this ratio is unknown, an efficiency frontier is constructed in order to visualize the trade-off between the two components. This was done by applying the algorithm for various cost ratios. Several dominating solutions in terms of both operational costs and user inconvenience were generated. In particular, the total journey time can be reduced by approximately 22% at the same operational cost. This represents shortening the average journey of a passenger from 65 to 51 minutes.

5. Conclusion

In this study, an integrated service oriented line planning and timetabling problem is introduced. Calculation of the total journey time is computationally involved and hence optimizing a line plan and timetable according to this measure is challenging. The Cross Entropy algorithm presented in this paper is shown to be effective in solving the problem. In addition, the ability of the CE to tackle problems with complicated structures allows readily extending the problem and incorporating additional component of the rail planning process.

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