Multi-objective periodic railway timetabling with overtaking optimization

2018 INFORMS Annual Meeting

Delft University of Technology

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November 4, 2018
Outline

1. Introduction
2. Model description
3. Pareto frontier
4. Computational experiments
5. Conclusions
1 Introduction

2 Model description

3 Pareto frontier

4 Computational experiments

5 Conclusions
Current railway traffic

- Constant growth of demand
- Heavily congested networks (some even oversaturated)
- Experiencing a lot of delays
Current railway traffic

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- Heavily congested networks (some even oversaturated)
- Experiencing a lot of delays

Need for better planning to provide a high level timetable
  - shortening travel time
  - robust for certain delays
  - best-utilize capacity
Railway timetabling

Input:
- Given line plan (stop pattern and frequency) and period length

Output:
- Departure and arrival time at each station

Goals:
- Efficient → shorter travel time & capacity
- Robust → less delay & better punctuality
- Regular → regular service (convenient to passengers)

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Network design → Line plan → **Timetable** → Rolling stock → Crew

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![Diagram](image)
Railway timetabling

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Multi-objective periodic railway timetabling model is proposed
1 Introduction

2 Model description

3 Pareto frontier

4 Computational experiments

5 Conclusions
Periodic event-activity network

- Event-activity network $\mathcal{N} = \mathcal{E}, \mathcal{A}$
- Cycle time: $T$
- Periodic events: arrival & departure times $\pi_i \in [0, T)$
- Process time constraints: $l_{ij} \leq \pi_j - \pi_i + z_{ij} \cdot T \leq u_{ij}$
- Period shift: $z_{ij}$ - binary variables (if $0 \leq u_{ij} \leq T - 1$)

\[
\begin{align*}
    a_1 & \xrightarrow{\text{A}_{\text{run}} \ [10,12]_T} a_2 \quad a_2 & \xrightarrow{\text{A}_{\text{run}} \ [12,14]_T} a_3 \quad a_3 & \xrightarrow{\text{A}_{\text{run}} \ [10,12]_T} a_1 \\
    a_1 & \xrightarrow{\text{A}_{\text{dwell}} \ [1,3]_T} d_1 \quad d_1 & \xrightarrow{\text{A}_{\text{run}} \ [16,19]_T} d_2 \quad d_2 & \xrightarrow{\text{A}_{\text{run}} \ [18,22]_T} d_3 \\
    a_2 & \xrightarrow{\text{A}_{\text{dwell}} \ [1,3]_T} d_1 \quad d_1 & \xrightarrow{\text{A}_{\text{run}} \ [16,19]_T} d_2 \quad d_2 & \xrightarrow{\text{A}_{\text{run}} \ [18,22]_T} d_3 \\
    a_3 & \xrightarrow{\text{A}_{\text{dwell}} \ [1,3]_T} d_1 \quad d_1 & \xrightarrow{\text{A}_{\text{run}} \ [16,19]_T} d_2 \quad d_2 & \xrightarrow{\text{A}_{\text{run}} \ [18,22]_T} d_3 \\
\end{align*}
\]
Single-objective models

- Train journey time model (PESP-TJT)
- Timetable regularity model (PESP-Reg)
- Timetable vulnerability model (PESP-Vnb)
- Flexible overtaking model (PESP-Ovt)
Train journey time model (PESP-TJT)

Objective: train journey time (TJT)

Minimize \[ \sum_{(i,j) \in A_{\text{run}} \cup A_{\text{dwell}}} \alpha_{ij} \cdot (\pi_j - \pi_i + z_{ij} \cdot T) \]

Basic constraints:

\[ l_{ij} \leq \pi_j - \pi_i + z_{ij} \cdot T \leq u_{ij} \quad \forall (i,j) \in A \]

\[ 0 \leq \pi_i < T \quad \forall i \in E \]

\[ z_{ij} \in \{0, 1\} \quad \forall (i,j) \in A \]
Timetable regularity model (PESP-Reg)

Regularity deviation

To improve service regularity, \( \theta_{ij} \) is introduced as *regularity deviation*, defined as:

\[
\theta_{ij} = |(\pi_j - \pi_i + z_{ij} \cdot T) - \left[ \frac{T}{f_l} \right] | \quad \forall (i, j) \in A_{reg}, l \in \mathcal{L}
\]

Objective:

Minimize \( \sum_{(i,j) \in A_{reg}} \theta_{ij} \)

Regularity constraints:

\[
- \theta_{ij} \leq a_{ij} - \left[ \frac{T}{f_l} \right] \leq \theta_{ij} \quad \forall (i, j) \in A_{reg}, l \in \mathcal{L}
\]

\[
0 \leq \theta_{ij} \leq \beta \quad \forall (i, j) \in A_{reg}, l \in \mathcal{L}
\]
Small buffer times between trains could lead the timetable vulnerable (sensitive to delays), hence \textit{timetable vulnerability} is introduced based on a designed headway penalty function to improve timetable robustness.

\[
\delta_{ij} = \begin{cases} 
    P_{\text{max}} - \phi \cdot (a_{ij} - l_{ij}) & \text{if } a_{ij} \leq h_{ij}^{p,\text{lower}} \\
    0 & \text{if } h_{ij}^{p,\text{lower}} < a_{ij} \leq h_{ij}^{p,\text{upper}} \\
    P_{\text{max}} - \phi \cdot (u_{ij} - a_{ij}) & \text{if } a_{ij} > h_{ij}^{p,\text{upper}}, \; \forall (i,j) \in \mathcal{A}_{\text{head}}
\end{cases}
\]
Timetable vulnerability model (PESP-Vnb)

Objective:

\[
\text{Minimize } \sum_{(i,j) \in A_{\text{head}}} \delta_{ij}
\]

Vulnerability constraints:

\[
\delta_{ij} = w'_{ij} \cdot (P_{\text{max}} - \phi \cdot (a_{ij} - l_{ij})) + w'''_{ij} \cdot (P_{\text{max}} - \phi \cdot (u_{ij} - a_{ij}))
\]

\[
w'_{ij} + w''_{ij} + w'''_{ij} = 1
\]

\[
w'_{ij}, w''_{ij}, w'''_{ij} \in \{0, 1\}
\]

\[
l_{ij} \leq a_{ij} \leq h_{ij}^{p,\text{lower}} + M_1 \cdot (1 - w'_{ij})
\]

\[
w''_{ij} \cdot h_{ij}^{p,\text{lower}} < a_{ij} \leq h_{ij}^{p,\text{upper}} + M_2 \cdot (1 - w''_{ij})
\]

\[
w'''_{ij} \cdot h_{ij}^{p,\text{upper}} < a_{ij} \leq u_{ij}
\]
Flexible overtakings model (PESP-Ovt)

Flexible overtakings

Flexible overtakings represent that a train can be overtaken (i) by more than one train at each station; (ii) multiple times along its path; (iii) at any station along its path.

Zhang and Nie (2016) proved that the overtaking occurs only if the sum of four modulo parameters of related dwell and headway activity equals 1 or 3, and introduced $p_{i,i'}^{j,j'}$ to present overtaking existence. A binary variable $y_{i,i'}^{j,j'}$ is introduced to represent whether activity $(i,i')$ is overtaken by activity $(j,j')$. 
Flexible overtakings

Flexible overtakings represent that a train can be overtaken (i) by more than one train at each station; (ii) multiple times along its path; (iii) at any station along its path.

Overtaking existence

Zhang and Nie (2016) proved that the overtaking occurs only if the sum of four modulo parameters of related dwell and headway activity equals 1 or 3, and introduced $p_{i'i'j'j'}$ to present overtaking existence.
Flexible overtakings model (PESP-Ovt)

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Overtaking existence

Zhang and Nie (2016) proved that the overtaking occurs only if the sum of four modulo parameters of related dwell and headway activity equals 1 or 3, and introduced $p_{ii'jj'}$ to present overtaking existence.

- A binary variable $y_{ii'jj'}$ is introduced to represent whether activity $(i, i')$ is overtaken by activity $(j, j')$.
Flexible overtakings model (PESP-Ovt)

Objective:

\[
\text{Minimize } \sum_{(i,j) \in A_{dwell}} y_{i'i'jj'}
\]

- Overtaking existence constraints

\[
z_{ij} + z_{i'j'} + z_{i'i'} + z_{jj'} = 2 \cdot o_{i'i'jj'} + p_{i'i'jj'}
\]

\[
y_{i'i'jj'} + y_{jj'i'i'} = p_{i'i'jj'}
\]

\[
o_{i'i'jj'}, p_{i'i'jj'}, y_{i'i'jj'}, y_{jj'i'i'} \in \{0, 1\}
\]

\[
-y_{i'i'jj'} \cdot d_{jj'} - (1-p_{i'i'jj'}) \cdot (u_{jj'} - l_{i'i'}) \leq a_{i'i'} - a_{jj'} \leq y_{i'i'jj'} \cdot d_{i'i'} + (1-p_{i'i'jj'}) \cdot (u_{i'i'} - l_{jj'})
\]

- Overtaking related activity time constraints
- Station capacity constraint
  - Fixed capacity constraint
  - Flexible capacity constraint (new overtaking facilities)
Multi-objective periodic railway timetabling (MOPRT)

\[
Z_{TJT} = \sum_{(i,j) \in A_{\text{run}} \cup A_{\text{dwell}}} \alpha_{ij} \cdot (\pi_j - \pi_i + z_{ij} \cdot T)
\]

\[
Z_{\text{Reg}} = \sum_{(i,j) \in A_{\text{reg}}} \theta_{ij}
\]

\[
Z_{\text{Vnb}} = \sum_{(i,j) \in A_{\text{head}}} \delta_{ij}
\]

\[
Z_{\text{Ovt}} = \sum_{(i,j) \in A_{\text{dwell}}} y_{ii'jj'}
\]

subject to above mentioned constraints.
Pareto frontier

- **ε-constraint method**
  - Optimize one chosen objective using the others as constraints
  - Different values of ε are used as the bound on the other functions
  - By varying ε-constraint bounds, obtain the Pareto frontier

- Adaptive ε-constraint method in our MOPRT model
  - Objective priority order: $Z_{TJT}$, $Z_{Reg}$, $Z_{Vnb}$, and $Z_{Ovt}$
  - Algorithm 1: compute adaptive payoff table
    - find the efficient range of each objective value
    - obtain ε-constraint values
  - Algorithm 2: explore the model feasibility
    - test feasibility of PESP-Vnb with ε-constraint
  - Algorithm 3: generate Pareto-optimal solutions
    - solve MOPRT model and obtain the Pareto-optimal set
Timetable evaluation

Evaluation criterions (solutions in Pareto-optimal set):

• Standardized Euclidean distance $\rho$
  • Normalization of objective values

• Capacity utilization $C = \frac{\lambda}{T}$
  • Minimum cycle time $\lambda$
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Computational experiments

- Theoretical case
- Real-world case
Computational experiments

- Theoretical case
- Real-world case

- Network layout and line plan:

```
A C EB D
f_1 = 4
f_2 = 2
f_3 = 3
```
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- Network layout and line plan:

```
A -- B -- C -- D -- E

f_1 = 4
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- Original payoff table and the updated payoff table:

<table>
<thead>
<tr>
<th>Models/Objectives</th>
<th>Z_{TJT}</th>
<th>Z_{Reg}</th>
<th>Z_{Vnb}</th>
<th>Z_{Ovt}</th>
<th>Z_{TJT}</th>
<th>Z_{Reg}</th>
<th>Z_{Vnb}</th>
<th>Z_{Ovt}</th>
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</thead>
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<td>13092</td>
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<td>30654</td>
<td>13092</td>
<td>46860</td>
<td>7</td>
</tr>
</tbody>
</table>
Theoretical case: Pareto-optimal set

- Algorithm 2: reduce 88 times computation for solving the model
- Algorithm 3: 83 non-dominant solutions out of 274 feasible solutions
Theoretical case: trade-off analysis
Pareto-optimal set for the MOPRT model in four sub-figures:
Theoretical case: trade-off analysis

Pareto-optimal set for the MOPRT model in four sub-figures:
Theoretical case: trade-off analysis

The relationship between the number of overtakings and capacity utilization:
Theoretical case: time-distance diagrams

Time-distance diagrams of single-objective models:

(a) Min TJT
(b) Min Reg
(c) Min Vnb
(d) Min Ovt

Top three solutions from Pareto-optimal set:
Real-world case

Dutch railway corridor: Utrecht central to Arnhem central

- Existing network: 1 ICE bi-hourly, 4 IC and 6 SPR per hour
- Future network: Extra trains and a new overtaking facility at DB (High frequency program)
Real-world case

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By using our MOPRT model,
- can we find a better timetable than the existing one?
- can we find an optimal station to be equipped with the new overtaking facility?
Real-world case: existing network

- Line plan:
Real-world case: existing network

- Line plan:

```
<table>
<thead>
<tr>
<th>Stations</th>
<th>Utvr</th>
<th>Bnk</th>
<th>Db</th>
<th>Mrn</th>
<th>Klp</th>
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Existing timetable:
Real-world case: existing network

- Line plan:

```
<table>
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<tr>
<th>Station</th>
<th>Utvr</th>
<th>Bnk</th>
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Existing timetable:

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```

Achieved optimal timetable:

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Real-world case: existing network

Comparison: existing timetable and solutions in Pareto-optimal set

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<th>$Z_{Vnb}$</th>
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Real-world case: existing network

Comparison: existing timetable and solutions in Pareto-optimal set

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Real-world case: future network

- Line plan: 2 extra IC trains and extend S7500

- Plan to construct a new overtaking facility
Real-world case: future network

- Line plan: 2 extra IC trains and extend S7500

- Plan to construct a new overtaking facility

- Achieved best timetables: overtaking facility at Wf and DB
Real-world case: future network

Comparison of solutions in the Pareto-optimal set (25 solutions)

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1 Introduction
2 Model description
3 Pareto frontier
4 Computational experiments
5 Conclusions
Conclusions

Main contributions:

• Each single-objective model performs well to find the optimal solution of the corresponding objective.
• The adaptive $\varepsilon$-constraint method is efficient to creat Pareto frontier.
• The MOPRT model could
  • achieve better solution than the existing timetable (with 18.54% reduction of capacity utilization in our case).
  • find the optimal location of a new overtkaing facility while optimizing the timetable.
  • provide flexible decision support to timetable planners.
Thank you for your attention!
Questions?

Fei Yan
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Delft University of Technology, the Netherlands