

This is a supplementary explanation of the answer for Question 2. Constraint (2), whose aim is to guarantee that each shipment can reach its destination, needs an auxiliary constraint (13) in Lin (2017) to avoid the example you presented in Fig. 1.

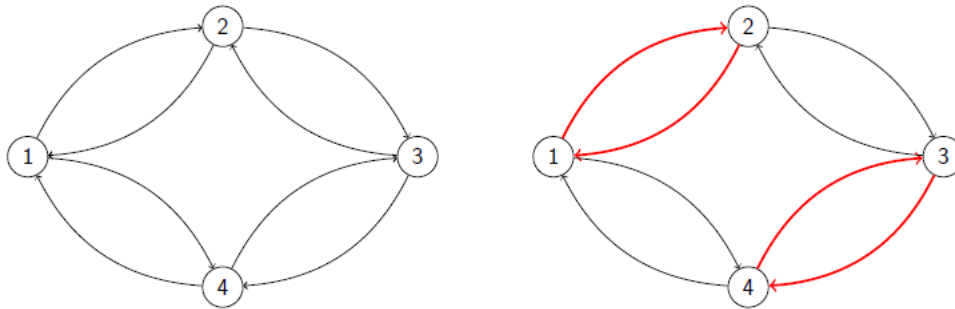


Figure 1: The rail network (left) and the selected car flow variables (in red) (right).

Taking  $N_{1,3}$  and  $N_{2,3}$  in Fig. 1 as an example, the traffic flow of  $N_{1,3}$  and  $N_{2,3}$  are positive according to your description, and  $x_{1,3}^2 = 1$  and  $x_{2,3}^1 = 1$  according to the computational results. Based on constraint (13), there are the following equations:

$$f_{2,3} = N_{2,3} + f_{1,3}x_{1,3}^2 \quad (a)$$

$$f_{1,3} = N_{1,3} + f_{2,3}x_{2,3}^1 \quad (b)$$

In accordance with Eq. (a) and (b), there is:

$$f_{2,3} = N_{2,3} + f_{1,3}x_{1,3}^2 = N_{2,3} + (N_{1,3} + f_{2,3}x_{2,3}^1)x_{1,3}^2 = N_{2,3} + N_{1,3}x_{1,3}^2 + f_{2,3}x_{2,3}^1x_{1,3}^2 \quad (c)$$

$$f_{2,3}(1 - x_{2,3}^1x_{1,3}^2) = N_{2,3} + N_{1,3}x_{1,3}^2 \quad (d)$$

$$0 = N_{2,3} + N_{1,3} \quad (e)$$

Apparently, Eq. (e) is contrary to the description that  $N_{1,3}$  and  $N_{2,3}$  are positive, which means it is impossible that  $x_{1,3}^2 = 1$  and  $x_{2,3}^1 = 1$  at the same time. With the auxiliary of constraint (13), the situation you mentioned in Fig.1 is avoided.