EXPLORING THE POTENTIAL OF “ALTERNATIVE GRAPHS” TO SOLVE RAILWAY OPTIMIZATION PROBLEMS

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Who is the speaker?

- Andrea D’Ariano is Associate Professor in Operations Research (OR)
- Background of knowledge in OR, Computer Science, Railway Engineering, Intelligent Transportation Systems
- Winner of Prizes given by IEEE, INFORMS, IAROR, AIRO, …
- Associate Editor for well-known intern. journals and conferences
- Participation in several research projects with Universities, Research Institutes, Railway Companies and Organizations
- Coordinator of AIRO (Italian Assoc. of Operat. Research) Chapter on “Optimization in Public Transport and Shared Mobility”
Goals of this presentation

- Introduce the alternative graph methodology (and some related technologies) to a general audience with a different background of knowledge (not necessarily OR/scheduling experts)
What represents an Alternative Graph (AG)?

- AG is a way of formulating job shop scheduling problems (JSSP)
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- **JSSP** is a class of scheduling problems in which:
  - Each *job* corresponds to a vehicle or person taking some actions
  - Each job is composed by a set of *operations* to be performed
  - The set of operations of each job can be pre-defined or flexible
  - Each operation is related to a job and a capacitated *resource*
  - Each resource is shared by different jobs in the schedule

[Shi Qiang Liu, Erhan Kozan, Transp. Science 2011]
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- JSSP can easily represent a train scheduling problem in which:
  - Each job corresponds to a specific train
  - Each resource corresponds to a piece of railway track
  - Each operation is a piece of track that is occupied by a train
  - The set of operations of a job is the train routing
What represents an Alternative Graph (AG)?

- AG is particularly suitable to model train scheduling problems. In AG, each node is an operation, while each arc is a constraint.
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- Each piece of track can only be occupied by one train at a time, thus requiring *blocking* (or *no-store*) constraints.
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- Each train has a traveling time window according to the timetable, i.e. minimum & maximum times to start processing an operation, requiring *release & due date* (soft) or *deadline* (hard) constraints.
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- Other types of constraints: service connection constraints, rolling stock constraints, arrival and departure time constraints, resource availability constraints, min and max travel time constraints, …
What about the modeling assumptions?

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- Each operation has a start time (i.e. a timing variable) and a duration time (input data), requiring a pre-defined processing time.

- **Train sequencing** can be partially or totally flexible (each alternative arc is a variable) between trains sharing resources.

\[ \sigma(j) = \begin{align*}
    h & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quad
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- Train sequencing can be partially or totally flexible (each alternative arc is a variable) between trains sharing resources.

- The routing of each train can be either fixed or flexible (each job can be a variable), with possibility of local or global re-routing.

[Source: Paola Pellegrini]
What about the modeling assumptions?

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- The **routing of each train** can be either fixed or flexible (each job can be a variable), with possibility of **local or global re-routing**.

- The train **arrival and departure times** can also be flexible.

- **Running/dwell times** are constrained between mix and max values.

- Assumptions on **time and resource granularities** must be set.
What about the modeling assumptions?

- The problem complexity (finding a feasible schedule is NP-hard) depends on the assumptions regarding the granularity, i.e. on the number of sequencing and routing variables (the timing variables are easy to handle, since modelled as shortest path problems).
What about the modeling assumptions?

- The problem complexity (finding a feasible schedule is \( NP\text{-}hard \)) depends on the assumptions regarding the granularity, i.e., on the number of sequencing and routing variables (the timing variables are easy to handle, since modelled as shortest path problems).

- The objective function is usually related to the **timing of operations**. There are powerful scheduling-theory-based techniques to minimize the maximum completion time or delay.

One critical path: \( Ar - Br - Cr - Cs - As \)
Makespan: \( 13+15+10+8+10 = 56 \)
What about the modeling assumptions?

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- The objective function is usually related to the timing of operations. There are powerful scheduling-theory-based techniques to minimize the maximum completion time or delay.

- Other objective functions are possible, but the resulting problems might be more difficult to handle with AG, while more general mathematical formulations can easily incorporate them (even if general solvers might be slow to converge to near-optimum).
How to optimize performance indicators?

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Corman et al.
TR part C 2012
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Corman et al.
JRTPM 2011
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- Lexicographic optimization of different indicators (e.g., train delay first and energy consumption second, or vice versa)
- Multi-class train scheduling (international, regional, freight trains)
- Multi-objective optimization (e.g., weighted sum or epsilon-constraint methods)
- Multiple decision makers and game theoretical approaches
Goals of this presentation

- Introduce the alternative graph methodology (and some related technologies) to a general audience with a different background of knowledge (not necessarily OR/scheduling experts)

- Give an overview of the literature on various railway simulation and optimization topics making use of this methodology
Which models exist in the literature?
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- A significant number of papers use AG for train scheduling:

![Survey on IEEE ITS Journal by Fang et al. 2015](image)

- Two main streams of research are based on either resource-dependent (e.g., MILP) or time-dependent formulations. Their complexity depends on the adopted resource and time granularity.
Which solving methods exist?

- General (commercial) solver:
  - Pros: easy to formulate business rules and objectives
  - Contros: very slow solving process when increasing problem size

- Smart (problem-dedicated) solver:
  - Pros: very good performance and scalability
  - Contros: some business rules and objectives require a lot of work

AG-based software uses heuristic, meta-heuristic, and exact algorithms to handle different types of variables. These algorithms need to be adapted when changing constraints and objectives.

Pre-processing is a key factor for any solver, e.g. filtering the train routes, pre-selecting variable values, reducing the variables.
Which types of problem decomposition?

- Decomposition is needed in practice and can be of different types:
  - **Temporal decomposition**, e.g., rolling horizon or MPC approaches;
  - **Spatial decomposition**, e.g., coordination or Benders approaches;
  - Decomposition based on the **different types of variables**, e.g., timing, sequencing, and routing approaches;
  - Decomposition based on **different decision layers**, e.g., the variables are grouped based on the definition of sub-problems.

All the decomposition methods are iterative and require to study **convergence**, **performance**, and **scalability** factors.
Goals of this presentation

- Introduce the alternative graph methodology (and some related technologies) to a general audience with a different background of knowledge (not necessarily OR/scheduling experts)

- Give an overview of the literature on various railway simulation and optimization topics making use of this methodology

- Discuss how to apply this methodology to solve various European problems from real-time railway traffic management to tactical train timetabling and capacity consumption
Real-time traffic management
Real-time traffic management

- Various types of local train dispatching: from a single line to a dispatching area, including a large and complex station area
Real-time traffic management

- Various types of **local train dispatching**: from a single line to a dispatching area, including a large and complex station area.

- Train dispatching needs to be coordinated with **train speed control**: this can be done either with an integrated approach or an iterative approach in which speed regulator is coordinated with reschedule.
Real-time traffic management

- Various types of local train dispatching: from a single line to a dispatching area, including a large and complex station area

- Global train dispatching: train dispatchers need to be coordinated by a high-level coordination methodology (also based on AG)

Corman et al. TRpE 2012
Real-time traffic management

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- Global train dispatching: train dispatchers need to be coordinated by a high-level coordination methodology (also based on AG)

- Different types of disturbances: from a single delayed train to multiple delayed trains, or even to disrupted trains or tracks

- Different types of networks and passenger demands
Tactical train timetabling

- This process is usually based on an existing train timetable
- Modify the timetable to insert new services, e.g. freight trains
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- Design timetables to deal with emergency situations, e.g. a serious capacity reduction, causing cancelling or globally re-routing trains
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- Modify the timetable to insert new services, e.g. freight trains
- Design timetables to deal with emergency situations, e.g. a serious capacity reduction, causing cancelling or globally re-routing trains
- Assessment of timetable robustness to disturbances (this can be achieved with coupling the scheduler with a traffic simulator)
- Investigating the best selection of train routes in the timetable
- Generating integrated train timetables with a pre-defined track maintenance schedule and/or rolling stock/personnel schedule
Assessment of capacity consumption

☐ AG can also be used to simulate traffic flows under normal and disturbed/disrupted traffic situations
Assessment of capacity consumption

- AG can also be used to simulate traffic flows under normal and disturbed/disrupted traffic situations.

- Dynamic infrastructure occupation assessment, based on the UIC concept of (scheduled) infrastructure occupation, to consider the following factors:
  - stochastic train delays,
  - response to the signaling/ATP system,
  - different signal configurations/positions,
  - traffic control actions to deal with conflicts.

![Diagram showing infrastructure occupation assessment](image-url)
Assessment of capacity consumption

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- Evaluation of traffic flows on newly built railway lines

- Evaluation of traffic flows when varying resource availability
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- Introduce the alternative graph methodology (and some related technologies) to a general audience with a different background of knowledge (not necessarily OR/scheduling experts)

- Give an overview of the literature on various railway simulation and optimization topics making use of this methodology

- Discuss how to apply this methodology to solve various European problems from real-time railway traffic management to tactical train timetabling and capacity consumption

- Present a train rescheduling application: mathematical models, advanced scheduling and routing algorithms, practical test case with consideration of railway business rules and objectives
Train rescheduling

- Introduction
- Mathematical models
- AGLIBRARY solver
- Alstom case study
- References
Recent research on train rescheduling

**Aim:** Development of novel rail traffic management systems for effective train traffic regulation

**Tool:** Flexible rail operations via advanced models and algorithms for train sequencing, routing and timing

**Applications:** Recover real-time railway traffic disturbances such as multiple delayed trains and blocked tracks; improving draft timetables with service intentions.
Timetable

- Station A
- Station B
- Station C
- Station D
- Station E
- Station F

- Minimum headway
- Buffer time
- Max speed
- Planned speed
- Recovery time

Time
Real-time

Station A
Station B
Station C
Station D
Station E
Station F

Current time
Time

Initial delay
Conflicts
Delay

Initial delay

Real-time
Real-time

Initial delay

Consecutive delay

Delay

Station A
Station B
Station C
Station D
Station E
Station F

Current time
Time

Real-time - Initial delay - Consecutive delay - FIFO sequence
Maximum consecutive delay for a typical instance is 16 min and the average initial delay is 1.24 min.

Combulative consecutive delay in all stations is 3093 min when using the timetable sequence and 1611 min if consecutive delays are minimized by optimal train rescheduling.
Train Rescheduling: Open issues

No advanced dispatching support tool exists to reschedule vehicle movements during operations.

In fact, there is still a lack of:

- **Precision**: Models and algorithms must include the variability of vehicle dynamics and must respect specific problem constraints;
- **Robustness**: Existing dispatching systems are able to provide viable solutions only for small instances and simple disturbances;
- **Quality**: A set of good solutions can be computed only if global conflict resolution is considered when computing orders, routes and times of the vehicles running in the studied railway area;
- **Efficiency**: The development of novel optimization algorithms must include the constraints due to limited computation times.
Presentation contents

- Introduction
- Mathematical models
- AGLIBRARY solver
- Alstom case study
- References
**Blocking time theory**

![Diagram of Blocking time theory](image)

- **Sight & Reaction time**
- **Running time**
- **Block sections**
- **Minimum headway time**
- **Clearing & Switching time**
- **Clearing point**

**Time**

**Space**
Conflict Detection and Resolution (CDR)
**Alternative Graph (AG)**

\[ G = (N,F,A) \]

- **N** = Set of nodes
- **F** = Set of fixed arcs
- **A** = Set of pairs of alternative arcs

**Selection** \( S \) = Choose at most one arc from each pair in \( A \), thus obtaining a graph \( G(S) = (N,F \cup S) \)

**Problem** = Find a complete selection \( S \) such that the longest path from 0 to \( n \) in \( G(S) \) is minimum

Max consecutive delay

[Mascis Pacciarelli EJOR 2002]
Min $f(t, x)$  s.t. 

$t_1 \geq w_{0,1} 
\quad t_7 \geq w_{0,7} 
\quad t_4 \geq w_{0,4} 
\quad t_{10} \geq w_{0,10} 
\quad t_2 \geq t_1 + w_{1,2} 
\quad \ldots 
\quad t_{12} \geq t_{11} + w_{11,12} 
\quad t_i \geq t_8 + w_{8,1} - M (1 - X_{8,1_2,7}) 
\quad t_7 \geq t_2 + w_{2,7} - M X_{8,1_2,7} 
\quad \ldots 

X_{8,1_2,7} = 1 
X_{9,2_3,8} = 1 
X_{10,3_4,9} = 1 
X_{11,4_5,10} = 1 
X_{12,5_6,11} = 1
MILP (Mixed-Integer Linear Programming) model

**FIXED ROUTES**

\[
\begin{align*}
\text{min} & \quad f(t, x) \\
\text{subject to} & \quad t_j \geq t_i + w_{ij} & \forall (i, j) \in F \\
& \quad t_j \geq t_i + w_{ij} - M (1 - x_{ij,hk}) & \forall ((i, j), (h, k)) \in A \\
& \quad t_k \geq t_h + w_{hk} - Mx_{ij,hk} \\
\end{align*}
\]

\[
x_{ij,hk} = \begin{cases} 
1 & \text{if } (i, j) \text{ is selected} \\
0 & \text{if } (h, k) \text{ is selected}
\end{cases}
\]
MILP (Mixed-Integer Linear Programming) model
FLEXIBLE ROUTES (D’Ariano et al. 2013)

\[
\begin{align*}
\min & \quad f(t, x, y) \\
\text{subject to} & \quad \sum_{r=1}^{ns} y_{rs} = 1 \\
& \quad t_j \geq t_i + w_{ij} - M (1 - y_{rs}) \\
& \quad t_j \geq t_i + w_{ij} - M (1 - x_{ij,hk}) - M (1 - y_{rs}) - M (1 - y_{uv}) \\
& \quad t_k \geq t_h + w_{hk} - Mx_{ij,hk} - M (1 - y_{rs}) - M (1 - y_{uv}) \\
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0 & \text{if } (h, k) \text{ is selected} 
\end{cases}
\]

\[
y_{rs} = \begin{cases} 
1 & \text{if route } r \text{ of train } s \text{ is selected} \\
0 & \text{otherwise} 
\end{cases}
\]

\[
ns: \text{ number of routes of train } s \quad nt: \text{ number of trains}
\]

\[
s = 1, ..., nt \quad \forall (i, j) \in F_{rs}, \quad \forall r, s \\
\forall ((i, j), (h, k) \in A
\]
Presentation contents

- Introduction
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Optimization software: AGLIBRARY

XML input file:
- Timetable
- Infrastructure Data
- Train routes
- Traveltimes

Train (Re)Scheduling

Infeasible Schedule

Feasible Schedule

Rerouting Alternatives?

No Rerouting or Time Limit Reached

Possible Improvements

New Routes

Train Rerouting

CDRFR* algorithms:
- Heuristics (e.g. FCFS, AMCC, JGH, ...)
- Branch and Bound (B&B)

CDR algorithms:
- Local Search (LS)
- Tabu Search (TS)
- VNS, VNTS, ...

Optimization software: AGLIBRARY

*Conflict Detection and Resolution with Fixed Routes
Illustrative example (1)

CDRFR formulation of a small example with three trains

Each alternative pair is used to order two trains on a block section
A conflict-free deadlock-free schedule is a complete consistent selection $S$. Optimal CDRFR solution computed by the B&B algorithm.
Illustrative example (3)

Optimal solution to the compound CDR problem

A new route for $T_A$ and a new complete consistent selection $S$ are shown

Max cons. delay = 0
CDRFR: Heuristics

- **Practical rules:**
  
  FIFO (First In First Out), ARI (route setting system)

**Greedy heuristics** (max consecutive delay minimization):

- **AMCC (Avoid Most Critical Completion time)**
  Chooses the pair containing the alternative arc which would cause the *largest increase* in consecutive delay.

- **AMSP (Avoid Max Sum Pair)**
  Chooses the pair with the *largest sum* of consecutive delays.

- **JGH (Job Greedy Heuristic)**
  Selects all the alternative arcs involving a *chosen job*, so that the sequencing of all the operations of this job are fixed in one step.
**CDRFR: Branch & bound method** [EJOR 2007]

**Branching rule**: Choose the most critical unselected alternative pair and branch on this pair.

**Hybrid search strategy**: Alternate $X$ repetitions of the depth-first visit with the choice of the open node of the search tree with smallest lower bound among the last $Y$ open nodes.

**Lower bound**: Generalization of the Jackson pre-emptive schedule [Carlier & Pinson MS 1989]. Implementation and evaluation of single and parallel machines [Brucker & Brinkkotter JS 2001].

**Implications rules**: Network topology and graph properties
Constraint Propagation: Static implications

One directional traffic flows

Bi-directional traffic flows

[ejor 2007]
CDRFR: Dynamic implications

Head: \( r_i = l^S(0,i) \)
Tail: \( d_i = l^S(i,n) - p_i \)
CDRFR: Dynamic implications

[((i,j),(h,k)) \in A
l^S(j,i) + a_{ij} > 0 \Rightarrow (h,k) implied by S

[((i,j),(h,k)) \in A
r_i^S + a_{ij} + d_j^S > UB \Rightarrow (h,k) implied by S

J: set of operations, c: operation, \{c\} \cup J must be processed on the same machine

\min_{j \in J\cup\{c\}} r_j + \sum_{j \in J\cup\{c\}} p_j + \min_{j \in J} d_j > UB \Rightarrow c must be processed after J

if \quad l^S(c,h) + a_{hk} + \max_{j \in J} \{l^S(k,j) + p_j\} > 0 \Rightarrow (h,k) is forbidden
CDRFR: Dynamic implications [EJOR 2007]

Evaluation expensive at each node of a search tree. Some of these implications can be evaluated in a preprocessing step, based on the network topology.

\[ ((i,j),(h,k)) \in A \]
\[ l^S(j,i) + a_{ij} > 0 \Rightarrow (h,k) \text{ implied by } S \]

\[ ((i,j),(h,k)) \in A \]
\[ r_i^S + a_{ij} + d_j^S > UB \Rightarrow (h,k) \text{ implied by } S \]

\( J \): set of operations, \( c \): operation, \( \{c\} \cup J \) must be processed on the same machine

\[ \min_{j \in J \cup \{c\}} r_j + \sum_{j \in J \cup \{c\}} p_j + \min_{j \in J} d_j > UB \Rightarrow c \text{ must be processed after } J \]

if
\[ l^S(c, h) + a_{hk} + \max_{j \in J} \{l^S(k, j) + p_j\} > 0 \Rightarrow (h,k) \text{ is forbidden} \]
We start from the solution obtained for the CDR problem with fixed routes. A local search for better routes is as follows:

- A move is to change one route and its evaluation is to solve the associated CDRFR problem;

- At each iteration the best (local) move is taken from a set of neighbours of a current CDR solution;

- **Neighbourhood**: It is well known that a solution can be improved by changing the critical path related to the current selection $S$ only;

- Our local search is based on a ramified critical paths in order to select potentially improving routes.
Illustrative example of ramified critical paths

(C(S))

Illustrative example of ramified critical paths

(B(S))

Illustrative example of ramified critical paths

(F(S))

waiting operation

6
The ramified critical paths are well focused on reducing the maximum consecutive delay but are not always opt-connected.

A novel tabu search (TS) algorithm escapes from local minima by taking a non-improving move and then forbidding the inverse move for a given number of iterations.

Another technique to escape from local minima is based on restarts (i.e., performing a few moves regardless they are good or bad).
Test on Utrecht-DenBosh dispatching area

- Utrecht-Den Bosch railway network (50 km long, including 21 station platforms)
- 40 running trains per hour (timetable 2007)
- Rolling stock connections are located in Zaltbommel and Den Bosch stations
- Rerouting is performed in stations and corridors (356 local routes)
Results on the compound CDR problem (1)  
[D’Ariano Transp. Science 2008]

Percentage of maximum consecutive delays for four ROMA(AGLIBRARY) config.

<table>
<thead>
<tr>
<th></th>
<th>Default Routing</th>
<th>Routing Optimization*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delay Max</td>
<td>Delay Avg</td>
</tr>
<tr>
<td>ARI + Default Routing</td>
<td>489.4</td>
<td>66.9</td>
</tr>
<tr>
<td>B&amp;B + Default Routing</td>
<td>279.8</td>
<td>50.4</td>
</tr>
</tbody>
</table>

*Routing Optimization by the local search algorithm
Results on the compound CDR problem (2)

Perturbations are multiple train delays

Disruptions are tracks which are blocked

*Routing Optimization by the tabu search algorithm
Presentation contents

- Introduction
- Mathematical models
- AGLIBRARY solver
- Alstom case study
- References
Alstom Strategy

Conflict Detection And Resolution
LOCAL

Current status

Timetable Manager
Infrastructure Manager
Rolling Stock Manager

ICONIS DSS

AGLibrary

Alternative routes near the conflicts detected

New schedule

Current status
Railway network (nearby London)
Example 1 of disruption
Test environment: East Coast Main Line

35 stations, 800 trains by day, 90 trains in peak hours
## Set of 10 small instances

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<th>Time horizon</th>
<th>Average number for resources per train</th>
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Computational results

Intel Core 2 Duo E6550 (2.33 GHz), 2 GB di RAM, Windows XP

Scheduling & routing problem (CDR problem) : 29 instances

**CPLEX (algorithm: 1 hour of computation):**
[MILP formulation solved by IBM LOG CPLEX MIP 12.0]
❖ 6 fails, 22 optimum, avg comp time (algo) best sol 1011.7 sec

**AGLIBRARY (algorithm: 20 sec of computation):**
[Branch & Bound (EJOR, 2007) + Tabu Search (TRpartB, 2010)]
❖ 0 fails, 21 optimum, avg comp time (algo) best sol 11.6 sec
CPLEX vs AGLIBRARY (scheduling & routing)

Maximum consecutive delay (in seconds)

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<th>medium instances</th>
<th>large instances</th>
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Legend:
- BB+TS
- CPLEX

Note: Instances marked with an asterisk (*) indicate failure.
**CPLEX vs AGLIBRARY (scheduling & routing)**

*Time solver to compute the best solution (in seconds)*

- **BB+TS**
- **CPLEX**

### Instance IDs

- **Small instances**
  - 1.1
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*Note: The table above shows the time taken by the solvers to compute the best solution for each instance ID, categorized by size (small, medium, large). The data points indicate whether the solution was successfully computed (BB+TS) or failed (CPLEX). The graph visualizes this data across different instance sets.*
Presentation contents

- Introduction
- Mathematical models
- AGLIBRARY solver
- Alstom case study
- References
A list of recent publications on railway operations research:


