Planning and Management in Urban Transit

Train rescheduling for urban rail transit systems under disruptions

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November 11, 2020

Outline

① Introduction

② Mathematical Formulation

③ Solution approach

④ Case study

⑤ Conclusions

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① **Introduction**

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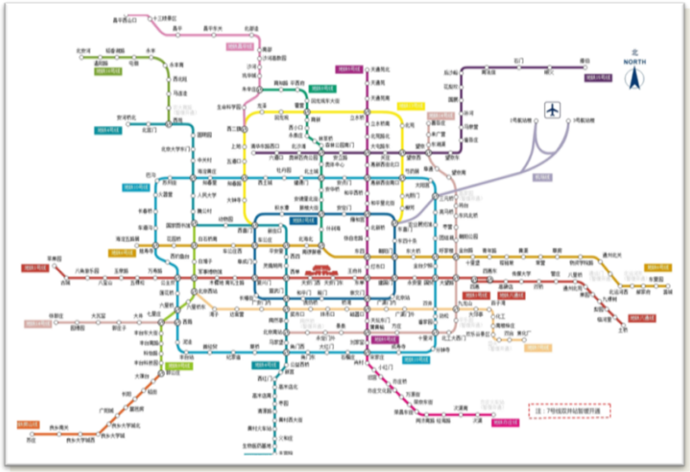
Introduction



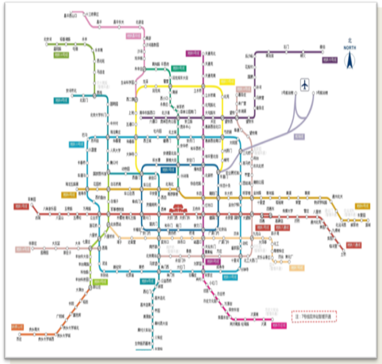
Lines operated separately

Massive passenger demand

High operational frequency

Low flexibility

Introduction



Lines operated separately

Massive passenger demand

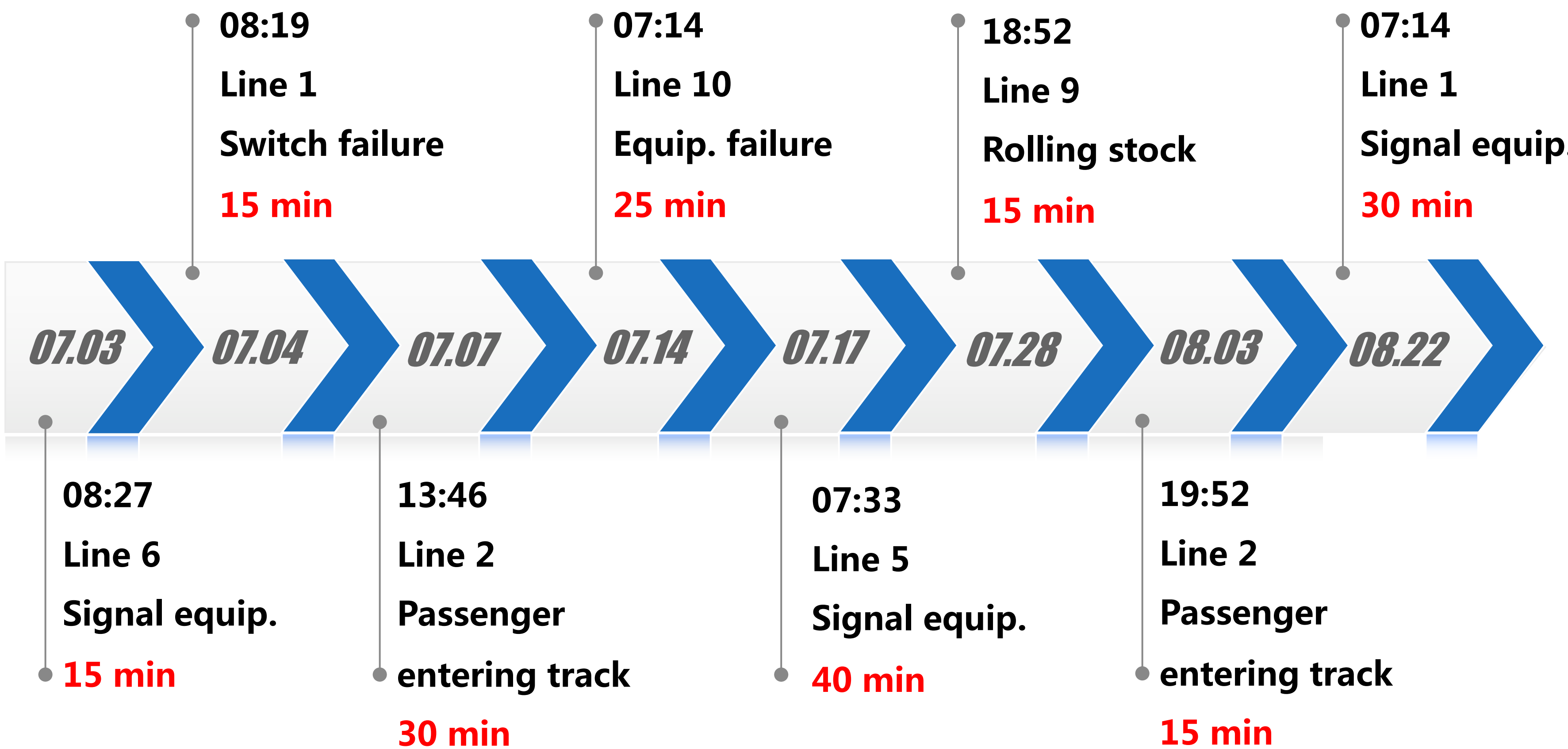
Disruption unavoidable
Serious affect on traffic

High operational frequency

Low flexibility

Introduction

8 disruptions (≥15 min) occurring at Beijing Metro in July and August , 2017



Introduction

Crowdedness

- Trains blocked in the middle of the track
- Passengers onboard may encounter safety issues

Passenger control

- Many passengers waiting in station
- Control measures needed to let passengers queueing outside the station

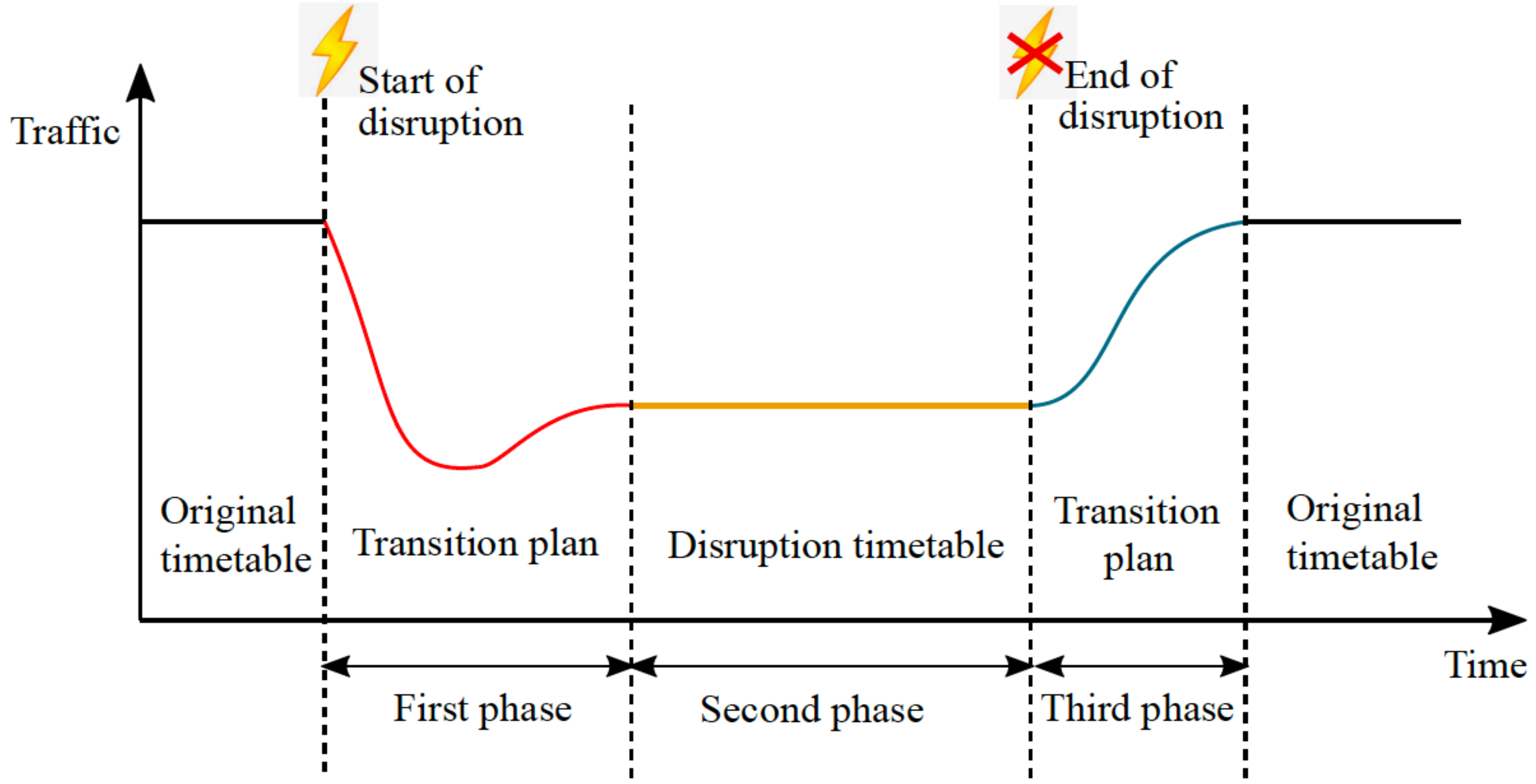
Paralyzed traffic

- Disorder in current line, adjacent lines, and metro networks
- Big pressure for bus systems and traffic jam in road networks

Effective rescheduling is important for disruption management

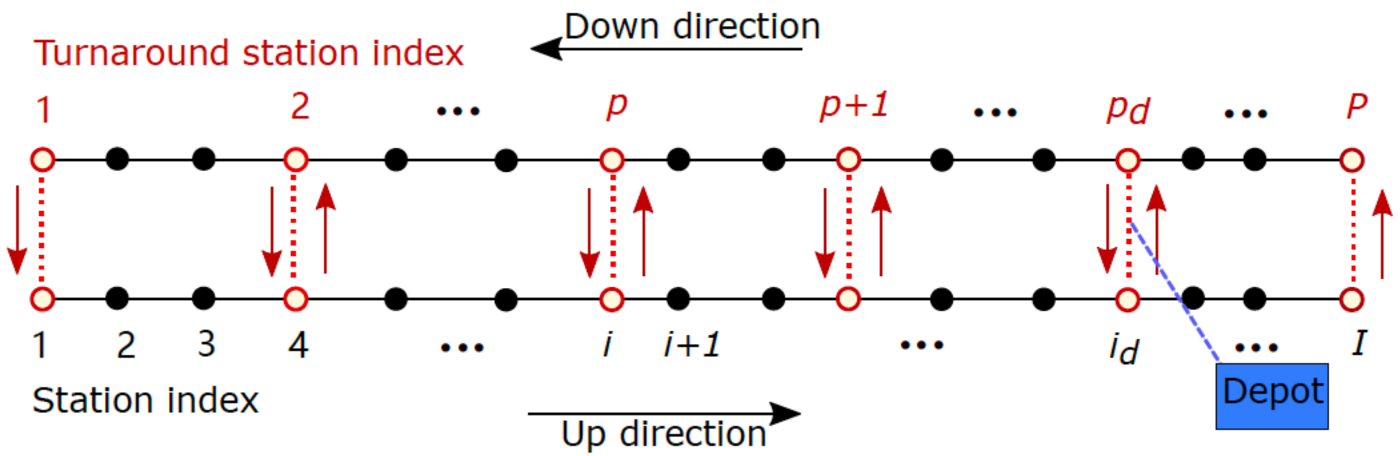
Introduction

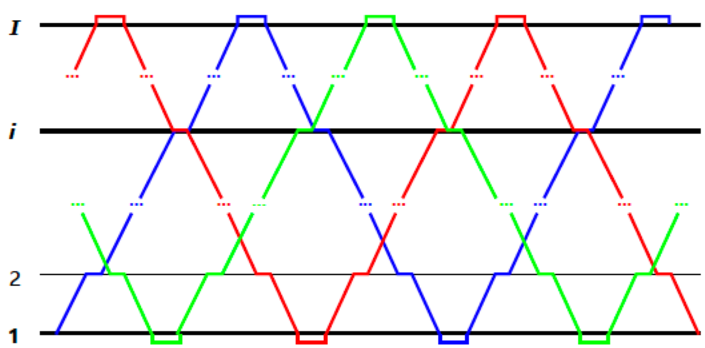
Three phase of the disruption management



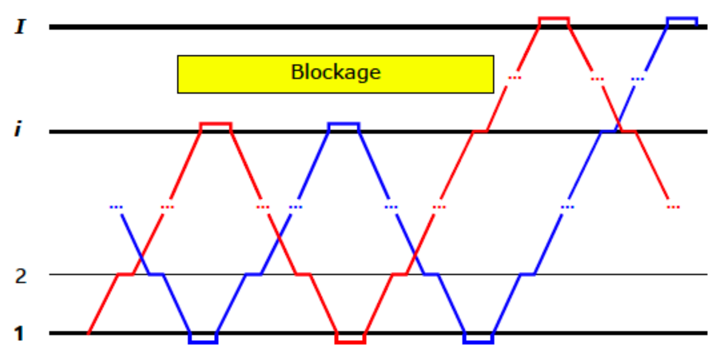
Outline

Mathematical formulation





Normal operation



Short-turning under disruption

Mathematical formulation

Assumptions

- Trains do not meet and overtake each other due to the station layout
- Each platform can only accommodate one train at a time
- Trains are not allowed to stop in the open tracks (tunnels)
- Trains that enter the blockage area before disruption can pass through the area
- Trains can arrive at and depart from stations earlier than the planned times
- Disruptions occur in off-peak hours and the metro line is not saturated

Mathematical formulation

Modeling

- **Objective functions**
  - **deviations w.r.t. planned timetable**
  - **Service cancellations (partial cancellations involved)**
  - **Headway variations between neighboring services**

$$\min \quad Z = w_1 \cdot \frac{Z_{\text{deviation}}}{Z_{\text{deviation,nom}}} + w_2 \cdot \frac{Z_{\text{cancel}}}{Z_{\text{cancel,nom}}} + w_3 \cdot \frac{Z_{\text{headway}}}{Z_{\text{headway,nom}}}$$

$$Z_{\text{deviation}} = \sum_{f \in \mathbf{F}} \sum_{i \in \mathbf{I}, i \neq 1} y^{\text{up}}_{f,i-1,i} \left| d^{\text{up}}_{f,i} - \bar{d}^{\text{up}}_{f,i} \right| + \sum_{g \in \mathbf{G}} \sum_{i \in \mathbf{I}, i \neq I} y^{\text{dn}}_{g,i+1,i} \left| d^{\text{dn}}_{g,i} - \bar{d}^{\text{dn}}_{g,i} \right|$$

$$+ \sum_{f \in \mathbf{F}} \sum_{i \in \mathbf{I}, i \neq 1} y^{\text{up}}_{f,i-1,i} \left| a^{\text{up}}_{f,i} - \bar{a}^{\text{up}}_{f,i} \right| + \sum_{g \in \mathbf{G}} \sum_{i \in \mathbf{I}, i \neq I} y^{\text{dn}}_{g,i+1,i} \left| a^{\text{dn}}_{g,i} - \bar{a}^{\text{dn}}_{g,i} \right|,$$

$$Z_{\text{cancel}} = \sum_{f \in \mathbf{F}} \sum_{p \in \mathbf{P}, p \neq P} \left(\bar{x}^{\text{up}}_{f,p,p+1} - x^{\text{up}}_{f,p,p+1} \right) + \sum_{g \in \mathbf{G}} \sum_{p \in \mathbf{P}, p \neq 1} \left(\bar{x}^{\text{dn}}_{g,p,p-1} - x^{\text{dn}}_{g,p,p-1} \right)$$

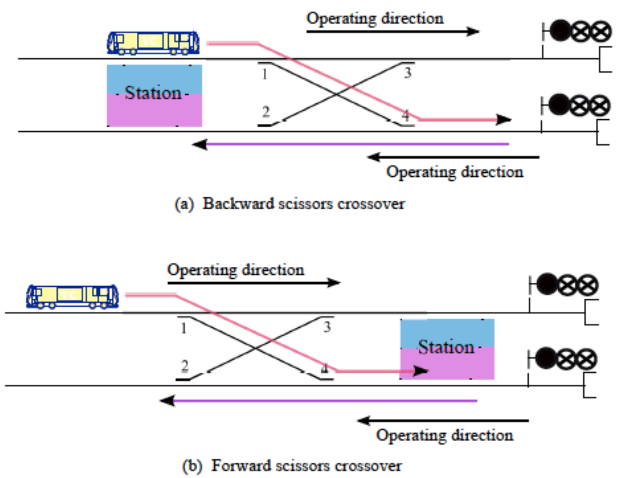
$$Z_{\text{headway}} = \sum_{f \in \mathbf{F}, f \neq 1, f \neq F} \sum_{i \in \mathbf{I}, i \neq 1} \left(y^{\text{up}}_{f-1,i-1,i} y^{\text{up}}_{f,i-1,i} y^{\text{up}}_{f+1,i-1,i} \left(d^{\text{up}}_{f+1,i} + d^{\text{up}}_{f-1,i} - 2 d^{\text{up}}_{f,i} \right) \right)$$

$$+ \sum_{g \in \mathbf{G}, g \neq 1, g \neq G} \sum_{i \in \mathbf{I}, i \neq I} \left(y^{\text{dn}}_{g-1,i+1,i} y^{\text{dn}}_{g,i+1,i} y^{\text{dn}}_{g+1,i+1,i} \left(d^{\text{dn}}_{g+1,i} + d^{\text{dn}}_{g-1,i} - 2 d^{\text{dn}}_{g,i} \right) \right)$$

Headway based timetabling

Modeling

- **Operational constraints**
  - Departure/arrival time constraints
  - Turnaround constraints
  - Headway constraints
  - Rolling stock circulation constraints
  - Number of available rolling stocks



$$a^{\mathrm{up}}_{f,i} = y^{\mathrm{up}}_{f,i-1,i}(d^{\mathrm{up}}_{f,i-1} + r^{\mathrm{up}}_{f,i-1,i})$$

$$a^{\mathrm{dn}}_{f,i} = y^{\mathrm{dn}}_{f,i+1,i}(d^{\mathrm{dn}}_{f,i+1} + r^{\mathrm{dn}}_{f,i+1,i})$$

$$d^{\mathrm{up}}_{f,i} = y^{\mathrm{up}}_{f,i-1,i}(a^{\mathrm{up}}_{f,i} + w^{\mathrm{up}}_{f,i})$$

$$d^{\mathrm{dn}}_{f,i} = y^{\mathrm{dn}}_{f,i+1,i}(a^{\mathrm{dn}}_{f,i} + w^{\mathrm{dn}}_{f,i})$$

$$a^{\mathrm{up}}_{f,i} = y^{\mathrm{up}}_{f,i-1,i}(d^{\mathrm{up}}_{f,i-1} + r^{\mathrm{up}}_{f,i-1,i})$$

$$a^{\mathrm{up}}_{f,i} = \sum_{g\in\mathbf{G}} \beta^{\mathrm{dn}}_{g,f,p}(d^{\mathrm{dn}}_{g,i} + t^{\mathrm{turn}}_{g,p})$$

$$t^{\mathrm{turn,min}}_{p} \le t^{\mathrm{turn}}_{g,p} \le t^{\mathrm{turn,max}}_{p}$$

$$d^{\mathrm{up}}_{f,i} = \sum_{g\in\mathbf{G}} \beta^{\mathrm{up}}_{f,g,p}(a^{\mathrm{up}}_{f,i} + w^{\mathrm{up}}_{f,i} + w_{\mathrm{c,p}})$$

$$c^{\mathrm{up}}_{f-\ell,i-1,i} c^{\mathrm{up}}_{f,i-1,i}(d^{\mathrm{up}}_{f,i} - d^{\mathrm{up}}_{f-\ell,i}) \ge c^{\mathrm{up}}_{f-\ell,i-1,i} c^{\mathrm{up}}_{f,i-1,i} h_{\min}$$

$$\sum_{g} \beta^{\mathrm{up}}_{f,g,p_d} + x^{\mathrm{up}}_{f,p_d,p_d+1} + \alpha^{\mathrm{up}}_{f,p_d} = c^{\mathrm{up}}_{f,p_d}$$

$$a^{\mathrm{up}}_{f',i_{\mathrm{d}}} - a^{\mathrm{up}}_{f,i_{\mathrm{d}}} \ge \varepsilon + (-a^{\mathrm{LB}}_{f} - \varepsilon)\cdot\delta^{\mathrm{up}}_{f,f'}$$

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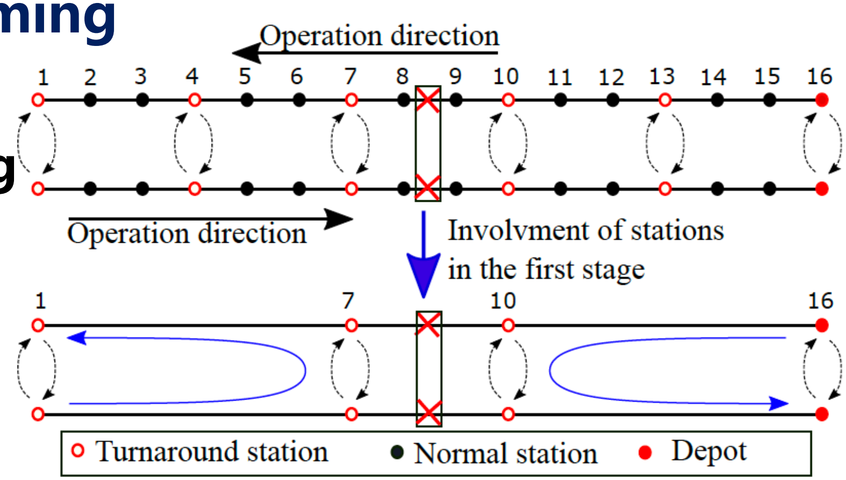
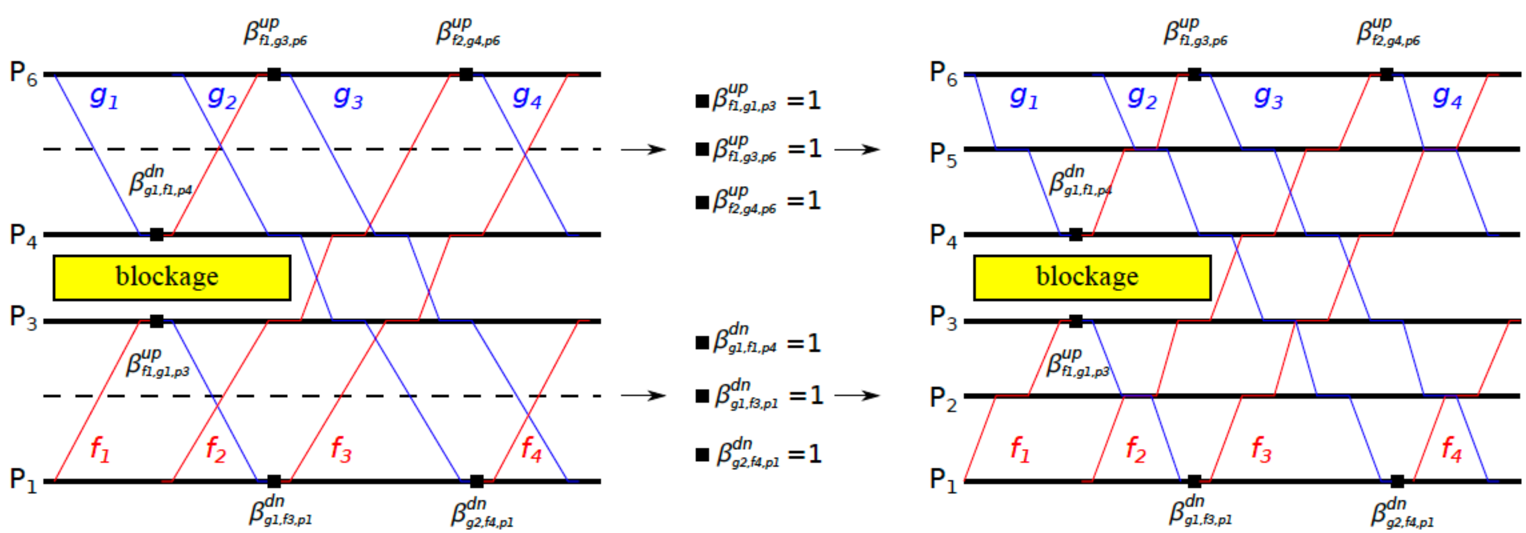
Solution approach

Mixed integer nonlinear programming

- Linearization transformations
- Mixed integer linear programming
- Two-stage approach
- Filtering constraints





First stage

Second stage

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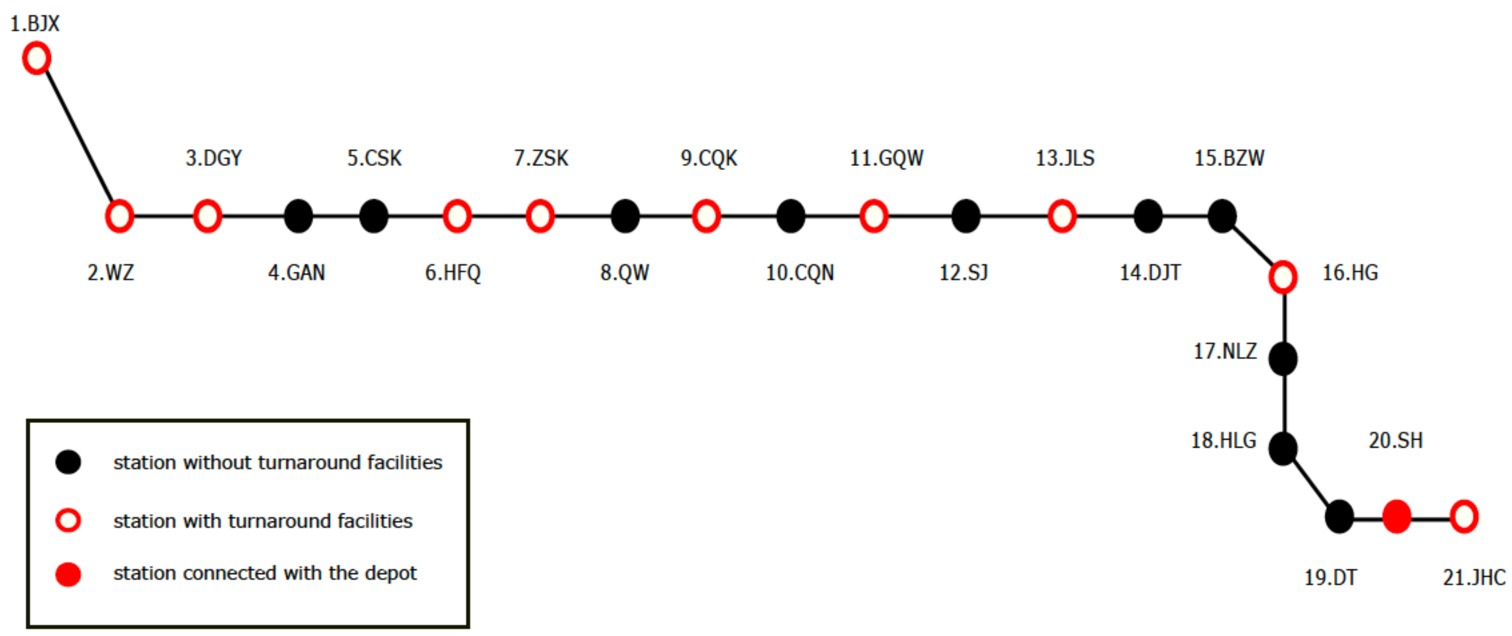
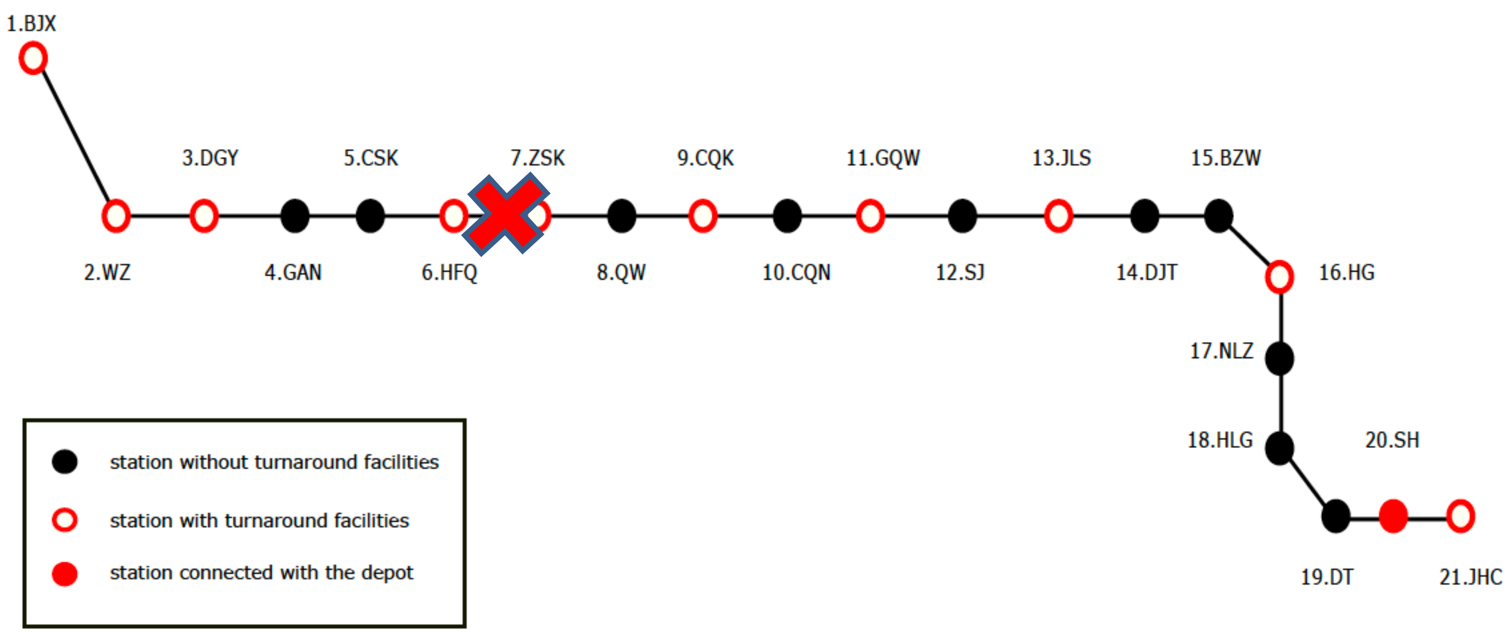
Case study

- **Beijing Subway Line 7**
  - 21 stations
  - 11 stations have turnaround facilities
  - BJX and JHC are terminal stations
  - Depot is connected with SH stations



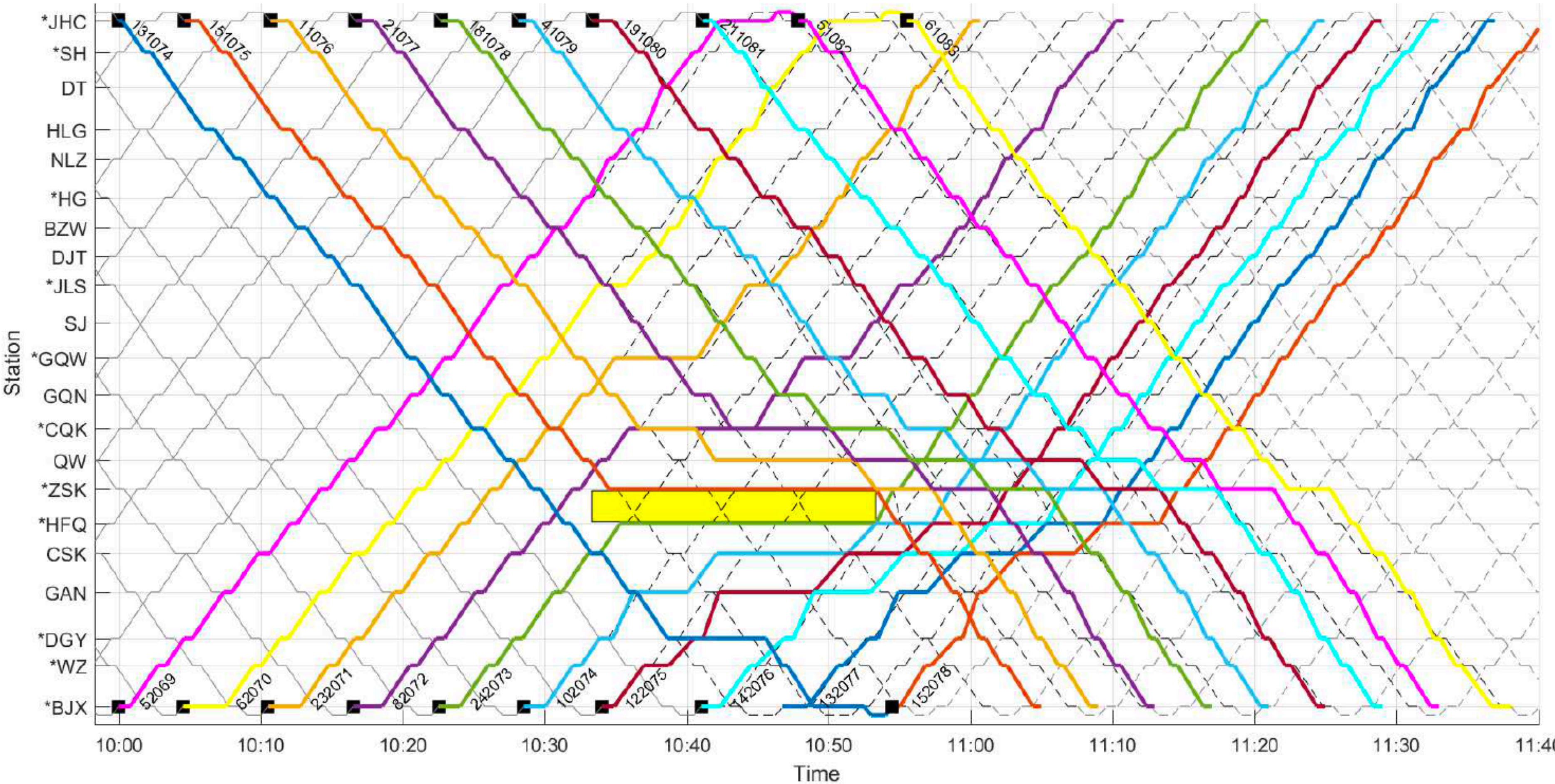
Case study

- Track blockage between HFQ and ZSK



Case study

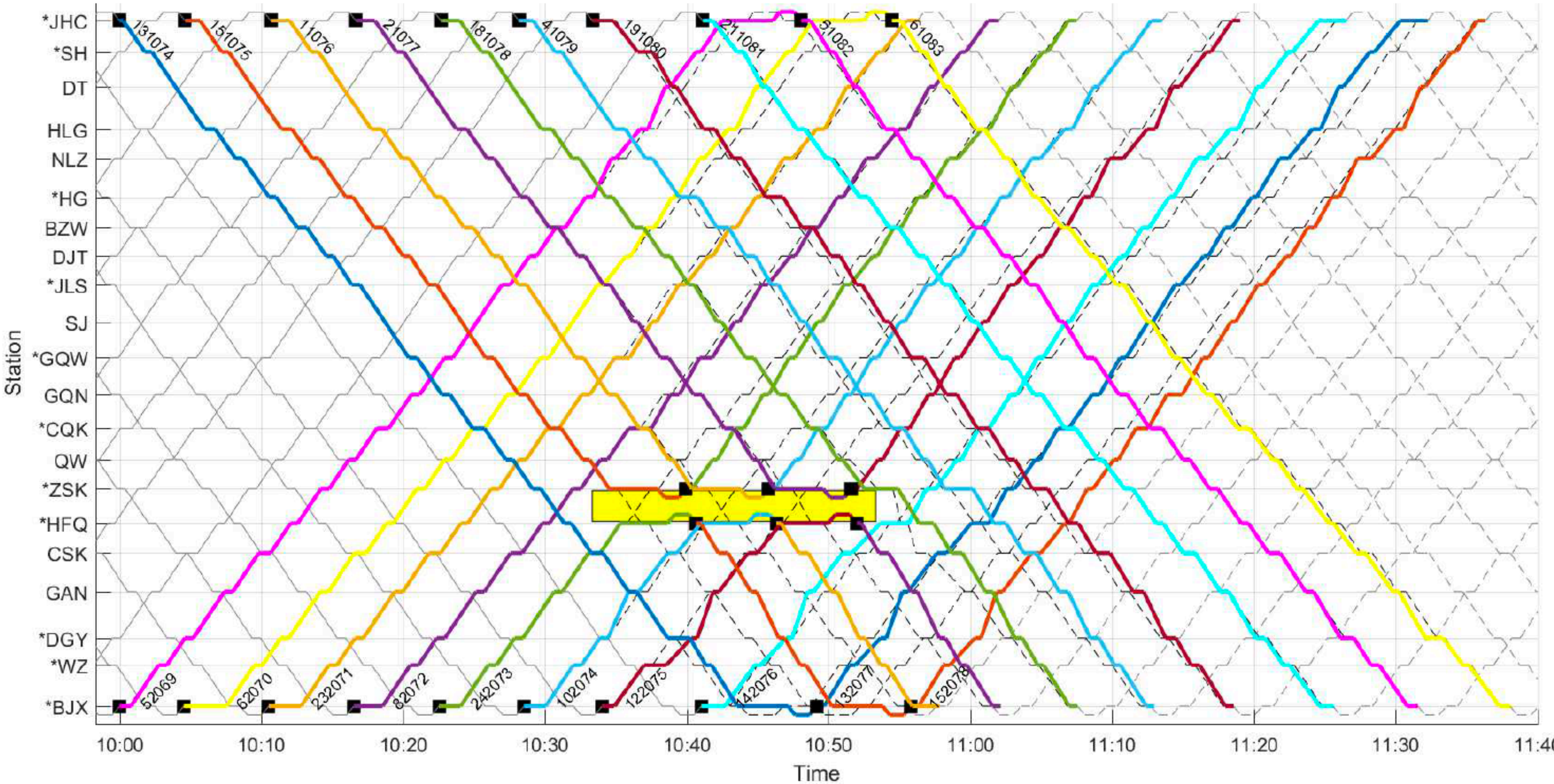
Track blockage between HFQ and ZSK



Train rescheduling solution obtained by holding strategy (holding all trains)

Case study

- **Track blockage between HFQ and ZSK**



Train rescheduling solution obtained by MILP strategy with filtering constraints

Case study

Track blockage between HFQ and ZSK

Performance comparison between different approaches

Solution approaches	Computation time (s)	Objective function value	Timetable deviations (s)	Headway variations (s)	Number of cancellations
MILP without filtering	2193	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6
MILP with filtering	1369	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6
Two-stage without filtering	250	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6
Two-stage with filtering	42	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6
Holding-4	10	5.507	$2.285 \cdot 10^4$	$7.602 \cdot 10^3$	4
Holding-2	69	11.563	$5.930 \cdot 10^4$	$1.452 \cdot 10^4$	2
Holding-0	76	21.570	$1.156 \cdot 10^5$	$2.710 \cdot 10^4$	0

Case study

Track blockage between HFQ and ZSK

Performance comparison for different disruption durations

Disruption time period	Solution approach	Objective func. values	Timetable deviations (s)	Headway variations (s)	Number of cancellations	Computation time (s)
(10:33, 10:43)	MILP without filtering	3.36	$1.234 \cdot 10^4$	$5.702 \cdot 10^3$	2	8604
	MILP with filtering	3.36	$1.234 \cdot 10^4$	$5.702 \cdot 10^3$	2	1045
	Two-stage without filtering	3.36	$1.234 \cdot 10^4$	$5.702 \cdot 10^3$	2	101
	Two-stage with filtering	3.36	$1.234 \cdot 10^4$	$5.702 \cdot 10^3$	2	16
(10:33, 10:53)	MILP without filtering	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6	6871
	MILP with filtering	4.232	$1.423 \cdot 10^4$	$5.563 \cdot 10^3$	6	2286
	Two-stage without filtering	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6	134
	Two-stage with filtering	4.232	$1.424 \cdot 10^4$	$5.560 \cdot 10^3$	6	42
(10:33, 11:03)	MILP without filtering	5.282	$1.723 \cdot 10^4$	$5.578 \cdot 10^3$	10	8202
	MILP with filtering	5.282	$1.723 \cdot 10^4$	$5.578 \cdot 10^3$	10	2927
	Two-stage without filtering	5.396	$2.103 \cdot 10^4$	$5.466 \cdot 10^3$	8	158
	Two-stage with filtering	5.396	$2.103 \cdot 10^4$	$5.466 \cdot 10^3$	8	110

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Conclusions

- Train rescheduling for completed blockage in metro lines
  - Integration of train rescheduling and rolling stock circulation planning
  - Mathematical models and effective solution approaches
  - Decision support for dispatchers

- Limitation of this research
  - Other types of disruptions, partial blockage, slowly moving train
  - Joint optimization of passenger control strategy and train rescheduling
  - …

Thank you!

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