A Mixed Integer Programming Model For Freight Train Travel-Time Estimation By Minimizing Delay of Trains

The 2020 RAS Problem Solving Competition

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Outline

Motivation

- Management of railway networks is extremely challenging
- Train timetabling or scheduling is a crucial problem
- Operational constraints and network structure should be considered
- Train delays occur due to various reasons
- Travel-time estimation by considering delays and restrictions

Literature

- Travel time and delay estimation Approaches:
  - **Optimization methods**: finding an optimal train timetable
  - **Simulation models**: Building a prototype of the network
  - **Data-driven methods**: Applying statistical and machine learning techniques

Our problem

- **Goal:** Estimating travel-time of trains by determining the arrival and departure time
- The followings are given:
  - Network characteristics
  - Business constraints
  - Resource availabilities
  - Planned train schedule
  - Delays

Business constraints

- Three type of delay: yard, crew and locomotive delay.
- There are single tracks, double tracks and 4-track sections
- Siding can occur at siding tracks and yard tracks
- High-priority trains are expected to have a smaller amount of delay
- Delay of a train has an impact on other trains departure/arrival

Important assumptions

- Yard tracks can be utilized for siding
- 5-minute penalty delay for siding
- Pick up/drop off activity at a non-yard station performed at a separate industrial spur (Only one train can use it at any time)
- Arrival/departure can occur earlier than planned time at intermediate stations
- Early arrival at stop stations is allowed while departure must be at or after planned time and imposing the stop duration
- distribution of the delays is assumed to be $LogN(\mu, \sigma^2)$. We assume that all trains have a delay equal to the mean of the corresponding distribution which is $e^{(\mu + \frac{\sigma^2}{2})}$.

Notations and parameters

Parameter	Description
I	Set of trains
S	Set of stations
J	Set of arcs in the railways network
H	Set of high priority trains
L	Set of low priority trains
$A^{(1)}$	Set of single-track arcs
$A^{(2)}$	Set of double-track arcs
$A^{(4)}$	Set of 4-track arcs
B	Set of stations having siding tracks
U	Set of non-yards stations having drop off/picking up activities
V_i	Set of the stations visited by train i ; $\forall i \in I$ (it does not include origin and destination)
C_i	Set of stations with a yard flag visited by train i; $\forall i \in I$
W_i	Set of the stations where train i changes the crew; $\forall i \in I$
$C_i^{'}$	Set of non-yard stations with a yard flag visited by train i; $\forall i \in I$
K_i	Set of the stop stations for train i; $\forall i \in I$
E_i	Set of arcs visited by train i; $\forall i \in I$
G_s	Set of trains passing through station s; $\forall s \in S$
$G_s^{'}$	Set of trains passing through station s and having a drop off/picking up order at s; $\forall s \in S$
O_s	Set of trains whose origin is station s; $\forall s \in S$
F_s	Set of trains whose final destination is station s; $\forall s \in S$
$T_j^{(1)}$	Set of trains passing through arc $j = s_1 s_2$ going from station s_1 to station s_2; $\forall j \in J$
$T_j^{(2)}$	Set of trains passing through arc $j = s_1 s_2$ going from station s_2 to station s_1; $\forall j \in J$

Notations and parameters

Parameter	Description
o_i	Original station of train i; $\forall i \in I$
f_i	Final destination of train i; $\forall i \in I$
$a_{i,s}$	Planned arrival of train i at station s; $\forall i \in I, s \in V_i \setminus \{o_i\}$
$d_{i,s}$	Planned departure of train i from station s; $\forall i \in I, s \in V_i \setminus \{f_i\}$
$t_{i,j}$	Travel time of train i on arc j; $\forall i \in I, j \in E_i$
n_s	Number of siding tracks at station s; $\forall s \in S$
m_s	Number of mainline tracks at station s; $\forall s \in S$
α	Penalty time delay due to siding
β	Average delay due to yard activity
γ	Average delay due to crew change
θ	Average delay due to locomotive unavailability
λ	Penalty considered for the delay of high priority trains
M	A sufficiently large number

Decision variables

- $x_{i,s}$: Arrival time of train i at station s; $\forall i \in I, s \in V_i \cup \{f_i\}$.
- $y_{i,s}$: Departure time of train i from station s; $\forall i \in I, s \in V_i \cup \{o_i\}$.
- $\delta^+_{i,s}$: Positive delay of train i at station s; $\forall i \in I, s \in K_i \cup \{f_i\}$.
- $\delta^-_{i,s}$: Negative delay of train i at station s; $\forall i \in I, s \in K_i \cup \{f_i\}$.
- p_{s,i_1,i_2}: A binary variable which is equal to 1 if train i_1 arrives at station s before arrival of train i_2, and it is equal to zero otherwise; $\forall s \in S, i_1, i_2 \in (G_s \cup F_s)|i_1 < i_2$.
- q_{s,i_1,i_2}: A binary variable which is equal to 1 if train i_1 departs from station s before departure of train i_2, and it is equal to zero otherwise; $\forall s \in S, i_1, i_2 \in (G_s \cup O_s)|i_1 < i_2$.
- r_{s,i_1,i_2}: A binary variable which is equal to 1 if train i_1 departs station s before arrival of train i_2, and it is equal to zero otherwise; $\forall s \in S, i_1 \in (G_s \cup O_s), i_2 \in (G_s \cup F_s)|i_1 \neq i_2$.

Decision variables

- $z_{s,i}$: A binary variable which is equal to 1 if train i does a siding at station s, and it is equal to zero otherwise; $\forall s \in S, i \in (G_s \setminus G_s^{'})$.
- h_{s,i_1,i_2}: A binary variable which is equal to 1 if train i_2 is on the mainline of station s when train i_1 arrives at s, and it is equal to zero otherwise; $\forall s \in B, i_1, i_2 \in (G_s \setminus G_s^{'}) | i_1 \neq i_2$.
- $w_{j,i}^{(1,1)}$: A binary variable which is equal to 1 if train i passing through arc j in direction 1 uses the first track assigned to this direction, and it is equal to zero otherwise; $\forall j \in A^{(4)}, i \in T_j^{(1)}$.
- $w_{j,i}^{(1,2)}$: A binary variable which is equal to 1 if train i passing through arc j in direction 1 uses the second track assigned to this direction, and it is equal to zero otherwise; $\forall j \in A^{(4)}, i \in T_j^{(1)}$.
- $w_{j,i}^{(2,1)}$: A binary variable which is equal to 1 if train i passing through arc j in direction 2 uses the first track assigned to this direction, and it is equal to zero otherwise; $\forall j \in A^{(4)}, i \in T_j^{(2)}$.
- $w_{j,i}^{(2,2)}$: A binary variable which is equal to 1 if train i passing through arc j in direction 2 uses the second track assigned to this direction, and it is equal to zero otherwise; $\forall j \in A^{(4)}, i \in T_j^{(2)}$.

Constraints

- **Departure and arrival time constraints:**

$$
\begin{aligned}
&y_{i,o_i} \geq d_{i,o_i} + \theta && \forall i \in I, && (1)\\
&y_{i,s} \geq x_{i,s} + (d_{i,s} - a_{i,s}) && \forall i \in I, s \in V_i, && (2)\\
&y_{i,s} \geq x_{i,s} + \beta && \forall i \in I, s \in C_i, && (3)\\
&y_{i,s} \geq x_{i,s} + \gamma && \forall i \in I, s \in W_i, && (4)\\
&y_{i,s} \geq x_{i,s} + \alpha z_{i,s} && \forall i \in I, s \in (V_i \cap B) \setminus C_i, && (5)\\
&y_{i,s} \geq d_{i,s} && \forall i \in I, s \in K_i, && (6)\\
&x_{i,s_2} = y_{i,s_1} + t_{i,j} && \forall i \in I, j = s_1 s_2 \in E_i. && (7)
\end{aligned}
$$

- **Delay calculation constraint:**

$$
\delta^+_{i,s} - \delta^-_{i,s} = x_{i,s} - a_{i,s} \qquad \forall i \in I, s \in K_i \cup \{f_i\}. \qquad (8)
$$

Constraints

- **Arrival and departure order constraints:**

$$x_{i_2,s} - x_{i_1,s} \leq Mp_{s,i_1,i_2} \qquad \forall s \in S, i_1, i_2 \in G_s \cup F_s | i_1 < i_2, \tag{9}$$

$$x_{i_1,s} - x_{i_2,s} \leq M\left(1 - p_{s,i_1,i_2}\right) \qquad \forall s \in S, i_1, i_2 \in G_s \cup F_s | i_1 < i_2, \tag{10}$$

$$y_{i_2,s} - y_{i_1,s} \leq Mq_{s,i_1,i_2} \qquad \forall s \in S, i_1, i_2 \in G_s \cup O_s | i_1 < i_2, \tag{11}$$

$$y_{i_1,s} - y_{i_2,s} \leq M\left(1 - q_{s,i_1,i_2}\right) \qquad \forall s \in S, i_1, i_2 \in G_s \cup O_s | i_1 < i_2, \tag{12}$$

$$x_{i_2,s} - y_{i_1,s} \leq Mr_{s,i_1,i_2} \qquad \forall s \in S, i_1 \in (G_s \cup O_s), i_2 \in (G_s \cup F_s) | i_1 \neq i_2, \tag{13}$$

$$y_{i_1,s} - x_{i_2,s} \leq M\left(1 - r_{s,i_1,i_2}\right) \qquad \forall s \in S, i_1 \in (G_s \cup O_s), i_2 \in (G_s \cup F_s) | i_1 \neq i_2. \tag{14}$$

Constraints

- **Siding and overtake constraints:**

$$z_{s,i} = 0 \qquad \forall s \in (S \setminus B), i \in (G_s \setminus G_s^{'}), \tag{15}$$

$$h_{s,i_1,i_2} \geq (1 - p_{s,i_1,i_2}) - r_{s,i_2,i_1} - z_{s,i_2} \qquad \forall s \in S, i_1, i_2 \in (G_s \setminus G_s^{'}) | i_1 < i_2, \tag{16}$$

$$h_{s,i_1,i_2} \geq p_{s,i_2,i_1} - r_{s,i_2,i_1} - z_{s,i_2} \qquad \forall s \in S, i_1, i_2 \in (G_s \setminus G_s^{'}) | i_2 < i_1, \tag{17}$$

$$z_{s,i_1} \geq 1 + \sum_{\substack{i_2 \in (G_s \setminus G_s^{'}) \\ i_2 \neq i_1}} h_{s,i_1,i_2} - m_s \qquad \forall s \in S, i_1 \in (G_s \setminus G_s^{'}), \tag{18}$$

$$z_{s,i_1} \leq n_s + m_s - \sum_{\substack{i_2 \in G_s \setminus G_s^{'} \\ i_1 < i_2}} (1 - p_{s,i_1,i_2}) - \sum_{\substack{i_2 \in G_s \setminus G_s^{'} \\ i_2 < i_1}} p_{s,i_2,i_1} + \sum_{\substack{i_2 \in G_s \setminus G_s^{'} \\ i_2 \neq i_1}} r_{s,i_2,i_1} \qquad \forall s \in B, i_1 \in (G_s \setminus G_s^{'}). \tag{19}$$

Constraints

- **Single-track capacity constraints:**

$$y_{i_1,s_1} \geq x_{i_2,s_1} - M(1 - r_{s_2,i_2,i_1}) \qquad \forall j = s_1 s_2 \in A^{(1)}, i_1 \in T_j^{(1)}, i_2 \in T_j^{(2)}, \qquad (20)$$

$$y_{i_2,s_2} \geq x_{i_1,s_2} - M(1 - r_{s_1,i_1,i_2}) \qquad \forall j = s_1 s_2 \in A^{(1)}, i_1 \in T_j^{(1)}, i_2 \in T_j^{(2)}, \qquad (21)$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M q_{s_1,i_1,i_2} \qquad \forall j = s_1 s_2 \in A^{(1)}, i_1, i_2 \in T_j^{(1)} | i_1 < i_2, \qquad (22)$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M(1 - q_{s_1,i_2,i_1}) \qquad \forall j = s_1 s_2 \in A^{(1)}, i_1, i_2 \in T_j^{(1)} | i_2 < i_1, \qquad (23)$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M q_{s_2,i_1,i_2} \qquad \forall j = s_1 s_2 \in A^{(1)}, i_1, i_2 \in T_j^{(2)} | i_1 < i_2, \qquad (24)$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M(1 - q_{s_2,i_2,i_1}) \qquad \forall j = s_1 s_2 \in A^{(1)}, i_1, i_2 \in T_j^{(2)} | i_2 < i_1. \qquad (25)$$

Constraints

- **Double-track capacity constraints:**

$$y_{i_1,s_1} \geq x_{i_2,s_2} - Mq_{s_1,i_1,i_2} \qquad \forall j = s_1 s_2 \in A^{(2)}, i_1, i_2 \in T_j^{(1)} | i_1 < i_2, \tag{26}$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M(1 - q_{s_1,i_2,i_1}) \qquad \forall j = s_1 s_2 \in A^{(2)}, i_1, i_2 \in T_j^{(1)} | i_2 < i_1, \tag{27}$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - Mq_{s_2,i_1,i_2} \qquad \forall j = s_1 s_2 \in A^{(2)}, i_1, i_2 \in T_j^{(2)} | i_1 < i_2, \tag{28}$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M(1 - q_{s_2,i_2,i_1}) \qquad \forall j = s_1 s_2 \in A^{(2)}, i_1, i_2 \in T_j^{(2)} | i_2 < i_1. \tag{29}$$

Constraints

- **4-track section capacity constraints:**

$$w_{j,i}^{(1,1)} + w_{j,i}^{(1,2)} = 1 \qquad \forall j \in A^{(4)}, i \in T_j^{(1)}, \tag{30}$$

$$w_{j,i}^{(2,1)} + w_{j,i}^{(2,2)} = 1 \qquad \forall j \in A^{(4)}, i \in T_j^{(2)}, \tag{31}$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M\left(2 + q_{s_1,i_1,i_2} - w_{j,i_1}^{(1,1)} - w_{j,i_2}^{(1,1)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(1)} | i_1 < i_2, \tag{32}$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M\left(2 + q_{s_1,i_1,i_2} - w_{j,i_1}^{(1,2)} - w_{j,i_2}^{(1,2)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(1)} | i_1 < i_2, \tag{33}$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M\left(3 - q_{s_1,i_2,i_1} - w_{j,i_1}^{(1,1)} - w_{j,i_2}^{(1,1)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(1)} | i_2 < i_1, \tag{34}$$

$$y_{i_1,s_1} \geq x_{i_2,s_2} - M\left(3 - q_{s_1,i_2,i_1} - w_{j,i_1}^{(1,2)} - w_{j,i_2}^{(1,2)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(1)} | i_2 < i_1, \tag{35}$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M\left(2 + q_{s_2,i_1,i_2} - w_{j,i_1}^{(2,1)} - w_{j,i_2}^{(2,1)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(2)} | i_1 < i_2, \tag{36}$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M\left(2 + q_{s_2,i_1,i_2} - w_{j,i_1}^{(2,2)} - w_{j,i_2}^{(2,2)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(2)} | i_1 < i_2, \tag{37}$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M\left(3 - q_{s_2,i_2,i_1} - w_{j,i_1}^{(2,1)} - w_{j,i_2}^{(2,1)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(2)} | i_2 < i_1, \tag{38}$$

$$y_{i_1,s_2} \geq x_{i_2,s_1} - M\left(3 - q_{s_2,i_2,i_1} - w_{j,i_1}^{(2,2)} - w_{j,i_2}^{(2,2)}\right) \qquad \forall j = s_1 s_2 \in A^{(4)}, i_1, i_2 \in T_j^{(2)} | i_2 < i_1. \tag{39}$$

Constraints

- **Dropping off/picking up capacity constraint for non-yard stations:**

$$x_{i_1,s} \geq y_{i_2,s} - Mp_{s,i_1,i_2} \qquad \forall s \in U, i_1, i_2 \in G_s^{'} | i_1 < i_2, \tag{40}$$

$$x_{i_1,s} \geq y_{i_2,s} - M(1 - p_{s,i_2,i_1}) \qquad \forall s \in U, i_1, i_2 \in G_s^{'} | i_2 < i_1. \tag{41}$$

Objective function

- The objective shown as (42) is minimizing the total penalized delay at the stop stations and final destination of trains:

$$\min \quad \sum_{i \in H} \sum_{s \in (K_i \cup \{f_i\})} \lambda \delta^{+}_{i,s} + \sum_{i \in L} \sum_{s \in (K_i \cup \{f_i\})} \delta^{+}_{i,s}. \tag{42}$$

Test instance

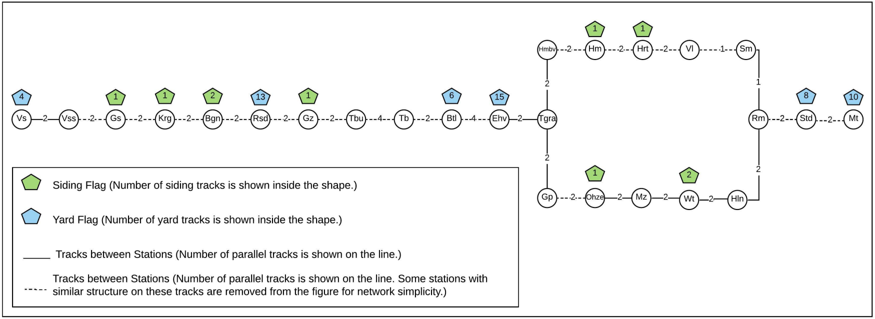

Figure: **Railway network diagram.** There are four routes with 61 stations the the provided network. Among 61 tracks connecting the stations, 4 are single-track, 6 are 4-track and the rest are double-track.

Numerical results

- Our model is coded in Julia using JuMP package and it is solved by Gurobi 8.1.1 on a Core-i7 computer with 8 GB RAM.
- MIPGap is set to 0.01 to terminate the solver at a desirable solution.
- Constraints (16)-(29) and (32)-(41) are considered as lazy constraints.

Day	Trains	Con vars	Bin vars	Total const	Lazy const	Time (s)	MIP Gap
1	211	14820	1050473	2202392	518096	4003	0.0025
2	212	15032	1073822	2251460	528983	2866	0.0081

Table: Results obtained by implementing our MIP model on the validation data set

Numerical results

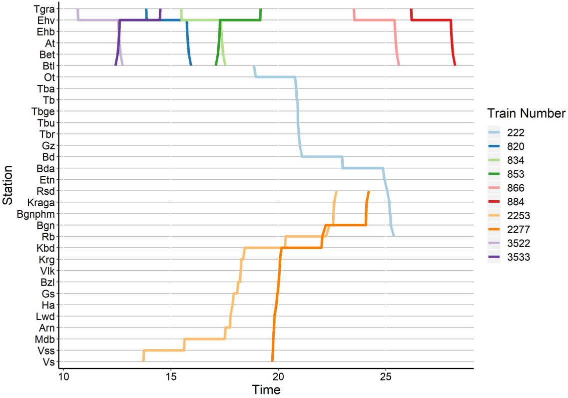

Figure: Train timetables of the west route for the first day.

Numerical results

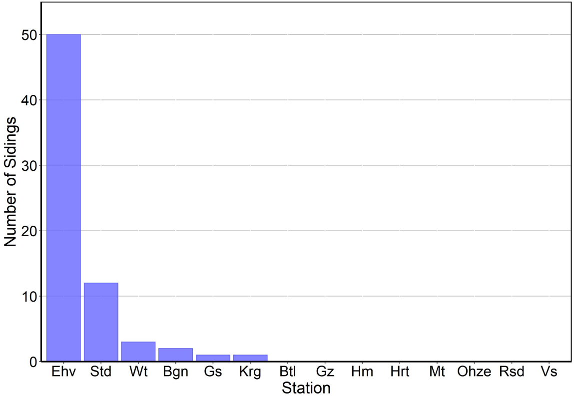

Figure: Number of sidings at the stations with siding or yard tracks in day 1.

Numerical results

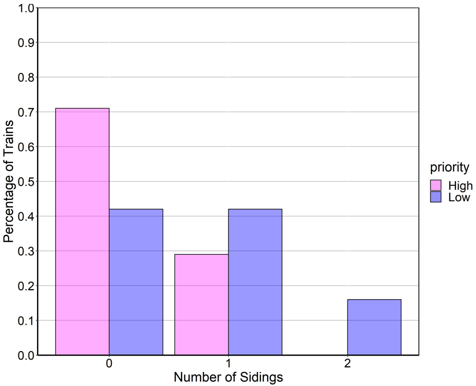

Figure: Percentage of trains with high and low priority which had different number of sidings in day 1.

Numerical results

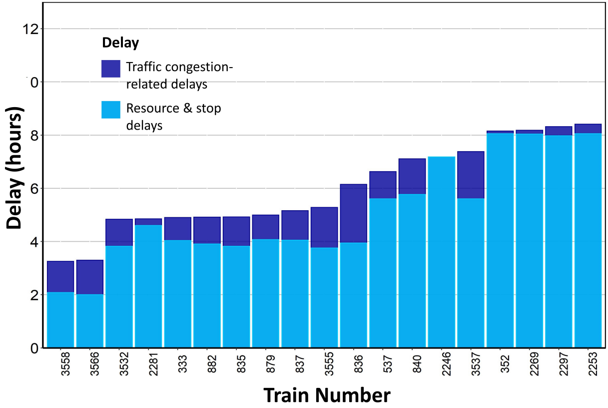

Figure: Total delay and delay due to resource unavailability and stops for trains in day 1.

Summary and future study

- Our optimization approach enables us to consider all restrictions in our model and can perfectly formulate this problem.
- We evaluate our model by implementing it on the test instance provided in this challenge.
- We observe that our approach can efficiently solve the model and acquire near-optimal solutions in a reasonable amount of time.
- Considering variable amount of delays depending on the stations, trains and time can be a future line of research to be considered in our next works.
- Other sources of delay such as weather conditions and planned or emergency maintenance can be considered.

References

- Brännlund, Ulf, et al. "Railway timetabling using Lagrangian relaxation." Transportation science 32.4 (1998): 358-369.
- Cordeau, Jean-Francois, Paolo Toth, and Daniele Vigo. "A survey of optimization models for train routing and scheduling." Transportation science 32.4 (1998): 380-404.
- Caprara, Alberto, et al. "A Lagrangian heuristic algorithm for a real-world train timetabling problem." Discrete applied mathematics 154.5 (2006): 738-753.
- Zhou, Xuesong, and Ming Zhong. "Single-track train timetabling with guaranteed optimality: Branch-and-bound algorithms with enhanced lower bounds." Transportation Research Part B: Methodological 41.3 (2007): 320-341.
- Barrena, Eva, et al. "Exact formulations and algorithm for the train timetabling problem with dynamic demand." Computers & Operations Research 44 (2014): 66-74.
- Yang, Lixing, Keping Li, and Ziyou Gao. "Train timetable problem on a single-line railway with fuzzy passenger demand." IEEE Transactions on fuzzy systems 17.3 (2008): 617-629.
- Khan, Muhammad Babar, and Xuesong Zhou. "Stochastic optimization model and solution algorithm for robust double-track train-timetabling problem." IEEE Transactions on Intelligent Transportation Systems 11.1 (2009): 81-89.
- Yang, Lixing, et al. "Collaborative optimization for train scheduling and train stop planning on high-speed railways." Omega 64 (2016): 57-76.
- Murali, Pavankumar, et al. "A delay estimation technique for single and double-track railroads." Transportation Research Part E: Logistics and Transportation Review 46.4 (2010): 483-495.
- Lu, Quan, Maged Dessouky, and Robert C. Leachman. "Modeling train movements through complex rail networks." ACM Transactions on Modeling and Computer Simulation (TOMACS) 14.1 (2004): 48-75.
- Wilson, Nigel HM, and Agostino Nuzzolo, eds. Schedule-based modeling of transportation networks: theory and applications. Vol. 46. Springer Science & Business Media, 2008.

References

- Jiang, Zhi-bin, et al. "A simulation model for estimating train and passenger delays in large-scale rail transit networks." Journal of Central South University 19.12 (2012): 3603-3613.
- Yalçınkaya, Özgür, and G. Mirac Bayhan. "A feasible timetable generator simulation modelling framework for train scheduling problem." Simulation Modelling Practice and Theory 20.1 (2012): 124-141.
- Gorman, Michael F. "Statistical estimation of railroad congestion delay." Transportation Research Part E: Logistics and Transportation Review 45.3 (2009): 446-456.
- Wang, Ren, and Daniel B. Work. "Data driven approaches for passenger train delay estimation." 2015 IEEE 18th International Conference on Intelligent Transportation Systems. IEEE, 2015.