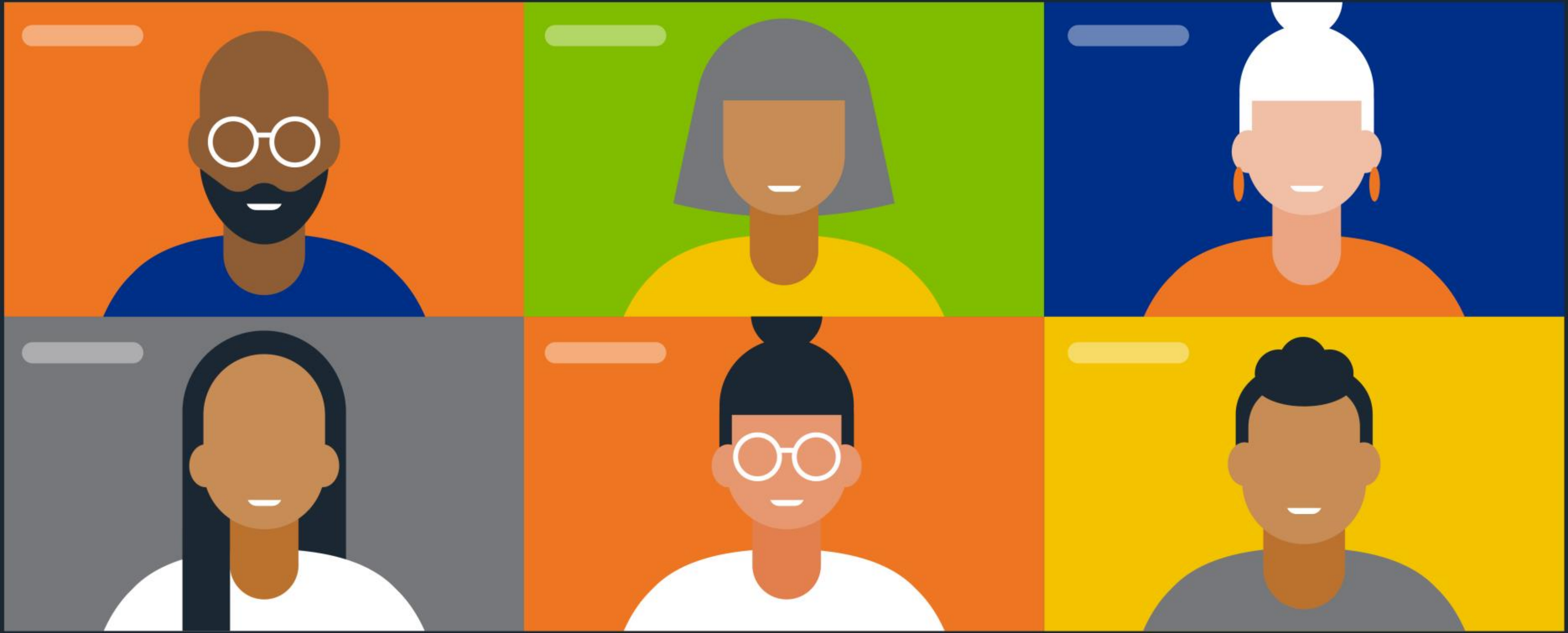

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Real-time Rolling Stock and Timetable Rescheduling in Urban Rail Networks: A Branch-and-Price Approach

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Introduction of this study

Mathematical model formulations

Solution methodologies

Conclusion

Introduction

- **Urban metro system** (or rapid transit) is a passenger railway system in an urban area with high capacity and frequency. (*Wikipedia*)

- Since the first metro line London Metropolitan Railway was put into use in 1863, urban metro systems have been widely established throughout the world in many large cities.

- During the COVID-19 pandemic, urban rail transit systems in China undertake great pressure to keep the crowdedness to avoid the spread of virus.

A large number of incidents happen in high-density metro systems

08:19 Line 1, switch failure (15 min)

07:14 Line 10, equipment failure (25 min)

18:52 Line 9, train shut down (25 min)

07:14 Line 1, signal failure (30 min)

07.03 07.04 07.07 07.14 07.17 07.28 08.03 08.22 2019

08:27 Line 6, signal failure (15 min)

13:46 Line 2, passenger invasion (30 min)

07:33 Line 5, signal failure (40 min)

19:52 Line 2, passenger invasion (15 min)

More than 60 incidents occur between Jul. to Aug.

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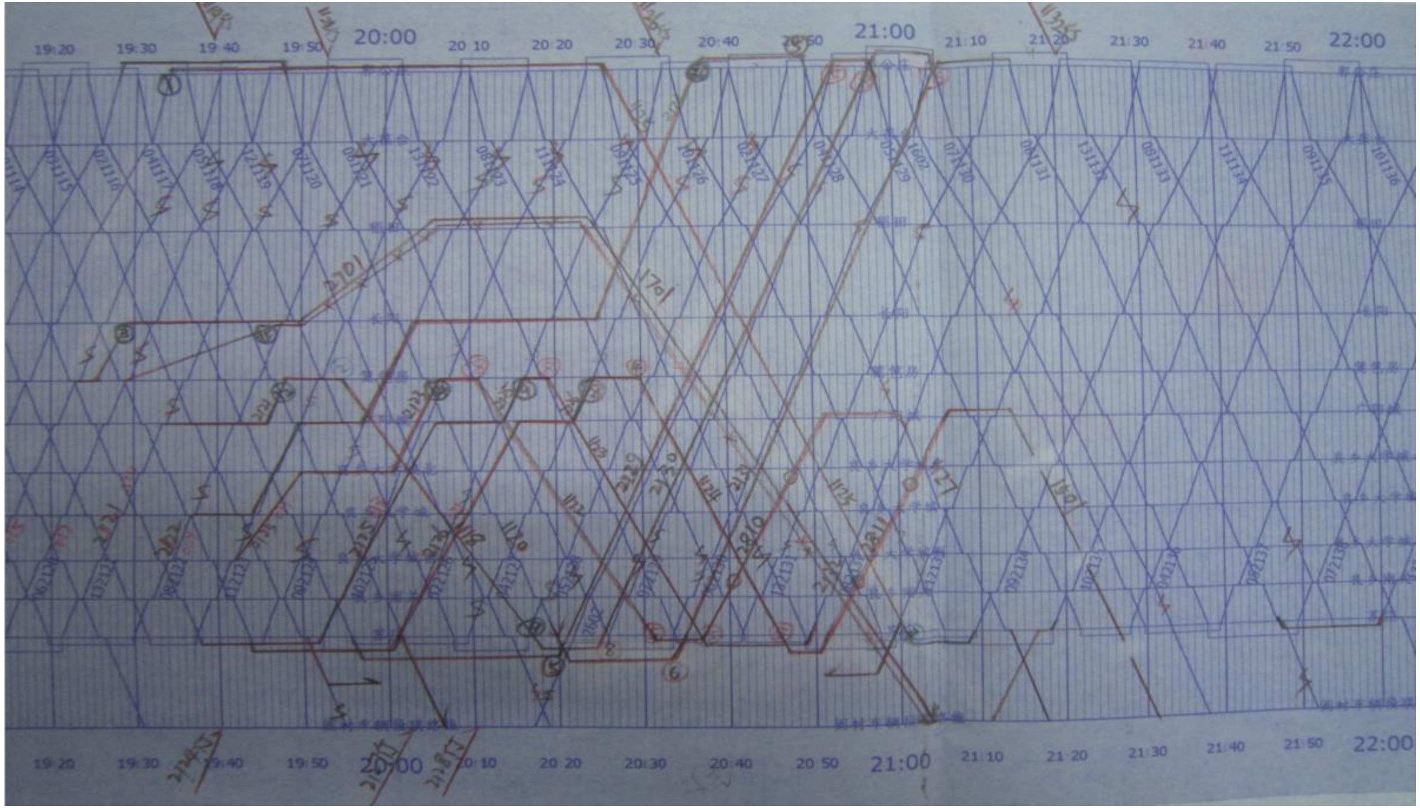
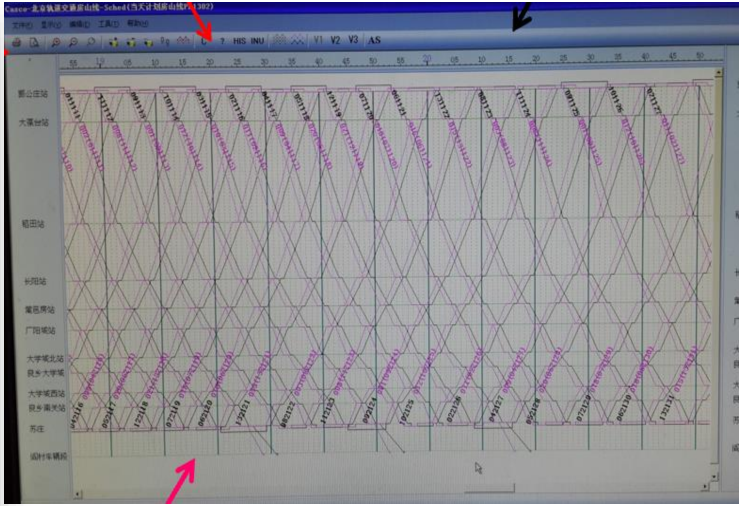


A real-world case of metro train rescheduling in Beijing subway

Introduction

➢ Modern urban metro systems

✓ ATS and dispatchers are in charge of train routing and re-timetabling



Literature review

Train timetable rescheduling

Rolling stock rescheduling

- ➢ Job-shop scheduling model
- ➢ Drawback: Big-M and large LP gap

- • Branch-and-bound
- • Benders decomposition
- • Methaheuristics

- ➢ Time-indexed formulation
- ➢ Drawback: large-number of integer variables

- • Lagrangian relaxation

- ➢ Event-activity formulation
- ➢ Drawback: large-number of binary variables with Big-M

- • Branch-and-cut

- ■ Most existing methodologies focus on mainline railways and require several minutes of computational time

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Contribution

- We first propose a path-based formulation for the integrated rolling stock and timetable rescheduling for urban rail systems

- We develop a branch-and-price approach for acquiring a near-optimal solution more efficiently

- Real-world case studies are conducted to illustrate the effectiveness of our approach

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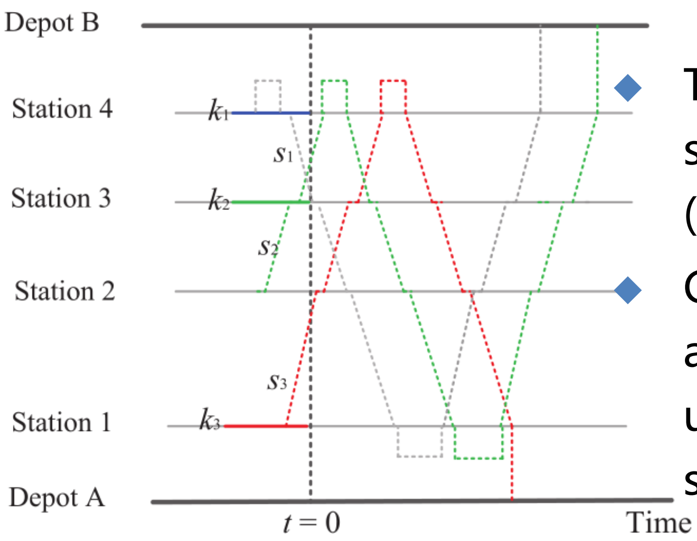
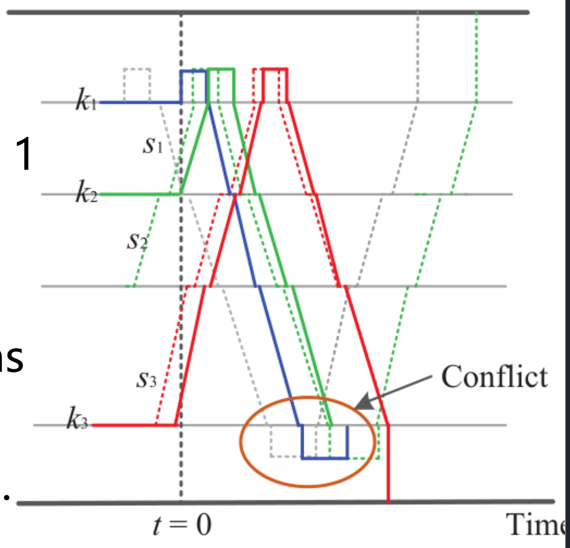
Mathematical formulation

Problem description

- We consider the train rescheduling problem for a bi-directional urban rail corridor with a set of stations (and depots) **I.**
- At the initial time t=0, the incident is resolved but the trains can no longer run to schedule.

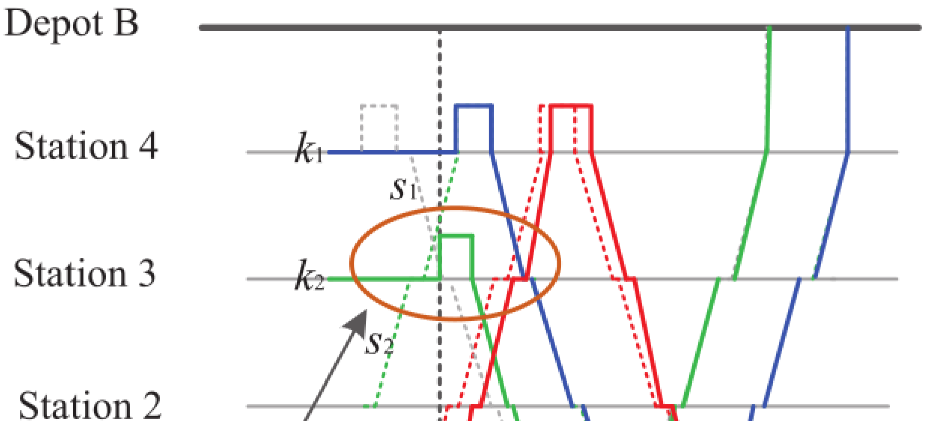


- Three trains at station 4, 3 and 1 (Left).
- Conflict may arise If the trains use existing schedule(Right).



- In practice, train dispatchers usually take four strategies to avoid conflict and reduce the negative effects for passengers, involving:

C. Short-turning or add new trains



- The strategies are required to be generated in real-time and still dependent on human experiences
- In practice, only experienced dispatchers have the ability and access to reschedule train timetable

Problem statement

- ◆ The issue can be modeled a mathematical optimization problem: Given a set of **K** trains located at stations and a planned schedule for each train, the aim is to generate a new schedule and rolling stock circulation plan for the trains (involving trains at depots), such that the delay of trains and canceling number of trains are minimized, with the following constraints satisfied:

- ➢ Each train in the system must be assigned to a service or back to depot
- ➢ Each service is covered at most one time
- ➢ Train following headway should be kept within a limitation
- ➢ At the end of time horizon (end of day), rolling stocks in each depot should be the same for the next day

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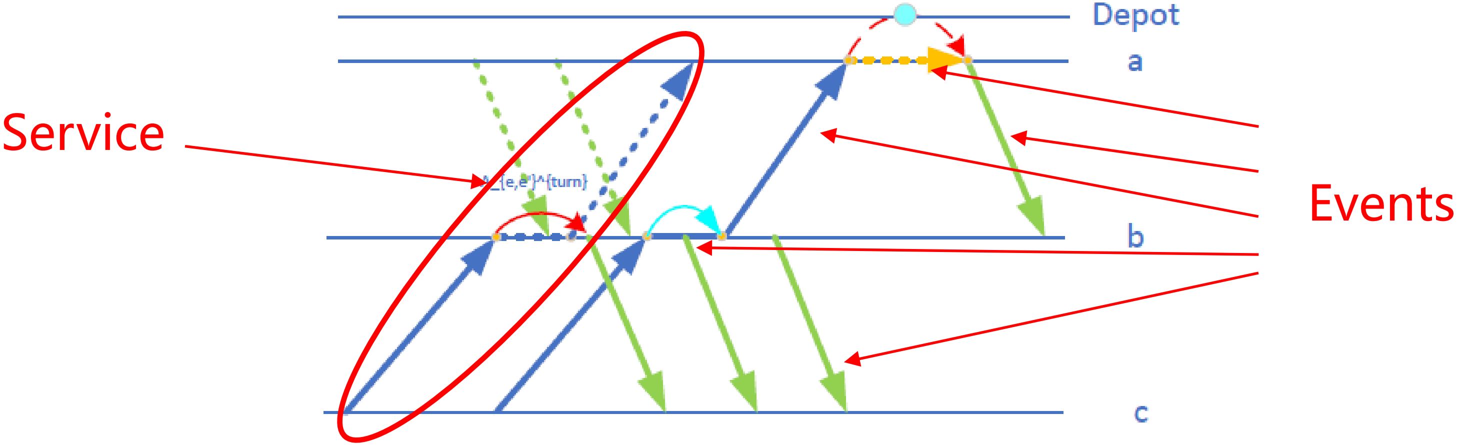
Event-activity network

1. The initial schedule defines a set of services as well as the arrival/depart time of these services at each station s.
2. The arrival/depart/short turn/back to depot of services are represented as an *event* $e \in E$
3. Each event is associated with two values: the penalty for canceling the event and penalty for delaying the event



➢ Zhu Y., Goverde R., 2019. Railway timetable rescheduling with flexible stopping and flexible short-turning during disruptions.

Path-based formulation

- The problem can be considered by: finding a set of paths in the event-activity network (originated from each train at different stations), such that the total cost is minimized.

Path-based formulation

$$
\begin{aligned}
\min_{x} \quad & \sum_{p\in P} c_p x_p && && (3)\\
\text{s.t.} \quad & \sum_{p\in P_k} x_p = 1, && \forall k \in K && (4)\\
& \sum_{p\in P^e} x_p \leq 1, && \forall e \in E && (5)\\
& \sum_{p\in \mathcal{P}} t_p^e x_p - \sum_{p\in \mathcal{P}} t_p^{e'} x_p \geq \Big(\sum_{p\in \mathcal{P}} r_p^e x_p - 1\Big)\bar{t}_{e'} + \bar{h} && \forall (e,e') \in A && (6)\\
& \sum_{p\in P} l_p^d x_p = N_d && \forall d \in D && (7)\\
& x_p \in \{0,1\}, && \forall p \in P && (8)
\end{aligned}
$$

Variable: x_p if path p is selected in the rescheduled timetable

Objective function:

$$\sum_{e\in E} q_e(1-\sum_{p\in P} r_p^e x_p) + \sum_{e\in E}(\sum_{p\in P} t_p^e x_p - \hat{t}_e)$$

$$= \sum_{e\in E} q_e - \sum_{e\in E} \hat{t}_e + \sum_{e\in E}\sum_{p\in P} t_p^e x_p - \sum_{e\in E}\sum_{p\in P} q_e r_p^e x_p$$

Reformulation

$$\sum_{e\in E}\sum_{p\in P} t_p^e x_p - \sum_{e\in E}\sum_{p\in P} q_e r_p^e x_p$$

$$= \sum_{p\in P}\sum_{e\in E} (t_p^e - q_e r_p^e) x_p$$

$$= \sum_{p\in P} c_p x_p$$

q_e	Penalty for canceling event *e*
r_p^e	If path *p* covers event *e*
t_p^e	Time of event *e* with path *p*
$\hat{t}_e$	Planned time of event *e*

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Constraints:

$$\sum_{p \in P_k} x_p = 1, \qquad \forall k \in K$$

Every train must be assigned with a path p

$$\sum_{p \in P^e} x_p \leq 1, \qquad \forall e \in E$$

Each even e should be covered at most once

$$\sum_{p \in \mathcal{P}} t_p^e x_p - \sum_{p \in \mathcal{P}} t_p^{e'} x_p \geq (\sum_{p \in \mathcal{P}} r_p^e x_p - 1)\bar{t}_{e'} + \bar{h} \qquad \forall (e, e') \in A$$

A minimum headway time should be guaranteed

$$\sum_{p \in P} l_p^d x_p = N_d \qquad \forall d \in D$$

Number of rolling stocks at depot d should equal to N_d

$$x_p \in \{0, 1\}, \qquad \forall p \in P$$

P_k Set of paths from train *k*

P^e Set of paths that may cover event *e*

$(e, e') \in A$ Set of event pairs

$\bar{h}$ Minimum headway

Model property

- Enumerating the complete set of the service path P is intractable and impractical since the number of possible event sequences increases exponentially with the number of events (i.e., the planned schedule)

- In particular, due to the computational time requirement (within one minute), we herein develop a column generation framework based branch-and-price solution methodology to generate high-quality service paths

Solution methodologies

- Model reformulation
- Define dual variables
- Define Restricted master problem and subproblems
- Column generation to generate paths
- Develop a branch-and-price framework

(1) Rewritten constraints

$$\sum_{p \in P_k} x_p = 1, \qquad \forall k \in K$$

$$\sum_{p \in P^e} x_p \leq 1, \qquad \forall e \in E$$

$$\boxed{\sum_{p \in \mathcal{P}} t_p^e x_p - \sum_{p \in \mathcal{P}} t_p^{e'} x_p \geq (\sum_{p \in \mathcal{P}} r_p^e x_p - 1)\bar{t}_{e'} + \bar{h} \qquad \forall (e, e') \in A}$$

$$\sum_{p \in P} l_p^d x_p = N_d \qquad \forall d \in D$$

$$x_p \in \{0, 1\}, \qquad \forall p \in P$$

$$\sum_{p \in \mathcal{P}} l_{pee'} x_p \geq \bar{h} - \bar{t}_{e'}, \qquad \forall (e, e') \in A$$

where $l_{pee'} = t_p^e - \bar{t}_{e'} r_p^e - t_p^{e'}$.

(2) Calculate the dual cost

Define dual variables

$\tau_k \ (k \in K)$	**Train-cover constraints**
$\pi_e \ (e \in E$	**Event-cover constraints**
$\phi_{e,e'} \ ((e,e') \in A$	**Headway constraints**
$\lambda_d \ (d \in D)$	**Rolling stock constraints**

(2) Calculate the dual cost

- Reduced cost for path p is given by

$$\bar{c}_p = c_p - \tau_k - \sum_{e \in E^p} \pi_e - \sum_{(e,e') \in A} l_{pee'} \phi_{e,e'} - \sum_{d \in D} l_p^d \lambda_d$$

$$= \sum_{e \in E} (t_p^e - q_e r_p^e) - \tau_k - \sum_{e \in E^p} \pi_e - \sum_{(e,e') \in A} (t_p^e - \bar{t}_{e'} r_p^e - t_p^{e'}) \phi_{e,e'} - \sum_{d \in D} l_p^d \lambda_d$$

$$= \sum_{e \in E} t_p^e - \sum_{e \in E^p} (\pi_e + q_e) - \sum_{(e,e') \in A} (t_p^e - \bar{t}_{e'} r_p^e - t_p^{e'}) \phi_{e,e'} - \sum_{d \in D} l_p^d \lambda_d$$

- The pricing problem is to find a event sequence with a negative reduced cost ($\bar{c}_p < 0$), which can be found by solving the following problem:

$$\boxed{\min_{p \in P_k} \bar{c}_p}$$

(2) Calculate the dual cost

The above objective function can be reformulated as follows

$$\bar{c}_p = \sum_{e\in E^p} [(1-\phi_{e,e'})t_e - \pi_e - q_e + \bar{t}_{e'}\phi_{e,e'}] + \sum_{e\in E_p} (t(e)\phi_{e_f,e})$$

$$= \sum_{e\in E^p} [(1-\phi_{e,e'})t_e - \pi_e - q_e + \bar{t}_{e'}\phi_{e,e'} + t(e)\phi_{e_f,e}]$$

$$= \sum_{e\in E^p} (q_e t_e + \xi_e)$$

where

$$q_e = 1 - \phi_{e,e'} + \phi_{e_f,e} \qquad \xi_e = \bar{h}\phi_{e,e'} - \pi_e - q_e$$

Minimizing the above problem is equivalent to finding a shortest path in the event network

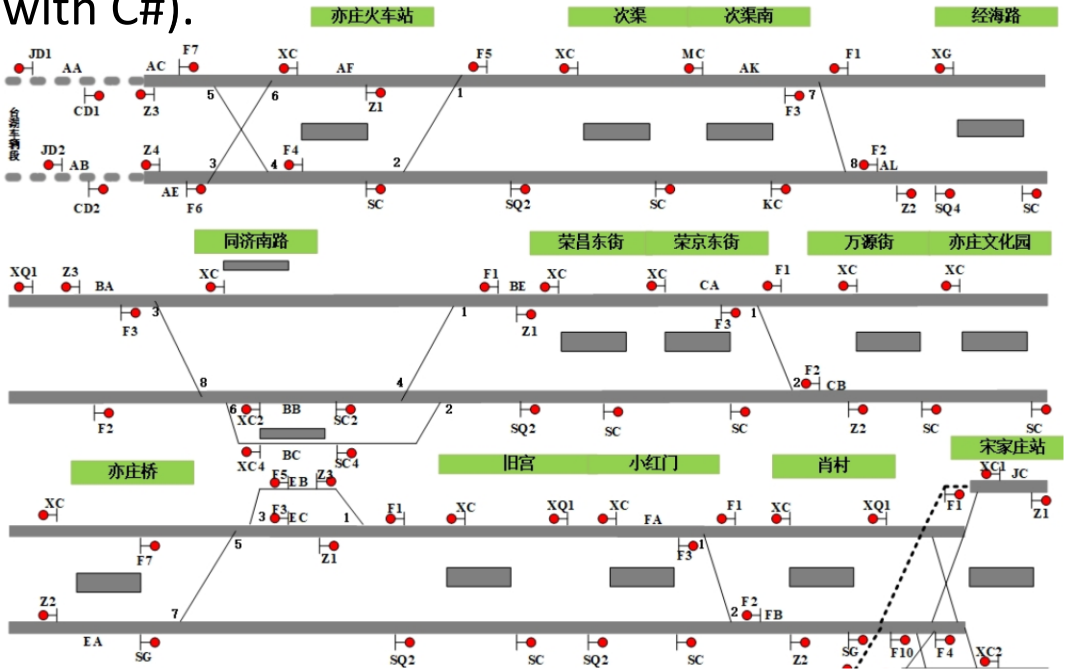
Branch-and-price procedure

Then, we can use standard branch-and-price procedure:

- Generate initial solution (a set of paths by heuristic strategies)
- Solve restricted master problem (RMP)
- Solving pricing subproblems for each train k
- Add the paths with negative cost to the RMP
- Stopping ceria: there is no path with negative cost; time exceeds the limit
- Branch on the LP relaxation, and iterate.

Results and conclusions

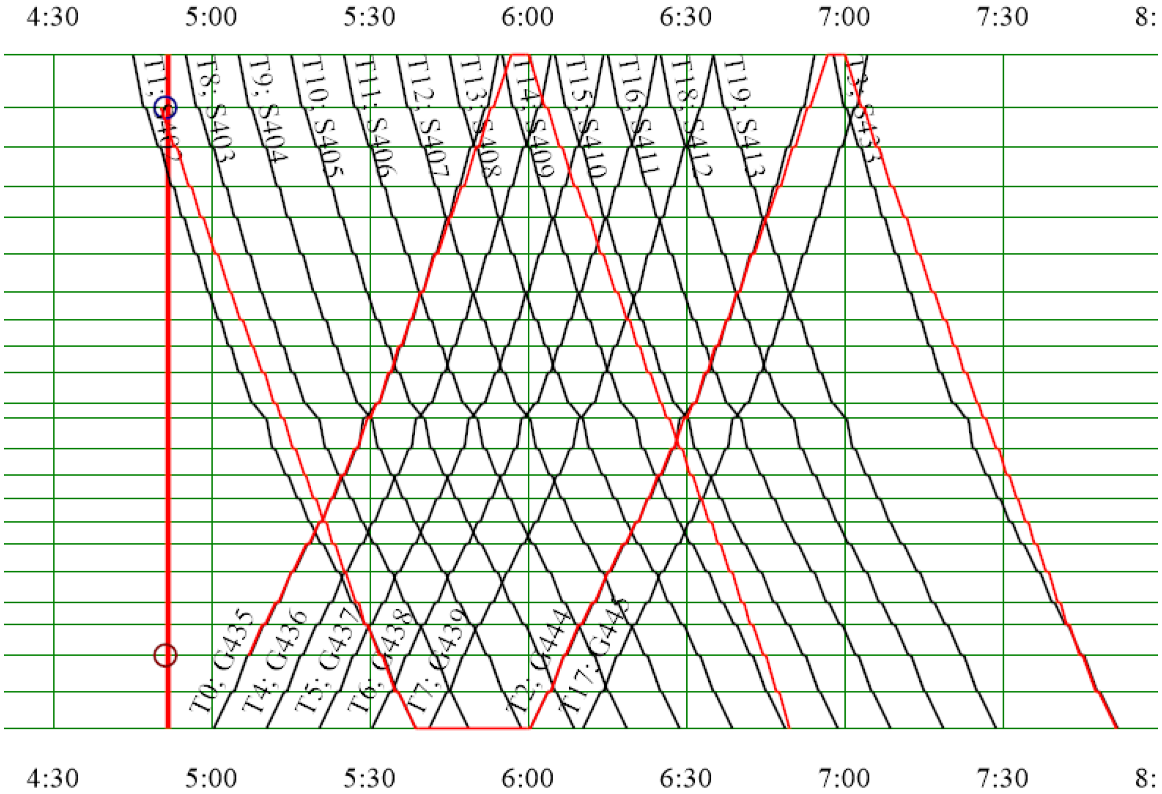
- We collaborated with Beijing subway and a signal company and developed a software embedded with our methodology (coded with C#).



Test line layout in Beijing subway

Results and conclusions

Some primarily results



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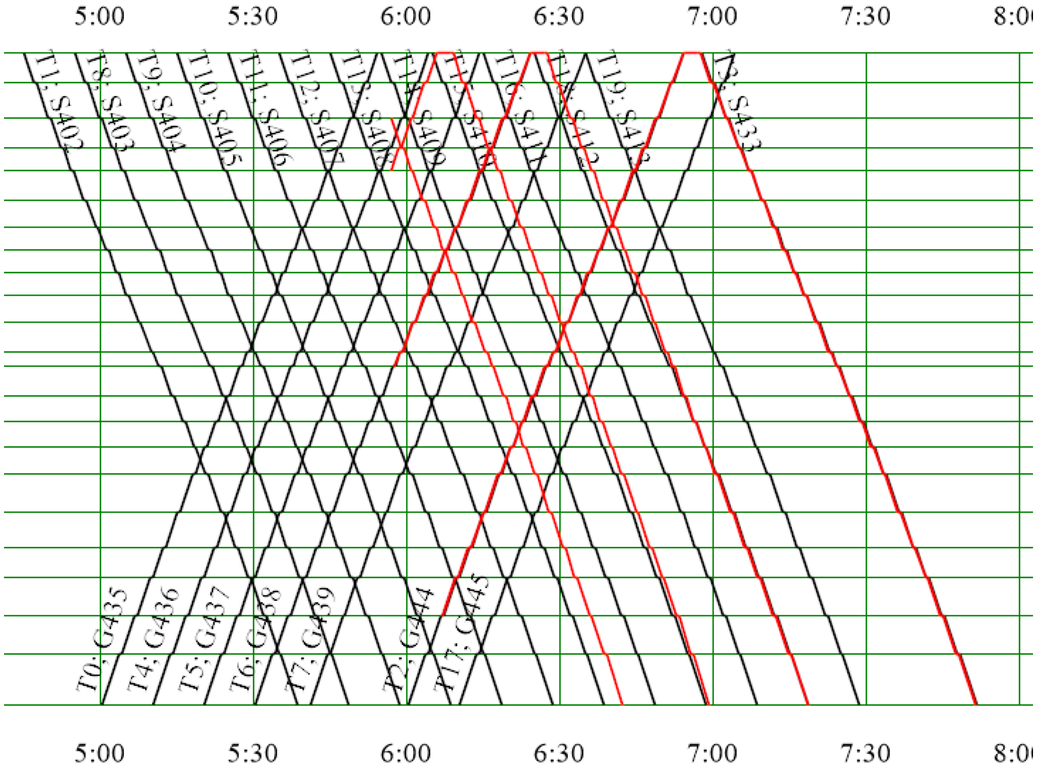
Results and conclusions

Some primarily results



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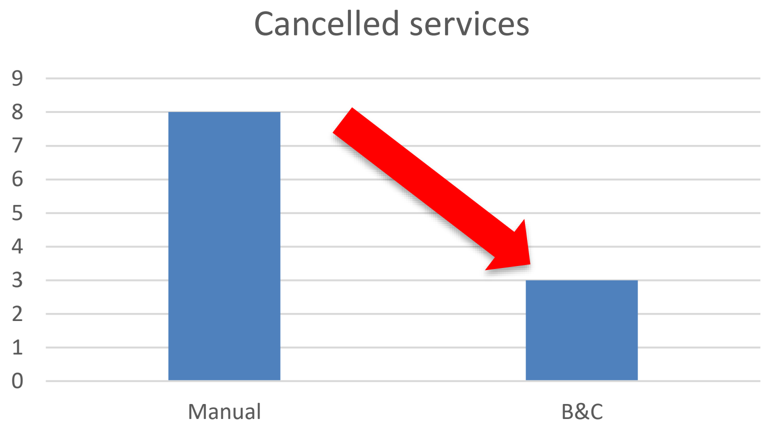
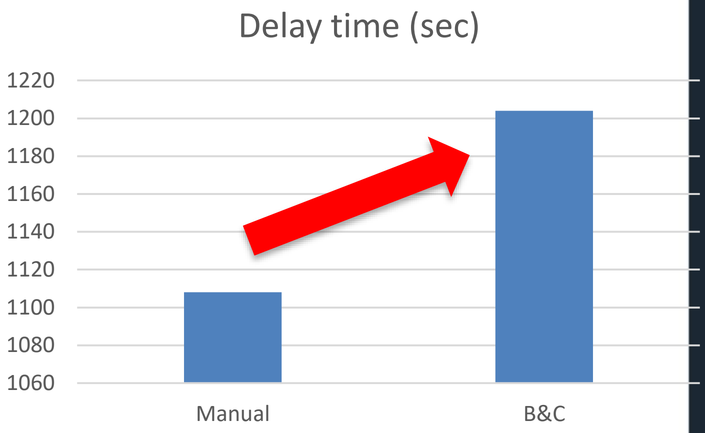
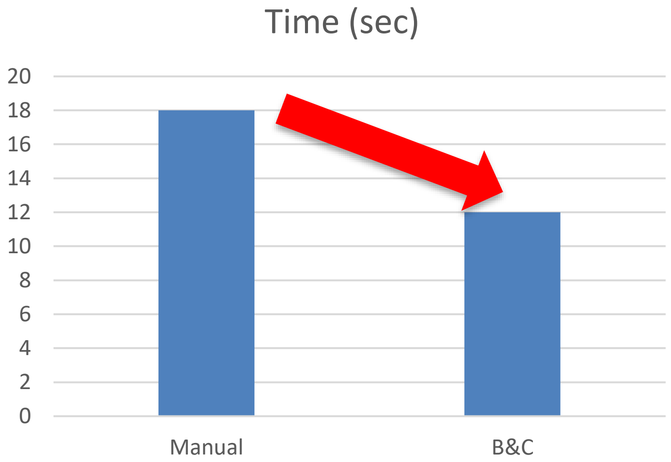
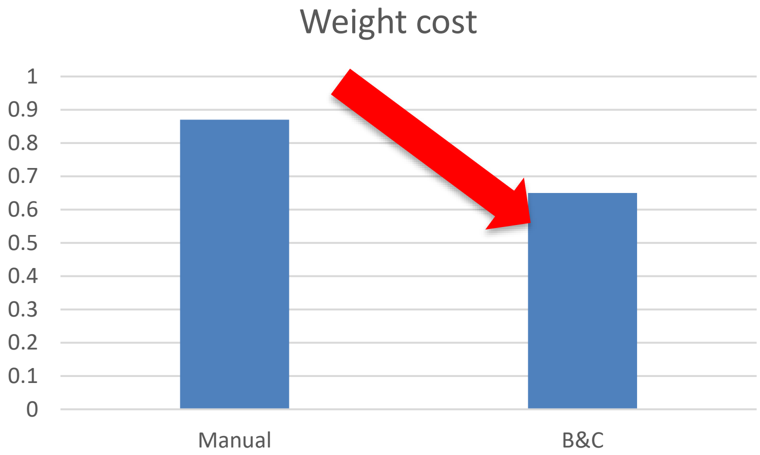
Comparison with a human dispatcher









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Conclusions

- Our study first developed a path-based formulation for the integrated rolling stock and timetable rescheduling problem
- We developed a branch-and-price methodology for solving the path-based formulation more efficiently
- We developed a software for the decision-making of rail dispatchers
- Our methodology outperforms human dispatchers in computational time and the total weight cost

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Thanks!

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