Joint Optimization of Track Maintenance and Renewal planning

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Outline

- Introduction
- Track Maintenance and Renewal Model
- Data-drive robust optimization
- Case study
- Results and Discussion

Introduction

- Maintenance and Renewal (MR) is crucial to guarantee the reliability, availability, and safety of a railway network.
- Railway assets/components:
  - Tracks, switches and crossings.
  - Signaling system: safety and telecommunication equipment.
  - Catenary systems: energy supply installations.
  - Vehicle
  - Bridges and tunnels.

Figure 2. Track geometry measurements

Track maintenance tasks

- **Tamping** conducted to restore track geometry irregularities, could be corrective or preventive.
- **Grinding:** the process to maintain a predetermined profile on the head of the rail in order to maximize rail life and minimize rolling resistance reducing wheel wear and improving fuel economy.
- **Renewal:** replacing the current track.

Motivation

- Maintenance tasks and renewal mostly studied separately.
- Renewal has been studied mostly from economical perspective
- Uncertainties in the maintenance effect have not been studied
- Dependence and relation between different maintenance tasks requires joint optimization of maintenance and renewal

Mathematical Programming Model

- Joint Optimization of track maintenance and renewal.
- Decisions Variables:

x_{ij}^{t}	1 if maintenance action/renewal j is performed in segment i at period t
w_{i}^{t}	Quality index time t
σ_{i}^{t}	TQI value of segment i at time t

- Objective Function:

$$\text{Max} \sum_{i=1}^{I} \sum_{t=1}^{T} \alpha^t w_i^t \pi_i$$

- Initialization and set up constraints:

$$w_i^{t_0} = W_i^0 \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (2)$$

$$\sum_{j=1}^{J} x_{ij}^t \leq L \qquad \forall i \epsilon I, \quad \forall t \epsilon T \qquad (3)$$

$$\sum_{t=1}^{T} x_{ij}^t \geq |I(i)| x_{ij}^t \qquad \forall i \epsilon I, \forall j \epsilon J \qquad (4)$$

$$y_{1j}^t \geq x_{1j}^t \qquad \forall j \epsilon J, \quad \forall t \epsilon T \qquad (5)$$

$$y_{ij}^t \geq x_{ij}^t - x_{i-1j}^t \qquad \forall j \epsilon J, \quad \forall t \epsilon T, \forall i \epsilon I, i > 1 \qquad (6)$$

➢ Budget, resource and time constraint

$$\sum_{i=1}^{I}\sum_{j=1}^{J}(c_{ij}^{t}x_{ij}^{t} + F_j y_{ij}^{t} + Ps_{ij}^{t}) \leq B_t \quad \forall t \epsilon T \qquad (6)$$

$$\sum_{t=1}^{T} x_{ij}^{t} \leq N_{ij} \quad \forall i \epsilon I, \forall j \epsilon J \qquad (8)$$

$$\sum_{i=1}^{I} b_{kj} x_{ij}^{t} \leq A_K^t \quad \forall j \epsilon J, \forall t \epsilon T, \forall k \epsilon K \qquad (9)$$

$$\sum_{i=1}^{I} R_j x_{ij}^{t} \leq g^t \quad \forall i \epsilon I, \forall t \epsilon T \qquad (10)$$

➢ Threshold constraints:

$$\omega_i^t \leq w^{a2} \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (12)$$

$$w_i^t \leq w^a x_{i1}^t \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (13)$$

$$M\left(x_{ij}^t - 1\right) \leq h_j^t - w_i^t \ \leq M x_{ij}^t \qquad \forall i \epsilon I, \forall \epsilon J, j \neq 2, \forall t \epsilon T \qquad (14)$$

$$\sigma_i^1 = \sigma_i^{t0} \qquad \forall i \epsilon I \qquad (15)$$

$$\sigma_i^t = \sigma_i^{t-1} + \rho_i^t - \theta_i^t x_{i2}^t \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (16)$$

$$M\left(x_{i2}^t - 1\right) \leq \sigma_i^t - \tau^t \ \leq M x_{i2}^t \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (17)$$

➢ Suppression constraint:

$$\sum_{te=1}^{TE_j} x_{ij}^{t+te_j} \leq te_j\left(1 - x_{ij}^t\right) + Ps_{ij}^t \qquad \forall i \epsilon I, \forall j \epsilon J, j \neq 2, \forall t \epsilon T, t + te < T \qquad (18)$$

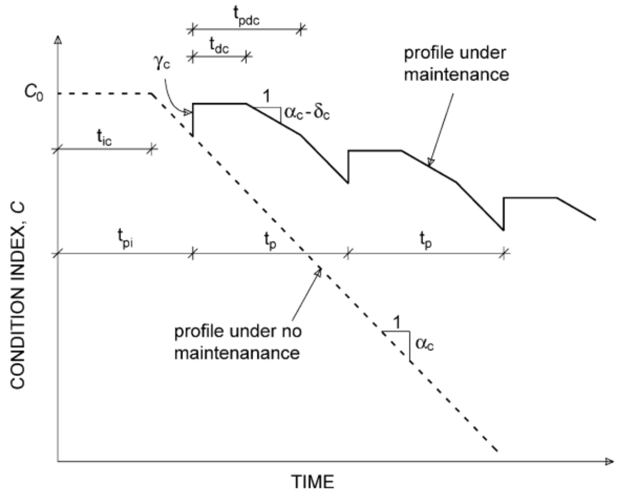
- Quality index equation:

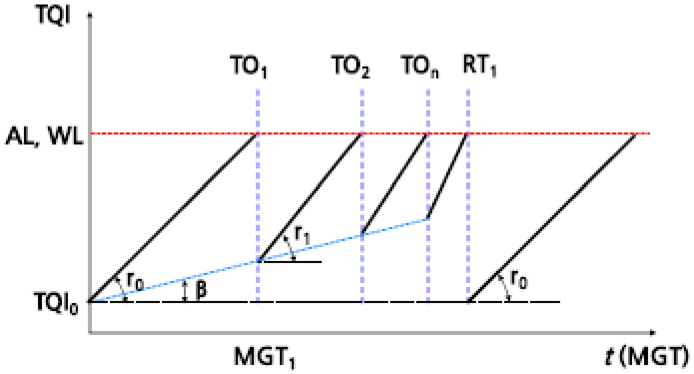
$$\sum_{j=1}^{J} x_{ij}^{t} \geq 2q_{ij}^{t} \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (19)$$

$$\omega_i^t = w_i^{t-1} - \lambda_i^t + \sum_{j=2}^{J} \delta_j x_{ij}^t + \gamma_{ij} q_{ij}^t \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (20)$$

$$w_i^t \leq \omega_i^t \qquad \forall i \epsilon I, \forall t \epsilon T \qquad (21)$$

Maintenance recovery rate





➢ Modeled the uncertainty in maintenance recovery rate through a data-driven robust optimization.

Robust Optimization

Deterministic Optimization

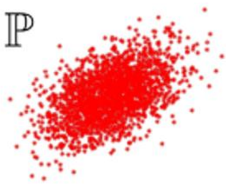
$$\inf_{\mathbf{x}} \; f(\mathbf{x}, \boldsymbol{\xi})$$
$$\text{s.t. } \mathbf{x} \in X$$

$\boldsymbol{\xi}$

Stochastic Optimization

$$\inf_{\mathbf{x}} \; \mathbb{E}_{\mathbb{P}}\{f(\mathbf{x}, \boldsymbol{\xi})\}$$
$$\text{s.t. } \mathbf{x} \in X$$

$\mathbb{P}$



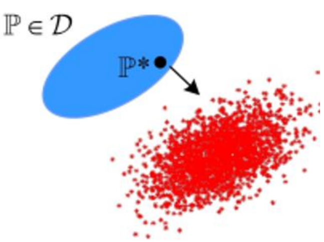
Robust Optimization

$$\inf_{\mathbf{x}} \; \sup_{\boldsymbol{\xi} \in U} \; f(\mathbf{x}, \boldsymbol{\xi})$$
$$\text{s.t. } \mathbf{x} \in X$$

$\boldsymbol{\xi} \in U$

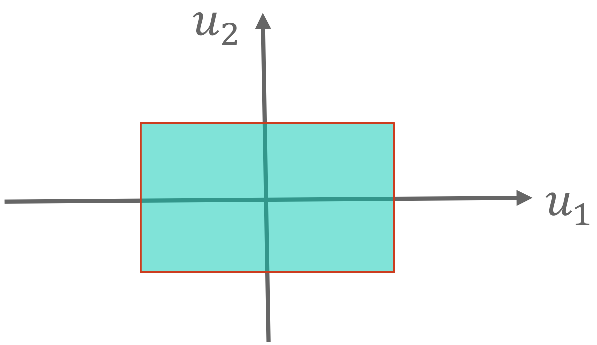
DR Optimization

$$\inf_{\mathbf{x}} \; \sup_{\mathbb{P} \in \mathcal{D}} \; \mathbb{E}_{\mathbb{P}}\{f(\mathbf{x}, \boldsymbol{\xi})\}$$
$$\text{s.t. } \mathbf{x} \in X$$

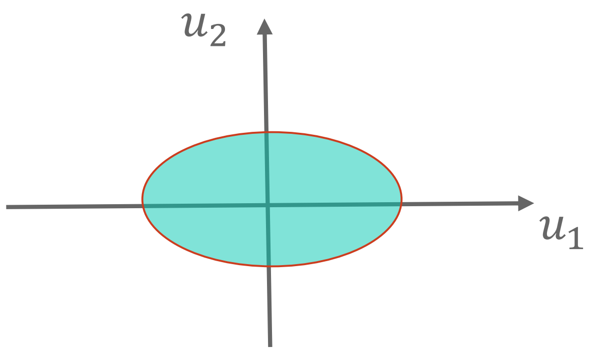


Classic Uncertainty Sets

- $U_{box} = \{u | u_i^L \leq u_i \leq u_i^U, \ \forall i \}$
  - Pros: Tractable
  - Cons: Very Conservative

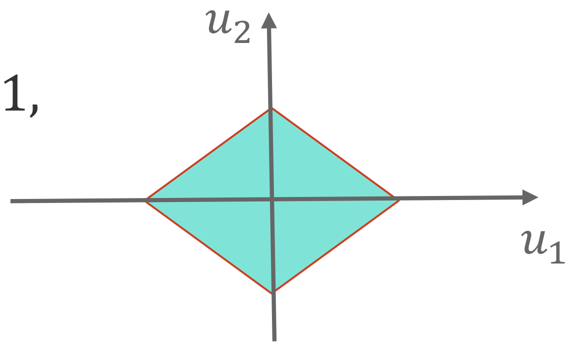


- $U_{Ellipsoidal} = \{\boldsymbol{U} | \boldsymbol{U}^T \sum \boldsymbol{U} \leq 1\} = \left\{\boldsymbol{U} \middle| \left\|\Sigma^{\frac{1}{2}} \boldsymbol{U}\right\|_2 \leq \boldsymbol{1}\right\}$
  - Pros: Control Conservatism
  - Cons: Nonlinearity

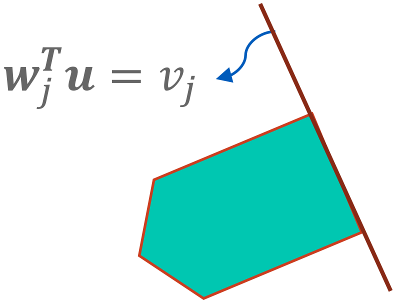


Classic Uncertainty sets

- Budget/Gamma Uncertainty:
  - Bertsimas and Sim (2003)
  - $U_{budgeted} = \{u_i | u_i = \bar{u} + \Delta u_i . z_i, -1 \leq Z_i \leq 1, \sum_i |z_i| \leq \Gamma_i, \forall_i\}$
  - Pros: Control Conservatism
  - Cons: Suitable for independent and symmetric uncertainty

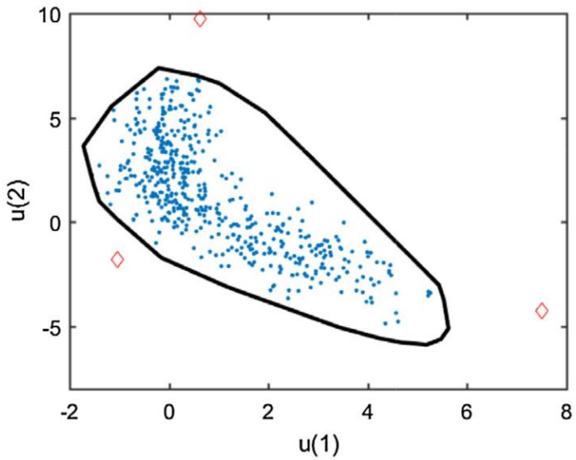


- Polyhedral Uncertainty (see this paper later)
  - Bertsimas and Ruiter (2016)
  - $U_{polyhedral} = \{\boldsymbol{u} | \boldsymbol{w}_j^T \boldsymbol{u} \leq v_j, \quad \forall_i = 1, \dots, s\}$
  - Pros: Flexible structure
  - Cons: Difficulty in optimal Polyhedral



Data-driven robust optimization

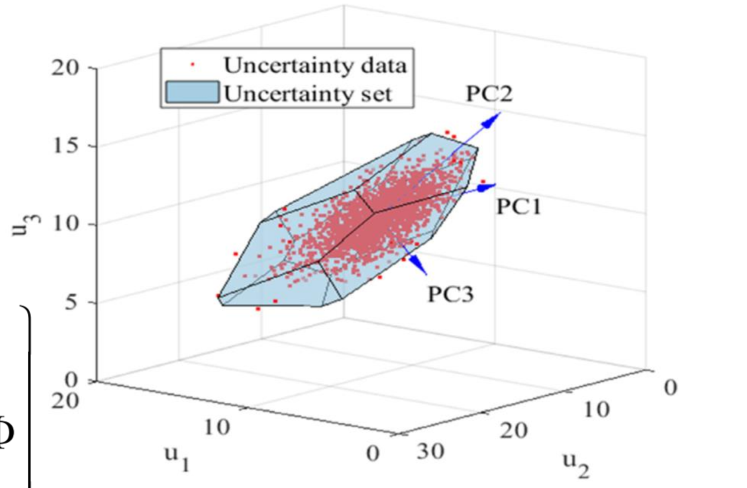
- Uncertainty set using machine learning or statistical inference methods.
- Kernel Density Estimation
  - Shang et al, (2017)



$$\mathcal{U}(\mathcal{D}) = \left\{ \mathbf{u} \,\middle|\, -2\sum_{i=1}^{N} \alpha_i K(\mathbf{u}, \mathbf{u}^{(i)}) \leq -2\sum_{i=1}^{N} \alpha_i K(\mathbf{u}^{(i')}, \mathbf{u}^{(i)}), \quad i' \in \mathrm{BSV} \right\}$$

Data-driven robust optimization

- Principal Component Analysis and Kernel Density Estimation:
  - Ning and You, (2018)
  - Pros: Very flexible
  - Cons: Intractable



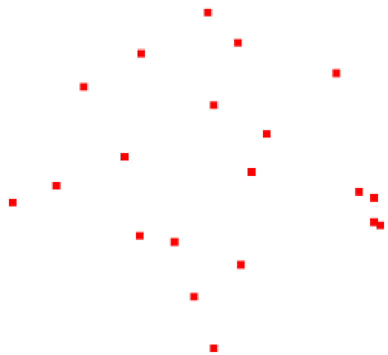
$$U_{\text{PCA+KDE}} = \left\{ \mathbf{u} \;\middle|\; \begin{array}{l} \mathbf{u} = \boldsymbol{\mu}_0 + \mathbf{P}\boldsymbol{\xi},\ \boldsymbol{\xi} = \underline{\boldsymbol{\xi}} \circ \mathbf{z}^- + \overline{\boldsymbol{\xi}} \circ \mathbf{z}^+, \\ \mathbf{0} \le \mathbf{z}^-,\ \mathbf{z}^+ \le \mathbf{e},\ \mathbf{z}^- + \mathbf{z}^+ \le \mathbf{e},\ \mathbf{e}^T\left(\mathbf{z}^- + \mathbf{z}^+\right) \le \Phi \\ \underline{\boldsymbol{\xi}} = \left[\hat{F}_{\text{KDE}}^{(1)\,-1}(\alpha), \ldots, \hat{F}_{\text{KDE}}^{(m)\,-1}(\alpha)\right]^T \\ \overline{\boldsymbol{\xi}} = \left[\hat{F}_{\text{KDE}}^{(1)\,-1}(1-\alpha), \ldots, \hat{F}_{\text{KDE}}^{(m)\,-1}(1-\alpha)\right]^T \end{array} \right\}$$

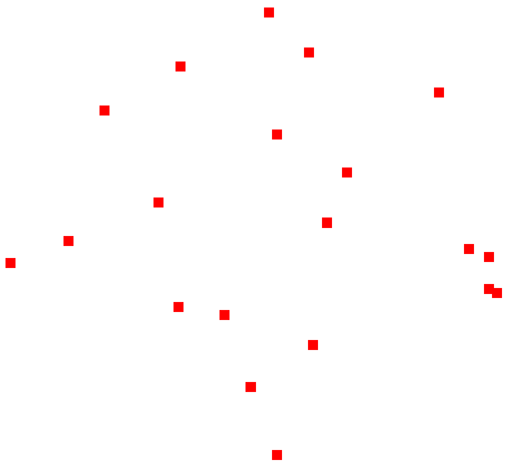
Proposed Uncertainty set construction methods

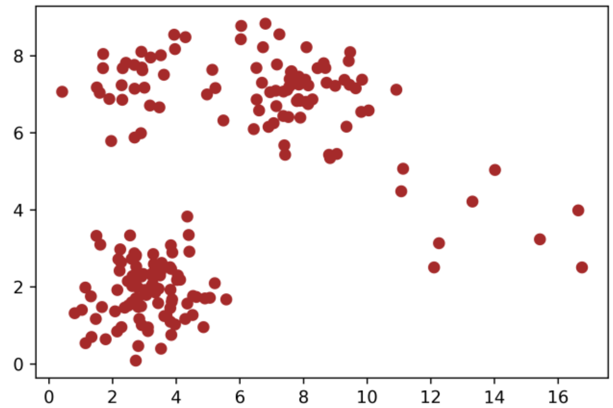
- Outer Approximation using a convex hull
- Outer Approximation OA-2; a tighter approximation
- Outer Approximation 3: Classic uncertainty set + cuts

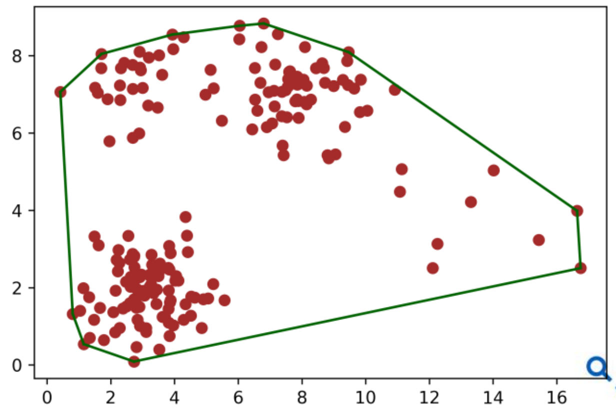
OA-1

- Used Qhull algorithm to find the convex hull

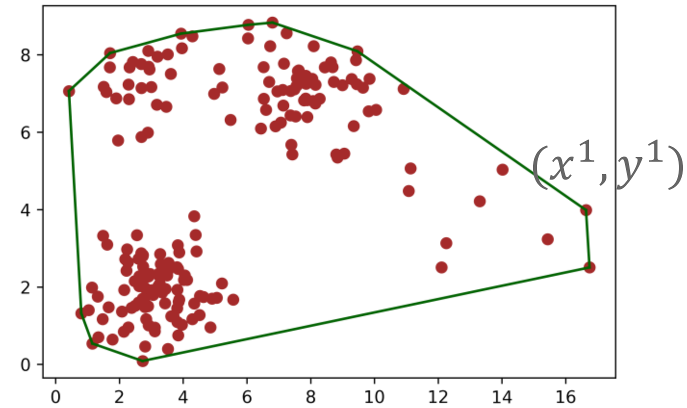








Uncertainty set for OA-1



$$(\mathrm{x}^{(1)}, \mathrm{y}^{(1)}), (\mathrm{x}^{(2)}, \mathrm{y}^{(2)}), \ldots, (\mathrm{x}^{(k)}, \mathrm{y}^{(k)})$$

$$\mathrm{U}^{\xi} = \mathbf{S}\xi + \mathbf{t} \geq 0$$

$$S = \begin{bmatrix} y^{(2)} - y^{(1)} & x^{(2)} - x^{(1)} \\ \vdots & \vdots \\ y^{(k+1)} - y^{(k)} & x^{(k)} - x^{(k+1)} \\ \vdots & \vdots \\ y^{(1)} - y^{(k)} & x^{(k)} - x^{(1)} \end{bmatrix}_{K\times 2}$$

$$t = \begin{bmatrix} y^{(2)} - y^{(1)} & x^{(2)} - x^{(1)} \\ \vdots & \vdots \\ y^{(k+1)} - y^{(k)} & x^{(k)} - x^{(k+1)} \\ \vdots & \vdots \\ y^{(1)} - y^{(k)} & x^{(k)} - x^{(1)} \end{bmatrix}_{K\times 1}$$

Robust Counterpart Model

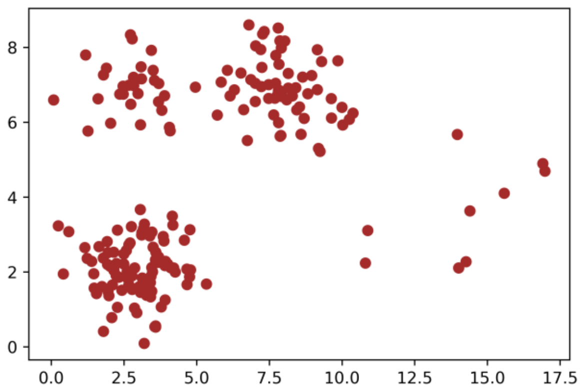
$$\omega_i^t = w_i^{t-1} - \lambda_i^t + \sum_{j=2}^{J} \delta_j x_{ij}^t + \gamma_{ij} q_{ij}^t \qquad \forall i \epsilon I, \forall t \epsilon T$$

$$w_i^{t-1} - \lambda_i^t + \sum_{j=2}^{J} \bar{\delta}_j x_{ij}^t + \gamma_{ij} q_{ij}^t + \sum_{l=1}^{L} \vartheta_l \mu_l \le \omega_i^t \qquad \forall i \epsilon I, \forall t \epsilon T$$

$$- \sum_{l}^{L} \varphi_l \mu_l = \hat{\delta}_j x_{ij}^t \qquad \forall i \epsilon I, \forall t \epsilon T, j = 2,3$$

$$\mu_l \ge 0$$

OA-2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2.5
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12.5
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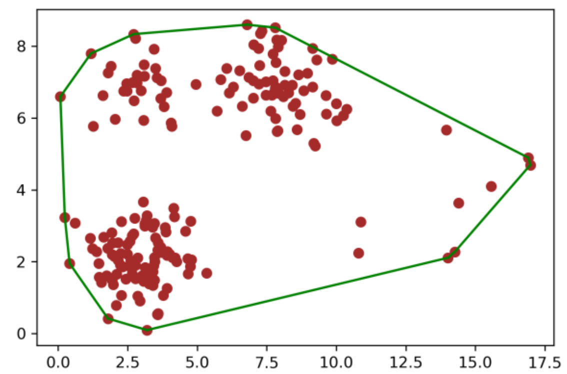
0
2
4
6
8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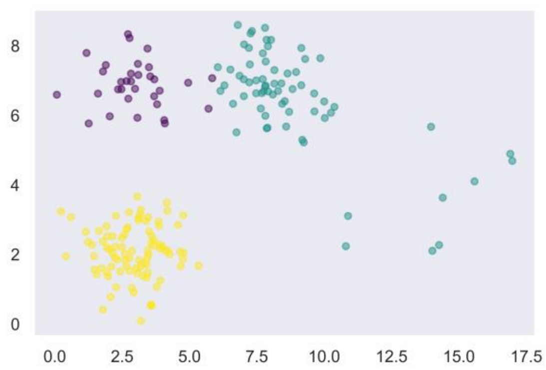
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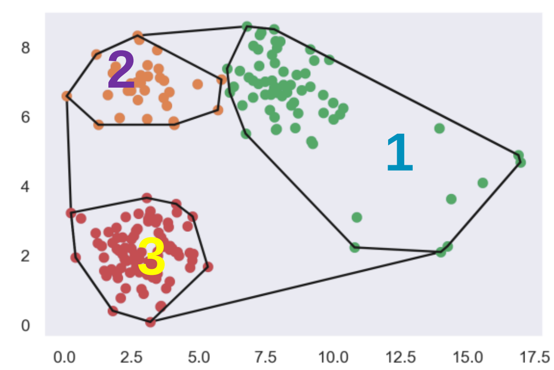
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1
3
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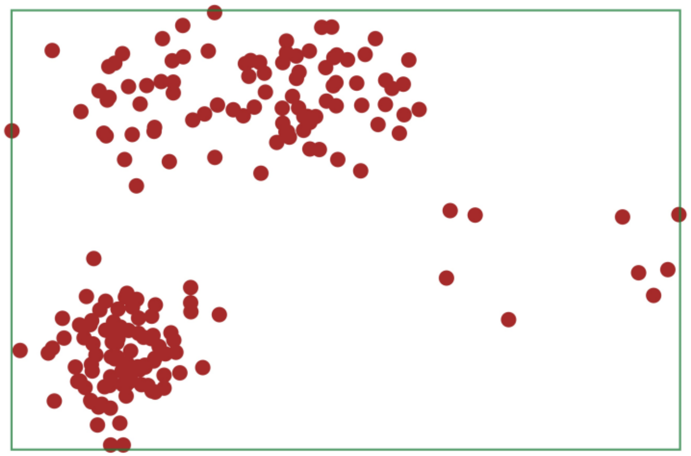
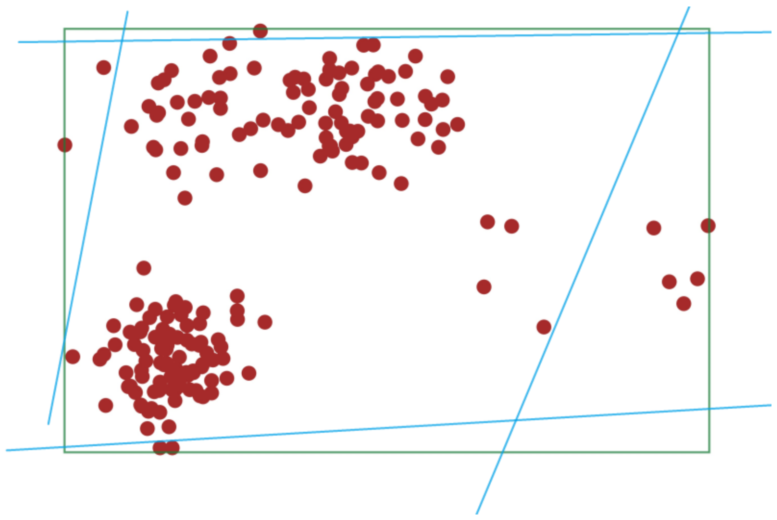
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0
2
4
6
8

OA-3

- Fit a classic uncertainty set
- Choose α% of the as outliers
- Cluster outliers again
- Fit linear regressions to generate some cuts





Robust Counterpart Model for OA-3

$$\max \qquad z = cx + dy$$

$$s.t. \quad \sum_j a_{ij}x_j + \sum_l b_{il}y_l + \max_{\xi \in U}\left\{\sum_{j\in J_i}\xi_{ij}\hat{a}_{ij}x_j + \sum_{l\in L_i}\eta_{il}\hat{b}_{il}y_l\right\} \le p_i \qquad \forall i$$

$$x_j \in X, y_l \in \{0,1\} \qquad \forall j,l$$

$$\sum_{h\in J_i \cup L_i} \beta_{kh}\zeta_{ih} + d_k \ge 0, \forall i, k = 1, 2, \ldots, q$$

Robust Counterpart Model for OA-3

$$\sum_{j} a_{ij}^{1-\gamma_i} x_j + \sum_{l} b_{il}^{1-\gamma_i} y_l + \sum_{j \in J_i} ua_{ij} + \sum_{l \in L_i} ub_{il} + \sum_{k=1}^{q} d_k \tau_k \leq p_i, \forall i$$

$$-ua_{ij} \leq c_{ij} \hat{a}_{ij}^{\max} x_j + c_{ij} \sum_{k=1}^{q} \beta_{kj} \tau_k \leq ua_{ij}, \forall j \in J_i$$

$$-ub_{il} \leq \mu_{il} \hat{b}_{il}^{\max} y_l + \mu_{il} \sum_{k=1}^{q} \beta_{k,l+|J_i|} \tau_k \leq ub_{il}, \forall l \in L_i$$

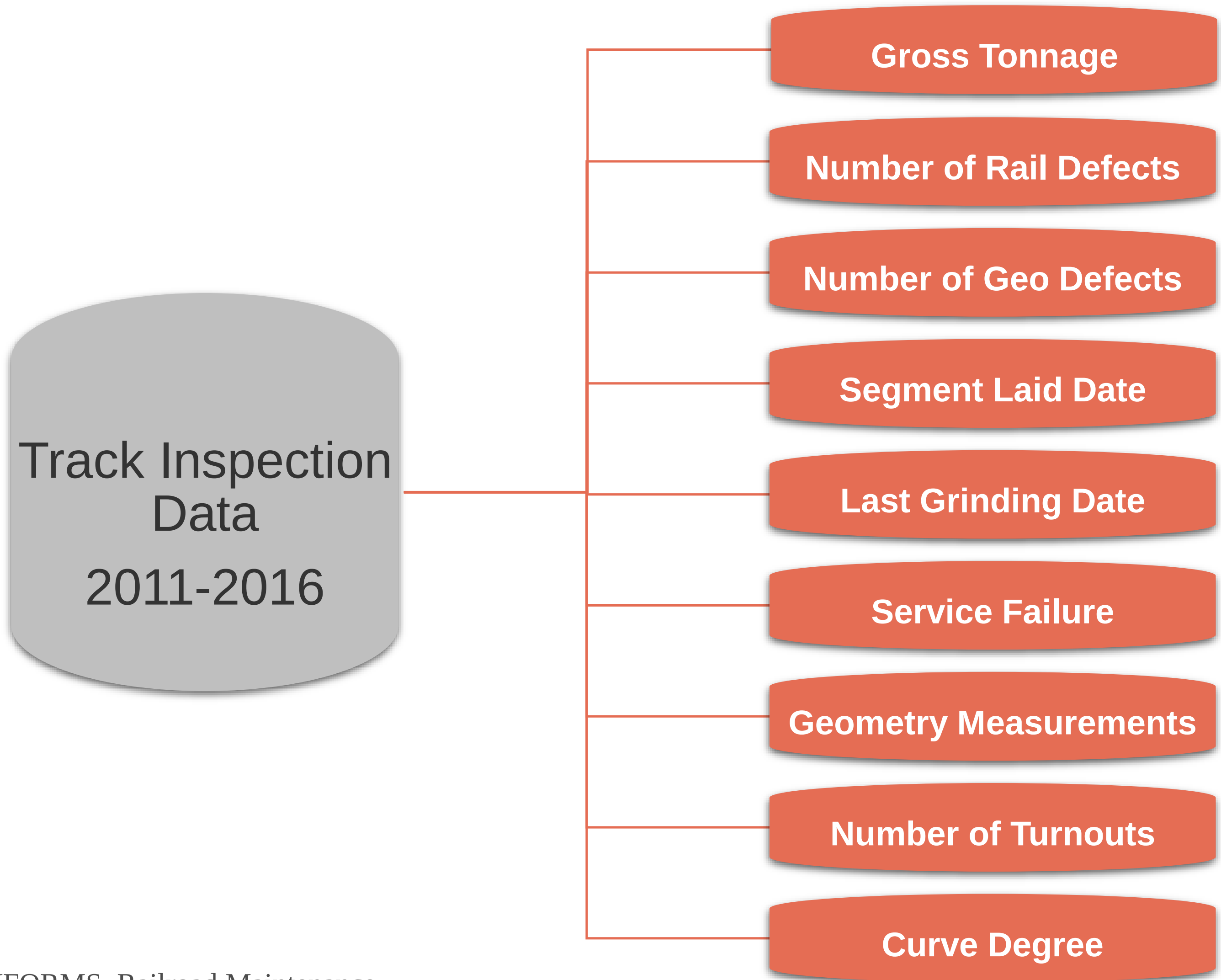
$$\tau_k \geq 0, \forall k$$

$$ua_{ij} \geq 0, \forall i, j \in J_i$$

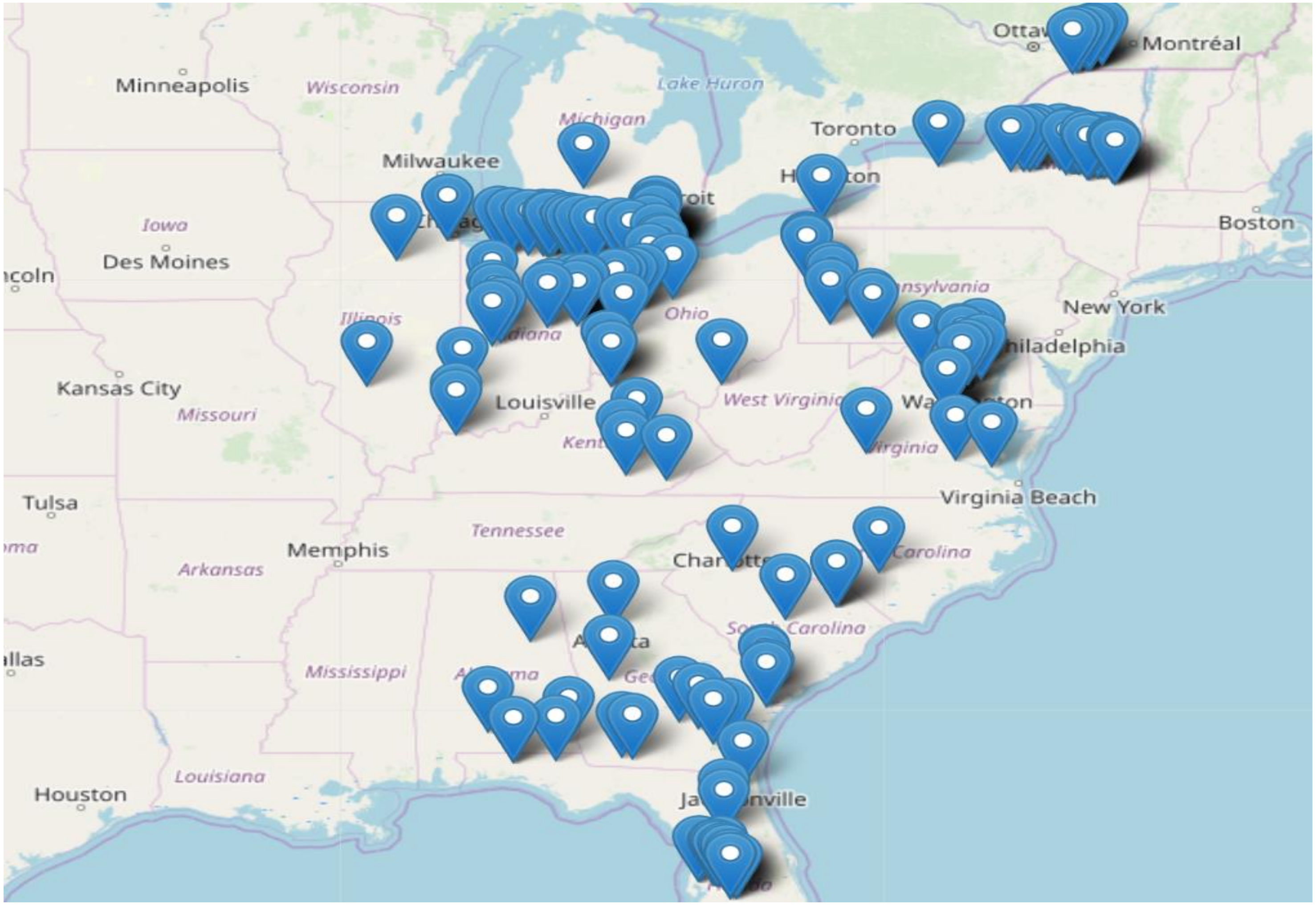
$$ub_{il} \geq 0, \forall i, l \in L_i$$

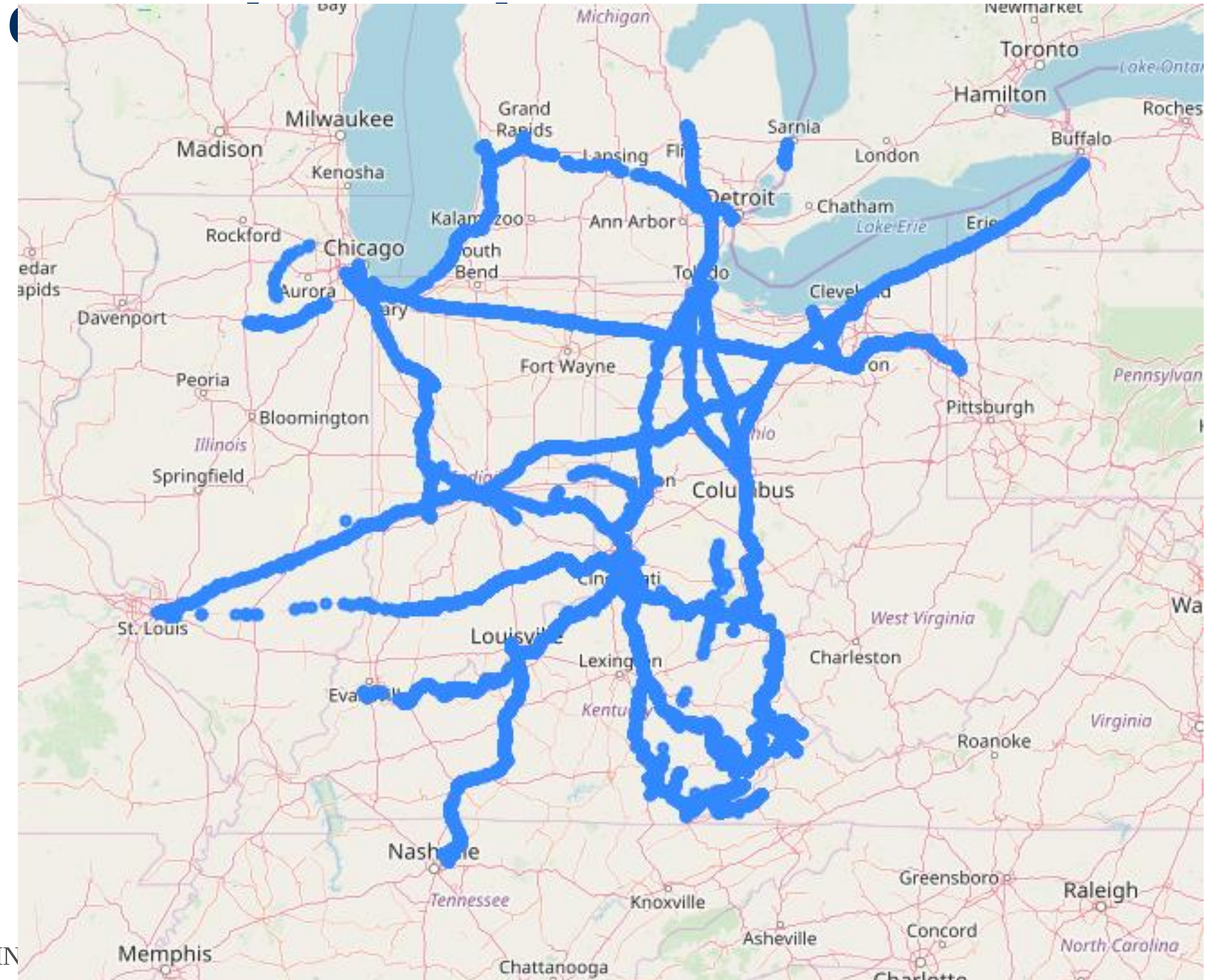
$$x_j \in X, y_l \in \{0,1\}, \forall j, l$$

Data description



Distribution of service failure

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Toronto
Lake Ontario
Hamilton
Roches
Milwaukee
Grand Rapids
Sarnia
Buffalo
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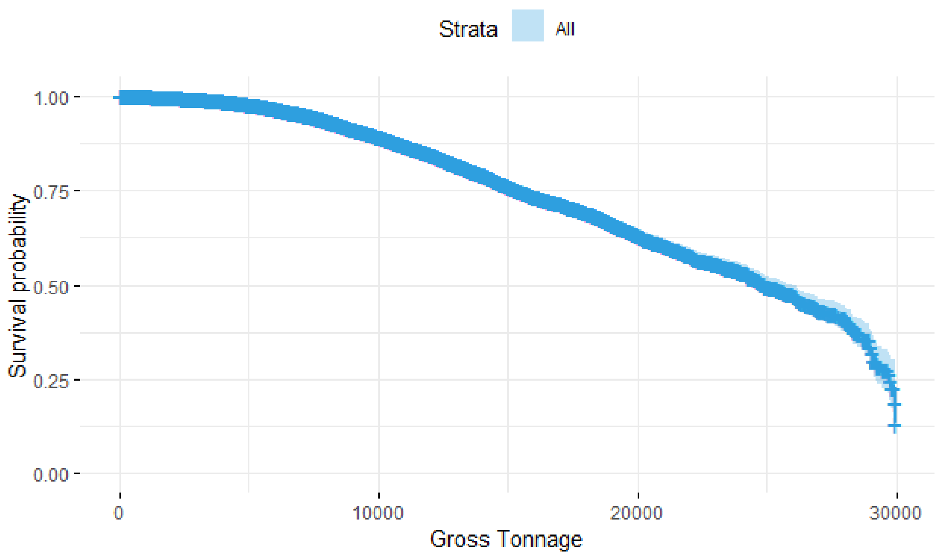
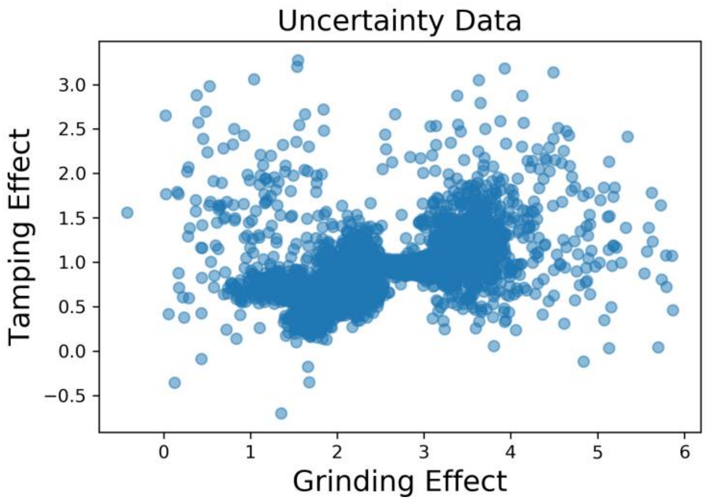
Chatham
Kalamazoo
Ann Arbor
Lake Erie
Erie
Rockford
Chicago
South Bend
Toledo
Cleveland
Aurora
Davenport
Fort Wayne
Pennsylvania
Peoria
Pittsburgh
Bloomington
Illinois
Springfield
Columbus
West Virginia
St. Louis
Louisville
Charleston
Lexington
Kentucky
Virginia
Roanoke
Nashville
Greensboro
Raleigh
Tennessee
Knoxville
Asheville
Concord
North Carolina
Memphis
Chattanooga
Charlotte

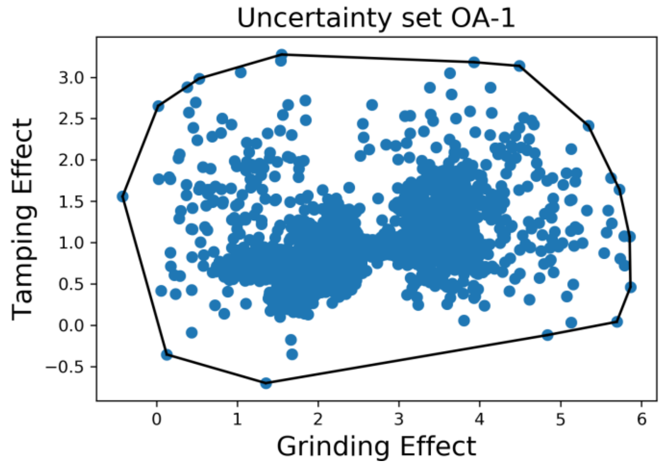
Model Parameter estimation

- Quality Index;
  - Fitted a Cox hazard model
  - Predicted the hazard rate for each segment
  - Inverse of the Hazard rate is used as a quality index

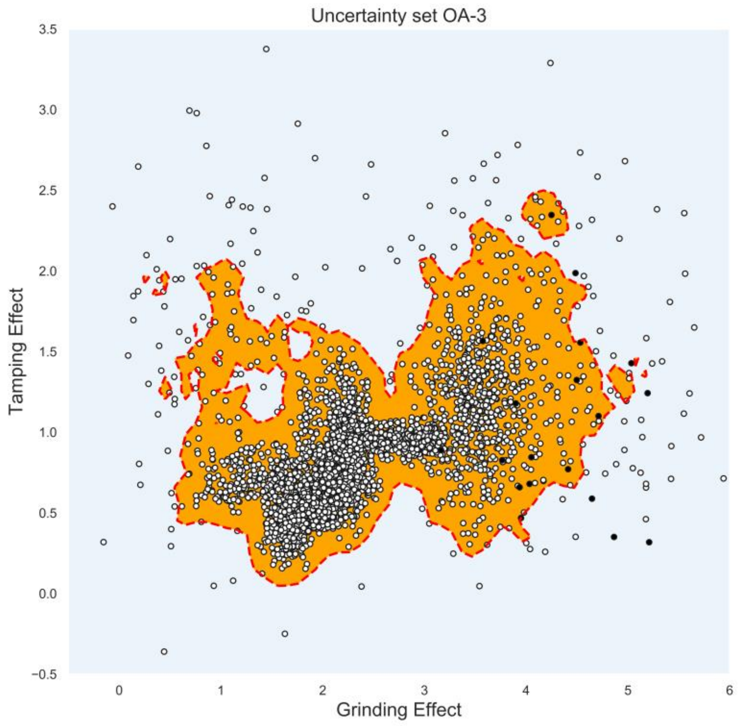


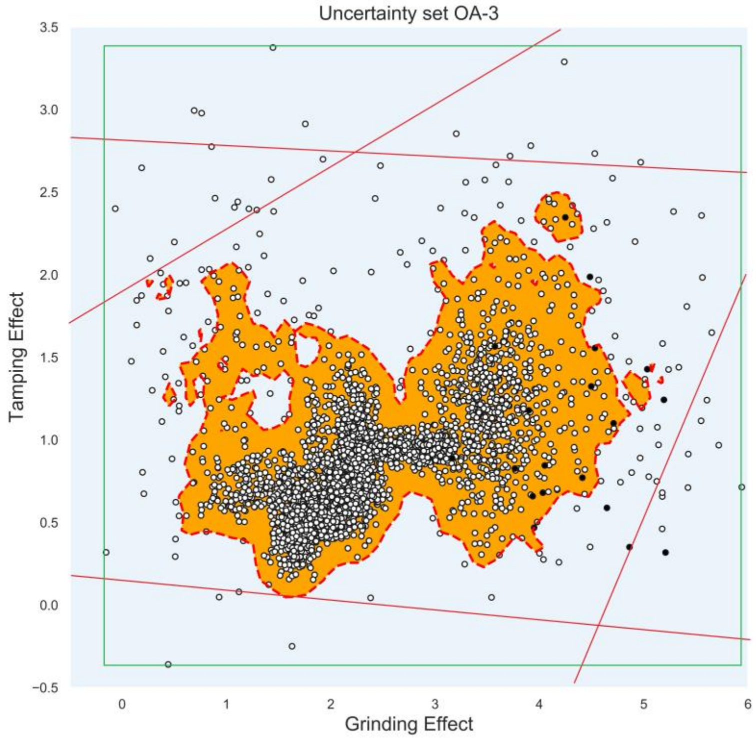
Uncertainty sets





Uncertainty set OA-3



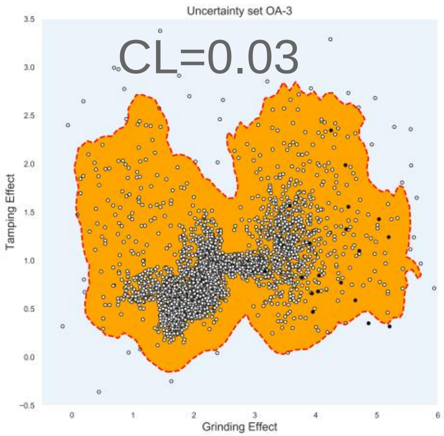


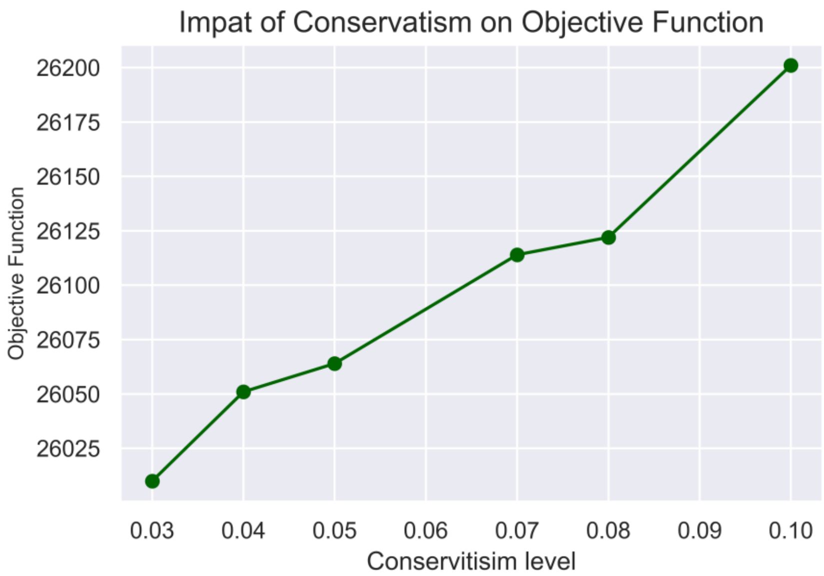
Computation results

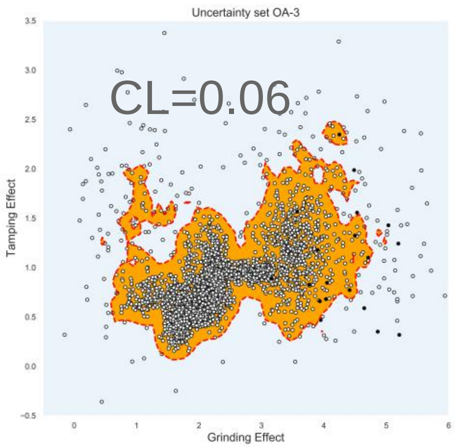
- Gurobi is used to solve the model.

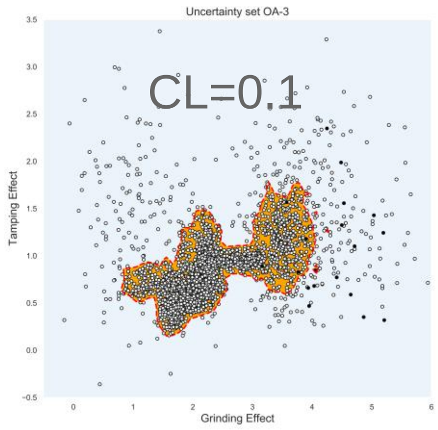
	Deterministic Model	OA-1	OA-2	OA-2
CPU Time	5802	7464	11400	8750
Objective Function	26430	25961	25961	26010
Gap%	2%	2%	2%	2%
Level of Conservatism	-	Worst Case	Worst Case	0.03

OA-3 Conservatism level

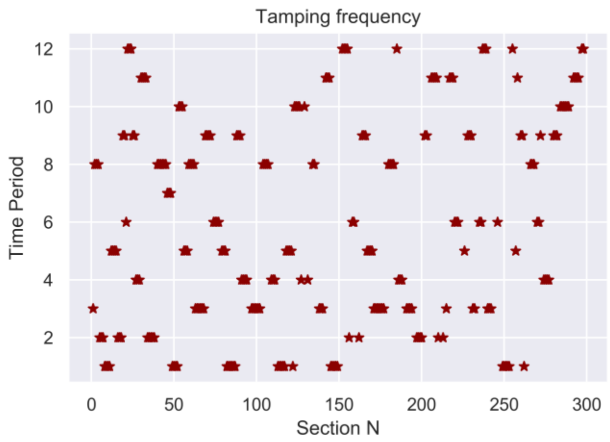


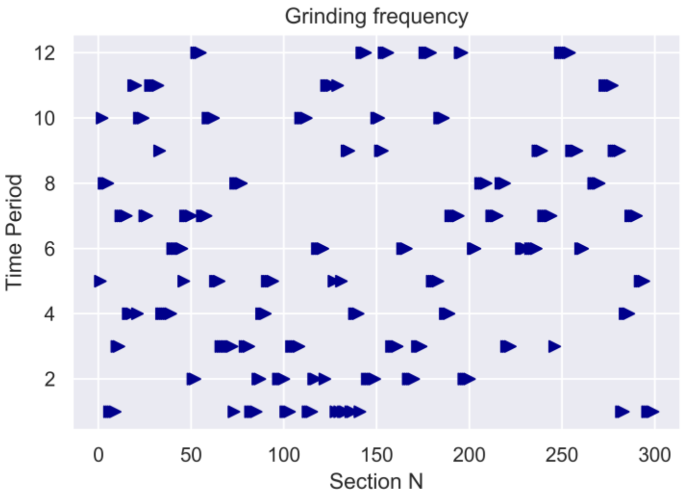


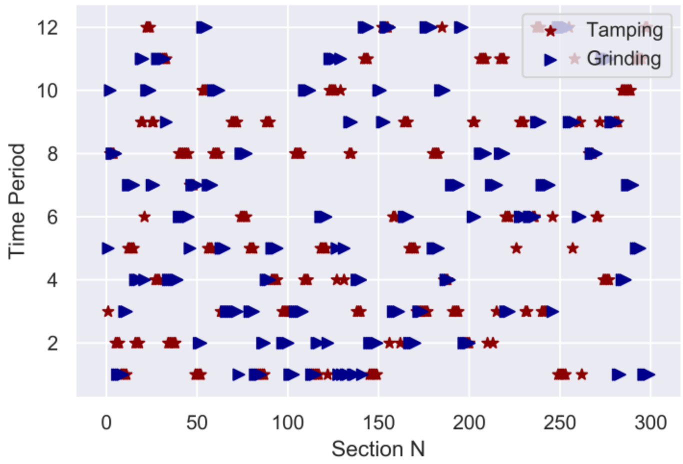




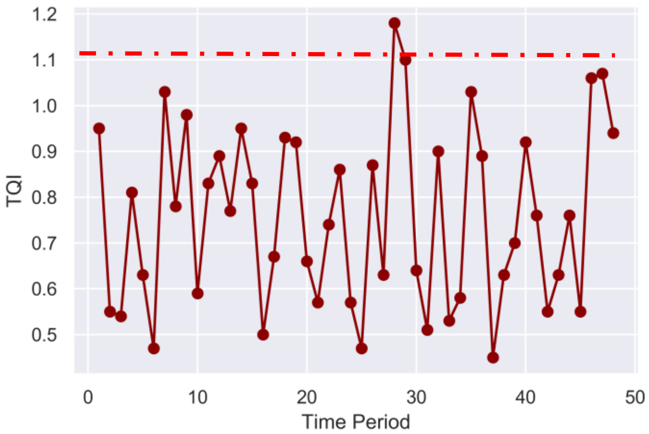
Tamping and grinding frequency

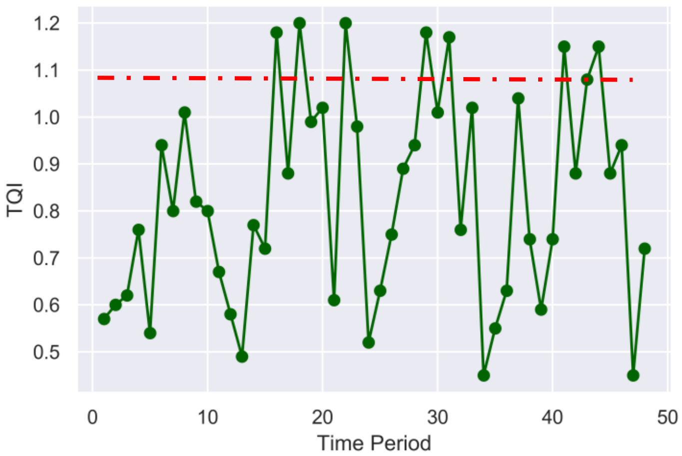


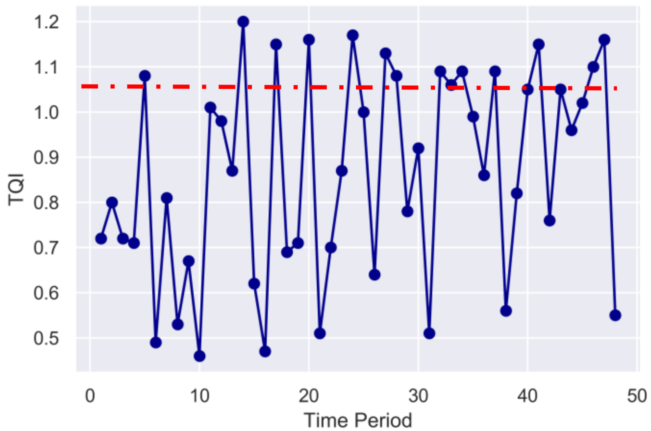


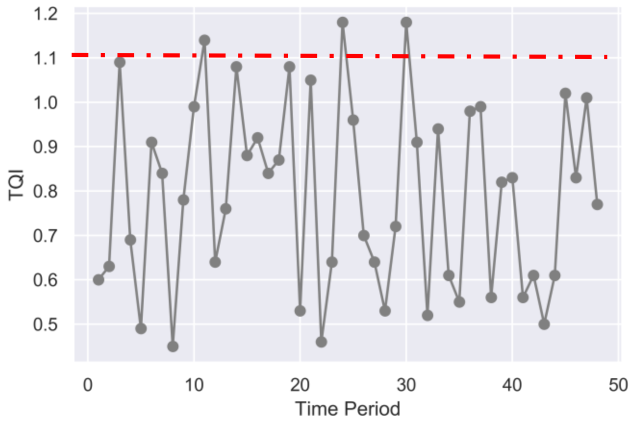


TQI results for selected segments









Conclusion

- Data-driven uncertainty modeling results in more reliable and robust track condition with the same amount of budget
- Level of uncertainty could be adjusted based on the importance of the segment
- Joint optimization of maintenance task results in quality index improvement
- Considering features such as suppression time and maintenance task correlation results in more realistic model

Questions?