Integrated Train Timetabling:
Problem Decomposition for Vehicle Routing based on an Alternating Direction Method of Multiplier framework

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Introduction

planning

Well-designed passenger train operation plans are critical ***for passengers*** to complete their trips in a fine manner and ***for operators*** to maximize their profits.

line planning → lines, types, stops and frequencies of passenger trains

↓

train timetabling → a set of train paths with detailed routes and time stamps

↓

rolling stock scheduling

↓

crew scheduling

In this paper, we aim to jointly optimize train stops, routes and time stamps of individual trains and service frequencies (in terms of accurate service interval time) involving a team of trains.

Literature review

- **line planning and demand-oriented train timetabling in railway systems**

publication	context	Objective function	Time-dependent passenger demand	Rolling stock capacity	Infrastructure capacity
Chierici et al. (2004)	N	Maximize the total demand captured by trains	T	N	H
Albrecht (2009)	U	Minimize operation cost and passenger travel time	T	T	H
Cordone and Redaelli (2011)	N	Maximize the total demand captured by trains	T	N	H

Literature review

- **line planning and demand-oriented train timetabling in railway systems**

publication	context	Objective function	Time-dependent passenger demand	Rolling stock capacity	Infrastructure capacity
Niu and Zhou (2013)	U	Minimize the total number of waiting passengers and weighted remaining passengers	T	T	H
Sun et al. (2014)	U	Minimize total passenger waiting time	T	T	H
Canca et al. (2014)	U	Minimize the total average waiting time	T	T	H

Literature review

- **line planning and demand-oriented train timetabling in railway systems**

publication	context	Objective function	Time-dependent passenger demand	Rolling stock capacity	Infrastructure capacity
Barrena et al. (2014a)	U	Minimize average waiting time of passengers	T	T	H
Kaspi and Raviv (2015)	N	Minimize passenger inconvenience and operational cost	T	F	T
Niu and Zhou (2015)	N	Minimize the total passenger waiting time	T	T	H
This paper	N	Maximize profits of railway operator(s)	T	T	T

Literature review

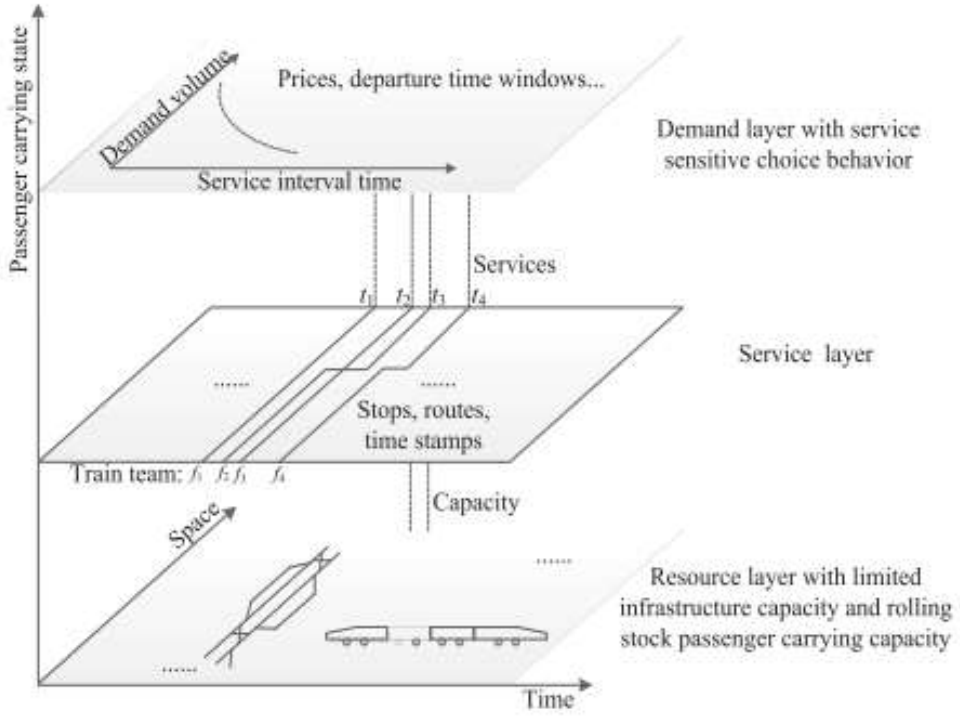
- **service network design in railway systems**

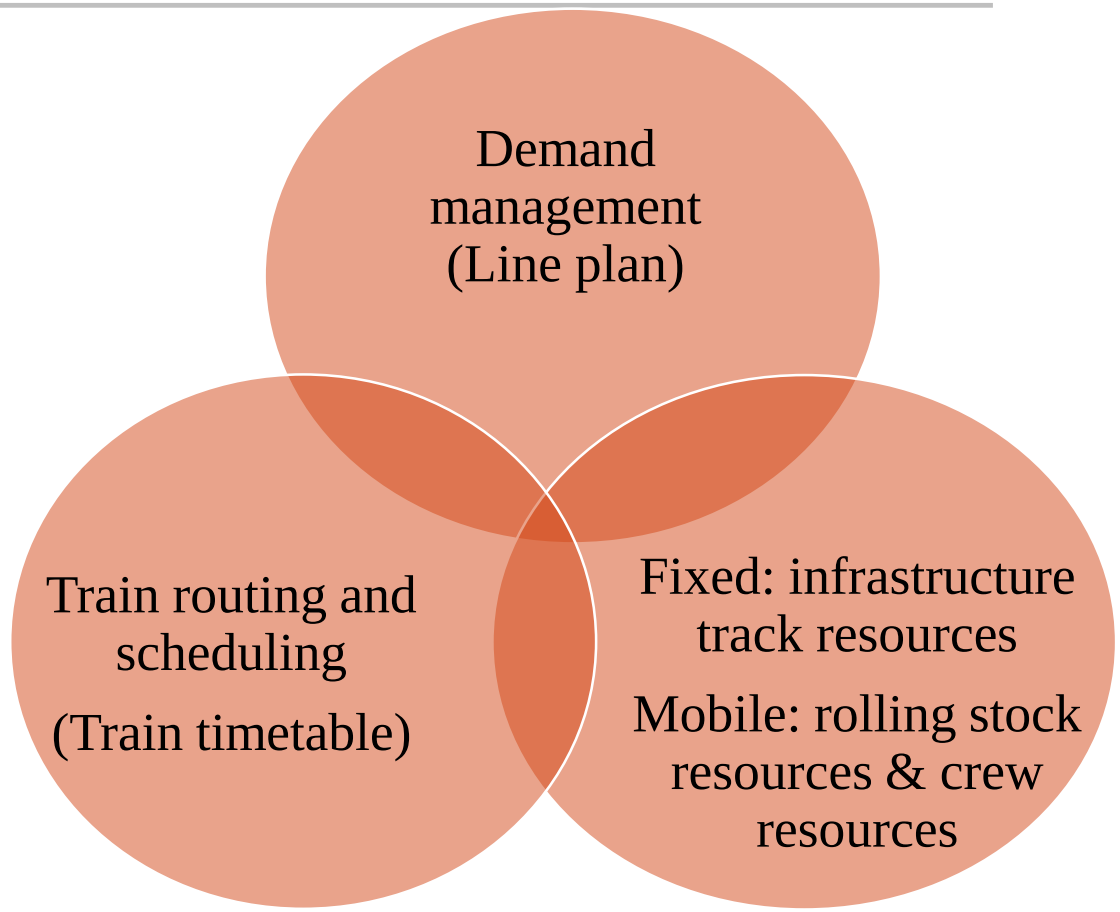
widely used in consolidation-based ***freight railway operations planning***, few studies on designing passenger service networks;

modeling ***usage of infrastructure capacity*** by trains makes designing a passenger train service network quite ***complicated***

Efficient implement of ADMM

Railway scheduling problem





Meng L, Zhou X*. An integrated train service plan optimization model with variable demand: a team-based scheduling approach with dual cost information in a layered network[J]. Transportation Research Part B, 2018.

CONTENTS

Background information on ADMM

The alternating direction method of multipliers (ADMM) was first introduced in the mid-1970s by Gabay, Mercier, Glowinski, and Marrocco, though similar ideas emerged as early as the mid-1950s. It is a simple but powerful algorithm and well suited to distributed convex optimization, and in particular to problems arising in applied statistics and machine learning.

Boyd S, Parikh N, Chu E, et al. Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers[J]. Foundations & Trends in Machine Learning, 2011, 3(1):1-122.

Background information on ADMM

01 Lagrangian relaxation

$$\text{minimize} \quad f(x)$$

$$\text{subject to} \quad Ax - c = 0$$

where x ∈ R^n, A ∈ $R^{p\times n}$, and c ∈ R^P, and $f(x)$ are assumed to be convex.

$$L(x,\lambda) = f(x) + \lambda^T(Ax - c)$$

$$x^{k+1} := \underset{x}{\operatorname{argmin}} L(x,\lambda^k)$$

$$\lambda^{k+1} := \lambda^k + \alpha^{k+1}(Ax^{k+1} - c)$$

The problem can be divided into two shortest-path problems for each vehicle. The relaxed constraint is ensured by adjusting Lagrangian multiplier λ_p.

Market Mechanism

Background information on ADMM

02 Augmented Lagrangian Method

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & Ax - c = 0 \end{aligned}$$

$$L(x, \lambda, \rho) = f(x) + \lambda^T (Ax - c) + \boxed{(\rho/2) \left\| Ax - c \right\|_2^2}$$

ρ is the parameter of quadratic penalty.

Augmented Lagrangian methods were developed in part to bring **robustness** to Lagrangian relaxation.

Market Mechanism + Administration Measure

The separability is lost.

Background information on ADMM

03 ADMM

ADMM approach can be viewed as an attempt to blend the decomposability of Lagrangian relaxation and the reliability of augmented Lagrangian method.

$$\text{minimize} \quad f(x) + g(z)$$

$$\text{subject to} \quad Ax + Bz - c = 0$$

$$L(x, z, \lambda, \rho) = f(x) + g(z) + \lambda^T (Ax + By - c) + (\rho/2) \left\| Ax + By - c \right\|_2^2$$

$$x^{k+1} := \underset{x}{\operatorname{argmin}} L(x, z^k, \lambda^k)$$

$$z^{k+1} := \underset{z}{\operatorname{argmin}} L_\rho(x^{k+1}, z, \lambda^k)$$

$$\lambda^{k+1} := \lambda^k + \rho\,(Ax^{k+1} + Bz^{k+1} - c)$$

Market Mechanism & Administration Measure Decomposable

03 ADMM

The necessary and sufficient optimality conditions：

primal feasibility: $Ax + Bz - c = 0$

dual feasibility: $\nabla f(x^*) + A^T\lambda^* = 0$

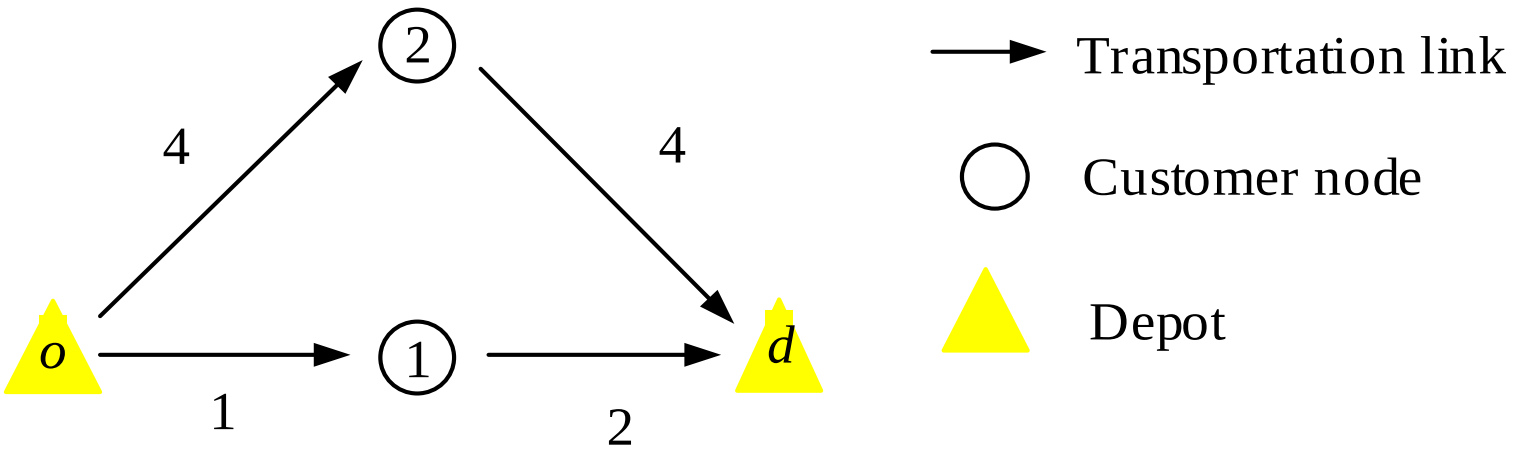
$\nabla g(z^*) + B^T\lambda^* = 0$

Eckstein J, Bertsekas D P. On the Douglas-Rachford splitting method and the proximal point algorithm for maximal monotone operators. *Mathematical Programming*, 1992.

CONTENTS

The advantage of ADMM

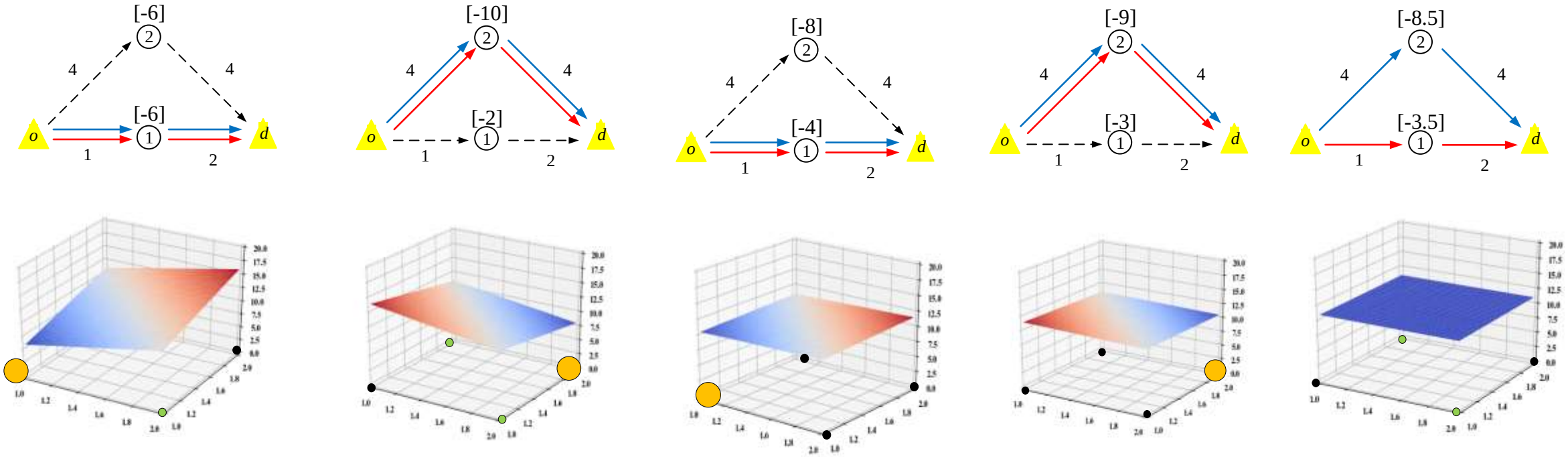
robust dual decomposition or **decomposable** method of multipliers



The advantage of ADMM

Lagrangian relaxation

Path of Vehicle 1 Path of Vehicle 2



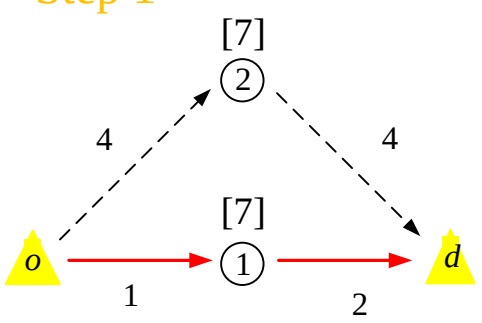
Two vehicles have the same Lagrangian multipliers in each iteration, two homogeneous vehicles will perform consistent route, which is infeasible in reality.

The advantage of ADMM

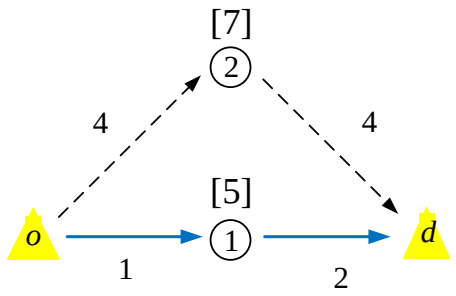
ADMM

Iteration 1

Step 1

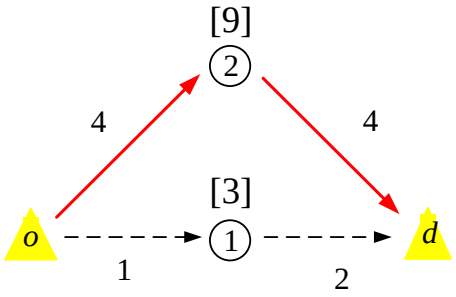


Step 2

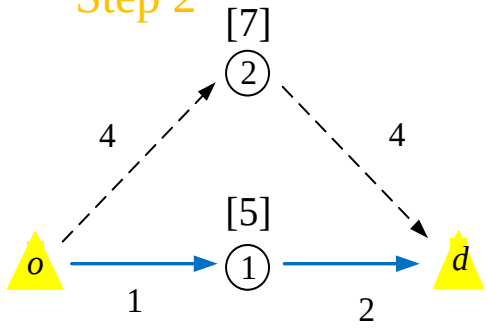


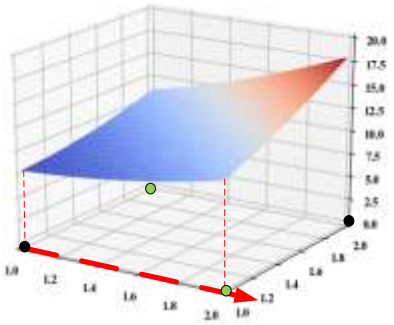
Iteration 2

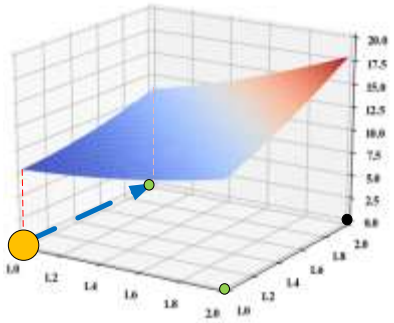
Step 1

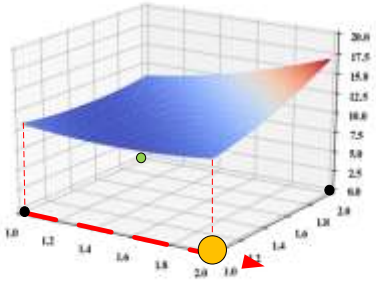


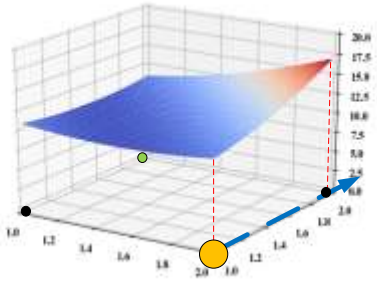
Step 2





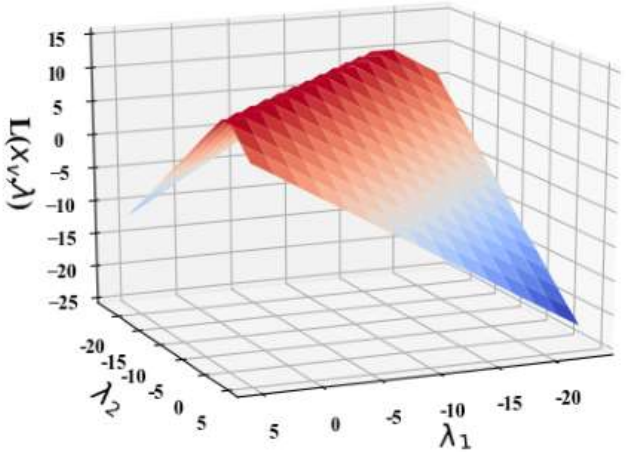




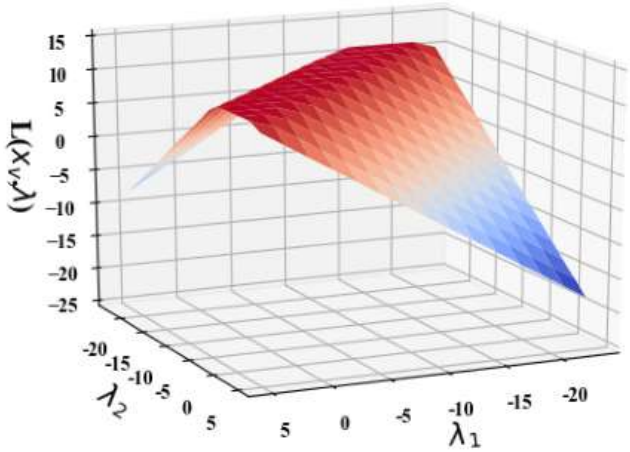


the service profit for each vehicle to serve customer p not only depends on Lagrangian multiplier λ_p but also the quadratic penalty terms related to parameter ρ.

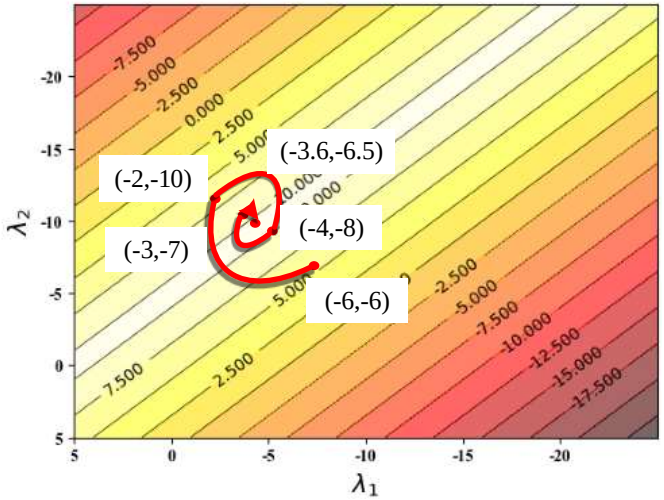
The advantage of ADMM



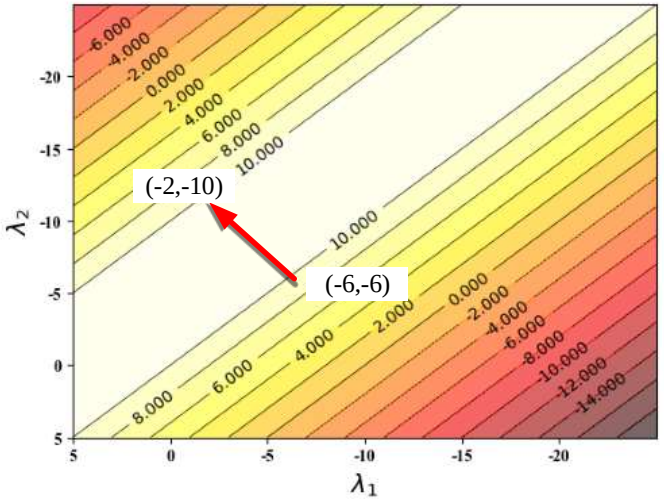
(a) surface of Lagrangian problem



(b) surface of augmented Lagrangian problem



(c) contour and solution process of LR



(d) contour and solution process of ADMM

The advantage of ADMM

$$L(x, z, \lambda, \rho) = f(x) + g(z) + \boxed{\lambda^T}(Ax + By - c) + (\boxed{\rho}/2)\left\|Ax + By - c\right\|_2^2$$

Primal and dual decomposition

Symmetry breaking

Reliable decomposition

CONTENTS

Vehicle Routing Problem with Time Windows (VRPTW)

Inputs：

- Customers' Location
- Customers' Preferred Delivery Time Windows

Outputs：

- Vehicle Routing
- Vehicle Scheduling
- Assigning Passengers to Vehicles (Many-to-One-Relationship)
- Price

Efficient implement of ADMM

Vehicle Routing Problem with Time Windows (VRPTW)

$$\min\ Z = \sum_{v\in(V\cup V^*)} \sum_{(i,j,t,s,w,w')\in A_v} c_{i,j,t,s,w,w'} x^v_{i,j,t,s,w,w'}$$

Minimize total costs

$$\sum_{(i,j,t,s,w,w')\in A_v} x^v_{i,j,t,s,w,w'} = 1 \quad i = o_v, t = e_v, w = w_0, \forall v \in (V \cup V^*)$$

Flow balance constraints

$$\sum_{(i,j,t,s,w,w')\in A_v} x^v_{i,j,t,s,w,w'} = 1 \quad j = d_v, s = l_v \ \ \forall v \in (V \cup V^*)$$

$$\sum_{(j,s,w'')} x^v_{i,j,t,s,w,w''} - \sum_{(j',s',w')} x^v_{j',i,s',t,w',w} = 0 \ \ (i,t,w) \notin \{(o_v, e_v, w_0), (d_v, l_v, w)\}, \forall v \in (V \cup V^*)$$

$$\boxed{\sum_{v\in(V\cup V^*)} \sum_{(i,j,t,s,w,w')\in \Psi_{p,v}} x^v_{i,j,t,s,w,w'} = 1 \quad \forall p \in P}$$

Request constraint

Coupling constraints

Efficient implement of ADMM

abbreviate (i, j, t, s, w, w') as a

$$\min L = \sum_{v \in (V \cup V^*)} \sum_{a \in A_v} c_a x_a^v + \sum_{p \in P} \lambda_p \left[\sum_{v \in (V \cup V^*)} \sum_{a \in \Psi_{p,v}} x_a^v - 1 \right] + \frac{\rho}{2} \sum_{p \in P} \left\| \sum_{v \in (V \cup V^*)} \sum_{a \in \Psi_{p,v}} x_a^v - 1 \right\|_2^2$$

and subject to constraints (4),(5) and (6).

Proposition. *Each subproblem can be converted in to an integer linear programming:*

$$L_v = \sum_{a \in A_v} \xi_a^v x_a^v + K$$

$$\mu_p^v = \sum_{v' \in (V \cup V^*)/\{v\}} \sum_{a \in \Psi_{p,v}} x_a^v \quad \forall p \in P$$

$$\xi_a^v = \begin{cases} c_a + \lambda_p + \rho \mu_p^v - \dfrac{\rho}{2} & a \in \Psi_{p,v} \\ c_a & otherwise \end{cases}$$

Efficient implement of ADMM

Proof.

$$\mu_p^v = \sum_{v' \in (V \cup V^*)/\{v\}} \sum_{a \in \Psi_{p,v}} x_a^v \quad \forall p \in P$$

$$L_v = \sum_{a \in A_v} c_a x_a^v + \sum_{p \in P} \sum_{a \in \Psi_{p,v}} \lambda_p x_a^v + \frac{\rho}{2} \sum_{p \in P} \boxed{\left\| \sum_{a \in \Psi_{p,v}} x_a^v + \mu_p^v - 1 \right\|_2^2}$$

$$\boxed{\left[\sum_{a \in \Psi_{p,v}} x_a^v \left(2\mu_p^v - 1\right) + \left(\mu_p^v - 1\right)^2 \right]}$$

$$\left\| \sum_{a \in \Psi_{p,v}} x_a^v + \mu_p^v - 1 \right\|_2^2 = \left\| \sum_{a \in \Psi_{p,v}} x_a^v \right\|_2^2 + 2 \sum_{a \in \Psi_{p,v}} x_a^v \left(\mu_p^v - 1\right) + \left(\mu_p^v - 1\right)^2$$

$$\left\| \sum_{a \in \Psi_{p,v}} x_a^v \right\|_2^2 = \sum_{a \in \Psi_{p,v}} x_a^v$$

$$\left\| \sum_{a \in \Psi_{p,v}} x_a^v + \mu_p^v - 1 \right\|_2^2 = \sum_{a \in \Psi_{p,v}} x_a^v \left(2\mu_p^v - 1\right) + \left(\mu_p^v - 1\right)^2$$

Efficient implement of ADMM

Proof.

$$L_v = \sum_{a \in A_v} c_a x_a^v + \sum_{p \in P} \sum_{a \in \Psi_{p,v}} \lambda_p x_a^v + \frac{\rho}{2} \sum_{p \in P} \left[\sum_{a \in \Psi_{p,v}} x_a^v \left(2\mu_p^v - 1\right) + \left(\mu_p^v - 1\right)^2 \right]$$

$$L_v = \sum_{a \in A_v} c_a x_a^v + \sum_{p \in P} \sum_{a \in \Psi_{p,v}} \left[\lambda_p \, x_a^v + \frac{\rho}{2} x_a^v \left(2\mu_p^v - 1\right) \right] + K \quad = \sum_{a \in A_v} \xi_a^v x_a^v + K$$

$$\xi_a^v = \begin{cases} c_a + \lambda_p + \rho \mu_p^v - \dfrac{\rho}{2} & a \in \Psi_{p,v} \\ c_a & otherwise \end{cases}$$

Efficient implement of ADMM

ADMM-based solution framework

//**Step 1**: Initialization

//**Step 2**: Solve the decomposed sub-problems sequentially

For each vehicle $v \in (V \cup V^*)$ do

Begin

Find time-dependent state-space-time shortest path for vehicle v based on the arc cost ξ_a^v by using the forward dynamic programming algorithm;

Update the arc cost $\xi_a^v \in \Psi_{p,v}$

//**Step 3:** Generate upper bound UB^*

//**Step 4**: Generate lower bound LB^*

Calculate lower bound LB^k by solving the Lagrangian dual problem.

//**Step 5:** Update Lagrangian multipliers using sub-gradient optimization procedure

Update Lagrangian multipliers by:

$\lambda_p^{k+1} = \lambda_p^k + \rho^k (\sum_{v \in V} \sum_{a \in \Psi_{p,v}} x_a^v - 1) \ \ \forall p \in P$,;

Update quadratic penalty by:

$$\rho^{k+1} = \begin{cases} \beta \rho^k & if \ \sum_{p \in P} (\sum_{v \in V} \sum_{a \in \Psi_{p,v}} x_a^{v\,k+1} - 1)^2 \geq \gamma \sum_{p \in P} (\sum_{v \in V} \sum_{a \in \Psi_{p,v}} x_a^{v\,k} - 1)^2 \\ \rho^k & otherwise \end{cases}$$

where $2 \leq \beta \leq 10$ and $\gamma = 0.25$ can be chosen.

//**Step 6:** Termination condition test

If the iteration number k is larger than the maximum iteration number, terminate the algorithm and output the best lower bound LB^* and best upper bound UB^*; otherwise, $k = k + 1$ and go back to Step 2.

Efficient implement of ADMM

ADMM-based solution framework

Remark 1: In the ADMM-based solution framework, the sub-problem is a state-space-time shortest path problem for single vehicle, which can be solved by efficient forward dynamic algorithm package.

Remark 2: In this VRP problem, all vehicles will perform consistent routes when a standard Lagrangian relaxation method is applied as all of the vehicles are homogeneous. We should use the cost of the shortest path to calculate the lower bound mathematically.

Remark 3: When we solve the subproblem for a single vehicle, the routes of other vehicles are known and need to be taken into account as shown in Figure . Thus, the routes of all vehicles is keep updating but do not need to reset.

Efficient implement of ADMM

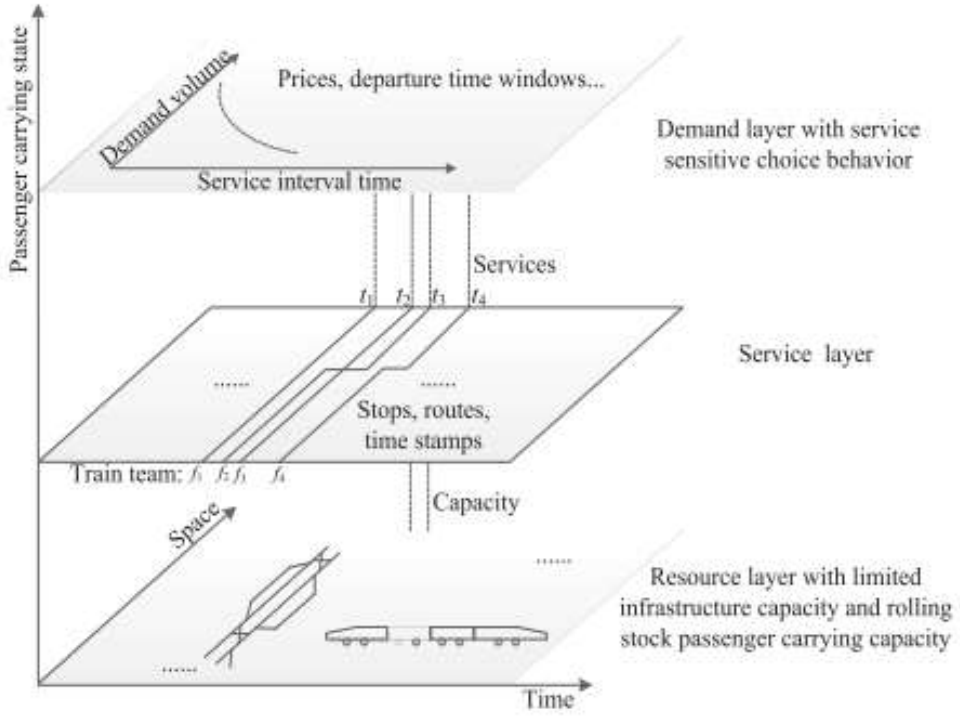
ADMM-based solution framework

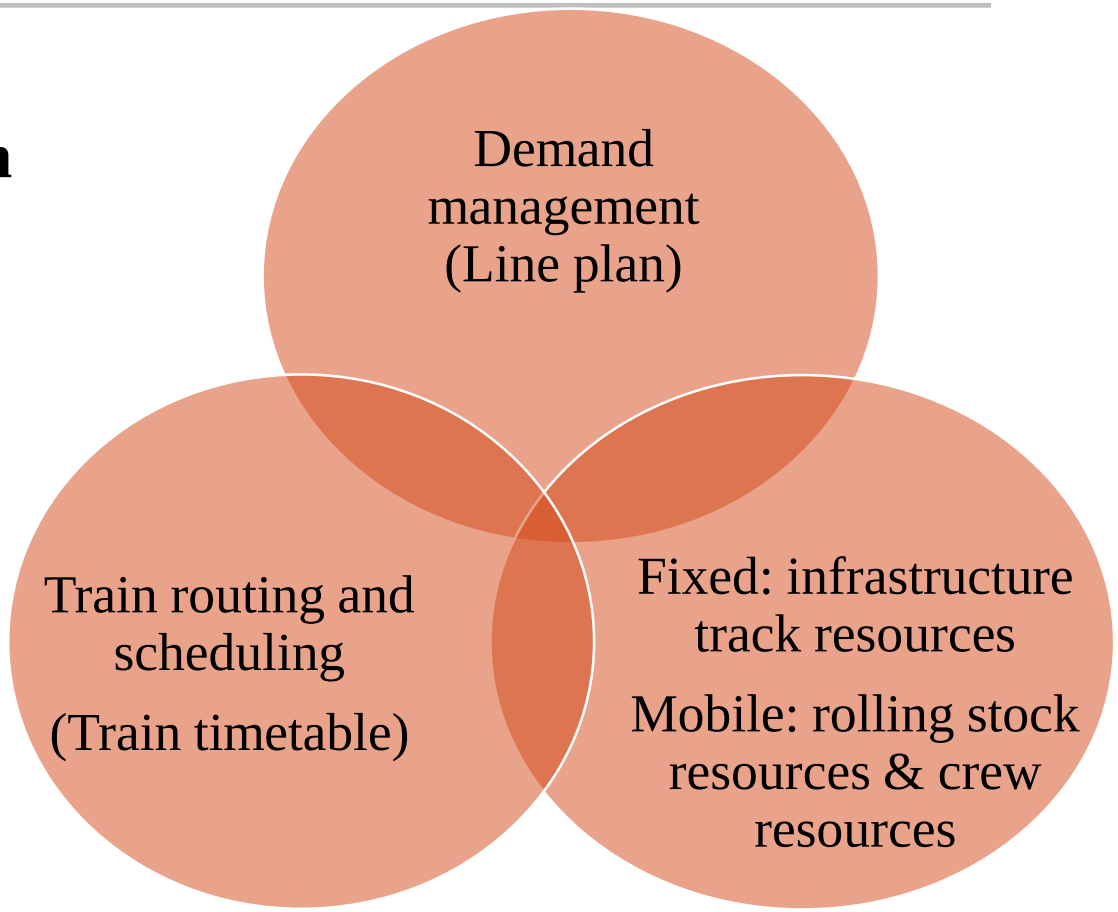
Remark 3: When we solve the subproblem for a single vehicle, the routes of other vehicles are known and need to be taken into account as shown in Figure . Thus, the routes of all vehicles is keep updating but do not need to reset.

Iteration k	Iteration $k+1$
$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr	$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr
$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr	$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr
$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr	$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr
……	……
$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr	$v1$ $v2$ $v3$ …… $v(r-2)$ $v(r-1)$ vr

Efficient implement of ADMM

Extension 1: Railway scheduling problem





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Extension 2: Assignment and Routing

Request assignment constraint:

$$\sum_{v \in (V \cup V^*)} y_p^v = 1 \quad \forall p \in P$$

Consistency constraint between assignment and routing:

$$\boxed{\sum_{(i,j,t,s,w,w') \in \Psi_{p,v}} x_{i,j,t,s,w,w'}^v = y_p^v \quad \forall p \in P, v \in (V \cup V^*)}$$

Coupling constraints

Extension 2: Assignment and Routing

subproblem x: routing (dynamic programming)
Subproblem y: generalized assignment (saving algorithm)

λ_p^v *and* ρ

Iteration k		Iteration $k+1$	
Assignment	*Routing*	*Assignment*	*Routing*
Assignment	*Routing*	*Assignment*	*Routing*
Assignment	*Routing*	*Assignment*	*Routing*

CONTENTS

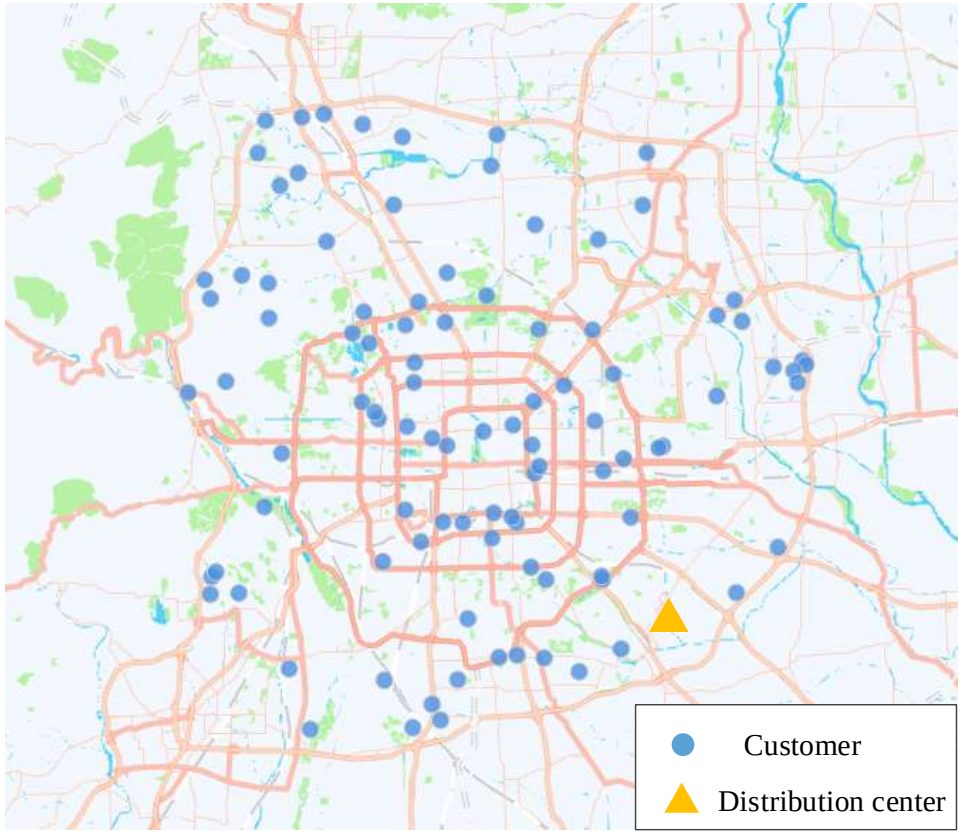
Numerical examples

Solomon benchmark instances

Dataset	Number of customers	Best known Solution	ADMM Solution	Computing time	Gap between best known and ADMM solution
R101	25	617.1	634.9	0.34s	2.80%
	50	1044.0	1083.6	3. 52s	3.65%
	100	1637.7	1724	48.49s	5.01%
C101	25	191.3	191.8	1.55s	0.26%
	50	362.4	363.2	12.11s	0.22%
	100	827.3	907	106.28s	8.78%
RC101	25	461.1	470.7	3.08s	2.04%
	50	944	1039.1	21.76s	9.15%
	100	1619.8	1813	143.26s	10.66%

Numerical examples

The real-world case study

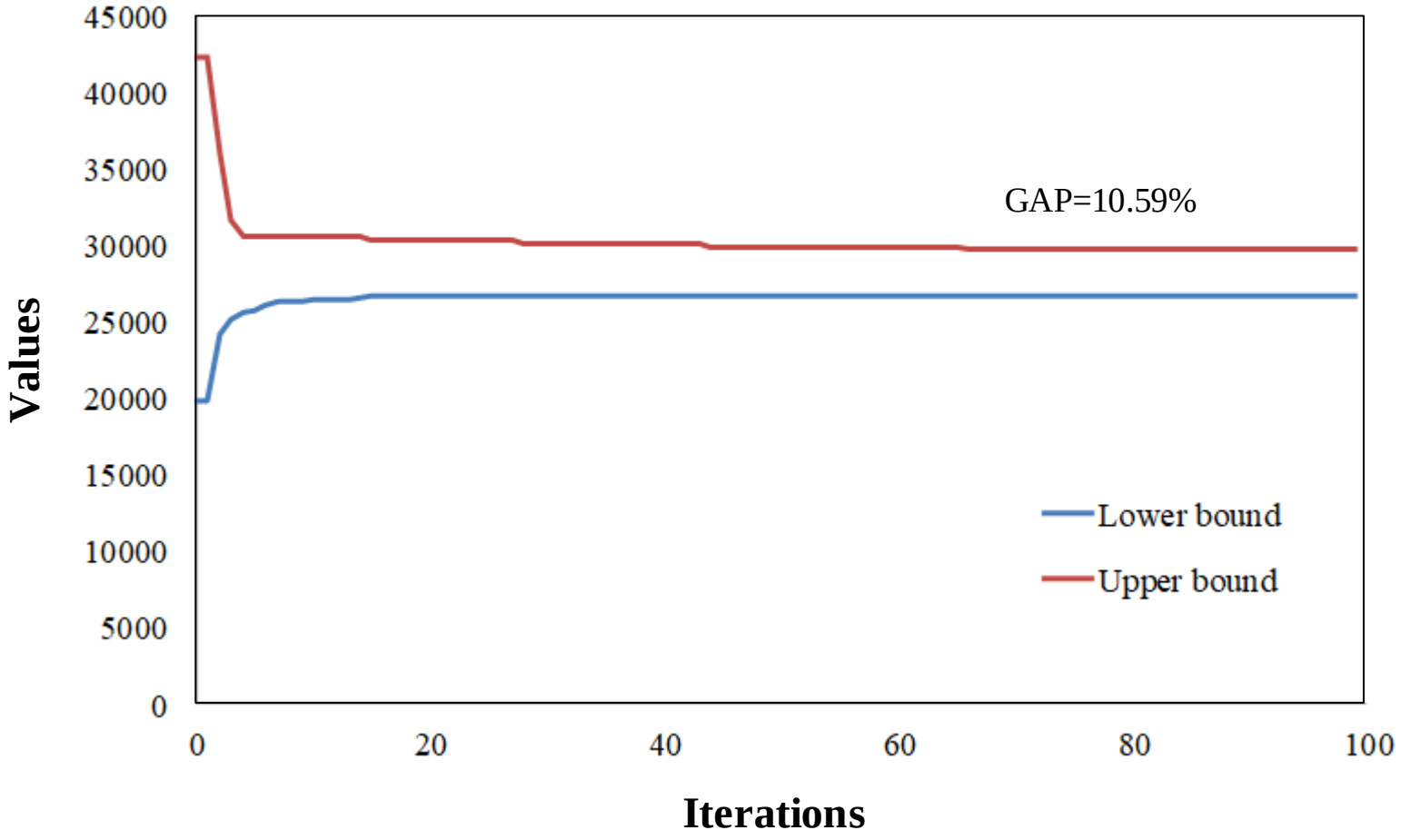


1. Each vehicle starts working after 8:00 and needs to return to the distribution center before 00:00.
2. The fixed usage charge is 200 yuan/day and the transportation cost is 12yuan/km. In addition, the number of backup vehicles is sufficient and there is no cost if the vehicle is not used.
3. The weight capacity of each vehicle is 12,000 kg and volume capacity is $12m^3$.
4. Each order or customer has a particular time window. The vehicle must arrive before the latest distribution time and cannot distribute until the earliest distribution time, the waiting cost is set as 24yuan/hour.

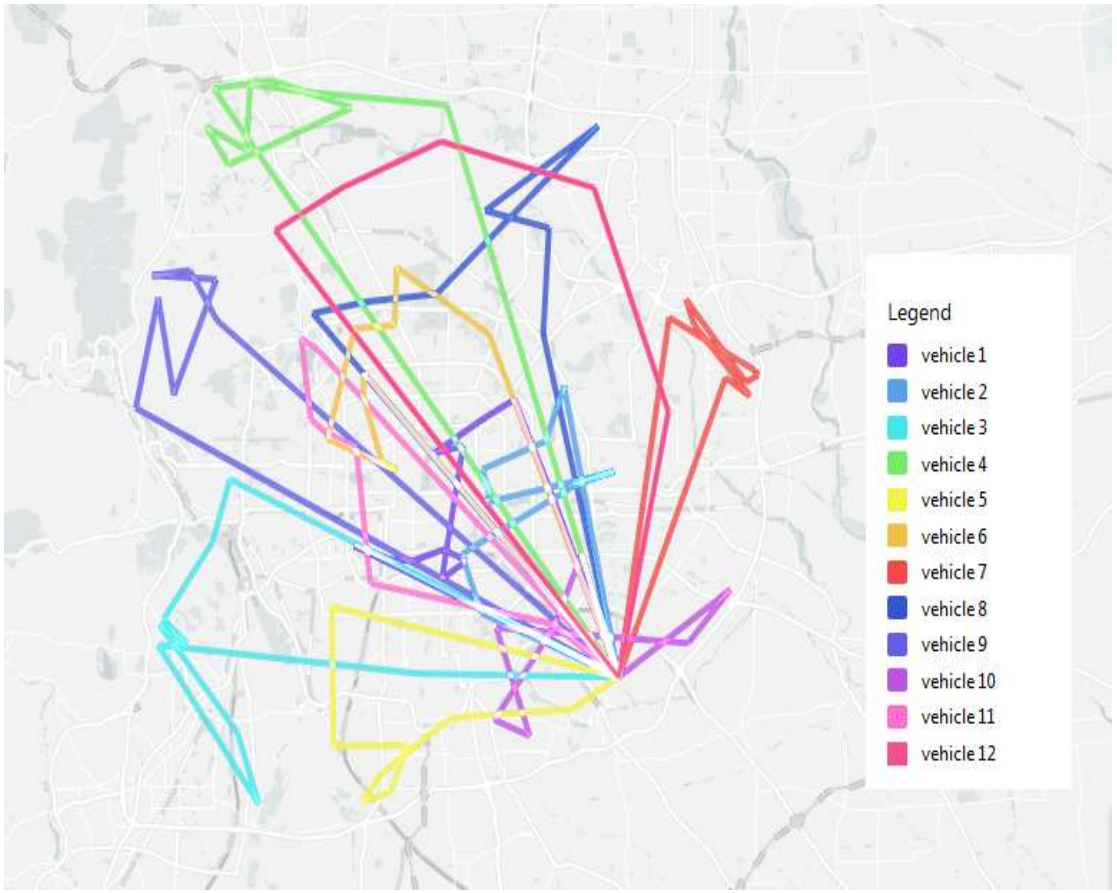
The objective is to minimize the total cost, including the transportation cost, the waiting cost and the fixed vehicle cost.

实际算例

The real-world case study



real-world case study



Value of objective function	29800.47
Total transportation cost	27154.87
Total waiting cost	45.6(114min)
Total number of vehicles needed	12

Thanks!

2018.11.04