Trade-off between efficiency and fairness in timetabling on a single urban rail transit line under time-dependent demand condition

State Key Lab of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing, China
Prof. Dewei Li E-mail: lidw@bjtu.edu.cn
Contents

1. Motivation
2. Efficiency and fairness in timetabling
3. Modeling and algorithm
4. Case study
5. Sensitivity analysis
6. Conclusions
1. Motivation

An optimal demand oriented timetable obtained.

When minimizing the total waiting time of all passengers during timetabling, a small number of passengers would have to endure very long waiting time.

Whether it is possible to achieve a reasonable amount of efficiency and guarantee a certain degree of fairness?
The efficiency of a train timetable can be defined by the ratio of the output to input.

Efficiency = output / input

\[
\eta_{\text{efficiency}} = \frac{P_m}{\alpha_1 \times T_{\text{waiting}} + \alpha_2 \times T_{\text{in-vehicle}} + N \times \lambda}
\]

\[
\text{obj}_{\text{efficiency}} = \min(\alpha_1 \times T_{\text{waiting}} + \alpha_2 \times T_{\text{in-vehicle}} + \lambda \times N)
\]
2. Efficiency and fairness in timetabling

Fairness in timetable

Two kinds of fairness in allocation

Equal allocation

On-demand allocation
2. Efficiency and fairness in timetabling

(1) Min-max fairness in train timetable (Equal allocation)

Reducing the differences of waiting times among different passengers by minimizing the maximum waiting time of passengers.

\[ f_{\text{minmax-fairness}} = \min \{ \max(\text{AWT}_{OD}) \} \]

(2) \(\alpha\)-fairness in train timetable (On-demand allocation)

The waiting time of the passengers is adapted to the distances of their travel OD pairs. The shorter the distance, the shorter the passenger waiting time should be. Defined by a disutility function.

\[ U(O_{od}) = \log_{10}(1 + \frac{\text{AWT}_{OD}}{\text{IVT}_{OD}}) \quad \iff \quad f_{\alpha-fairness} = \min \sum_{OD} \log(1 + \frac{\text{AWT}_{OD}}{\text{IVT}_{OD}}) \]
2. Efficiency and fairness in timetabling

The disutility value change with the waiting time.
3. Modeling and algorithm

Objective functions

Efficiency

$$obj_e = \min(\alpha_1 \times T_{waiting} + \alpha_2 \times T_{in\text{-}vehicle} + \alpha_3 \times N)$$

Efficiency and min-max fairness

$$obj_{e\&m-f} = \min[(\alpha_1 \times T_{waiting} + \alpha_2 \times T_{in\text{-}vehicle} + \alpha_3 \times N) + \beta \max(AWT_{OD})]$$

Efficiency and $\alpha$ fairness

$$obj_{e\&\alpha-f} = \min[(\alpha_1 \times T_{waiting} + \alpha_2 \times T_{in\text{-}vehicle} + \alpha_3 \times N) + \beta \sum \log_{10} (1 + \frac{AWT_{OD}}{IVT_{OD}})]$$
Modeling and algorithm

Constraints

\[ x_0^i = 0, \ i \in S \]
\[ x_{k+1}^i = p, \ i \in S \]
\[ p(1-y_m) \leq x_m^i \leq p, \ i \in S, m \in M \]
\[ N = \sum y_m < k + 2, m \in M \]
\[ x_m^{i+1} = x_m^i + s_m^i l_{i+1} + w_m^{i+1}, \ i \in S, m \in M \]

\[ y_m s_{min} \leq s_m^i \leq y_m s_{max}, \ i \in S, m \in M \]
\[ x_{m+1}^i \geq x_m^i + h_{min} y_m + w_m^{i+1}, \ i \in S, m \in M \]
\[ x_m^i \in N^+, \ i \in S, m \in M \]
\[ y_m \in \{0,1\}, \ m \in M \]
Algorithm

Simulated annealing controlled adaptive large neighborhood search algorithm (ALNS)

The main idea of ALNS is based on a number of destroy and repair operators. (Barrena in 2014)

Randomly insert train, insert trains in the maximum interval, inserts train between the interval with maximal demand.

Randomly delete trains, delete trains in the smallest interval, inserts train between the interval with the smallest demand.
Algorithm

Convergence of ALNS

In the algorithm, the number of iterations is set to 150000 times.

The algorithm is convergent at 100000 iterations through multiple experiments.
Case study

Rail transit line and passenger demand

Beijing subway Changping line

The cumulative passenger demand
Case study

Results

trains in the timetable considering the efficiency and fairness are more evenly distributed in time
Case study

Results

Timetable with efficiency & min-max fairness

Timetable with maximizing efficiency

KO
**Case study**

To ensure a fairer solution, the efficiency is sacrificed more or less (29.2% and 7.9%).

A “price of fairness” has always to be paid. Min-max fairness has more negative impact on efficiency than the $\alpha$-fairness counterpart.
Case study

Results

Timetable with maximizing efficiency  
Timetable with efficiency and min-max fairness
Case study

To ensure the equal fairness, under the goal of min-max fairness, the maximum passenger waiting time is reduced by 14.5%.
In the $\alpha$-fairness the maximum waiting time is also reduced by 9.6%.

Comparing the promotion of the fairness (9.6%) with the reduction of efficiency (7.9%), timetable considering the $\alpha$-fairness is a compromise timetable. We can say that $\alpha$-fairness timetable can achieve a reasonable amount of efficiency while the timetabling remains reasonably fair.
Case study

Results

Timetable with maximizing efficiency

Timetable with efficiency and $\alpha$ fairness
Case study

Fairness performance

Distribution of passenger waiting time for each OD pair

Passenger waiting times among different OD pairs can be adjusted according to the in-vehicle times when considering the $\alpha$-fairness of the train timetable.
Sensitivity analysis

Weight of fairness

The equilibrium can be obtained while setting the weight of fairness around $2.1 \times 10^5$

Note: the point is change with the number of trains
The efficiency value increases as the number of trains increases. When the number of trains continue to increase, the efficiency decreases.

More trains will bring more fairness effect, however, there is no significant change after the number of trains reaches a certain value.
Conclusions

1. An “efficient” timetable does not always perform well from perspective of fairness and vice versa.

To ensure a fairer solution, the efficiency is sacrificed more or less (29.2% for min-max fairness and 7.9% for $\alpha$-fairness).

2. Timetable considering the $\alpha$-fairness achieve a reasonable amount of efficiency while remains reasonably fair
Promotion of the fairness (9.6%) with the reduction of efficiency (7.9%)

3. An equilibrium can be found between efficiency and fairness
With the increasing weight of fairness, the efficiency decreases. The equilibrium of two point can be obtained while setting the weight of fairness around $2.1 \times 10^5$. 
Thank you for Attention

Dawei Li, Beijing Jiaotong University, Beijing, China