Warranty Cost Models of Renewable Risk-Free Policy for Multi-Component Systems

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Abstract: This paper presents several system warranty cost models of a renewable risk-free policy for multi-component products with system structure such as series, parallel, series-parallel, and parallel-series. Warranty cost distributions, expectations, variances, and prediction intervals are then derived to facilitate practical applications.

Notations

w = length of a warranty period.
T = length of a warranty cycle.
N = number of replacements during a warranty cycle T.
C = System warranty cost within a cycle T.
$F(t)$, $R(t)$, $h(t)$ = Cumulative distribution function, reliability function and hazard rate (failure rate) function respectively.
p_i = probability that the failure of a series system is due to component i.
p_{ij} = probability that the failure of i^{th} subsystem in series is due to j^{th} component in the subsystem.

1. Introduction

Warranty is an obligation attached to products that requires manufacturers (vendors or sellers) to provide per-specified compensation for consumers when the products fail to perform their designed functions under normal usage within the warranty coverage. One key issue of interest that arises from warranty analysis is the modeling of warranty cost. It involves a consideration of characteristics of warranty policies, replacement/repair cost upon each failure, and products' failure time distributions.

The characteristic of warranties is an essential factor in warranty cost. There are numerous types of warranties in practice. Usually warranty cost models heavily depend on the structure of underlying warranty policies. A slight change in a policy may result in a totally different cost model. For a comprehensive discussion on variations of warranty policies, we refer to Blishchke and Murthy [2,3].

Replacement/repair cost for products under warranty is often assumed to be constant. It can be justified when the structure of a product is simple, i.e., a single-component item for which the diagnosis and labor cost dominates other cost upon failure. However, for a multi-component product, the constant cost assumption may not be appropriate for the reasons such as: (1) Often such a product as well as its components are quite expensive, i.e., the powertrain of automobiles; (2) It is often the replacement/repair cost of the failed component(s) that determines the warranty cost. Although one can still assume constant replacement/repair cost at component level, which is adopted throughout this paper, it is impossible to foresee exactly which component will fail that may cause a system failure. As a result, models of warranty cost of complicated products should incorporate the information of product structure and failure mechanism.

Failure time distribution can be characterized by the corresponding reliability function, or equivalently, the hazard rate function. Previously, researchers dealing with a multi-component system often assume a specific type of failure time distribution for the system based on a black box approach. In this paper, instead of assuming a system failure time distribution, we only require a general hazard rate function for components/subsystems in series. No restriction is imposed for components/subsystems in non-series structures.

According to Blishchke and Murthy's taxonomy on warranties, the warranty policy considered in this paper is a Renewable Free Replacement (risk-free) Warranty (RFRW). Our main focus is on modeling warranty cost of multi-component items base on system structure. As described in reference [4, p.544], warranty analysis for complex systems is a new topic and there are few systematic and explicit analyses upon warranty policies for multi-component products although the topic has been addressed in a few papers. Ritchken [8] provides an example of two-component parallel system under a two-dimensional warranty. Hussain, etc. [5] also discuss warranty cost estimation for parallel systems. A Markovian approach to the analysis of warranty cost for a three-component system can be found in [1]. Chukova and Dimtrov [4] provide several warranty cost models for simple series systems and parallel systems based on renewal theory, but some important statistical properties of warranty cost such as warranty cost variance and prediction intervals, which are essential in evaluating warranty risks, are not addressed.

In this paper, we will focus on RFRW policies for complex products with non-repairable components. Warranty cost models under four different system structures: series system, parallel system, series-parallel system and parallel-series system, will be discussed. The corresponding warranty cost expectations and variances will be derived. The outline of the paper is as follows: In section 1, we provide a brief overview of warranty cost analysis. Section 2 introduces the warranty policy under study. Model assumptions and the justification of assumptions are also provided here. In section 3, a warranty cost model for q-component-in-series systems is derived. In section 4, we show a cost model for parallel systems. Section 5 generalizes the ideas in section 3 and 4, and presents warranty cost models for complex systems with series-parallel structure or parallel-series structure. Two numerical examples are given in section 6 to demonstrate our approach. Finally, we conclude this paper with a brief discussion of some of the topics currently under study and additional topics for future research.

2. Model Consideration

The warranty policy under study is a RFRW with a pre-specified warranty period denoted by w. For a multi-component system (product) under such a policy, whenever it fails within w, the failed component(s) will be replaced without any charge to consumers, and the repaired system will automatically carry the same warranty as for the original system. For example, in figure 1 t_1 is the first system failure time. Since $t_1<w$, the system will be repaired free of charge to consumers, but it will cost the manufacture c_1. Starting from t_1, the repaired system will carry the same warranty with period w again.

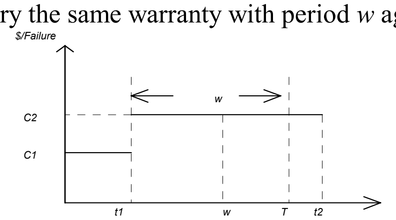

Figure 1: Warranty Cost Per Failure

Let's define the warranty cycle, T, as follows: T is the time interval from the time point of a system sold, to the time point when the warranty policy expires. For a renewable policy with warranty period w, it will expire if and only if the system under the warranty coverage survives a period w. For the example shown in figure 1, $T=t_1+w$ since the inter-arrival time of the second failure is t_2-t_1, which is longer than w, therefore, the warranty expires exactly at the time point t_1+w.

Assumptions

1. All components in a system under study are s-independent.
2. The replacement cost at component level is constant.
3. Upon any system failure, the failed component(s) will be identified and replaced instantly. The repaired system is assumed to be as good as new.
4. For i^{th} component (or subsystem) in series, the hazard rate function has the form $h_i(t)=\lambda_i g(t)$, where $\lambda_i>0$ and $g(t)$ can be any non-negative integrable function of t.

Justification of Assumptions

For components/subsystems in series, we assume that their hazard rate functions have the form of $\lambda_i g(t)$. The assumption is quite general, and many well-known distributions satisfy the assumption, for example, exponential, Rayleigh and Weibull distribution, etc.. Furthermore, there is no restriction on the failure time distributions of components that are in parallel. This makes our model more flexible and with more applicability.

It is also assumed that the repaired system is `as good as new.' The assumption is appropriate when components' failure times are independent and exponentially distributed because of the memoryless property of exponential distribution. Nevertheless, it might raise some concern when the failure times follow a non-exponential distribution. In such cases a minimal-repair (as-good-as-old repair) assumption may be adopted [7]. However, the `as-good-as-new' assumption is often reasonable for highly reliable systems for which usually no more than one system failure will happen under warranty. Actually, for such systems, even if a system failure happens within a warranty period, after the failed component(s) being replaced, the deterioration of other components is usually quite small. Hence the repaired system can still be treated approximately as good as new.

For simplicity, we assume instant diagnosis and replacement upon any system failure. The replacement cost at the component level is assumed to be constant, but at the system level, the repair cost incurred to manufactures is actually a random variable, whose values depend on the system failure mechanism and the components' reliability.

3. Cost Model of RFRW Policy for Series Systems

In this section, we will discuss cost models for series systems under RFRW policies. We will derive the distribution of warranty cost C, and then provide the expectation, variance and prediction intervals of C.

Figure 2: q-component series system

For the q-component series system shown in figure 2, it is easy to see that for a RFRW policy with warranty length w, the warranty cost C within a warranty cycle T, is given by:

$$C=\sum_{i=1}^{q} c_i N_i(w). \tag{1}$$

As a result, the system warranty cost distribution can be determined as long as one knows the distribution of $N_i(w)$. Since w is predetermined, we will suppress it later on to simplify our notations whenever appropriate.

Let us define $N=\sum_{i=1}^{q} N_i$, where N can be interpreted as the number of system failures within a warranty cycle T, which is also the number of replacement at component level within T. It is easy to see that N follows a Geometric distribution with parameter $F_s(w)$, where $F_s(w)$ is the system unreliability function evaluated at w. Therefore, the distribution mass function of N is:

$$P[N=k]=F_s(w)^k(1-F_s(w)), k=0,1,2,... \tag{2}$$

Lemma 1: *Given that hazard rate functions of component i in the series system has the form $h_i(t)=\lambda_i g(t)$, the conditional probability, p_i, that component i fails provided the system has failed is $\lambda_i/\sum_{i=1}^{q}\lambda_i$, a constant, not depending on the system failure time t, for i=1,2,…,q.*

Proof: Denote the random failure times of the system and of component i as T_s, T_i respectively, then we have the following result:

$$\begin{aligned} p_i(t) &= \lim_{\Delta t\to 0} P[T_i\in(t,t+\Delta t), T_j>(t,t+\Delta t), j\neq i \mid T_s\in(t,t+\Delta t)] \\ &= \lim_{\Delta t\to 0}\frac{f_i(t)\Delta t\prod_{j\neq i}^{q} R_j(t+\Delta t)/R_s(t)}{f_s(t)\Delta t/R_s(t)} \\ &= h_i(t)/\sum_{j=1}^{q} h_j(t), \end{aligned} \tag{3}$$

where the last step comes from the definition of hazard rate function and $h_s(t)=\sum_{i=1}^{q} h_i(t)$ for the q-component series system. Replacing $h_i(t)$ by $\lambda_i g(t)$, the result follows. □

Based on lemma 1, it is obvious that the conditional distribution of N_i given $N=n$ is Binomial with parameters (n, p_i), $\forall i$, $i=1,2,…,q$.

Theorem 1: *For the series system described above,* N_i *follows a Geometric distribution with parameter* $p_i F_s(w)/[1-(1-p_i)F_s(w)],\ \forall i,\ i=1,2,...,q$ *. The distribution mass function is:*

$$P[N_i = k] = [\frac{p_i F_s(w)}{1-(1-p_i)F_s(w)}]^k \frac{R_s(w)}{1-(1-p_i)F_s(w)}, k=0,1,2... \quad (4)$$

In addition, N_i and N_j *are statistically independent, for* $i \neq j$.

Proof: For the case of q=2, the joint distribution of N_1 and N_2 is:

$$P[N_1 = n, N_2 = m] = P[N_1 = n, N_2 = m \mid N = m+n] P[N = n+m]$$
$$= \binom{n+m}{n} p_1^n p_2^m [F_s(w)]^{n+m} R_s(w)$$

Therefore, the distribution mass function of N_1 is:

$$P[N_1 = n] = \sum_{m=0}^{\infty} \binom{n+m}{n} p_1^n p_2^m [F_s(w)]^{n+m} R_s(w)$$
$$= [p_1 F_s(w)]^n R_s(w) \sum_{m=0}^{\infty} \binom{n+m}{n} [p_2 F_s(w)]^m \frac{[1-p_2 F_s(w)]^{n+1}}{[1-p_2 F_s(w)]^{n+1}}$$
$$= (\frac{p_1 F_s(w)}{1-p_2 F_s(w)})^n \frac{R_s(w)}{1-p_2 F_s(w)},$$

where the last step comes from $\sum_{m=0}^{\infty} \binom{n+m}{m} [p_2 F_s(w)]^m [1-p_2 F_s(w)]^{n+1} = 1$, which is given by the fact that $\binom{m+(n+1)-1}{m} [p_2 F_s(w)]^m [1-p_2 F_s(w)]^{n+1}$ is the distribution mass function of a Negative Binomial distribution with parameter ($1-p_2 F_s(w), n+1$). The proof for N_2 is similar. The independence between N_1 and N_2 comes directly from the independence assumption between components.

Now consider the general case of q>2. Actually the proof is essentially the same as long as one realizes that he/she can always construct a new two-component series system equivalent to the original q-component series system, for which the hazard rate function of the first component is $\lambda_i g(t)$, and the hazard rate function of the second component is $\sum_{j \neq i}^{q} \lambda_j g(t)$. □

Starting from theorem 1, we can then derive the expectation, variance and the prediction intervals of the system warranty cost.

Corollary 2 *The expected warranty cost for a q-component series system under a RFRW policy with period w is:*

$$E[C] = \frac{F_s(w)}{R_s(w)} \sum_{i=1}^{q} c_i p_i \quad (5)$$

And the corresponding variance follows:

$$Var(C) = \sum_{i=1}^{q} c_i^2 \frac{p_i F_s(w)[1-(1-p_i)F_s(w)]}{R_s^2(w)} \quad (6)$$

Proof: From equation 1, the system expected warranty cost is: $E[C] = \sum_{i=1}^{q} c_i E[N_i]$. Since N_i and Nj, $i \neq j$, are independent, the variance of warranty cost follows: $Var[C] = \sum_{i=1}^{q} c_i^2 Var(N_i)$ From theorem 1 and given that the expectation and variance of a Geometric distribution with parameter p are $p/(1-p)$ and $p/(1-p)^2$ respectively, the results then follow. □

Corollary 3 *The* $1-\alpha$ *prediction interval of the warranty cost for the q-component-in-series system under a RFRW policy is* $[q^c_{\alpha/2}, q^c_{1-\alpha/2}]$, *where* $q^c_{\alpha/2}$ *and* $q^c_{1-\alpha/2}$ *are the* $\alpha/2$ *quantile and the* $1-\alpha/2$ *quantile of C respectively, which is distributed as a mixture of q Geometric distributions.*

4. Cost Model for Parallel Systems

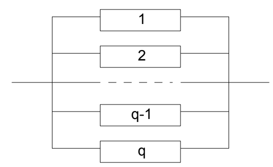

Figure 3: q-component parallel system

For a parallel system, it won't fail unless all the components in the system fail. As a result, the repair cost of a parallel system upon a system failure is simply $\sum_{i=1}^{q} c_i$. Again let N be the number of system failures within a warranty cycle, then the corresponding system warranty cost within a cycle for a RFRW policy with period w is $N \sum_{i=1}^{q} c_i$. Obviously, the distribution of N is Geometric with parameter $F_s(w)$.

Corollary 4 *Under a RFRW policy with period w, the expected warranty cost of the q-component parallel system is* $E[C] = \frac{F_s(w)}{R_s(w)} \sum_{i=1}^{q} c_i$. *And the corresponding warranty cost variance is:* $Var(C) = \frac{F_s(w)}{R_s^2(w)} (\sum_{i=1}^{q} c_i)^2$.

5. Cost Models for Series-Parallel and Parallel-Series Systems

The warranty cost analysis for multi-component systems with series-parallel structure or parallel-series structure is a natural extension from the analyses for systems with simple series structure or parallel structure. In this section, we will formulate the warranty cost for series-parallel and parallel-series systems under RFRW policies, and derive the corresponding expected warranty cost and the cost variance.

5.1 Series-Parallel Systems

Consider a series-parallel system composed of q subsystems in series drawn in figure 4. For the i^{th} subsystem, denote the number of components in the subsystem, which are in parallel, as r_i. Let C_i be the repair cost for i^{th} subsystem upon failure, and let c_{ij} be the replacement cost of component j in subsystem i. One should notice that $C_i = \sum_{j=1}^{q} c_{ij}, \forall i, i=1,2,...,q$.

Similar to equation 1, the total system warranty cost within a warranty cycle is:

$$C = \sum_{j=1}^{q} [N_i \sum_{j=1}^{r_i} c_{ij}] \tag{7}$$

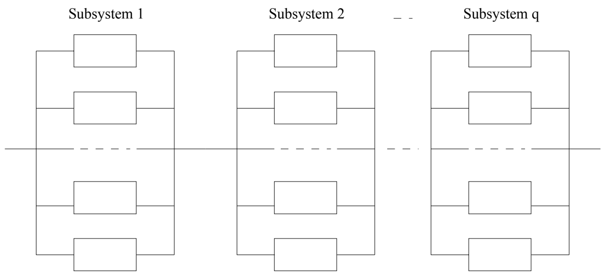

Figure 4: A series-parallel system

Corollary 5 *For the series-parallel system described above, suppose the hazard rate function of the failure time of subsystems i is* $\lambda_i g(t), \forall i, i = 1,2,...,q,$ *then the expected system warranty cost under a RFRW policy with length w is:*

$$E[C] = \frac{F_s(w)}{R_s(w)} \sum_{i=1}^{q} (p_i \sum_{j=1}^{r_i} c_{ij}) \tag{8}$$

And the corresponding cost variance is:

$$Var[C] = \sum_{i=1}^{q} \frac{p_i F_s(w)[1-(1-p_i)F_s(w)]}{R_s^2(w)} (\sum_{j=1}^{r_i} c_{ij})^2 \tag{9}$$

where $p_i = \lambda_i / \sum_{j=1}^{q} \lambda_j$.

Proof: The proof is similar to that for corollary 2, hence omitted here. □

5.2 Parallel-Series Systems

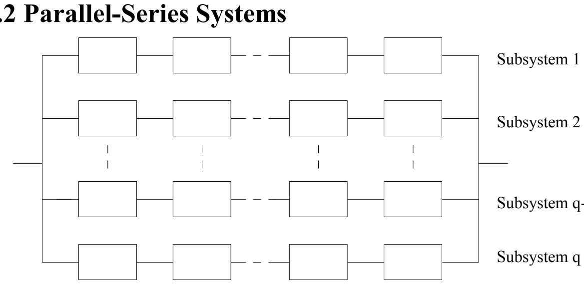

Figure 5: A Parallel-series system

For a parallel-series system as shown in figure 5, let r_i be the number of components in the i^{th} subsystem, and let N and N_i be the number of system failures and the number of failures of subsystem i within a cycle T. Obviously we have that $N \equiv N_i, \forall i, i = 1,2,...,q$ since the system is composed of q independent subsystems in parallel, and all of them must fail in order to observe a system failure. Obviously N as well as N_i follow a Geometric distribution with parameter $F_s(w)$.

Let C_i be the repair cost for the i^{th} subsystem upon failure. One should note that here C_i is not a constant, but a random variable, which can take r_i different values, $c_{ij}, j = 1,2,...,r_i$ depending on which component in subsystem i causes the subsystem failure.

Using the notations above, the total system warranty cost within a warranty cycle is:

$$C = N \sum_{i=1}^{q} C_i \tag{10}$$

Before deriving the system warranty cost distribution under a RFRW policy with period w, first let us derive the distribution of C_i and the expectation and the variance of C_i.

Let p_{ij} be the probability that the j^{th} component in subsystem i is the cause of the failure of i^{th} subsystem. According to this definition and from lemma 1, it is easy to see that $P[C_i = c_{ij}] = p_{ij}$, where $p_{ij} = \lambda_{ij} / \sum_{j=1}^{r_i} \lambda_{ij}$.

As a result, the expected value of C_i follows:

$$E[C_i] = \sum_{j=1}^{r_i} c_{ij} p_{ij} \tag{11}$$

And the variance of C_i is given by:

$$Var[C_i] = \sum_{j=1}^{r_i} p_{ij} c_{ij}^2 - (\sum_{j=1}^{r_i} p_{ij} c_{ij})^2 \tag{12}$$

Corollary 6 *For a RFRW policy with period w, the expected warranty cost within a cycle T for the parallel-series system described above is:*

$$E[C] = \frac{F_s(w)}{R_s(w)} \sum_{i=1}^{q} (\sum_{j=1}^{r_i} p_{ij} c_{ij}) \tag{13}$$

And the warranty cost variance is:

$$\begin{aligned} Var(C) = {} & [\frac{F_s(w)}{R_s^2(w)} + \frac{F_s^2(w)}{R_s^2(w)}] \{ \sum_{i=1}^{q} [\sum_{j=1}^{r_i} p_{ij} c_{ij}^2 - (\sum_{j=1}^{r_i} p_{ij} c_{ij})^2] \} \\ & + (\sum_{i=1}^{q} \sum_{j=1}^{r_i} p_{ij} c_{ij})^2 \frac{F_s(w)}{R_s^2(w)} \end{aligned} \tag{14}$$

Proof: The derivation of the expected warranty cost is straightforward. To derive the warranty cost variance, from equation 10, we have:

$$\begin{aligned} Var(C) &= E(\sum_{i=1}^{q} C_i N)^2 - [E(\sum_{i=1}^{q} C_i N)]^2 \\ &= E(\sum_{i=1}^{q} C_i)^2 EN^2 - (E[\sum_{i=1}^{q} C_i])^2 EN^2 \\ &\quad + (E[\sum_{i=1}^{q} C_i])^2 EN^2 - (E(\sum_{i=1}^{q} C_i)^2 (EN)^2 \\ &= EN^2 [\sum_{i=1}^{q} Var(C_i)] + Var(N) (\sum_{i=1}^{q} E[C_i])^2 \end{aligned} \tag{15}$$

Plug equation 11 and 12 into equation 15 and use the fact that N is Geometrically distributed with parameter $F_s(w)$, the result follows. □

6. Numerical Examples

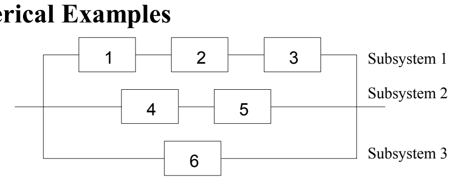

Figure 6: A *6*-component parallel-series system

First let us consider the 6-component parallel-series system shown above. Suppose that for all components in subsystem one, the failure times follow Weibull distribution, for components in the second subsystem, the failure times have Rayleigh distribution, and for component 6, the failure time is exponentially distributed. Components' cost and the reliability functions are given in table 1. The unit of time t is day.

From equation 13 and 14, for a RFRW policy with period of 180 days, the expected system warranty cost within a cycle is *6.29*, the corresponding warranty cost standard deviation is *114.03*, which is much higher than the cost expectation. However, this is what one should expect since the distribution of warranty cost is a mixture of Geometric distributions, whose standard deviation is always larger than its expectation.

Table 1: Parameters of the parallel-series system

NO.	1	2	3	4	5	6
c_i	300	400	500	450	460	1200
R(t)	$e^{\frac{-t^{.012}}{36.8}}$	$e^{\frac{-t^{.012}}{18.42}}$	$e^{\frac{-t^{.012}}{27.42}}$	$e^{\frac{-(3.26E-4)t^2}{2}}$	$e^{\frac{-(3.26E-4)t}{2}}$	$e^{\frac{-(3.26E-4)t^2}{2}}$

To illustrate how the expected warranty cost and the standard deviation change while the warranty period varies, we consider w in the range of [1,60] months (up to 5 years). From figure 7, one can see that the expected system warranty cost increases monotonically over w. Furthermore, the rate of change of E[C] increases first, then decreases, which makes the expected warranty cost curve *S*-shaped. Similar to the expected warranty cost curve, the curve of the standard deviation of warranty costs is also *S*-shaped. It is worth noting that in this case, the standard deviation curve is always above the expectation curve, which indicates that the risks embedded in the system warranty cost of a RFRW policy is quite high, therefore, using the expected warranty cost alone is far from sufficient for the purpose of evaluating such a warranty policy.

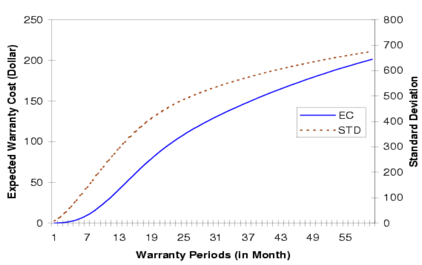

Fig. 7: E[C] and V(C) over w.

CV (Coefficient of Variance) is a standardized measure often used in evaluating risks. From figure 8, we can see that the CV decreases monotonically over w, indicating that as w increases, the relative risk for the RFRW policy actually declines. However, the relative risk is quickly stabilized at 3.4 after $w=12$ months. We also plot the system reliability curve in figure 8. As expected, the system reliability decreases steadily over w. At $w=5$ years, the system reliability is 0.91.

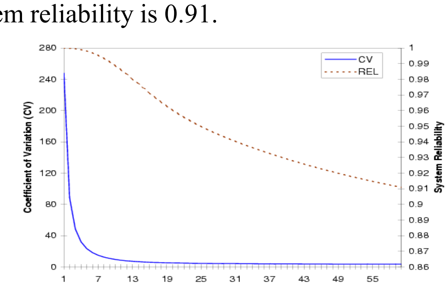

Fig. 8: CV and reliability over w.

Theoretically speaking, computing prediction intervals is equivalent as computing quantiles. Since most practitioners are interested in the upper bound for system warranty cost with confidence level of $1-\alpha$, we decide to provide upper warranty cost quantiles for different $w, w\in[1,60]$ months at $\alpha=5\%$ and 1%. Since C follows a mixture of some Geometric distributions as shown in the paper, one can tabulate the exact quantiles of C based on its distribution. Here for computational convenience, we only provide estimates of the desired quantiles based on Monte Carlo simulation [9]. For each w, we simulate *400,000* runs for the system to obtain *400,000* realizations of the system warranty cost within a warranty cycle T. The corresponding estimated quantiles, or equivalently, the upper prediction confidence bounds are then computed. The results are reported in table 2.

Table 2: Upper prediction intervals of the parallel-series system.

w	Q_{95}	Q_{99}	w	Q_{95}	Q_{99}	w	Q_{95}	Q_{99}	w	Q_{95}	Q_{99}
1	0	0	16	0	2060	31	1950	2150	46	2050	2160
2	0	0	17	0	2150	32	1960	2150	47	2050	2160
3	0	0	18	0	2150	33	1960	2150	48	2050	2160
4	0	0	19	0	2150	34	1960	2150	49	2050	2160
5	0	0	20	0	2150	35	2050	2150	50	2050	2160
6	0	0	21	0	2150	36	2050	2150	51	2050	2160
7	0	0	22	0	2150	37	2050	2150	52	2050	2160
8	0	0	23	0	2150	38	2050	2150	53	2050	2160
9	0	0	24	0	2150	39	2050	2150	54	2050	2160
10	0	1950	25	1950	2150	40	2050	2150	55	2050	2160
11	0	2050	26	1950	2150	41	2050	2150	56	2050	2160
12	0	2050	27	1950	2150	42	2050	2150	57	2050	2160
13	0	2050	28	1950	2150	43	2050	2150	58	2050	2160
14	0	2050	29	1950	2150	44	2050	2160	59	2050	2160
15	0	2060	30	1950	2150	45	2050	2160	60	2050	2160

The second example is about a 6-component series-parallel system drawn in figure 9. Again first let us consider a RFRW policy with a warranty period of *180* days. Assume the hazard rate functions of the failure times of subsystems are as follows: for subsystem one, $h_1(t)=0.00012t$, for subsystem two, $h_2(t)=0.000008t$, and for subsystem three, $h_3(t)=0.0000042t$. The replacement costs for components 1 to 6 are *200, 200, 200, 800, 2000, 2000* respectively. Also suppose that the system cost is the sum of all components' cost, which is 5400. As a result, the expected system warranty cost within a cycle is *8.57*, and the standard deviation is *100.30*. The corresponding expected warranty cost to system cost ratio is *0.16%.*

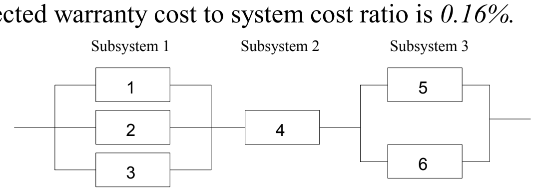

Figure 9: A 6-component series-parallel system

Similar to the analysis in the previous example, we perform sensitivity analysis of the expected warranty cost with regard to warranty period w for w in [1,60] months. The resulting expected warranty cost curve and the standard deviation curve are shown in figure 10. The system reliability curve and the CV curve are shown in figure 11. Again simulation is used to compute the estimated prediction intervals for the system warranty cost. The result is in table 3.

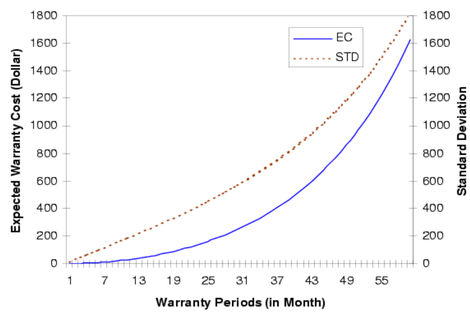

Fig. 10: E[C] and V(C) over w.

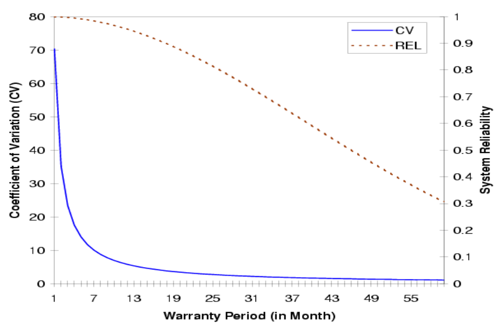

Fig. 11: CV and reliability over w.

Table 3: Upper prediction intervals of the series-parallel system.

w	Q_{95}	Q_{99}	w	Q_{95}	Q_{99}	w	Q_{95}	Q_{99}	w	Q_{95}	Q_{99}
1	0	0	16	600	800	31	1200	4000	46	3000	5600
2	0	0	17	600	1200	32	1200	4000	47	3200	5800
3	0	0	18	600	1200	33	1200	4000	48	3400	5800
4	0	0	19	600	1200	34	1400	4000	49	3800	6200
5	0	0	20	600	1200	35	1400	4000	50	4000	6400
6	0	600	21	600	1200	36	1800	4000	51	4000	6600
7	0	600	22	600	1400	37	1800	4400	52	4200	7000
8	0	600	23	600	1400	38	1800	4600	53	4600	7200
9	0	600	24	800	1800	39	1800	4600	54	4600	7600
10	0	600	25	800	1800	40	2000	4600	55	4800	7800
11	0	600	26	800	2000	41	2200	4600	56	5000	8200
12	0	600	27	1200	2000	42	2400	4800	57	5200	8600
13	600	800	28	1200	2400	43	2400	5200	58	5400	8800
14	600	800	29	1200	2800	44	2600	5200	59	5800	9200
15	600	800	30	1200	3200	45	2800	5200	60	6000	9600

7. Conclusion Remarks

In this paper, we provide a systematic approach on a RFRW policy for complex products considering four types of system structure. Warranty cost models, warranty cost distributions as well as the cost expectation, variance, and prediction intervals are derived. As many products such as complex machinery, mechanical equipment etc., are composed of several components and have one of the structure studied in the paper, the models can be very useful in warranty applications in industry.

There are several potential extensions to our warranty cost models for multi-component systems. The first is to consider different warranty policies. Instead of modeling renewable warranty policies, one may be interested in non-renewable warranty policies, which are more popular among researchers and practitioners. Also one may study a pro-rata policy, or a combined policy (combination of FRW and Pro-rata policy) based on our framework. Two-attribute warranty policies should also be considered for complex systems. The authors have made some progress in those topics, and the report will be available soon. The second extension is the consideration of time discounting effect in warranty cost that is essential in estimating warranty reserve funds. We are currently working on the topic. The third extension is to incorporate more complicated system structure in our modeling besides the four types of structure addressed in this paper, for example, cold-standby systems, k-out-of-n systems, bridge systems, or even systems with network structure. Due to the complexity of system structure, it might be difficult to obtain analytical result of statistical properties of warranty cost such as cost variance, quantiles, etc. The extension is still open for future research.

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