Pricing and Portfolio Management at HP

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AGENDA

Introduction
Business problem
Integrated Suite of Models
Business Use Cases and Impact
Conclusions
Q&A
HP

Fortune 10 – U.S.
Fortune 28 – Global
Operating in approximately 170 countries
300K+ employees
177,000 partners worldwide
#1 or #2 in most markets
HP Financials

Q2FY12 net revenue: $30.7B

By segment

- Personal Systems Group: 30%
- Imaging & Printing Group: 19%
- Software: 28%
- Other: 17%
- Services: 5%
- Enterprise Servers, Storage & Networking: 2%

By region

- Americas: 45%
- EMEA: 35%
- Asia Pacific: 20%
Vision & Strategy

HP Labs
Delivering innovations that transform business, life, and society

DELIVER
breakthrough technologies

CREATE
opportunities for our business and customers

ADVANCE
fundamental science

ENGAGE
with customers and partners

LEADING EDGE RESEARCH AGENDA • OPEN INNOVATION • CO-INNOVATION & DEMONSTRATORS • TECHNOLOGY COMMERCIALIZATION
HP Labs

Message from Chandrakant Patel
HP Senior Fellow, HP Labs’ Director
Business Problem

Challenges

Competition
Product configuration complexity
Product variety
Product life cycles
Global scope

This Talk

About pricing and portfolio decisions in HP PC business
• Suite of models integrated with business processes
• Overview of models
• Business implementation
Integrated Approach

**Demand**
How do consumers value products?

**Product Selection and Pricing**
What products should we offer?
What is the right pricing?

**Competitive Product Similarity**
What products are we competing with on the market?

**Leveraging Intelligence**
Can we infer market intelligence from current prices, and learn?
How do consumers value products?

- Demand Model
- Product Selection & Pricing
- Product Similarity
- Quasi Equilibrium Pricing
Demand Modeling

**PCs are feature based**
Consumers value brand and features of the product

**Marketing data**
Market sales data reveals customer selection
Aggregated mobile PC sales
- **Brands, country, region, attributes, period, channel, price, volume**

**Complexity of modeling estimation**
40+ different key features (memory, CPU, display, storage, OS, ...)
Price sensitivity varies with attributes, time, and region
High-dimensional prediction problem
Discrete-Choice Model

Modeling demand via discrete consumer choice models
Choice set
Utility depends on brand, attributes and price
Other factors are aggregated in the random error term

Utility function specification
Non-linear
• Utility of (4GB-2GB) RAM is not twice the utility of (2GB-1GB)
Non-additive
• Utility of (4GB-1GB) of RAM differs between brands

Semi-parametric DCM
Flexibly models product utility without specifying a functional form of utility
Varying-Coefficient DCM

Single market with $M$ products, $i = 1, 2, \ldots, M$, with volume $(n_1, \ldots, n_M)$

Utility of product $i$

$$U_i = u_i(x_i, p_i) + \varepsilon_i = \beta_0(x_i) + \beta_1(x_i)p_i + \varepsilon_i$$

$x_i$ contains (period, location, channel, attributes)

$$d_i(S, p_s) = \Pr(U_i \geq \max \{U_s : s \in S^+\}), \forall i \in S^+$$

Under Gumbel distribution errors

$$d_i(S, p_s) = \frac{e^{u_i(x_i, p_i)}}{1 + \sum_{s \in S} e^{u_s(x_i, p_s)}}, \forall i \in S$$

Log multinomial likelihood criteria
Boosted trees

Tree partitioning method
Segment products into homogeneous utility-function groups

Boosting method
Iteratively fits simple trees to explain errors not captured in the previous iteration
- Incrementally builds better models
Model Performance

Outperforms state-of-the-art methods (e.g., linear choice model)

Boosting DCM out-sample R2: 50 ~ 60%
Linear choice model out-sample R2: ~ 20%

Extensions

Other than Gumbel errors
Other specifications of utility function
What products should we offer, and at what prices?

- Demand Model
- Product Selection & Pricing
- Product Similarity
- Quasi Equilibrium Pricing
Joint Price and Product Selection

Pricing and Assortment: MIP, non linear formulation

$$\max_{S \subseteq N, \mathbf{p}_s} R(S, \mathbf{p}_s) := \sum_{i \in S} (p_i - c_i) d_i(S, \mathbf{p}_s)$$

s.t. $$|S| \leq C$$ \hspace{1cm} \text{Cardinality Constraints}

$$\sum_{i=1}^{N} a_i d_i(S, \mathbf{p}_s) \geq b$$ \hspace{1cm} \text{General constraint}

$$d_i \leq d_i(S, \mathbf{p}_s) \leq \overline{d}_i, \forall i$$ \hspace{1cm} \text{Market share constraints}

$$p_i \leq p_i \leq \overline{p}_i, \forall i$$ \hspace{1cm} \text{Pricing constraints}
Joint Price and Product Selection

Pricing and assortment, relaxed formulation

\[
\max_{S \subseteq N, p_s} R(S, p_s) \overset{\text{def}}{=} M \sum_{i \in S} (p_i - c_i) d_i(S, p_s)
\]

s.t.

\[
x \leq |S| \leq \bar{x}
\]

\[
\underline{p}_i \leq p_i \leq \overline{p}_i
\]

Literature handles special cases (Chen and Hausman 2000; Schon 2010)

Extensions
Cardinality disjoint sets
Market share constraint on subset
Transform to optimization w.r.t. market share
Constrained Price Optimization

Optimization w.r.t. market share:

\[
\max_p R(p) := \sum_{i=1}^{N} (p_i - c_i)d_i(p)
\]

s.t.

\[
\sum_{i=1}^{N} a_i d_i(p) \geq b
\]

\[
d_i \leq d_i(p) \leq \bar{d}_i, \forall i
\]

\[
p_i \leq p_i \leq \bar{p}_i, \forall i
\]

Main Results

One-to-one mapping between price and market share

Transform price optimization to market share optimization

Simplify to concave optimization problem with linear constraints
Unconstrained Pricing Optimization

Summary of results

**MNL**

Constant markup

- Gallego and Stefanescu (2010)

\[
p_j - c_j - \frac{1}{\beta_j} = \theta, \text{ for product } j
\]

**Nested Logit model**

The markup is constant upper nests

\[
p_{ij} - c_{ij} - \frac{1}{\beta_{ij}} = \theta_i, \text{ for product } j, \text{ nest } i
\]

The markup is constant at nest level

\[
\theta_i + (1 - \frac{1}{\gamma_i})w_i(\theta_i) =: \phi
\]

Reduces to single dimensional optimization

- Gallego and Wang 2012 (earlier talk)

\[
\max_{\phi} R(\phi)
\]
What products are we competing with in the market?

- Demand Model
- Product Selection & Pricing
- Product Similarity
- Quasi Equilibrium Pricing
## Product Similarity Analysis

Product dissimilarity metric is total “distance” between features

<table>
<thead>
<tr>
<th>Product x</th>
<th>Brand A</th>
<th>Intel i3</th>
<th>2GB</th>
<th>14in</th>
<th>...</th>
<th>GPU Intel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>↑ 40</td>
<td>↓ -63</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Product y</td>
<td>Brand B</td>
<td>Intel i3</td>
<td>4GB</td>
<td>15.6in</td>
<td>...</td>
<td>GPU ATI</td>
</tr>
</tbody>
</table>

Develop a customer feature valuation in monetary units
- Homogeneous metric across features, but regional

**Competitive product comparison**

Adjust competitor’s prices to account for feature gaps
- Current price minus dissimilarity due to feature differences
Feature Valuation

Average unit price regression model
Dependent variable: average unit street price
Independent variables: features and brand dummies
Observations weighted by market share, time dependent

Model’s betas give dollar valuation for each feature level

\[
\text{Price} = \beta_{\text{HP}} \text{I}_{\text{HP}} + \beta_{\text{ACER}} \text{I}_{\text{ACER}} + \ldots + \beta_{\text{dispsize}} \text{I}_{\text{dispsize}} + \beta_{\text{RAM}} \text{I}_{\text{RAM}} + \ldots + \epsilon
\]

Penalized constrained regression
Co-linearity across features and dimensionality reduction

\[
\min_{\beta} (p - X\beta)^T (p - X\beta) + \lambda \left( \frac{1 - \alpha}{2} \| \beta \|_2^2 + \alpha \| \beta \|_1 \right)
\]
Leveraging Market Intelligence

- Demand Model
- Product Selection & Pricing
- Product Similarity
- Quasi Equilibrium Pricing

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Leveraging Market Intelligence

Motivation
Parameters estimation and data quality
Industry competition dynamics

Idea
If prices were at equilibrium, no need to do anything
If a firm priced randomly, it will go out of business
Market prices are somewhere in-between

Approach
Side step demand estimation
Find equilibrium prices
Move prices closer to equilibrium
Learn from market!
Price Improvement Heuristic

Equilibrium prices for products \( j \in N_i \) of firm \( i \) are of the form

\[
p_j - c_j = \frac{\alpha}{1 - s_i}, \quad j \in N_i, \forall i
\]

- \( s_i \): market share
- \( c_j \): cost
- \( \alpha \): estimated through regression given \( p_j, c_j \), for \( j \in N_i, s_i, \forall i \),

Use convex combination of equilibrium (recommended) and old price

We use other results for different attraction models
Simulation of Equilibrium Price

Method leads to more benefits when market use of more intelligence

Mean Profit Benefit

More intelligence in market

More benefits
Business Implementation
Use cases

Weekly review of competitive landscape
Assessing marketing coverage per segment in the market
Outdated products
Assess new offerings and competing products
Alerts on features importance in each market segment
Pricing guidance
Comparative pricing
Cluster Coverage

Find optimal location of new products to maximize HP’s coverage of a given cluster

- Competitor product
- HP product
- Proposed new HP product
Multidimensional Scaling Graph – India Market
MDS Clusters – Product Variety
Balanced Variety
Efficient Frontier: Market Share vs. Portfolio Size

How much market share is at stake for every product we drop from the portfolio?

![Market Share Change Diagram]

Min market share loss from 62 to 60 prods
Efficient Frontier: Pricing vs Portfolio

Optimal pricing guidance per assortment

<table>
<thead>
<tr>
<th>Markup increase</th>
<th>Average Price Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount -12.0%</td>
<td>92</td>
</tr>
<tr>
<td>-11.8%</td>
<td>89</td>
</tr>
<tr>
<td>-11.6%</td>
<td>86</td>
</tr>
<tr>
<td>-11.4%</td>
<td>83</td>
</tr>
<tr>
<td>-11.2%</td>
<td>80</td>
</tr>
<tr>
<td>-11.0%</td>
<td>77</td>
</tr>
<tr>
<td>-10.8%</td>
<td>74</td>
</tr>
<tr>
<td>-10.6%</td>
<td>71</td>
</tr>
<tr>
<td>-10.4%</td>
<td>68</td>
</tr>
<tr>
<td>-10.2%</td>
<td>65</td>
</tr>
<tr>
<td>-10.0%</td>
<td>62</td>
</tr>
<tr>
<td>-9.8%</td>
<td>59</td>
</tr>
<tr>
<td>-9.6%</td>
<td>56</td>
</tr>
<tr>
<td>-9.4%</td>
<td>53</td>
</tr>
<tr>
<td>-9.2%</td>
<td>50</td>
</tr>
</tbody>
</table>

Pricing differential per portfolio capacity decrease
Pricing Guidance
Product level detail

Product mapping to markup adjustments

Product Pricing Guidance

overpriced

discount

underpriced
Discontinuance Candidates

**Products selected/discontinued in the optimal portfolio with min**

![Participation Rate in Opt Portfolios](image)

- **This set of products are selected in every scenario.**
- **This set of products are drop from the opt portfolio as Portfolio Size decreases.**

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Implementation and Deployment

**PCs** → **Printers** → **ESS**

- Personal Systems Group: 30%
- Imaging & Printing Group: 19%
- Services: 28%
- Enterprise Servers, Storage & Networking: 17%

**China** → **APJ** → **EMEA** → **AME**

- Americas: 45%
- Asia Pacific: 35%
- EMEA: 20%
Benefits

Speed up the decision process
Profitability of product strategies
Best response to new competing products
Expand scenario analysis based on recent news

Marketing
Life cycle decisions (introduction and discontinuance)
Promotions

Supply chain
Products and components
Conclusions

This innovation is about
Analytics
Products portfolio and pricing decisions
Influence product design
Improve business competitiveness

Scope
Tactical to strategic
Aggregated to detail

Impact complex system with global scale, very large product line
Business role is crucial to test model’s assumptions
Flexibility
Data limitations
Business Impact

Message from Tony Prophet
HP Sr. V.P. Printing and Personal System Group

IP
6 Patents, invention disclosures filed
3 Papers to be submitted
6+ Internal reports
Thank you

Team
Estimating Intelligence in the Real Market

Market Data
- Australia laptops
- 15 products, 5 brands
- 10 weeks of data
- 5 periods in-sample 5 periods out-sample

Simulation and
Quantal Price Equilibrium

Quantal Response Equilibrium
Relationship to HP Edelman Award


Complexity ROI analysis framework for product/feature launch decisions

Post-launch variety management tool to find product portfolios along efficient frontier of historical revenue coverage and portfolio size

Independent demand model: no substitutions

**HP Labs wins 2012 INFORMS RMP Award**

Pricing

Product substitution

Competition

Portfolio Efficient Frontiers
Linear Transformation – Efficient Algorithm

LP with variable Transformation

\[ w : = \frac{1}{a_0 + \sum_{j=1}^{N} \sum_{r=1}^{m_j} e^{w_j - \beta s_{jr} x_{jr}}}, \]

\[ y_{ir} : = x_{ir} w. \]

\[
\max_{\omega, y} M \sum_{i=1}^{N} \sum_{r=1}^{m_i} (s_{ir} - c_i) e^{w_i - \beta s_{ir} y_{ir}}
\]

subject to

\[ a_0 w + \sum_{i=1}^{N} \sum_{r=1}^{m_i} e^{w_i - \beta s_{ir} y_{ir}} = 1, \]

\[ \sum_{r=1}^{m_i} y_{ir} \leq w, \quad i = 1, 2, \ldots, N, \]

\[ x w \leq \sum_{i=1}^{N} \sum_{r=1}^{m_i} y_{ir} \leq Tw, \]

\[ 0 \leq y_{ir} \leq w, \quad \forall i, r. \]
Nested Logit model: correlated U

- Two stage structure
  - Upper stage: select nest $i$ with probability $Q_i$
  - Lower stage: select a product $k$ within nest $i$ with $q_{k|i}$.

- Nested Logit model:

$$Q_i(p_i, p_{-i}) = \frac{e^{\gamma_i I_i}}{1 + \sum_{l=1}^{n} e^{\gamma_l I_l}},$$

$$q_{k|i}(p_i) = \frac{e^{\alpha_{ik} - \beta_{ik} p_{ik}}}{\sum_{s=1}^{m_i} e^{\alpha_{is} - \beta_{is} p_{is}}},$$

$$\pi_{ik}(p_i, p_{-i}) = Q_i(p_i, p_{-i}) \cdot q_{k|i}(p_i),$$

where $I_l = \log \sum_{s=1}^{m_l} e^{\alpha_{ls} - \beta_{ls} p_{ls}}$ represents the attractiveness of nest $l$, $\gamma_l$ refers to degree of interfirm heterogeneity.
Nested Logit Model

- **Single Stage Model: Multinomial Logit (MNL) Model**
  - “blue bus/red bus” paradox
  - suffer from “independence of irrelevant alternatives” (IIA)
  - not take into account of interdependence among alternatives

- **Multi-Stage Choice Model: Nested Logit (NL) Model**
  - Two stage: (1). select a nest of products. (2). select one within the nest
  - alleviate IIA property
  - generalize MNL model
  - more flexibility, fit data better

- **Optimization**
  - constant adjusted markup
  - constant nested-level markup
  - simplify to single dimensional problem
  - easy to solve
Optimization in a Nest

Firm $i$’s total expected profit:

$$R_i(p_i, p_{-i}) \overset{\text{def}}{=} \sum_{k=1}^{m_i} (p_{ik} - c_{ik}) \pi_{ik}(p_i, p_{-i}).$$  \hspace{1cm} (1)

Proposition

It is optimal to offer all the products, and at optimal prices the adjusted markup $p_{ij} - c_{ij} - \frac{1}{\beta_{ij}}$ is constant for all $j$, denoted by $\theta_i$. 
Optimization in a Nest, cont.

Multi-product price optimization (1) can be simplified to an optimization problem with a single decision variable:

\[ R_i(\theta_i, \theta_{-i}) = Q_i(\theta_i, \theta_{-i})(\theta_i + w_i(\theta_i)) \]  \hspace{1cm} (2)

where

\[ q_{k|i}(\theta_i) = \frac{e^{\alpha_{ik} - \beta_{ik} \theta_i}}{\sum_{s \in F_i} e^{\alpha_{is} - \beta_{is} \theta_i}}, \quad Q_i(\theta_i, \theta_{-i}) = \frac{e^{\gamma_i I_i}}{1 + \sum_{l=1}^{n} e^{\gamma_l I_l}}, \]

\[ \pi_{ik}(\theta_i, \theta_{-i}) = q_{k|i}(\theta_i) \cdot Q_i(\theta_i, \theta_{-i}), \quad w_i(\theta_i) = \sum_{k \in F_i} \frac{1}{\beta_{ik}} \cdot q_{k|i}(\theta_i). \]

**Proposition**

(a) Function \( R_i(\theta_i, \theta_{-i}) \) is strictly unimodal with respect to \( \theta_i \) if \( \gamma_i \geq 1 \) or \( \max_s \beta_{is} \leq \frac{1}{1 - \gamma_i} \).

(b) The optimal \( \theta^*_i \) is in a bounded interval.
Optimization in multiple Nests

Maximize total expected profit,

\[
\max_{p_1, \ldots, p_n} R(p) \overset{\text{def}}{=} \sum_{i=1}^{n} \sum_{k=1}^{m_i} (p_{ik} - c_{ik}) \pi_{ik}(p_i, p_{-i}).
\]

Simplify to maximizing in \textit{adjusted markups} \( \theta = (\theta_1, \ldots, \theta_n) \):

\[
\max_{\theta_1, \ldots, \theta_n} R(\theta) \overset{\text{def}}{=} \sum_{i=1}^{n} Q_i(\theta_i, \theta_{-i})(\theta_i + w_i(\theta_i)).
\]
Problem Simplification

Proposition

The adjusted nested markup $\theta_i + (1 - \frac{1}{\gamma_i})w_i(\theta_i)$, called adjusted nested markup, is constant for all $i$, denoted as $\phi$.

Thus, problem (4) can be reduced to an optimization problem with respect to single variable $\phi$,

$$\max_{\phi} \quad R(\phi) \overset{\text{def}}{=} \sum_{i=1}^{n} Q_i(\theta_i, \theta_{-i})(\theta_i + w_i(\theta_i)),$$

where $\theta_i + (1 - \frac{1}{\gamma_i})w_i(\theta_i) = \phi$, $\forall i = 1, 2, \ldots, n.$ (5)

Proposition

Function $R(\phi)$ is strictly unimodal in $\phi$ if $\gamma_i \geq 1$ or $\frac{\max_{s} \beta_{is}}{\min_{s} \beta_{is}} \leq \frac{1}{1 - \gamma_i}$ for all $i$. 
Portfolio Optimization with Market Constraints

\[
\begin{align*}
\max_x & \quad M \sum_{i=1}^{N} \sum_{r=1}^{m_i} (s_{ir} - c_i) d_{ir}(x) \\
\text{s.t.,} & \quad \sum_{r=1}^{m_i} x_{ir} \leq 1, \quad i = 1, 2, \ldots, N, \\
& \quad \underline{x} \leq \sum_{i=1}^{N} \sum_{r=1}^{m_i} x_{ir} \leq \overline{x}, \\
& \quad d \leq \sum_{i=1}^{N} \sum_{r=1}^{m_i} d_{ir}(x) \leq \overline{d}, \quad i = 1, 2, \ldots, N. \\
& \quad x_{ir} = 0, 1, \quad \forall i, r.
\end{align*}
\]
Efficient Frontier: Profit vs Portfolio

• We set a cardinality constraint on portfolio (minimum number of configurations)

![Graph showing Efficient Frontier: Profit vs Portfolio]

Best Profit change = $1.5M vs status quo, with best 62 prods

Horizontal axis is the Portfolio Size Constraints
Product Selection Map and Optimal Assortment

Prod 350 is selected in the portfolio whenever we allow more than 54 products.
Title (28 pt. HP Simplified bold)
Subtitle (18 pt. HP Simplified)

Heading (18 pt. HP Simplified bold HP blue)

Body copy (16 pt. HP Simplified)

• Put your first-level bullet here. Try to keep bullet lists simple. (14 pt. HP Simplified)
  – Put your second-level bullet here. Use no more than you need to explain your point. (14 pt. HP Simplified)
• Put your third-level of copy here. Use no more than you need to explain your point. (14 pt. HP Simplified)