DYNAMIC PRICING OF OMNICHANNEL INVENTORIES

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To survive, retailers need strategies to compete online.

- **Brick-and-mortar**
- **E-commerce**

Source: U.S. Census Bureau
OMNICHANNEL RETAILING
New online-offline interactions

OMNICHANNEL ORDER FULFILLMENT

PRICE-SAVVY CUSTOMERS

X STORE
$699.99

X.COM
$729.99

$719.00

$699.00

$709.99
DYNAMIC PRICING OF LIMITED INVENTORIES

Lifecycle pricing or clearance optimization
Legacy pricing system treats channels separately

Store inventory

Legacy pricing system

Store prices

EFC inventory

Legacy pricing system

Online price

Store demand

Online demand

IBM Commerce
Oracle Retail
JDA
SAS
Legacy pricing system treats channels separately

- Store demand
- Online demand
- Store inventory
- EFC inventory

Legacy pricing system

Store prices (LOW)
Margin erosion

Online price (HIGH)
Cannibalization

Substitution
Ship-from-store
Omnichannel Retailer
• Top 15 retailer in U.S.
• Operates > 1,000 physical stores
• Operates one e-fulfillment center

“Due to extensive ship-from-store (SFS) fulfillment for many SKUs, there is significant margin erosion at our physical stores using a traditional MDO solution…”

<table>
<thead>
<tr>
<th># SKUs</th>
<th>Clearance revenues</th>
<th>% Sales from online</th>
<th>% Online sales fulfilled from store</th>
</tr>
</thead>
<tbody>
<tr>
<td>195</td>
<td>$107 M</td>
<td>11%</td>
<td>94%</td>
</tr>
</tbody>
</table>
THE PRICE OPTIMIZATION MODEL
JOINT OPTIMIZATION OF ONLINE AND OFFLINE PRICES

A network optimization model is required

OMNICHANNEL INVENTORY

OMNICHANNEL DEMAND

\[ d_{bz}(p_e, p_{bz}) \]

\[ d_{ez}(p_e, p_{bz}) \]
OMNICHANNEL DYNAMIC PRICING OPTIMIZATION

Maximize network-wide expected revenues

\[ \xi^t, y^t \]
Randomness from
Demand uncertainty at period \( t \)
Order Fulfillment at period \( t \)

Inventory levels at period \( t \)
\[ x^t = (x^t_e, x^t_{b1}, \ldots, x^t_{bn}) \]

Pricing system

\[ x^{t+1} \]

Price vector at period \( t \)
\[ p^t = (p^t_e, p^t_{b1}, \ldots, p^t_{bn}) \]

Expected channel demand (for all locations)

\[ d_{bz}(p_e, p_{bz}) \]
\[ d_{ez}(p_e, p_{bz}) \]
PROPOSED PRICING POLICY

The optimal store price depends on future fulfillment

STORE Z INVENTORY

ZONE B FULFILLMENT
ZONE A FULFILLMENT
ZONE Z FULFILLMENT

Set a LOW store price
PROPOSED PRICING POLICY

The optimal store price depends on future fulfillment

Future fulfillment is difficult to predict!
Statistical prediction ~ 70-80% MAPE

Set a HIGH store price

We approximate with inventory partition variables

\[ Y_{zb} \]
\[ Y_{za} \]
\[ Y_{zz} \]

\[ x_z = \sum_{z' \in Z} Y_{zz'} \]

Prediction accuracy ~ 20% MAPE
OMNICHANNEL MARKDOWN OPTIMIZATION

Jointly optimize prices and inventory partitions

TWO PROPOSED MODELS
- Deterministic
- Robust

TRACTABILITY
- Under discrete prices, we can prove equivalence to a mixed integer program (MIP)
- For multiple real problem instances
  - up to 10K binary variables and 100K constraints
  - 80% solves < 40 secs
  - 95% solves < 3 mins

Nonlinear, non-convex

\[
\begin{align*}
\text{maximize} & \quad \sum_{k=t}^{T} \sum_{z \in Z} (p_k^k s_{ez}^k + p_{bz}^k s_{bz}^k) - \sum_{z \in Z} c_{ez} Y_{ez} - \sum_{z \in Z} \sum_{z' \in Z} c_{zz'} Y_{zz'} + q \left( \theta_e + \sum_{z \in Z} \theta_{bz} \right) \\
\text{subject to} & \quad s_{ez}^k \leq d_{ez}^k(p_e^k, p_{bz}^k), \quad k = t, \ldots, T, \quad \forall z \in Z, \\
& \quad s_{bz}^k \leq d_{bz}^k(p_e^k, p_{bz}^k), \quad k = t, \ldots, T, \quad \forall z \in Z, \\
& \quad \sum_{z \in Z} Y_{ez} + \theta_e = x_e^t, \\
& \quad \sum_{k=t}^{T} s_{bz}^k + \sum_{z' \in Z} Y_{zz'} + \theta_{bz} = x_{bz}^t, \quad \forall z \in Z, \\
& \quad \sum_{k=t}^{T} s_{ez}^k = Y_{ez} + \sum_{z' \in Z} Y_{z'z}, \quad \forall z \in Z, \\
& \quad s \geq 0, \quad Y \geq 0, \quad \theta \geq 0, \\
& \quad p_k^k \in \Omega, \quad k = t, \ldots, T.
\end{align*}
\]

SALES LESS THAN DEMAND

EFC INVENTORY PARTITION

STORE Z INVENTORY PARTITION

ONLINE SALES = PARTITION FOR ONLINE

Omnichannel Network Demand Model

Customer Locations
Represented by discrete non-overlapping zones

Zone Customer Decision
Represented by a choice model

Channel Demand = Market Size * Choice probability

\[
d_{bz}(v_{bz}, v_{ez}) = M_z \times \frac{f_{bz}(v_{bz})}{1 + f_{bz}(v_{bz}) + f_{ez}(v_{bz})}
\]

\[
d_{ez}(v_{bz}, v_{ez}) = M_z \times \frac{f_{ez}(v_{ez})}{1 + f_{bz}(v_{bz}) + f_{ez}(v_{bz})}
\]

**FITTING DEMAND MODEL TO DATA**

**Weekly cadence**

Channel Demand = Market Size * Choice probability

\[
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\]

**MARKET SIZE INFLUENCERS**
- Product life cycle
- Seasonality, holidays

**ATTRACTION MODEL INFLUENCERS**
- Channel price (store or online)
- Channel Promos, Ads
- Prices of 18 competitors

Estimation challenge:
“No purchase” data is unobserved!
Censored Demand Estimation Model

Integrated estimation of market size and share

Piecewise linear channel attraction imputation

Piecewise linear market size imputation

Top N competitor subset selection model

ELASTICITIES AND PREDICTION ACCURACY

OUT-OF-SAMPLE VALIDATION
Category-level accuracy (Tablets)
Sales weighted MAPE = 22%

PRICE ELASTICITY OF CHANNEL DEMAND (TABLETS)

<table>
<thead>
<tr>
<th></th>
<th>Store price</th>
<th>Online price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Store sales</td>
<td>-1.3</td>
<td>+0.7</td>
</tr>
<tr>
<td>Online sales</td>
<td>+2.8</td>
<td>-3.9</td>
</tr>
</tbody>
</table>

Elasticities and prediction accuracy

Price elasticity of channel demand (Tablets)

- Store price
  - Actual: [-1.3]
  - Predicted: [0.7]

- Online price
  - Actual: [2.8]
  - Predicted: [-3.9]
COMPETITOR RISK VISUALIZATION

Tables -- Revenue-at-risk
(Annual loss for 1% decrease in competitor price)

Legend:
- Retailer A
- Retailer B
- Retailer C
- Retailer D
- Retailer E
- Retailer F
- Retailer G
- Retailer H
- Retailer I
- Retailer J
- Retailer K
- Retailer L
IMPLEMENTATION
Timeline of Partnership

- **Nov 2014:** Data collection and preprocessing
- **Jan 2015:** Initial results presented
- **April 2015:** Business value assessment presented
- **Dec 2015:** System Integration Test
- **May 2016:** Commercial release
- **Mar 2016:** Preproduction starts (beta release)
- **May 2016:** Commercial release
- **Mar 2017:** Production data analyzed
- **Nov 2015:** Development of system
Effects on prices and sales

Projections estimated from historical data

1. **Better Price Management**
   - 10% Higher Store prices
   - 5% Lower Online price
   - 7% Less Store price variation

2. **Higher Sell-through**
   - Use of total inventory
     - Sold in-store: 64%
     - Sold online: 21%
     - Unsold: 15%

   - Optimized
     - Sold in-store: 66%
     - Sold online: 24%
     - Unsold: 10%

Actual

Optimized
The more store inventory is used for online fulfillment, the higher is the projected revenue gain.
**Timeline of Partnership**

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**Notes:**
- Data collection and preprocessing were completed in November 2014.
- Initial results were presented in January 2015.
- Business value assessment was presented in April 2015.
- System integration test was conducted in December 2015.
- Preproduction started in March 2016 (beta release).
- Commercial release took place in May 2016.
- Production data was analyzed in March 2017.
A CAREFULLY DESIGNED CONTROLLED PILOT EXPERIMENT CANNOT BE PERFORMED BECAUSE:

A. Unable to apply pre-treatment and post-treatment on the same SKU, since a SKU is only on clearance once.
B. Unable to identify close substitutes with the same network characteristics (demand, supply).

OUR APPROACH:

• Estimate treatment effect using statistical models calibrated using observational data collected from the pilot implementation.

• Partner retailer used the omnichannel system on SKUs across 34 product categories.

• SKUs (with and without treatment) with end dates in Q1 2017.

ACTUAL BENEFITS
We let the data tell us
CAUSAL MODEL

Controlling for pre-treatment predictors

DEPENDENT VARIABLE  Weekly average markdown revenue
TREATMENT VARIABLE  Whether omnichannel model was used to determine prices
PRE-TREATMENT PREDICTORS  Online share in regular season,
                            Weekly average revenue in regular season,
                            Initial markdown inventory normalized by number of weeks in clearance

\[
\ln(\text{Avg-Weekly-MD-Rev}_i) \\
\sim \alpha_0 + \alpha_1 \text{Treatment}_i \times \text{Reg-Online-Share}_i \\
+ \alpha_2 \text{Reg-Online-Share}_i + \alpha_3 \ln(\text{Avg-Weekly-Reg-Rev}_i) \\
+ \alpha_4 \text{Avg-Weekly-MD-Inventory}_i, \quad \forall i \in \text{SKUs}.
\]
REVENUE IMPACT

AN AVERAGE OF 12% INCREASE IN MARKDOWN REVENUE DUE TO OCPX

\[
\begin{align*}
\ln(\text{Avg-Weekly-MD-Rev}) & \quad \text{w/o markdown depth} & \quad \text{with markdown depth} \\
\text{Constant} & \quad 1.820^{**} & \quad 1.486^{***} \\
       & \quad (0.178) & \quad (0.230) \\
\ln(\text{Avg-Weekly-Reg-Rev}) & \quad 0.597^{**} & \quad 0.596^{***} \\
       & \quad (0.021) & \quad (0.021) \\
\text{Avg-Weekly-MD-Inventory} & \quad 0.004^{**} & \quad 0.004^{***} \\
       & \quad (0.001) & \quad (0.001) \\
\text{Treatment} \times \text{Reg-Online-Share} & \quad 1.481^{**} & \quad 1.483^{***} \\
       & \quad (0.469) & \quad (0.436) \\
\text{Control} \times \text{Reg-Online-Share} & \quad 0.565^{**} & \quad 0.394^{*} \\
       & \quad (0.263) & \quad (0.248) \\
\text{Avg-Online-MD-Depth} & \quad -0.865^{***} & \quad \text{Avg-Store-MD-Depth} & \quad 2.070^{***} \\
       & \quad (0.220) & \quad (0.318) \\
\text{R-sq} & \quad 0.832 & \quad 0.855 \\
\text{# obs} & \quad 275 & \quad 275 \\
\text{# treatment} & \quad 57 & \quad 57 \\
\text{Improvement due to OCPX} & \quad 12.3\% & \quad 14.8\% \\
\end{align*}
\]

* **p < .001, **p < .05, *p < .1, \( \frac{p}{2} = .11 \). SKUs with very small durations and rate of sales were eliminated.
Innovation in large-scale network price optimization and demand forecasting

- **5 Patents**
- **3 Journal Papers**

**Shared inventory systems allocation, fulfillment**

**Omnichannel Prices:**
- Rebalance network inventory
- Better manage channel demand

**Industry-first scalable omnichannel pricing solution**

For a retailer with $1B in markdown revenue p.a., a projected $120M increase in revenue.

*Part of IBM Watson Commerce MDO solution offering*
SUMMARY OF OUR APPROACH
Network-based models

PRICE OPTIMIZATION

Challenges
• New online-offline interactions
• Order fulfillment is difficult to predict
• Network model is large-scale, nonlinear, and non-convex
• Demand is uncertain

Techniques
• Network optimization of online-offline prices
• Endogenous “inventory partition” variables
• Novel linear reformulation techniques
• Robust dynamic pricing formulation

DEMAND FORECASTING

Challenges
• Customers substitute across channels
• Potentially many competitors
• Online treated as a single store (can’t predict the impact of multiple stores on online)
• “No purchase” data is censored

Techniques
• Channel substitution using choice modeling
• Online is composed of multiple virtual stores
• Novel single step censored data estimation using imputation of lost sales probability
• Top N competitor identification
Innovation in Large-Scale Network Price Optimization and Demand Forecasting

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THANK YOU!

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Main Paper:

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