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Optimization Model of Supporting the Small Business Enterprises

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ABSTRACT

This paper is investigated some credit allocation models for borrowers-representatives small and middle buseness and support problem of the poor part of these clients. The constructed algoritms are based on finding the optimal solutions of considering models. The model will be easily applicable to a similar situation that the management encounters.

Keywords: SMEs, Credit allocation, Credit risk, Gibbs' Lemma, Maxmin problem, Dynamic programming, Penalty function method

INTRODUCTION

In applying of mathematical methods for research financial operations in the Mongolia to confined the linear approach: linear programming and linear regression and etc. Necessary to use powerful facility nonlinear problem of mathematical programming and game theory. Great attention is paid to microfinance on the part of international financial organizations as well as with side governments. That is why necessary became question about construct and the researches to construct mathematical models of the rational conduct of microfinance operations in Mongolia [1,7,9].

It is possible has the problem of the distribution of credit money by bank-monopolist which is keen in maximization the incomes of the borrowers-representatives small and middle business, and when the purpose of credit organization or the project of supporting middle and small business consists in the support of the poor part of clients that we must decide maxmin problem. In above cases interests of borrowers and supporting creditors almost are coincided.

In this work are developed the decisions of the problems of the allocation resources with limitations. These algorithms are used for deciding of the various problems of maximization the summary income of borrower's the-representatives of small business and the problems of the minimization of credit risk. We have writen two algorithms an optimal solution to the two problems when the total amount of the loan size is given and the loan size will accept as a collateral of small business enterprises and bounded upper;
- Optimal allocation credits of the small business enterprises when the maximal income depending on their received loan.
- Optimal allocation credits the maximal income of the small business enterprises with the smallest lowest income.

In these problems when they received loan size evaluated for R^{+} we are using Gibbs expanded lemma, if the loan size evaluated for finite number of nonnegative and fixed values we are using a dynamic programming and penalty function method. These cases are named as continuous nonlinear resourse allocation problem and discrete nonlinear resource allocation problem, respectively [3,5,10].We have special case of common problem when constraints are linear, but utility functions are concave.

A. Algoritms For Continuous Nonlinear Resourse Allocation Models.
In paper [10] is presented extensive survey on the continuos nonlinear resource allocation problem. The quoting paper surveted the history and applications of the problem, as well as algorithmic approaches to this solution. The most common tecniques were based on finding the optimal most often through use of a type of line search procedure [10,11, 12,13,14].

But our algorithms are finite and finding solution exactly for considereng special problems. Necessary and sufficient condition for solution of continuous problem (9)-(10) is named as Gibbs'Lemma [3,10]. The name " Gibbs' Lemma" was coined by John M. Danskin [15]. The Lemma helps to find solution of problem (9)-(10) exactly, if utility functions are strong concave and growing. Maxmin problem as similar game but of a different origin than above allocation problem was utilited Gibbs' Lemma in the investigation of following form by John M. Danskin [15]:

$$\max_{x\in X} imize \min_{y\in Y} imum \sum_{j=1}^{n} \upsilon_j \left(1-\alpha_j e^{-k_j x_j / y_j}\right)^{y_j},$$

where $X := \left\{x \in R_{+}^{n} \middle| \sum x = b\right\}$ and $Y := \left\{y \in R_{++}^{n} \middle| \sum y = c\right\}$, all constants b, c, υ_j, k_j are greater than zero and $\alpha_j \in (0,1)$. But transformation of maxmin problem (11)-(12) to problem (9)-(10) is realized by J.M.Germeier [3].

The extented Gibbs'Lemma was formulated [10] for more common problem than continuous problem (18)-(19) where every x_i is bounded from above. For example, suppose that for $j=1,...,n$ the functions $\phi_i : R \to R$ and $g_j : R \to R$ are convex and differentiable and that $-\infty \le l_j < u_j \le +\infty$ holds. Let $b \in R$.

Then continuous nonlinear resource allocation problem has the following general statement:

$$\min_x imize\, \phi(x) = \sum_{j=1}^{n} \phi_j(x_j),$$

$$\text{subject to } g(x) = \sum_{j=1}^{n} g_j(x_j) \le b,$$

$$x_j \in X_j := \lfloor l_j, u_j \rfloor, \; j=1,...,n.$$

Introducing the Lagrange multiplier $\lambda \ge 0$ for the problem we obtain the following conditions for the optimality of x^* in our problem:

$$\lambda \ge 0, \quad g(x^* \le 0), \quad \lambda g(x^*) = 0, \quad x_j^* \in X_j, \quad j=1,...,n,$$

$$x_j^* = l_j, \quad if \quad \phi_j^i(x_j^*) \ge -\lambda g_j^/(x_j^*), \quad j=1,...,n,$$

$$x_j^* = u_j, \quad if \quad \phi_j^/(x_j^*) \le -\lambda g_j^/(x_j^*), \quad j=1,...,n,$$

$$l_i < x_j^* < u_j, \quad if \quad \phi_j^/(x_j^*) = -\lambda g_j^/(x_j^*), \quad j=1,...,n.$$

Indeed, Lagrange multiplier techniques for our problem are order, dating back at least to the mid 1950s, if not earlier: the earlist reference found so far is to [13], although the Lagrange multiplier algorithm there in is a simple grid search method. In present time except of Lagrange multiplier techniques [17,18,19,20], we have so-called primal "pegging algorithms [21,22,23].

B. Algorithms For Discrete Resource Allocation Models

In this case every variable can take finite number of nonnegative values and so algorithms must be also finite and finding solution exactly. Such situation is characteristic for credit allocation problems. Their main computation method for resource allocation problem is dynamic programming [2,4,5,6,8].

Dynamic programming method is order, dating back at least to the mid 1950s, its application to our problem application with computation schema for resource allocation discrete models appears in almost literature of operational research methods in economics [1,2,4,5,6,8]. Discrete variant of problem (11)-(12), as we known, can be transformed to problem (14)-(15).

Concerning discrete variant of model (18)-(19), where every variable x_i bounded from above A_i, we well use penalty function method. Namely, assume that $\varphi_i(x_i) = -C$ if $x_i > A_i$, where C is sufficiently greater number. After this procedure we well use classical method of dynamic programming [8].

THEORY AND MOTIVATION

The General Statement of Mathematical Programming

Let $f_0,..., f_s : R^n \to R$ convex functions and $M \subseteq R^n$ the exclusive subset Euclidean spaces R^n. Then a following problem names the general problem mathematical programming.

$$f_0(x_1,...,x_n) = f_0(x) \to \max(\min), \qquad (1)$$

$$f_j(x_1,...,x_n) = f_j(x) \le 0, \quad j = 1,...,k \ , \tag{2}$$

$$f_i(x_1,...x_n) = f_j(x) = 0, \quad j = k+1,...,s \ , \tag{3}$$

$$x = (x_1,...,x_n) \in M \subseteq R^n \ . \tag{4}$$

Problems of Linear Programming

In case of when functions $f_0(x),..., f_s(x)$ are linear and $M = R_+^n$ a problem (1) - (4) name a problem of linear programming. The problem of linear programming can be solved completely, for example, by means of a simplex-method. In introduction we have resulted examples of known linear models of optimization, namely, two models of optimum planning of actives and one model of optimum planning of system of portfolios i.e. planning of actives and passives, which written as problems of linear programming. The general problems

$$f_0(x) = \sum_{i=1}^{n} c_i x_i \to \max(\min) \ , \tag{5}$$

$$f_j(x) = \sum_{i=1}^{n} a_{ij} x_i \le b_j \ , \quad j = 1,..,k \ , \tag{6}$$

$$f_j(x) = \sum_{i=1}^{n} a_{ij} x_i = b_j \ , \quad j = 1,..,s \ , \tag{7}$$

$$x_i \ge 0, \quad i = 1,..,n \ . \tag{8}$$

But the basic mathematical models of optimum control are formed by system of portfolios of bank-monopolist as a nonlinear problem of mathematical programming which in mathematical economy can be named problems of storekeeping. Especially it concerns optimum distribution of credits for financing small and middle business.

Problems of Optimum Allocation Of Resources

The problem of optimum allocation of resources which is a special case of a problem (1) - (4), has a following appearance [3,5,10].

$$f_0(x) = f_0(x_1,...,x_n) = \sum_{i=1}^{n} \varphi_i(x_i) \to \max , \qquad (9)$$

$$\sum_{i=1}^{n} x_i = A; \quad x_i \geq 0, \quad i = 1,...,n , \qquad (10)$$

Where x_i the-quantity of a resource allocated for i-th sector, A-quantity of resources, $\varphi_i(x_i)$-function of profitableness of i-th sector.
The problem of distribution of credit money bank-monopolist which has such kind is interested in maximization of incomes of the borrowers-representatives small and middle business. When the purpose of the credit organization or subscriber the project of financing middle and small business consists in support of a poor part of clients we should solve following maxmin a problem.

$$f_0(x_1,..., x_n) = f_0(x) = \min_{1 \leq i \leq n} \{\psi_i(x_i)\} \to \max , \qquad (11)$$

$$\sum_{i=1}^{n} x_i = A; \quad x_i \geq 0, \quad i = 1,...,n \ , \qquad (12)$$

where $\psi_i(x_i)$- function of profitableness of i-th sector.
Though the problem (11) - (12) does not belong to a classical problem of mathematical programming (1) - (4), the theorem 1 shows, that it can be translated to a problem of type (9) - (10) by means of transformation

$$\varphi_i(x_i) = -\int_0^x \psi_i(y_i) dy_i \ .$$

Theorem 1. The set of leveling decisions of a problem (11) - (12) coincides with set of decisions of a problem

$$\min_x \sum_{i=1}^{n} \int_0^{x_i} \psi_i(y_i) dy_i \ ,$$

$$\sum_{i=1}^{n} x_i = A; \quad x_i \geq 0, \quad x = (x_1,..., x_n), \quad i = 1,...,n .$$

The set of decisions of a problem which will turn out from (9) - (10) if integer functional (9) aspires to a minimum, coincides with set of leveling decisions of a problem

$$\max_{x} \min_{1 \le i \le n} \{\varphi_i' \ (x_i)\},$$

$$\sum_{i=1}^{n} x_i = A; \quad x_i \ge 0, \quad x = (x_1, ..., x_n); \quad i = 1, ..., n.$$

For the decision of a problem of type (9) - (10) use a following Gibbs' lemma, who is a basis for construction of algorithm of search of optimum decisions.

Lemma1. (Gibbs' Lemma)[3,5,10] The vector $x^* = (x_1^*, ..., x_m^*)$ is the decision of a problem (9) - (10) in only case when there is such number λ, that

$$x_i^* > 0, \ \varphi_i'(x_i^*) = \lambda$$

$$x_i^* = 0, \varphi_i' \ (x_i^*) \le \lambda \ .$$

Now we shall result the assumption which carries the name a principle of equalizing of J.M.Germeier which can be useful to construction of algorithm of search of optimum decisions of a problem (11) - (12).

Lemma 2. The problem (11) - (12) has decisions of a kind

$$x_i^* > 0, \psi_i(x_i^*) = \lambda \tag{13}$$

$$x_i^* = 0, \ \psi_i(x_i^*) \ge \lambda .$$

Any vector $x^* = (x_1^*, ..., x_m^*)$, satisfying (13) and to restrictions of a problem (11) - (12), is maxmin strategy of a problem (11) - (12).

Lemmas 1-2 for problems (9) - (10) and (11) - (12) can be used only when products infinitely allocated i.e. when everyone x_i can accept any non-negative natural value. But in many problems of distribution of resources, everyone x_i can accept only final number of non-negative values. It concerns quantity the goods and distribution of bank money.

In this case instead of (9) – (10) and (11) – (12) consider compliance problems

A. $$\sum_{i=1}^{n} \varphi(x_i) \to \max, \tag{14}$$

$$\sum_{i=1}^{n} x_i = A, \quad x_i = y_{l_i}, \quad i = 1,\dots,n; \quad l_i = i_1,\dots,i_{p_i}. \tag{15}$$

B. $$\min_{1 \le i \le n} \{\psi_i(x_i)\} \to \max, \tag{16}$$

$$\sum_{i=1}^{n} x_i = A, \quad x_i = y_{l_i}, \quad i = 1,\dots,n; \quad l_i = i_1,\dots,i_{p_i}. \tag{17}$$

y_{l_i} - the fixed non-negative numbers.

For the decision of problems (14) - (15) and (16) - (17) it is convenient to accept methods of dynamic programming about which it will be told in the following paragraph. In problems (9) - (10) and (11) - (12) quantities of resources x_i unbounded from upper. But in the majority offered in this work models they are limited from above by the set numbers:

$$f_0(x) = f(x_1,\dots,x_n) = \sum_{i=1}^{n} \varphi_i(x_i) \to \max, \tag{18}$$

$$\sum_{i=1}^{n} x_i = A, \quad 0 \le x_i \le A_i, \quad i = 1,\dots,n. \tag{19}$$

In this case the following lemma is fair

Lemma 3. (Generalized Gibbs'Lemma)[10,13,17] Vector $x^* = (x_1^*,\dots,x_n^*)$ the-decision of a problem (18) - (19) in only case when there is such number λ, that

$$A_i > x_i^* > 0, \varphi_i'(x_i^*) = \lambda$$
$$x_i^* = A_i, \ \varphi_i' \ (x_i^*) \ge \lambda$$

$$x_i^* = 0\ ,\ \varphi_i'\ (x_i^*) \le \lambda\ \ .$$

The Conversion Of The Problem Of The Allocation Resources To The Problem Of Dynamic Programming And Building Of Computational Scheme

Let problem (9)-(10) describes of problem the optimal distribution of credit money between n borrowers. Here are supposed fulfilled following conditions:

1. income received of borrower Z_k , is not dependent on another borrower ;
2. received income different borrowers, is expressed in similar units;
3. total income is equal to the amount of incomes received from the distribution of all money as to all borrowers.

We shall pass on to problem definition (9)-(10) in the form of the model of dynamic programming. The inner property of the process of the distribution of money between n borrowers let's to consider it how n-step process. For the number of k step shall accept the number of borrower whom are distinguished of money x_k. On first step distinguish to the first borrower of money x_1, on second step to-second borrower distinguish of money x_2 from stayed and etc. It is evident that variable x_k, $(i = 1,...,n)$ it is possible consider how controlling variable ones. The initial state of system features the value of ξ_0 money being subject distribution. After allotment x_1 stays $\xi_1 = \xi_0 - x_1$ money and etc. Values $\xi_0, \xi_1,..., \xi_n$ characterizing the remainder of money after distribution on former steps, will consider how the parameters are of state. By equations conditions serve equalities [2,5,6]

$$\xi_k = \xi_{k-1} - x_k \quad (k = 1,...,n)$$

Summary income for n steps composes

$$Z = \sum_{k=1}^{n} f_k(x_k)$$

and presents the measure of effectiveness of process having, as it is obvious from this equality, additive form.
If to the onset of k step the remainder of money is equal ξ_{k-1}, then income which can be received on stayed $n-k+1$ steps, will compose $Z_k = \sum_{i=k}^{n} f_i(x_i)$.
Maximal income for these $n-k+1$ steps is dependent on that, how many of money remained from previous $k-1$ steps, from value ξ_{k-1}. That is why will be it to designate through $Z_k^*(\xi_{k-1})$. It is evident that $Z_1^*(\xi_0) = Z_{\max}$, $Z_1^*(\xi_0)$ presents summary maximal income for n steps. We shall consider any k step. It is evident that x_k can be chosen from condition $0 \le x_k \le \xi_{k-1}$. Significance x_k satisfying to this double inequality is called permissible. Principle optimization in this concrete case means that having distinguished value x_k and having received from k borrower income $f_k(x_k)$, we must to dispose stayed money by $\xi_k = \xi_{k-1} - x_k$ advantageous image and receive from borrower $Z_{k+1},...,Z_n$ maximum income $Z_{k+1}^*(\xi_k)$. It is explicit that value g follows to determine from the condition of maximization amount $f_k(x_k) + Z_{k+1}^*(\xi_k)$. Thus, gain equation

$$Z_k^*(\xi_{k-1}) = \max_{0 \le x_k \le \xi_{k+1}} \left\{ f_k(x_k) + Z_{k+1}^*(\xi_k) \right\}, \qquad (20)$$

which presents Bellman's equation (5) for this problem (9)-(10) [8].

We shall pass on to computational scheme. We are interested in $Z_1^*(\xi_0)$; but if to begin with first step, from deciding of task h that necessary to know $Z_2^*(\xi_1)$. In its turn, in identification $Z_2^*(\xi_1)$ necessary to know $Z_3^*(\xi_2)$ and etc. However there is step, for which no subsequent. Such is n step, on which are distinguished of money to last borrower Z_n. For him inequality (20) has aspect

$$Z_{n-1}^*(\xi_{n-1}) = \max_{0 \le x_n \le \xi_{n-1}} \{f_n(x_n)\}, \tag{21}$$

We will to consider that function of income $f_n(x_n)$ monotonously grows, that is why deciding of this task is provisory optimal control $x_n^*(\xi_{n-1})$, in which is achieved provisory maximum $Z_n^*(\xi_{n-1}) = f_n(x_n^*)$ .Hence, to borrower Z_n are distinguished all stayed money ξ_{n-1} which brings income $f_n(\xi_{n-1})$ .We shall return to previous, to $(n-1)$ step, at the beginning which there is the remainder of money ξ_{n-2}. Equation (20) in this instance will accept type

$$Z_{n-1}^*(\xi_{n-2}) = \max_{0 \le x_{n-2} \le \xi_{n-2}} \left\{f_{n-1}(x_{n-1}) + Z_n^*(\xi_{n-1})\right\}.$$

Here optimal choice x_{n-1} not so evident, how in deciding of previous problem (21). It is first of all having expressed from the equation of state ξ_{n-1} through $\xi_{n-2} - x_{n-1}$, shall receive

$$Z_{n-1}^*(\xi_{n-2}) = \max_{0 \le x_{n-1} \le \xi_{n-2}} \left\{f_{n-1}(x_{n-1}) + Z_n^*(\xi_{n-2} - x_{n-1})\right\}. \tag{22}$$

Both resign in decorative brackets-known functions being dependent on controlling variable x_{n-1}. Parameter ξ_{n-2} is initial state for this problem. Having fulfilled research on the maximum of function $Z_{n-1}(x_{n-1}, \xi_{n-2}) = f_{n-1}(x_{n-1}) + Z_n^*(\xi_{n-2} - x_{n-1})$ from one variable x_{n-1}, shall receive provisory optimal control $x_{n-1}^*(\xi_{n-2})$ and corresponding provisory maximum of summary income $Z_{n-1}^*(\xi_{n-2})$. In language of this problem this decision means that if before the accentuation of money to

borrower Z_{n-1} in our disposal there is remainder ξ_{n-2}, to then borrower Z_{n-1} necessary to distinguish of $x^*_{n-1}(\xi_{n-2})$ money. Where at the amount of incomes from borrower Z_{n-1} and Z_n achieves maximum.
Having ended deciding of problem (22), shall pass on to following from end to $(n-2)$ step, shall determine similarly provisory optimal control to $x^*_{n-2}(\xi_{n-3})$ and corresponding remainder ξ_{n-3} provisory maximum $Z^*_{n-2}(\xi_{n-3})$ and etc.

As a result passing step by step all steps from the end of the process of distribution to it onset, shall receive two the sequences of functions:

$$Z^*_n(\xi_{n-1}),\ Z^*_{n-1}(\xi_{n-2}),...,\ Z^*_2(\xi_1), Z^*_1(\xi_0)$$

and

$$x^*_n(\xi_{n-1}),\ x^*_{n-1}(\xi_{n-2}),...,\ x^*_2(\xi_1), x^*_1(\xi_0)\ .$$

This terminates the first and principal stage of computational process receiving the title of conditional optimization. Now proceed to the second stage of the computational scheme of-unconditional optimization. At that stage of first of all knowing function $Z^*_1(\xi_0)$, as to given significance ξ^*_0 determine $Z_{\max} = Z^*_1(\xi^*_0)$.

It is further on; apply to succession $x^*_k(\xi_{k-1})$ which passes from onset by the end of process. We distinguish to $x^*_1 = x^*_1(\xi^*_0)$ first borrower; then for distribution stays $\xi^*_1 = \xi^*_0 - x^*_1$. As to this value determine the optimal quantity of money $x^*_2 = x^*_2(\xi^*_1)$ distinguished to second borrower. Again find $\xi^*_2 = \xi^*_1 - x^*_2$, after what determine x^*_3, and etc. While will not be determined being looked for optimal up $(x^*_1, x^*_2,..., x^*_n)$.

AN EXAMPLE APPLICATION

Maximization Of The Total Income Of Small Business

In this paragraph one simple model of maximization of the total income small business and it is illustrated is offered as problems such are solved by means of a method of dynamic programming.

Statement of Mathematical Model

Let N is available representatives of small and middle business who require financial support. We shall admit, that any credit organization took for credit these businessmen, having allocated thus sum M of money. Usually such work is carried out on target to the program or under the project that to credit the organizations let out the same products. Let k -th borrower, having taken on loan in quantity x_k, can let out production in cost $f_k(x_k)$, having spent thus $(c_k + \mu)x_k$ money, where - $\mu > 0$ the credit rate average, for credit term. We shall be limited to a case when the price of capitals /or we here to consider is known, that potential borrowers have opportunities to substitute something or to receive a guarantee of reliable sponsors and etc./, substituted on the security of borrowers E_k, $k = 1,....,N$. Then we shall have a following problem of optimization.

$$\sum_{k=1}^{N}\left[f_k(x_k) - (c_k + \mu)x_k \right] \to \max , \tag{23}$$

$$\sum_{k=1}^{N} x_k = M, \quad 0 \le x_k \le \alpha \cdot E_k, \quad k = 1,...,N ,$$

where $\alpha > 0$ it is set by the credit organization. This problem at natural can be solved by means of the algorithm using generalized Gibbs'Lemma. In case of when accepts final number of values, it is necessary to use a method of dynamic programming. Applications of a method of dynamic programming demonstrate on a following example.

A Numerical Example

In practical activities credit the quantity of given out money can accept the organizations only multiple, means, final values. Because so it is more convenient as to the creditor, and borrowers. Therefore in this item the example similar to volume is resulted, that us was is investigated and solved.

Let the credit organization allocates 3million.tug for credit four organizations small business. We shall be to consider, that money can allocate only in the sizes 0, 0.5 million, 1 million, 1.5 million, 2 million, 2.5 million, 3 million. Through x_i we shall designate quantity of credit money, issued for i-th borrowers. $f_i(x_i)$- function of utility i-th borrowers which represents a difference between the net profit $R_i(x_i)$and expenses $C_i(x_i)$, connected with commercial activity and borrowers payment of credit percent. Let functions of incomes and expenses have following kinds

$R_1(x)=5x_i^{0.1} \quad ,C_1(x)=0.3x_i$

$R_2(x)=4x_i^{0.2} \;,\; C_2(x)=0.37x_i$

$R_3(x)=2x_i^{0.4} \quad ,C_3(x)=0.4x_i$

$R_4(x)=3.5x_i^{0.1},\quad C_4(x)=0.2x_i$.

Then for corresponding functions of utility it is had

$f_1(x)=5x_i^{0.1}-0.3x_i$

$f_2(x)=4x_i^{0.2}-0.37x_i$

$f_3(x)=2x_i^{0.4}-0.4x_i$

$f_4(x)=3.5x_i^{0.1}-0.2x_i$.

Let's admit, that value of function of utility of everyone borrowers depending on the credit received by it is set following tables.

Each of functions $f_i(x_i)$ is convex and is had a unique point of a maximum x_i. Simple calculation shows, that maxima are reached accordingly in points $\bar{x}_1 = 1.75$, $\bar{x}_2 = 2.62$, $\bar{x}_3 = 5.07$, $\bar{x}_4 = 1.86$. i-th borrowers to receive the credit x_i which is more than $\bar{x}_i$ thoughtless as the same income can be received at the smaller credit. Therefore $\bar{x}_i$ it was possible to consider upper as a limit of the credit which can to receive i-th borrowers. On the other hand, upper the limit of the credit depends on a condition of a fixed capital borrower, or from the size, the capital substituted in the mortgage.

Table 1. Each of functions $f_i(x_i)$

	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
0	0	0	0	0
0.5	4.5	3.3	1.3	3.16
1	4.7	3.6	1.6	3.3
1.5	4.756	3.78	1.75	3.34
2	4.758	3.85	1.83	3.35
2.5	-100	3.87	1.88	-100
3	-100	-100	1.9	-100

We shall admit that upper limits A_i of the credit for borrowers are equal to following values:
$A_1 = 2$ million, $A_2 = 2.5$ million, $A_3 = 3$ million, $A_4 = 2$ million.
Above we have a following problem of allocation of money:

$$\sum_{i=1}^{4} f_i(x_i) \to \max;$$

$$\sum_{i=1}^{4} x_i = A = 3, \quad 0 \le x_k \le A_i, \quad i = 1,...,4. \tag{24}$$

This problem from considered in (24), variables x_i bounded from above numbers A_i. At construction of the computing scheme we should consider it. It is reached by use of a so-called method of "penalty".

Table 2. Results of calculation with optimum distribution (k = 3, 2, 1)

$k = 3, 2, 1$			Step 3 $(k = 3)$			Step 2 $(k = 2)$			Step 1 $(k = 1)$		
ξ_{k-1}	x_k	$\xi_k = \xi_{k-1} - x_k$	$f_3(x_3)$	$Z_4^*(\xi_3)$	$Z_3(\xi_2, x_3)$	$f_2(x_2)$	$Z_3^*(\xi_2)$	$Z_2(\xi_1, x_2)$	$f_1(x_1)$	$Z_2^*(\xi_1)$	$Z_1(\xi_0, x_1$
1	2	3	4	5	6	7	8	9	10	11	12
0.5	0	0.5	0	3.16	3.16	0	3.16	3.16	0	3.3	3.3
	0.5	0	1.3	0	1.3	3.3	0	3.3	4.5	0	4.5
1	0	1	0	3.3	3.3	0	4.46	4.46	0	6.46	6.46
	0.5	0.5	1.3	3.16	4.46	3.3	3.16	6.46	4.5	3.3	7.8
	1	0	1.6	0	1.6	3.6	0	3.6	4.7	0	4.7
1.5	0	1.5	0	3.34	3.34	0	4.76	4.76	0	7.76	7.76
	0.5	1	1.3	3.3	4.6	3.3	4.46	7.76	4.5	6.46	10.96
	1	0.5	1.6	3.16	4.76	3.6	3.16	6.76	4.7	3.3	8
	1.5	0	1.75	0	1.75	3.78	0	3.78	4.75	0	4.75
2	0	2	0	3.35	3.35	0	4.91	4.91	0	8.06	8.06
	0.5	1.5	1.3	3.34	4.64	3.3	4.76	8.06	4.5	7.76	12.26
	1	1	1.6	3.3	4.9	3.6	4.46	8.06	4.7	6.46	11.16
	1.5	0.5	1.75	3.16	4.91	3.78	3.16	6.94	4.75	3.3	8.05
	2	0	1.83	0	1.83	3.85	0	3.85	4.758	0	4.758
2.5	0	2.5	0	-100	-100	0	5.05	5.05	0	8.36	8.36
	0.5	2	1.3	3.35	4.65	3.3	4.91	8.21	4.5	8.06	12.56
	1	1.5	1.6	3.34	4.94	3.6	4.76	8.36	4.7	7.76	12.46
	1.5	1	1.75	3.3	5.05	3.78	4.46	8.24	4.75	6.46	11.21
	2	0.5	1.83	3.16	4.99	3.85	3.16	7.01	4.758	3.3	8.06
	2.5	0	1.88	0	1.88	3.87	0	3.87	-100	0	-100
3	0	3	0	-100	-100	0	5.13	5.13	0	8.54	8.54
	0.5	2.5	1.3	-100	-98.7	3.3	5.05	8.35	4.5	8.36	12.86
	1	2	1.6	3.35	4.95	3.6	4.91	8.51	4.7	8.06	12.76
	1.5	1.5	1.75	3.34	5.09	3.78	4.76	8.54	4.75	7.76	12.51
	2	1	1.83	3.3	5.13	3.85	4.46	8.31	4.758	6.46	11.22
	2.5	0.5	1.88	3.16	5.04	3.87	3.16	7.03	-100	3.3	-96.7
	3	0	1.9	0	1.9	-100	0	-100	-100	0	-100

The idea consists in when value of a variable x_i surpasses A_i value $f_i(x_i)$ we shall enough consider small, for example, it equally-100. Below two tables are presented, which are used in the computing scheme.

Table3. Results of calculation with optimum distribution (z = 4, 3, 2, 1)

	Step 4		Step 3		Step 2		Step 1	
ξ	$Z_4^*(\xi_3)$	$x_4^*(\xi_3)$	$Z_3^*(\xi_2)$	$x_3^*(\xi_2)$	$Z_2^*(\xi_1)$	$x_2^*(\xi_1)$	$Z_1^*(\xi_0)$	$x_1^*(\xi_0)$
0.5	3.16	0.5	3.16	0	3.3	0.5	4.5	0.5
1	3.3	1	4.46	0.5	6.46	0.5	7.8	0.5
1.5	3.34	1.5	4.76	1	7.76	0.5	10.96	0.5
2	3.35	2	4.91	1.5	8.06	0.5	12.26	0.5
2.5	-100	2.5	5.05	1.5	8.36	1	12.56	0.5
3	-100	3	5.13	2	8.54	1.5	12.86	0.5

As a result calculation we have received following optimum distribution of credit money for borrowers: $x_1^* = 0.5$, $x_2^* = 1$, $x_3^* = 1$, $x_4^* = 0.5$. At such distribution the total income borrowers is maximal and equal to $\sum_{i=1}^{4} f_i(x_i^*) = 12.86$.

Support of A Needy Part of The Population By Means of The Micro Credit

In last year, in our country more and more wide scope get so-called micro financial services poor to a layer of the population. In such activity are engaged not only the bank organizations, but also not bank financial organizations and credit cooperative societies. Environments of micro financial services the main place borrow needy i.e. poor parts of the population. And in this question the state gives the most serious attention. Confirmation to that organized the national congress on micro financing in November, 2004, in Ulaanbaatar. Here we shall briefly characterize these terms using works of the Mongolian researchers [9].

Micro Enterprise

96 % from organizations of our country engaged by business are made by representatives of medium and small business, and they make 60 % GDP. In turn, the majority of them is engaged in micro business, so they work only to have a financial source for maintenance of the family with the most necessary needs. Save up surplus money they cannot and consequently not only not in a condition to expand the business, but often get in difficulty with a turn of the capital. Results of interrogation show, the majority representatives of micro business considered that they are on a poverty line or engaged in the service which is not demanding greater work and the capital. However, the majority in a country side is engaged in animal industries, agriculture, hunting, etc. On the basis of results of public opinions, in work the following recommendation for distinction of micro business from medium and small business in present economic conditions of Mongolia is offered.

The main parameters are the number of workers and the revenue that is accepted in a world practice. It is offered to define micro business as follows.

a. The business carries out individually, or within the limits of family, or the number working does not surpass 10.
b. In all questions of business itself owner (head) of facilities accepts direct participation.
c. The weight does not surpass the capital of facilities 55000 $.
d. The volume of the industrial capital does not surpass 10000 $.

According to this definition, the overwhelming majority of economic units of Mongolia represent micro business.

Table 4. Business characteristics

Kind of business	Branch	Concrete kind of work	Number of workers	The revenue
Microbusiness	Trade	Retail and wholesale trade	to 2	to 6
	Service	Repair, work on service family or persons, transport, small hotel or cafe.	to 5	to 5
	Agriculture	Agriculture	to 5	to 5
small and medium business	Service	Repair, service of family or the person, service in social or medical spheres, work of the manager in warehouses, in small finance, in transport, in formation, in hotel and café	to 20	6-8
	Trade	Retail and wholesale trade	to 10	6-24
	Manufacture	Almost all kinds of manufacture, especially in mining and light industry	to 30	to 120
	Agriculture	Animal industries, agriculture, hunting, preparation wooden	to 20	4-48
	Construction	Construction, repair, roads	to 50	to 240

Microfinance

In a broad sense microfinance covers the various financial services rendered poor and poor parts of the population. Therefore in following definition was accepted. Complex service, such as the savings, the credit, computing for payment, translation, insurance, leasing and others which are intended for poor and low income parts of the population, refers to microfinance.

The Micro Credit

If above named service is carried out in the form of the loan it name the micro credit. According to the interrogation, the poor layer of the population lead the environment in 2005, the majority interrogated considers that it is favorable to them to take the credit at a rate of 1-2 million.tug. For 1 year, but the main thing barrier is the high credit rate. Among foreign sponsors for support microfinance, including the micro credit in Mongolia it is possible to name the following organizations: USAID, Word Bank, CGAP, ADB, IFC, JICA, GTZ, IFAD and UNDP. And the state policy of support microfinance is carried out mainly through Монголбанк.

When the project provide is carried out under the governmental program or under the initiative of any foreign organization of good will most of all the attention is given a needy part of medium and small business to micro business. Thus speech does not go about decrease in credit rates and granting of other privileges needy, and have in view of support of commercial activity of those who most of all requires it. Usually poor men have no capital which could be pawned, but is quite capable to lead commercial activity. At such statement of a question, we should solve following maxmin problem.

$$\min_{1 \le k \le N} \left(f_k(x_k) - (c_k + \mu) x_k \right) \to \max , \tag{25}$$

$$\sum_{k=1}^{N} x_k = M, \quad 0 \le x_k \le \alpha \cdot E_k, \quad k = 1,\dots,N,$$

where x_k - the quantity of the credit allocated to k-th borrower.
Here E_k - quantity of the capital of k-th borrower, $0 < \alpha < 1$ a share of the capital E_k, such, that the credit organization cannot give k-th borrower of more money, than αE_k.
Now we shall consider a problem of distribution of credit money for support low income parts of four borrowers.

$$\min_{1 \le i \le 4} f_i(x_i) \to \max; \tag{26}$$

$$\sum_{i=1}^{4} x_i = 3, \quad 0 \le x_i \le A_i, \quad i = 1,\dots,4.$$

According to a principle of equalizing of M.Germeiera, the problem (2) will be transformed to a problem of type (26) to next images

$$-\sum_{i=1}^{4} \int_0^{x_i} f_i(t)dt = \sum_{i=1}^{4} F_i(x_i) \to \max;$$

$$\sum_{i=1}^{4} x_i = 3, \quad 0 \le x_i \le A_i, \quad i = 1,\dots,4.$$

Such as, all over again we should find primary $F_i(x)$ from functions $f_i(t)$, then use computing the scheme from the previous item. For primary we have formulas

$$F_1(x) = 4.54x^{1.1} - 0.15x^2, \quad F_2(x) = \frac{10}{3}x^{1.2} - 0.185x^2, \quad F_3(x) = \frac{10}{7}x^{1.4} - 0.2x^2,$$

$$F_4(x) = \frac{35}{11}x^{1.1} - 0.1x^2$$

At $x_i > A_i$ we shall considered, that $F_i(x_i) = -1000$. Then for $F_i(x_i), i = 1,\dots,4$ it is had the following table

Table 5. Computing scheme

	$F_1(x)$	$F_2(x)$	$F_3(x)$	$F_4(x)$
0	0	0	0	0
0.5	-2.08	-1.4	-0.49	-1.46
1	-4.39	-3.15	-1.23	-3.08
1.5	-6.75	-5	-2.07	-4.74
2	-9.13	-6.92	-2.97	-6.42
2.5	-1000	-8.85	-3.9	-1000
3	-1000	-1000	-4.85	-1000

Applying the computing scheme from the previous item, we have found, that optimum is allocation (0.5, 0.5, 1.5, 0.5) with corresponding prizes (4.5, 3.3, 1.75, 3.16). At such distribution the total income borrowers is maximal and equal $\sum_{i=1}^{4} f_i(x_i^*) = 12.71$.

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