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Send your comments and feedback to the Editor:

Shabbir Ahmed
School of Industrial & Systems Engineering
Georgia Tech, Atlanta, GA 30332
(sahmed@gatech.edu)

Chair's Column

Jon Lee
Industrial and Operations Engineering Department
University of Michigan, Ann Arbor, Michigan
(jonxlee@umich.edu)

The present issue of our INFORMS Optimization Society newsletter, "INFORMS OS Today," features articles by the 2011 OS prize winners: Kees Roos and Jean-Philippe Vial (Khachiyan Prize for Lifetime Accomplishments in Optimization), Andrew V. Goldberg (Farkas Prize for Mid-career Researchers), Tobias Achterberg (Prize for Young Researchers), and Daniel Dadush (Student Paper Prize). Each of these articles describe the prize-winning work in a compact form. Also, in this issue, we have announcements of key activities for the OS: Calls for nominations for the 2012 OS prizes, and a call for nominations of candidates for OS officers. Please consider being active in the nomination process.

Of course, the raison d'être of the OS is to ensure a strong and organized presence for optimization at the annual INFORMS meetings, the next one being at the Phoenix Convention Center, Phoenix, Arizona, October 14–17, 2012. Our participation is organized via the OS sponsored clusters. As usual, the OS clusters and cluster chairs for that meeting mirror our list of Vice Chairs:

- Brian Borchers, Computational Optimization and Software (borchers@nmt.edu)
- Oleg A. Prokopyev, Global Optimization (prokopyev@engr.pitt.edu)
- Santanu Dey, Integer Programming (santanu.dey@isye.gatech.edu)

- Mohammad Oskoorouchi, Linear Programming and Complementarity (moskooro@csusm.edu)
- Baski Balasundaram, Networks (baski@okstate.edu)
- Frank E. Curtis, Nonlinear Programming (frank.e.curtis@gmail.com)
- Huseyin Topaloglu, Stochastic Programming (ht88@cornell.edu)

The hard work of our Vice Chairs is the means by which we have a very strong presence within INFORMS — a presence that reflects the very large membership of the OS. Please contact appropriate Vice Chairs to get involved.

I want to remind you that Pietro Belotti is the OS webmaster, and he is always pleased to get your feedback on our website: `www.informs.org/Community/Optimization-Society`.

All of the OS officers and I look forward to seeing you at Phoenix in October — in particular, at the OS Prize Session and at the OS Business Meeting. The latter is always one of the highlights of an INFORMS meeting for me — a great opportunity to have some light food and refreshments, meet with old friends and make new ones.

Seduced by Optimization

Kees Roos
Delft University of Technology
(C.Roos@tudelft.nl)

Introduction

Having received the Khachiyan prize of the INFORMS Optimization Society for Lifetime Accomplishments, it seems appropriate to present a brief history of the scientific part of my life in this paper. Of course, this part closely interferes with other aspects of my life. Therefore, I first want to emphasize that I owe a lot to other people. First my parents, my brothers, my wife and family, my colleagues, many of whom became close friends, and last but not least, many great scientists from the past who led the scientific foundation for our generation. Special thanks go to the Khachiyan prize committee members George Nemhauser, Yurii Nesterov, Lex Schrijver and Tamás Terlaky, and my friends and colleagues Dick den Hertog, Aharon Ben-Tal and Frédéric Babonneau who nominated Jean-Philippe Vial and me.

My path to optimization

After having obtained my Master's degree in mathematics at TU Delft, in 1969, I was offered a permanent position at TU Delft. The Master's thesis was about upper bounds for cyclic codes, and was supervised by professor Frans Loonstra (1910-1989), who was an algebraist, and who also became my PhD supervisor. In 1975 I defended my PhD thesis, on a subject in pure algebra (more specifically, ring theory). But soon after, Loonstra gave the advise to change my research topic, because he saw no future for algebra in Delft. So I returned to the topic of my Master's thesis, algebraic coding theory. At that time I started to participate in seminars organized by professor Jack H. van Lint (1932-2004) at TU Eindhoven. He was a world-leading expert in this field and very energetic and inspiring. Among the participants were also Hendrik W. Lenstra and Alexander Schrijver. Compared to their contributions in these seminars mine were modest, but after all I learned a lot and it was an exciting time. If something was worth being published, Jack was very supportive. Once I submitted a paper to a prestigious IEEE journal, but it was declined. A few months later, Jack attended a conference in the US where the editor who handled my submission presented a paper that was identical to my submission. Jack was astonished, and made this clear during the discussion. Back home, being convinced that the editor had cheated, he wrote a letter to the editor-in-chief of the journal, asking him to intervene. Without further delay the paper was accepted and published [15]. In some of the seminars we studied Philip Delsarte's PhD thesis on Association Schemes [4]. One of the highlights in this thesis was a revolutionary bound for codes based on the use of linear optimization (LO). This impressive result was not only extremely elegant, but it turned out to be a very strong computational tool in deriving new bounds for codes. During my study I had taken an introductory LO course, but this result made me understand how powerful LO can be. It became the bridge to my next and final research topic. It happened in 1982 that I was invited by professor Freerk Lootsma (1936-2003), whose office was opposite to mine, to join his chair in Operations Research and to take responsibility for the courses and research in LO. Although at that time my knowledge of optimization was quite limited, for more than one reason it was not hard to take a decision: I accepted the invitation. It came as a surprise to my colleagues in coding theory, because just around that time I found a new bound for cyclic codes (the topic of my Master's thesis!) that is still cited [16, 17]. It was a generalization of the well-known BCH bound for cyclic codes and became known as the 'Roos bound'. Nevertheless, I never had to regret my decision. Even more excitement was waiting.

In 1984 it became known that Narendra Karmarkar had found a new polynomial-time method for LO. I soon got a preprint of his paper [10]. I was struck by his approach, which differed in many ways from everything before. It was not only elegant from a mathematical point of view, but Karmarkar claimed that it was about 100 times faster than the Simplex method, at that time the only computationally efficient method. It took a long time to get the claim verified, with turbulent discussions during conferences. Many experts in the field were skeptical and advised not to spend much time on it. They could not believe that after almost 30 years of research on LO something new could happen; as a research topic they considered LO 'dead'. On the other hand, at foreign conferences leading researchers from the US and Japan appeared to take Karmarkar's invention seriously. It stimulated me to not quit the topic.

Optimization Group at Delft

Only in 1989 my first paper on the new approach was published [18]. In the meantime a Master's student became interested in the new so-called *interior-point methods* (IPMs), namely Dick den Hertog. His Master's thesis became the basis for a second paper [5]. Also Jean-Philippe Vial, at the University of Geneva, and I started to cooperate. His reading of [18] gave rise to a much simplified version of it [21], emphasizing the relation of the new IPMs with the classical logarithmic barrier approach. After the arrival (on September 2, 1989) of dr. Tamás Terlaky as a research fellow, the Optimization Group in Delft had three members, because Dick den Hertog started one day earlier as a PhD student. This marked the beginning of an exciting time, with new PhD students, and many foreign visitors. The cooperation with PhD students[1] and visitors[2] resulted in tens of publications in prestigious journals and eight books on IPMs [7, 8, 9, 11, 12, 13, 20, 22]. Lectures by foreign colleagues were influential for the curriculum that we offered, thus attracting many Master's students. Some of these contributed to the research on IPMs, others did their Master's projects externally in companies as Philips, Shell, Heineken, Schiphol, ORTEC and Paragon. In total about 80 students completed their Master's thesis in our group.

In 2000 Terlaky announced that McMaster University (Hamilton, Canada) offered him a chair in Optimization. With his departure a fruitful cooperation of more than ten years came to an end. His position in the group was given to dr. Hans Melissen. Since October 2000 also dr. Etienne de Klerk was a member of the group, but he departed in 2003 to the University of Waterloo, Canada. Because after my retirement, in May 2006, dr. Melissen moved to the Systems Theory group (at Delft), I was left behind with ten PhD students. Figure 1 shows nine of them. Eight of them have already defended their thesis.[3] The tenth PhD is Henk N. Post, who combines his PhD research with a fulltime job at a transportation company; since 2010 he is jointly supervised by

[1] Besides Dick den Hertog, Benjamin Jansen, Etienne de Klerk, Arie J. Quist, Jiming Peng, Mohamed Elghami, Diah Chaerani, Gamal Elabwabi, Manuel Vieira, Ivan Ivanov, Hossein Mansouri, Guoyong Gu, Bib P. Silalahi, Alireza Asadi, Maryam Zangiabadi, and Henk N. Post.

[2] Among others, Erling Andersen, Kurt Anstreicher, Yanqin Bai, Florian Barb, Aharon Ben-Tal, Immanuel Bomze, Ilya Dikin, Robert Freund, Francois Glineur, Clovis Gonzaga, Maria Gonzalez, Harvey Greenberg, Osman Güler, Margareta Halická, Alexander Hipolito, Allen Holder, Tibor Illés, John Kaliski, Goran Lesaja, Tom Luo, John Mitchell, Renato Monteiro, Arkadi Nemirovski, Yurii Nesterov, Mike Todd, Miklós Újvári, Lieven Vandenberghe, Jean-Philippe Vial, Yinyu Ye.

[3] In two cases dr. de Klerk (who is now at the University of Tilburg) was co-supervisor.

Figure 1: From left to right: Diah Chaerani, prof.dr. Yanqin Bai (visitor), Kees Roos, Alireza Asadi, Hossein Mansouri, Maryam Zangiabadi, and their daughter, Guoyong Gu, Manuel Vieira, Ivan Ivanov, Bib P. Silalahi, and Gamal Elabwabi.

Karen Aardal (my successor) and me.

Form 1984 on, the research in our group focussed on algorithms for the solution of linear and later also non-linear optimization problems. The paper of Karmarkar initialized a revolution in the field of optimization. Initially IPMs were designed for linear optimization, but around the early nineties it became clear that they could also be used for nonlinear problems, especially convex problems. This was of great importance for many application areas. For example, in system theory models linear matrix inequalities (LMIs) were used since the late seventies, but there were no effective solution methods. Now it is clear that LMIs are just the type of inequalities that occur in semidefinite optimization problems, which can be solved efficiently by standard software based on IPMs. It became even more interesting when it turned out that for some hard combinatorial problems (NP-complete problems) one can get very good solutions in polynomial time. The method consists of constructing a semidefinite relaxation whose solution can be rounded to a solution of the original problem, and whose objective value is not worse than an a priori known percentage (e.g., 13 %) of the optimal value.

This is not the place to go into the theory that lies behind the developments that were sketched above. As a branch of applied mathematics, for me it has always been an attractive aspect of the field of optimization that it is so closely connected to problems that arise in real life. Before my retirement, in 2006, I was involved in many real life applications, due to the Master's thesis projects in companies that I supervised, but my main focus was theoretical research. Only after my retirement I was able to spent more time on some applied projects. Below, I will present a few of such projects.

Optimal safe dike heights

In the history of The Netherlands, the fight against the water of sea and rivers has always been of vital importance, because about 55% of the country is submersible. For example, the village where I live is about 5 meters below sea level. During my lifetime, serious situations occurred twice. In 1953 there was a serious flood in the south-west part of the country; almost 2,000 people were killed by drowning. The second time was in 1995. While in 1953 the danger came from the sea, in 1995 it was the main rivers that rose to dangerous levels and as a consequence 200,000 people had to be evacuated. Protection against floods is realized by the dunes (mainly at the sea coast and sometimes artificially enforced), and dikes (mainly along the rivers). The submersible part of the country is divided into more than 50 so-called *dike rings*. A dike ring is a region that is surrounded by dikes and/or dunes. Figure 2 shows these dike rings. At present, by law a safety norm has been fixed for every dike ring: it is prescribed that on the average a flood should occur not more than once every 1,250 to 10,000 years; the number varies per dike ring. On the basis of historical data, predictions are made for future sea and river levels, and ground levels. Also effects of climate change are taken into account. These predictions are used in a cost-benefit analysis. The objective is to find an optimal balance between investment costs and the benefit of reducing flood damages, both as a result of heightening dikes. The results of our research are reported in [3, 6].

Dynamic positioning

The second project concerns the problem of keeping a ship in position, in the offshore industry. A

Figure 2: Dike rings in The Netherlands.

dynamic positioning (DP) system uses several types of thrusters that generate forces to compensate the external forces by wind, waves and currents. The thrusters have usually two degrees of freedom: the amount of thrust delivered and the direction in which this thrust is delivered. They are driven by electric motors; the required electricity is generated by diesel or gas turbines.

Figure 3: Dynamic positioning in action.

Ships that are equipped with a DP system can operate at deep seas, where it is impossible to use chains to fix the position of the ship. Since the DP system operates continuously, the fuel use of the turbines may be significant. A PhD student at the Department of Marine and Transportation Technology, TU Delft, is studying the problem of how to control the thrusters in such a way that the fuel consumption is minimized. Thus it becomes an optimization problem. When we discussed this problem, it turned out to be a highly nonlinear problem. But a closer inspection made clear that it can be modeled as a second-order cone optimization (SOCO) problem, which can be solved efficiently by IPMs. A case study revealed that under mild weather conditions the new approach decreases the fuel consumption by 6% and, moreover, under heavier weather circumstances, it keeps the ship in position better than existing DP systems. A manuscript is ready for publication [23], but its submission is postponed, because a company wants to patent this application of SOCO.

Robust electrical resistor networks

Resistor networks are electrical networks that contain only resistors. Their behavior is completely reigned by the laws of Ohm (1787-1854) and Kirchhoff (1824-1887). Figure 4 shows a network with four nodes. Input currents are given at the nodes, their sum being zero. One easily verifies that the currents

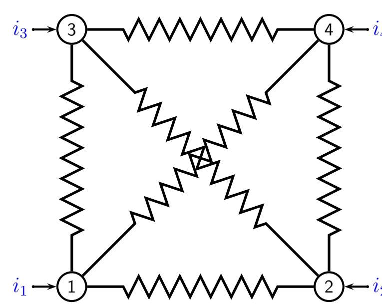

Figure 4: A resistor network with four nodes.

in the resistors uniquely follow from the aforementioned laws. We want to find resistor values that minimize the dissipation in the network. Hereby we allow infinite resistor values. Furthermore, we require that the sum of the inverse resistor values does not exceed a prescribed value. The latter constraint, as well as the objective function, is nonlinear. Surprisingly enough, the problem can be reformulated as a linear optimization problem, and hence it can be solved efficiently, yielding values of the resistors that minimize the dissipation. For more details I refer to [19].

However, the optimal network can be very unstable, in the sense that a small perturbation in one or more of the input currents yields an unexpectedly high dissipation value. In [19] we found an example where the optimal value of the dissipation is 10.0, but a 10% perturbation may increase the dissipation to about 1035.2. In practice the resulting heating might burn the network.

In order to get a more stable network we applied the Robust Optimization methodology as introduced by A. Ben-Tal, L. El Ghaoui and A. Nemirovski.[4] Their approach uses a suitably chosen *uncertainty set*, which contains all possible perturbations of the nominal input currents at the nodes. Given the resistor values, we maximize the dissipation over all possible perturbed input currents. If the uncertainty set has a simple enough structure, e.g., a box or ellipsoid, then this maximization is easy. The resistor network for which the maximal dissipation is minimal is likely to be most insensitive to perturbations. This network can be found by solving a semidefinite optimization problem. For more details on this methodology, I refer to [1] and [2]. When we apply this technique to the example mentioned above, it yields a network that is surprisingly stable. For the nominal input currents the dissipation is no longer optimal, it becomes 11.4. But note that the nominal input values occur with probability zero. More important is that we gained a remarkable increase in stability: the new network survives perturbations up to 10% of the nominal input, since under such perturbations the dissipation will never exceed 14.4 (instead of 1035.2).

Concluding remarks

In this short paper I wanted to demonstrate how and why I have been seduced by optimization. It is amazing how many opportunities this part of mathematics offers to describe and/or to optimize phenomena in nature and daily life. To conclude, with consent I cite one of our great predecessors, Ludwig Wittgenstein (1889-1951):

> Bach wrote on the title page of his *Orgelbüchlein*, 'To the glory of the most high God, and that my neighbor may be benefited thereby'. That is what I would have liked to say about my work [14, p. 181-182].

[4]Ben-Tal and Nemirovski spent sabbatical leaves as guest professors in Delft.

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Optimization for Decision Making

Jean-Philippe Vial
Em. Professor, University of Geneva and ORDECSYS, Geneva
(jpvial@ordecsys.com)

I cannot resist telling how I learned about the award of the Khachiyan Prize. Looking at my e-mails after having been away for some time, I clicked on the message at the top of the list and read an enigmatic word from Ronny Ben-Tal: “My poor professor, now you will have to sing for 25 minutes in Charlotte, North Carolina...”. Ronny loves to joke at my expense, and I thought he was mocking my new hobby of singing in a choir. But why in Charlotte? I love Ronny’s humor, but the confusion between the INFORMS meeting and a concert was beyond my understanding. Leaving the enigma aside, I went down the list of older messages and got the one by George Nemhauser who announced that I was awarded the Khachiyan prize. I did not start to sing but to shout “what? what?” so loudly that my wife rushed to me, fearing that something terrible had happened. Indeed, I was so surprised and so honored by the distinction that after the shout I remained voiceless. Still now, I cannot fully believe that the jury chaired by George Nemhauser and composed of Yurii Nesterov, Lex Schrijver and Tamás Terlaky have elected me for the prize upon the nomination by F. Babonneau, Aharon Ben-Tal and Dick den Hertog. I am very thankful to all of them. Sharing the prize with Kees Roos is also an honor and a great pleasure.

Prolegomenon

After graduating from, École Centrale, a French engineering school, I was very unclear about my future. Like many of my schoolmates, I felt attracted by social sciences and management, with the presumptuous feeling that my engineering training in hard sciences was a sufficient asset. Actually, what I wanted to do was Operations Research, but I did not even know the name of that discipline. The much compartmented French system of education in those times between universities and schools of engineering did not give many opportunities to cross curriculum boundaries. The world was not yet a global village, and the United States looked as a remote but fascinating place. The dream brought me to the University of Michigan, where I graduated in industrial engineering. This stimulating experience led me to spend a year at the Center of Operations and Econometrics, a research institute that four young professors of the University of Louvain had just launched. This was supposed to be a maturing year before joining my new employer, in the baby diaper business, but eventually turned to be the start of an academic career.

J. Drèze and G. de Ghellinck had gathered young people and prestigious visitors, in a casual and friendly atmosphere which seduced me. I was also very much attracted by what we would call now the mission of CORE: better serve society by favoring interaction between three disciplines, Economic theory to give a framework for actions, Econometrics to give quantitative flesh to the analysis, and Operations Research to provide the tools to transform the analyses into quantified recommendations. My personal problem, that I inherited from the rigid engineering training I received in France, was the fear of not being able to produce any respectable piece of research work. The highly stimulating intellectual atmosphere at CORE and my very positive experience at the University of Michigan helped me to overcome those negative feelings, though never completely.

Jon Lee, Jean-Philippe Vial, Kees Roos and George L. Nemhauser

The formation years

In retrospect, I more or less followed Drèze's program of mixing disciplines that all dealt with decision making. I earned my PhD in 1970 on the topic of the Cash Balance problem. It can be viewed as an extension of the well-known inventory problem, in which positive and negative transfers from cash to savings are allowed at the expense of fixed costs. In standard inventory theory, when only positive transfers can take place, the discrete time version of the problem received a nice solution through Scarf's K-convexity concept. But when negative transfers can also occur, the analysis in a discrete time framework breaks down. Having read an article on hydro-reservoir management by Bather, who used a continuous time model, I thought of using the same framework for cash balance. It turns out that the magic of integration dramatically simplifies the problem and that the solution takes the form of a computable (S, s) policy. In 1970, it was an early contribution [1] to continuous time Finance, but I knew too little in Finance to anticipate the major developments of the continuous time models. Following the program of CORE, I contributed to Mathematical Economics [2] and Game Theory [3]. Even though these pieces of work were intellectually satisfactory and are frequently quoted, they did not meet my expectations. I was looking for more practical decision-making problems, something in the vein of the cash-balance problem but applied to more tangible objects, as my engineering training had taught me to value.

I entered some years of doubts. The gap between the nice theoretical progress in Operations Research and the practical applications I heard about, seemed to widen, at least from my own limited perspective. I could see that many a time the models for real problems were too big to be handled by the existing computational tools, software as well as hardware. Economic and Game theory provided nice tools for analysis but did not seem to be that powerful in shaping decisions for the industrial world. I started to focus on the more specialized topic of nonlinear programming. While not making decisive progresses, the work made me familiar with two concepts, path-following [4] and trust regions [5]. I also extended the concept of convexity, by means of geometrical arguments [6]. In that period, I was narrowing my interests to algorithms and convexity, at my own slow pace, but it made me ready and receptive to the major innovations that the Interior Point Methods introduced soon after.

The interior point method (IPM) saga

In 1984, the community of optimizers became greatly excited. Khachiyan's proof that the ellipsoid method solved LP in polynomial time was a major breakthrough, but it soon appeared that the new method did not hold all its promises, at least as far as solving numerical instances of LP. The successful analysis of the most popular optimization problem in continuous variables by means of complexity arguments was a major theoretical step forward, but not yet a practical one. In 1984, the announcement by Karmarkar that his new projective method for LP was polynomial and would beat the Simplex by orders of magnitude stirred great excitement. I was part of that excitement and started to work hard on that method with G. de Ghellinck. In late 1985, I presented an interpretation of the projective method as a parameterized Newton method [7] at a meeting on complexity at the Lawrence Livermore Lab in Berkeley. I met there the small community that started to work on interior point methods and originated fruitful cooperations. Soon after, I left CORE in the spring of 1986 and moved to the University of Geneva, which was definitely closer to the Alps and would make it easier for me to exercise my favorite sport.

In that new position, I was fortunate enough to obtain, right from the start and up to my retirement, constant support of the FNS[1]. I organized a workshop in 1987 around K. Anstreicher, D. Goldfarb, N. Karmarkar and M. Todd. A few others joined, and Kees Roos was one among them. This originated a fruitful and friendly cooperation with Kees. Having a different background—Kees had been working in coding theory up to that time— we were forced to clarify our views on the new IPM. Eventually, we came up with a neat presentation of a primal IPM as a path-following method [8]. The key point in this paper was that one could give an explicit description of a neighborhood around the central trajectory within which Newton's method converges quadratically. In a sense, the method is a perfect trust region method, in which the trust region is analytically identified and quadratic convergence makes possible a complexity analysis. D. den Hertog, a young PhD student of Kees and T. Terlaky, who arrived soon after in Delft, embarked on that program for a few years of exciting cooperation that culminated in the book [9].

Our work in Delft, focused on the barrier definition of the central path defined by the minimization of the objective augmented by a logarithmic barrier term. But points on the central path can also be defined as maximizers of the barrier function over the polytope associated with the problem constraints and a hyperplane that bounds the objective function value. We later named that polytope the *localization set* for the obvious reason that it contains the optimal solutions. Convergence is achieved by moving gradually the bound on the objective and shrinking the localization set. This approach, which underlies [7], turned out to be very appropriate to tackle non-differentiable convex optimization. The argument is as follows. At a given iteration, the set of generated subgradients and the best recorded objective value determine a localization set, and the analytic center is a natural point at which to construct a new hyperplane. The algorithm was formalized in [10]. This initiated a close cooperation with J.-L.Goffin and A. Haurie. The latter had just joined me at the University of Geneva. The new method turned out to be remarkably efficient and stable. A great deal of the activity of the PhD students in Geneva was devoted to developing the so-called Analytic Center Cutting Plane Method (ACCPM) [11] and turning it into an open-source software. Several of my PhD students, O. Bahn, O. du Merle, R. Sarkissian, O. Péton, and F. Babonneau, have been involved, at one time or another, in the development of ACCPM and in its numerous applications. The most striking results were obtained with F. Babonneau [12, 13], who was able to solve nonlinear multicommodity flow problems with 2 millions commodities, on a network with around 40,000 arcs. The complexity analysis of the method is not easy. In 1995, Yu. Nesterov analyzed a variant of the original method, and Goffin, Luo and Ye worked out the case of the original method

[1] I am grateful to Fonds National Suisse de la Recherche Scientifique, an institution with goals similar to those of the NSF.

in 1996. Later, Nesterov and I [14] revisited the problem: by embedding the problem in a homogeneous framework, we could derive its efficiency in a much simplified way. The implementation of the homogeneous version can be done via ACCPM without modification of the code.

Decision making under uncertainty

Stochastic programming was an obvious application for ACCPM. The so-called deterministic equivalent, that is the model one obtains by plugging the deterministic problem on the event tree, is usually very large, but it also has a structure that is amenable to decomposition techniques. However, stochastic programming suffers from severe limitations, particularly on multistage models. The less severe, but still important one, is the practical construction of the deterministic equivalent. Commercial codes offer packaged solutions, but to keep control of the model and on the solution techniques, we developed with C. van Delft and J. Thénié [15] an automatic generator based on open-source software. A much more serious issue is numerical tractability, since it is known that multistage problems are inherently NP-hard. Simpler problems, e.g., 2-stage ones, are amenable to complexity analysis. With Y. Nesterov [16], we proposed a version of the stochastic gradient method, which computes a solution that is ϵ-optimal with a guaranteed probability β. This is still not adequate for multistage problems, but I became convinced robust optimization is a most reasonable substitute, as we could verify it by applying it to supply chain management [17]. Robust optimization has become my favorite tool in the more recent part of my career.

Back to my beginner's dreams

A few years before the end of my official academic career, I felt I had not made applications to real-life decision-making so far. To meet my beginner's dream, I had to rebound after my soon-to-come retirement and start a new adventure. In 2002, together with my colleague at the University of Geneva, A. Haurie, a renowned scientist in Game Theory and OR, we launched a consulting company to apply the techniques we both taught to reluctant business students. The energy sector of the economy, in particular in its relation to global warming, offered many opportunities to apply optimization. ACCPM proved to be very useful to coordinate the exchange of information between models of totally different nature: a physical model of climate and an energy-economy model [18]. A typical world energy-economy model of the so-called Markal type is a very large bottom-up LP model with integer variables. Clearly, most coefficients in the model are uncertain, and robust optimization is a very appropriate tool. To keep up with numerical tractability one has to concentrate on some type of uncertainty. In [19], we studied the impact of failures on the European energy supply channels. As expected, the robust solution forecasts more diversified supplies and quantifies it. The unit commitment problem for electricity generation is typically a dynamic problem under uncertainty. The rise of new decentralized technologies requires new models and solution techniques. Once again robust optimization is a tool of choice, as we experienced it, for example in modeling the management of a hydro valley.

Notwithstanding the exciting challenges of these applications, the piece of applied work that brought me the greatest satisfaction has nothing to do with business. In the early 80's I met at Berkeley a man of exception, Gilles Corcos, who was professor of fluid mechanics. He was, and of course still is, a great humanitarian who believes that science and advanced technologies should also serve the poor. In the late 80's he started a NGO in Nicaragua, one of the poorest country in the world, with the goal of bringing potable water to rural communities. Many options exist, but distribution through protected pipes is the only one which really guarantees sustained water quality and also frees people from the burden of carrying water from a central distribution location, especially when dwellings are very dispersed. But those systems involve expensive equipment, such as pumps and valves, that require careful maintenance. G. Corcos came to the conclusion that the only way to get around the obstacles was to use gravity, a free energy in a hilly countryside. But flow control would have to be achieved through friction in the pipes to avoid the use of any mechanical device. The design of such system is not easy. The literature on gravity-driven fluids in pipes is not adequate, and new techniques had to be developed. We talked about the subject in the early 90's, and it appeared to me that the stationary flows could be derived from a minimum energy principle. This amounted to minimizing a convex function amenable to SOCP formulation under linear constraints. Those efforts resulted in NeatWork [20], a piece of software that has been used in Nicaragua through its successive versions since the mid 90's. It has permitted the design of many networks that are all still in use, without any significant maintenance, except for accidental destructions like those following Hurricane Mitch. It is surprising to see how the Nicaraguan villagers, who have been trained in the APLV (Agua Para la Vida) school, master the tool and are able to design projects on their own.

It turns out that a similar reasoning works for the design of gas transportation networks. Even though gas is compressible and water is not, the equations representing the dissipation of energy in stationary flows are very closely related. With F. Babonneau and Y. Nesterov, we built on [20] to propose a scheme for gas transportation [21]. There is some intellectual satisfaction that a work designed for poor people, applies to projects that cost more by many orders of magnitude.

In retrospect

The jury, and those who nominated me, deemed I deserved the award of the prize for "life-time achievements in the area of optimization". I felt compelled to look back on what my activity in this domain taught me. From a methodological point of view, I convinced myself that numerical tractability is something that should always been kept in mind. It has shaped modern convex optimization theory and suggests that it is better to work with numerically tractable approximations of the problem at hand than with unsolvable accurate models. On a more personal note, my first observation is that hazard has played a major role in shaping my career. But even though I had no clear perception of my motivations, especially for entering Academia, I realize that I made coherent choices that eventually met my aspirations. I have been guided by the faith that decision making under uncertainty is an essential paradigm in management, and that optimization is one of the best mathematical tools to support it. The other important element that shaped my work is a taste for learning: it brings great satisfaction to better understand a mathematical theory and to be able to apply it to a real-life situation. It also taught me to be humble, in particular as I had the privilege of working with outstanding scholars. A good deal of my scientific activity has certainly been the one of a go-between, who helps in disseminating new ideas: this has merits of its own. Finally, I must pay tribute to my doctoral students. The relation between an advisor and his students is one of the richest professional rewards one can have. It is deeply personal, but in the meantime facing together the high demands of research makes both parties give the best of themselves. I can't speak for my former students, but I acknowledge how much I've been enriched throughout our cooperation.

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Highway Dimension: From Practice to Theory and Back

Andrew V. Goldberg
Microsoft Research Silicon Valley,
1065 La Avenida, Mountain View, CA 94043, USA
(http://research.microsoft.com/∼goldberg/)

Introduction

This paper is based on the talk I gave at the Optimization Society Prizes session at the INFORMS 2011 annual meeting where I had the great honor of receiving the Farkas prize.

The scientific method starts from empirical observations and is followed by a theory that explains the observations. Then the theory makes a prediction that is verified by experiments, which validate or invalidate the theory.

I am a strong supporter of the use of the scientific method in the area of the algorithm design. The method leads to a relevant theory and theoretically justified algorithms. My recent work on the shortest path algorithms for road networks is a good example of how theoretical analysis and experimental evaluation can complement each other.

The proliferation of on-line map services and GPS devices motivated an intensive study of routing algorithms for road networks. Advances in this area benefited from the scientific method: efficient heuristics motivated theory which explained their good performance in practice. The theory derived a better bound for an algorithm that had never been tested in the context of road networks, and an implementation of this algorithm performed better in practice as well.

Background

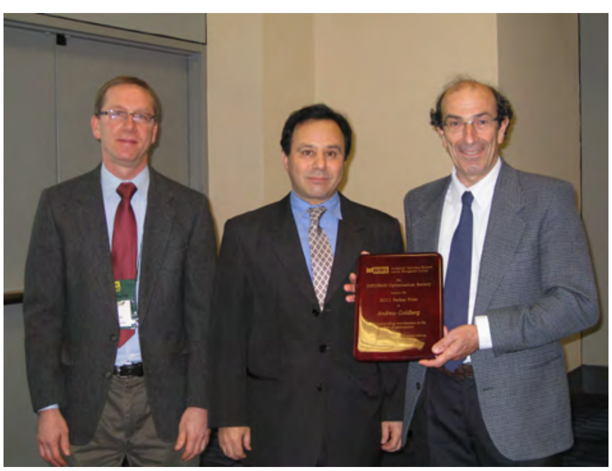
The shortest path problem is a classical combinatorial optimization problem. In this paper we discuss the following point-to-point variant of the problem: given a graph $G = (V, E)$, a non-negative length function $\ell : E \to \mathcal{R}$ and two vertices s (the origin), and t (the destination), find the shortest path from s to t.

In the context of road networks, the vertices of G correspond to road intersections and the edges – to road segments between intersections. The length function ℓ models the particular version of the problem we want to solve. For example, $\ell(a)$ can be the length of the corresponding road segment. In this paper, $\ell(a)$ is the transit time for the corresponding road segment. Then the shortest path corresponds to the fastest way to get from s to t.

The classical algorithm of Dijkstra [14] solves the problem in near-linear time in theory and in practice (see e.g., [20]). Intuitively, the algorithm grows a ball around s until t is reached. The bidirectional version of the algorithm grows balls around s and around t simultaneously.

Practical applications require faster solutions: one needs to solve the problem without searching the full graph. One potential approach to achieve this is A* search [15, 26], which uses lower bounds on distances to direct Dijkstra's search towards the goal. A* search performance depends on the lower bound quality. Significant improvement over Dijkstra's algorithm requires fairly tight bounds.

For road networks, Euclidean distances can be used as distance lower bounds [33]. Euclidean distances divided by the fastest possible speed can be used as the transit time bounds. However, due to obstacles such as bodies of water, mountains, and international borders, Euclidean bounds can be much smaller than the true distances. Speed variations add additional inaccuracy to the transit time bounds. By modern standards, the performance of A* search with Euclidean bounds is poor.



Jon Lee, Andrew V. Goldberg and Gérard Cornuéjols

algorithm	**reference**	**time** [μs]
Dijkstra	[11]	2 008 300.0
ALT	[10]	24 656.0
RE	[23]	2 444.0
HH	[32]	462.0
CH	[19]	94.0
TN	[5]	1.8
HL	[3]	0.3

Table 1: Distance queries on Western Europe.

A more powerful approach is to use *preprocessing*, which computes auxiliary data off-line. This data can be used to speed up on-line queries. For small graphs, one can precompute and store all pairs of shortest paths. Continental road networks, however, have tens of millions of vertices, and storing all pairs of shortest paths is infeasible. We assume that the amount of space that can be used for auxiliary data is not much greater than the input network size.

Early approaches that use preprocessing include separator-based methods (e.g., [28, 29]) and arc flags [30]. The original implementations of these approaches, however, were impractical for continental-size networks. Recently, practical variants of these methods have been developed [12, 27].

One can improve the A* search algorithm's performance by selecting a small number of landmarks and precomputing distances between every landmark and every vertex in the network. These distances can be used in combination with the triangle inequality to obtain distance lower bounds [21]. The resulting algorithm, *ALT*, performs much better than A* with Euclidean bounds. Although not fast enough for continental-size networks, ALT has been used in combination with the other methods and in different applications (e.g., [9, 10, 23]).

Experimental Results

To put the algorithms we discuss in perspective, we give running times for some of them in Table 1. The times are for random queries on the network of Western Europe [13] with 18 million vertices and 42 million edges. The times are on a server with two 3.33 GHz Intel Xeon X5680 processors, or scaled appropriately. The citations in the table are for the most efficient implementation of each algorithm, which are not necessarily the original implementations.

Note that the times are for computing distances only and do not include the time to compute the shortest path. As the path usually contains thousands of arcs, the time to compute the path dominates the fastest algorithms. Some applications, however, need only the distances, and the distance-only times emphasize performance differences among the algorithms.

The table shows that ALT improves upon Dijkstra's algorithm by almost two orders of magnitude, but there are much faster algorithms.

Practical Algorithms

Shortest paths in road networks tend to have hierarchical structure. They usually go from a local intersection on a minor road to more and more global ones, such as a highway, and then to more and more local intersections. Sanders and Schultes [31, 32] introduced the *highway hierarchies* algorithm (HH) which precomputes several levels of highway hierarchy and uses them to speed up queries. They also introduced the notion of *shortcuts*, edges that are added to the graph with the length equal to the distance between their endpoints. Addition of a relatively small number of shortcuts can drastically improve the algorithm performance. This results in a highly practical algorithm.

Gutman [25] came up with a definition of *reach*, a mathematical notion that measures the locality of intersections. Reach values can be precomputed and used in queries for pruning Dijkstra's search when far from the origin and the destination. For example, when driving from Mountain View to Berkeley, we know that we will not take the Treasure Island exit off Bay Bridge because all off-highway intersections on the island are local. We have shown that one can use the shortcuts to obtain a practical implementation of reach (RE) [24, 22].

Geisberger at al. [18] developed the *contraction hierarchies* algorithm. This algorithm is both simpler and more efficient than HH. Instead of partitioning the network into a small number of levels, as HH does, the algorithm orders all vertices according to their "importance". CH also adds shortcuts, which form an integral part of the algorithm. Ideas behind CH have been used in many follow-up papers, and are important for our theoretical results.

Table 1 shows that CH is faster than HH, which is faster than RE.

Bast et al. [4], introduced an even faster *transit node* algorithm (TN). They observed that, for any region of the graph, all long shortest paths out of the region pass through a small number of *access nodes*. Their preprocessing algorithm partitions the graph into regions, computes access nodes for every region, as well as distances from every vertex of the region to its access nodes. The union of all access nodes is the set of *transit nodes*. Preprocessing also computes all pairs of the shortest paths between the transit nodes. Note that the number of transit nodes is moderate, so the latter computation is feasible. An improved version of TN [5] uses ideas from CH for both preprocessing and queries.

TN queries are divided into local and global. If s and t is sufficiently far away, the shortest path between them passes through their access nodes. One can find the shortest path by trying all paths of the form s-v-w-t, where v and w are access nodes of s and t, respectively. If s and t are close, one uses an algorithm such as CH to compute the shortest path. Note that CH performance for local queries is much better than for global queries, so on global queries TN is faster than CH.

Theory: Highway Dimension

CH, RE and TN perform very well on road networks, answering exact distance queries while examining only a few hundred vertices. However, until recently, there was no theoretical justification for their good performance in practice. For such an explanation, one needs to develop a formal model of some of the road network properties, and to show that under this model, one can get polynomial-time preprocessing, polylogarithmic space overhead, and sublinear query times. Note that these algorithms do not perform well on some other graphs, even those with a relatively simple structure such as 2-dimensional grids with random edge weights. Therefore one needs stronger properties than planarity or a small doubling dimension.

We define such a graph class in [1] as graphs with a small *highway dimension* h. The definition of h captures the intuition behind TN. Intuitively, we require that at every scale r, the shortest paths of length comparable to r touching a ball of radius r can be hit by a set of vertices of size at most h. We develop a preprocessing algorithm motivated by CH and show that with this preprocessing, CH and RE queries take $O((h \log n \log D)^2)$ time, where n is the number of vertices and D is the diameter of the network. Recall that we assume that h is small: $h \ll n$.

In [2], we refine the above result to replace $\log n$ by $\log h$. We do so by using the notion of VC-dimension [34]. VC-dimension is heavily used in the learning theory and computational geometry, but has not been used much in the algorithm design. We show that the shortest paths induce set systems with small VC-dimension, and use a hitting set algorithm for the small VC-dimension case [7, 6, 16] in our preprocessing algorithm to obtain the desired result.

Hub-Labeling Algorithm

In [1] we show a better bound of $O(h \log n \log D)$ for the hub-labeling algorithm [17, 8] (HL). (As above, the $\log n$ factor can be improved to $\log h$.) To describe HL, we need to define the notion of *labels*.

For a vertex v, its label $L(v)$ is a set of vertices (*hubs*) with distances $d(v, w)$ for every hub $w \in L(v)$. Furthermore, the labels must obey the cover property: for every $s, t \in V$, the intersection of $L(s)$ and $L(t)$ contains a vertex on the shortest path from s to t. HL preprocessing computes the labels.

HL distance query is very simple. Given s and t, we intersect their labels and find the vertex w in the intersection that minimizes $d(s, w) + d(w, t)$. Query time is linear in the label size.

Note that the cover property is quite strong, and one would not expect small labels for road networks with tens of millions of vertices. However, motivated by theory, we obtained a practical implementation of HL [3]. The theoretical preprocessing algorithm, although polynomial time, is impractical for large networks. We use theory-guided heuristics to obtain a practical preprocessing algorithm. The algorithm produces labels with the average size of 85. This is much smaller than we expected.[1]

[1]Recently we reduced the label size to 80.

A careful implementation of the algorithm performs as stated in Table 1. HL distance queries are extremely fast, taking the time roughly equal to five random accesses to the main memory of the machine. HL outperforms other algorithms, including CH and RE, as predicted by theory.

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SCIP: Solving Constraint Integer Programs and Beyond

Tobias Achterberg
IBM R&D Germany
(achterberg@de.ibm.com)

This article gives a brief overview on constraint integer programming (CIP) and in particular on the software package SCIP (Solving Constraint Integer Programs) [29]. It summarizes the paper [3], which in turn is a condensed version of the thesis [2]. We focus on mixed integer programming (MIP), leaving out chip design verification, which is the other main application covered in [2] and [3]. Moreover, the article briefly reports on some new and exciting features and applications that the SCIP team and SCIP users have developed since I left for CPLEX five years ago.

The starting point in 2001 was to combine MIP and constraint programming (CP) in order to develop a software framework that allows to exploit the strengths of both paradigms in a tightly integrated fashion. Previously, I worked with the MIP solver SIP of Alexander Martin [26], mostly on branching [5]. My first idea was to extend SIP towards constraint programming, but soon I realized that SIP's matrix oriented data structures, which are actually similar to those in CPLEX, are too specialized towards MIP and not appropriate for my goal of designing an open, extensible, plug-in based framework. Thus, I decided to develop a new software from scratch, which I called SCIP.

The theoretical concept behind SCIP is quite simple. The central definition is the one of a *constraint integer program* (CIP), which basically covers all optimization and constraint satisfaction problems that can be solved by linear programming (LP) based branch-and-bound, i.e., enumerating an integer search space plus solving linear programs. It is a special case of CP that includes search and optimization problems such as MIP, finite domain CP, satisfiability problem solving (SAT), and pseudo-Boolean programming (PB). The fundamental notions of CP, MIP and SAT, namely relaxation (LP relaxation, variable domains, implication graph), restriction (branching, probing), and inference (cutting plane separation, domain propagation, conflict analysis), easily translate to CIP, see also Hooker [19] and the references therein for a similar theoretical foundation.

The main contribution of SCIP is its practical usefulness. For instance, as a black-box MIP solver, SCIP 1.1 is only 1.87 times slower [3] than CPLEX 10.2, which was one of the fastest MIP solvers at the completion of [2] in early 2007. This property allows to conduct computational research in a sophisticated environment and to produce experimental results that are somewhat more meaningful than when obtained with solvers that are orders of magnitude slower than the state-of-the-art.

Some advances in MIP solving have been established with SCIP, which had later been incorporated into CPLEX 11 and CPLEX 12:

- The *hybrid reliability/inference* branching rule [4] improves over the previously used *pseudo cost branching with strong branching initialization* by 9% w.r.t. solving time (measured in shifted geometric mean over some reasonable test set).

- Using the product instead of a weighted sum for combining the objective estimates of the two child nodes into a branching variable score leads to another 14% performance improvement.

- Filtering cutting planes w.r.t. their orthogonality [6] decreases solve times by 22% on average.
- Generalizing conflict analysis from SAT solvers to MIP [1] yields a 12% performance improvement.
- Root node restarts to trigger additional presolve reductions provide a speed-up of 8%.
- Extensions to other ingredients like node selection and primal heuristics [9] result in additional enhancements.

In addition to its pure MIP solving capabilities, SCIP as a framework allows the implementation of efficient branch-and-cut as well as hybrid CP/MIP algorithms. My thesis [2] introduces a first application of this type, namely solving the property checking problem on arithmetic circuits, which arises in chip design verification. Further applications and enhancements to the framework have since then been developed by my colleagues at Zuse Institute Berlin and other research groups. Feedback from SCIP users lead to many improvements in the design, performance, and applicability.

One can find numerous applications of SCIP in the literature, and I can only list a few of them here. See also [28] for further references.

Joswig and Pfetsch [22] developed a branch-and-cut approach to compute *optimal Morse matchings* of discrete Morse functions. Armbruster et al. [7, 8] combined SCIP with a semi-definite programming relaxation to solve the minimum bisection problem in graphs. Kaibel et al. [23, 24] investigated a special case of symmetry breaking using SCIP. Pfetsch [27] reimplemented his code for finding *maximum feasible subsystems* of infeasible linear programs on top of SCIP, obtaining noticeable performance improvements due to the arsenal of MIP components that are available in SCIP. Berthold, Heinz, and Pfetsch [11] turned SCIP into a pseudo-Boolean solver, which won in several categories of the pseudo-Boolean competitions 2009, 2010, and 2011 [25]. Gamrath and Lübbecke [16] solved integer programs by column generation through a generic Dantzig-Wolfe decomposition. Berthold et al. [10] and Heinz and Beck [17, 18] integrated MIP and CP techniques to solve resource allocation and project scheduling problems by adding constraint handlers to SCIP. Berthold, Heinz, and Vigerske [13] extended SCIP to be able to solve nonconvex mixed integer quadratically constrained programs (MIQCPs) to global optimality, with applications in gas [15] and water [20] supply network planning. Moreover, Berthold et al. [12] introduced a generic CIP version of generalized large neighborhood search heuristics as can be found in MIP solvers, which nicely specializes to MIQCPs, nonlinear pseudo-Boolean optimization, and scheduling problems. Januschowski and Pfetsch [21] implemented a branch-and-cut algorithm for solving the *maximum k-colorable subgraph problem*. Finally, Cook et al. [14] produced a first version of an exact rational integer programming solver based on SCIP, which is able to find truly optimal solutions for MIPs and does not suffer from numerical errors due to floating point calculations.

Jon Lee, Tobias Achterberg and James Renegar

I am very grateful to see that the SCIP development continues even five years after I left for CPLEX and that its usage in academia and industry still grows. Many thanks to all the numerous people who contributed to this success!

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On the Chvátal-Gomory Closure of a Compact Convex Set

Daniel Dadush
Industrial and Systems Engineering
Georgia Institute of Technology, Atlanta GA 30332
(dndadush@gatech.edu)

The study of cutting planes, i.e. valid linear inequalities for the integer points within the continuous relaxation of an integer program, has been an important component of integer programming (IP) research since its inception in the 1950's. In 1958, Gomory [13] introduced the Gomory fractional cuts, also known as Chvátal-Gomory (CG) cuts, to design the first finite cutting plane algorithm for integer linear programs. Since that time tremendous advancements have been made in our understanding of how to use cutting planes in practice as well as of their underlying mathematical structure. From the practical standpoint, they are now central tools for modern mixed-integer linear programming (MILP) solvers such as CPLEX and Gurobi [16, 18, 2, 17], often enabling them to close the integrality gaps of continuous relaxations very quickly. From the theoretical perspective, numerous classes of cutting planes have been introduced including CG, lift and project, mixed integer rounding cuts and many others. Global properties of these families have been studied, including the relative strengths and polyhedrality of the associated closures, as well as methods to effectively separate over them. Furthermore, the form of all valid cutting planes for generic relaxations, such as the group and corner relaxations, and of certain IP substructures, such as mixing sets, has been characterized.

MILP is the current dominant modeling paradigm for most real world applications of IP. However, over the past decade, much effort has been focused on IP models with non-linear (or non-convex) constraints and objectives, such as bilinear or polynomial programming, known generally as integer non-linear programming (INLP). Given the effectiveness of cutting plane techniques for MILP, one important challenge is to understand whether their utility carries over to the INLP setting.

In our paper [8], we make progress in this effort by developing theoretical tools and techniques for analyzing the structure of cutting planes in the non-linear setting. We focus on the analysis of Chvátal-Gomory (CG) cuts for convex INLPs, i.e. the class of IPs whose continuous relaxations are general convex optimization problems. However, we believe the developed techniques should be useful for many other cutting plane families over convex INLPs. Convex INLPs occur naturally as the convexifications of general non-linear problems, and hence are important to study from the perspective of obtaining strong bounds for non-linear problems. Regarding the relevance of CG cuts, they are the first class of cutting planes introduced for IP, and yield many important classes of facet-defining inequalities for combinatorial optimization problems. For example, the classical blossom inequalities for general matching [11] - which yield the integer hull - and comb inequalities for the traveling salesman problem [14, 15] are both CG cuts over the base linear program. CG cuts have also been effective from a computational perspective; see for example [3, 12].

Although CG cuts have traditionally been defined with respect to rational polyhedra for ILP, they straightforwardly generalize to the setting of convex INLP. CG cuts for non-polyhedral sets were considered implicitly in [5, 19] and more explicitly in [4, 7, 9].

Main Result

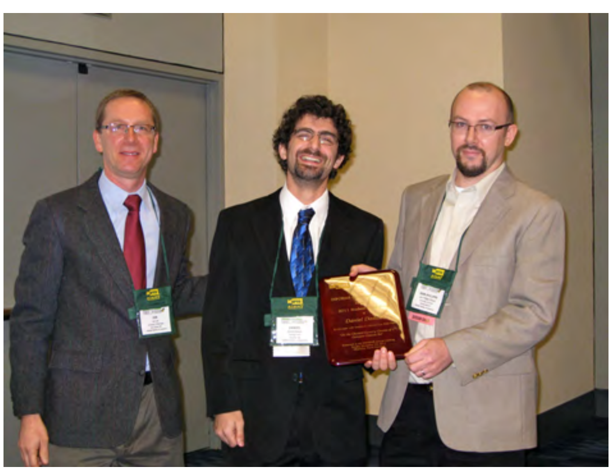
Let $K \subseteq \mathbb{R}^n$ be a closed convex set and let h_K represent its support function, i.e. $h_K(a) = \sup\{\langle a, x\rangle : x \in K\}$. Given a subset $S \subseteq \mathbb{Z}^n$, we define the CG cuts for K derived from S as the valid inequalities

$$\mathrm{CG}(K,S) = \{x \in \mathbb{R}^n : \langle s, x\rangle \leq \lfloor h_K(s)\rfloor \;\forall s \in S\}.$$

The **CG closure** of K is $\mathrm{CG}(K) \equiv \mathrm{CG}(K, \mathbb{Z}^n)$, i.e. the convex set obtained by the intersection of all CG cuts for K. A classical result of Schrijver [19] states that the CG closure of a rational polyhedron is a rational polyhedron. This is a crucial property, since it is a mathematical guarantee that there exists a 'relatively important' finite subset of CG cuts that defines the CG closure. Recently, we were able to verify that the CG closure of a compact convex set obtained as the intersection of a strictly convex set (a convex set containing no lines on its boundary) and a rational polyhedron is a rational polyhedron [7]. Here we develop significantly different techniques than those used in [19], and exhibit qualitative differences in the behavior of the CG closure for strictly convex sets and polytopes. In particular, we prove that for a strictly convex set K the CG closure separates all non-integral points on the (relative) boundary of K, a fact which does not hold in general for polytopes.

Even with the previous results, we remained far from understanding the structure of the CG closure for many sets that appear in the context of convex INLP. The main difficulty in extending these results to more general settings lay with the complex boundary structure of general convex sets. In particular, such sets can contain infinitely many faces of arbitrary dimensions, and faces whose supporting hyperplanes are irrational (i.e. whose defining equations cannot be expressed using rational data). When working with ILPs, it is reasonable to assume that the set is defined by rational data and that all the extreme points and rays of the feasible set are rational. However, when dealing with general convex INLPs, this assumption breaks down in a natural way. For example, the Lorentz cone [1] has irrational extreme rays and second order representable sets naturally (not always) inherit irrational generators. Perhaps the simplest setting to consider when tackling these issues is that of irrational polytopes (i.e. defined by irrational data). Schrijver [19] considered this question. In a discussion section at the end of the paper, he writes that[1]:

> "We do not know whether the analogue of Theorem 1 is true in real spaces. We were able to show only that if P is a bounded polyhedron in real space, and P' has empty intersection with the boundary of P, then P' is a (rational) polyhedron."



Jon Lee, Daniel Dadush and Jean-Philippe Richard

In our paper [8], we affirmatively answer Schrijver's question[2] and furthermore show that the polyhedrality of the CG closure holds in nearly full generality[3]. Our result is stated as follows:

Theorem 1. *If $K \subseteq \mathbb{R}^n$ is a compact convex set, then the CG closure of K is finitely generated. That is, there exists a finite set $S \subseteq \mathbb{Z}^n$ such that* $\mathrm{CG}(K) = \mathrm{CG}(K,S)$. *In particular, the CG closure of K is a rational polyhedron.*

Proof Sketch

We sketch our proof of Theorem 1. Given a compact convex set $K \subseteq \mathbb{R}^n$, we must find a finite set $S \subseteq \mathbb{Z}^n$, whose associated CG cuts $\mathrm{CG}(K,S)$ yield the CG closure $\mathrm{CG}(K)$. We build the set S in three separate steps, adding CG cuts in each step to get successively finer approximations of the full closure. For simplicity, let us assume that K is full dimensional. The steps we use are as follows:

[1]Theorem 1 in [19] is the result that the CG closure is a polyhedron. P' is the notation used for CG closure in [19]

[2]For the special case of irrational polytopes, our result was also independently obtained by Dunkel and Schulz[10].

[3]For general unbounded convex sets even the convex hull of integer points can be non-polyhedral. Therefore the CG closure cannot always be rational polyhedral.

1. Achieve Containment: initialize S to a finite subset of $\mathbb{Z}^n$ satisfying $\mathrm{CG}(K, S) \subseteq K$.

2. Finish up the Boundary: update S to guarantee that $\mathrm{CG}(K, S) \cap \partial K = \mathrm{CG}(K) \cap \partial K$.

3. Complete the Closure: add all cuts which separate a vertex of $\mathrm{CG}(K, S)$.

We now discuss some the above outline in more detail. We go through the steps in reverse order. As a base assumption, we inductively assume that each proper face of K has a finitely generated CG closure. For the third step, we first show that any CG cut that separates a vertex of $\mathrm{CG}(K, S)$ must separate one lying in the interior of K (as the boundary is correct by step 2). Given the finiteness of these vertices, they must all be at least some distance $\delta > 0$ away from the boundary of K. This allows us to show for any $a \in \mathbb{Z}^n$ of norm $\|a\|_2 \geq \frac{1}{\delta}$, the associated CG cut cannot separate an interior vertex. Therefore all remaining useful CG cuts are of bounded norm and hence finite in number. This type of argument is relatively standard and has appeared in previous polyhedrality proofs [6, 9]. For the second step, the main insight is that the intersection of $\mathrm{CG}(K, S)$ and the boundary of K is contained inside a finite number of proper faces of K. Using the induction hypothesis on these faces, we lift the necessary CG cuts from the faces to K using a crucial lifting lemma (see below) to finish the boundary.

Somewhat surprisingly, the most challenging part of the paper is building the first approximation (which is trivial for rational polyhedra). The main reason is that the inner approximation we build for K is rational polyhedral, and hence must "kill" all irrational parts of the boundary. Note this implies that if K contains a facet F whose affine hull does not intersect the rationals, then the CG closure must separate all of F from $\mathrm{CG}(K)$ irrespective of how large F is. The crucial lemma which tackles this issue is the following lifting lemma (classical for rational polyhedra):

Lemma 1. *Let $K \subseteq \mathbb{R}^n$ be a compact convex set, and let F be a proper exposed* [4] *face of K. Then*

$$\mathrm{CG}(K) \cap F = \mathrm{CG}(K \cap F).$$

[4] admits a supporting hyperplane H satisfying $K \cap H = F$

The standard approach to prove such lemma is to take a valid CG cut $\langle b, x\rangle \leq \lfloor h_F(b) \rfloor$ for F, $b \in \mathbb{Z}^n$, and "tilt" b about F until the new cut is valid for K and identical to the original on F. For rational polyhedra such tilting is achieved by scaling the normal vector ν to F until $\nu \in \mathbb{Z}^n$, and then adding a sufficiently large integer multiple of ν to b. A first difficulty in our setting is that there maybe no scaling of ν in $\mathbb{Z}^n$. To overcome this, we use fine rational approximations of ν (Dirichlet approximates) to perform the tilting (a technique first used in [9]). From here, we prove that the CG cut induced by a large enough tilt of b comes "very close" to $\mathrm{CG}(F, b)$ on F though is not necessarily identical. Next we show that this inexact lifted cut is in fact identical to $\mathrm{CG}(F, b)$ when restricted to the rational part of F (the affine hull of $F \cap \mathbb{Q}^n$).

The final problem we are left with is therefore what to do with the irrational parts of F. In the most technical part of the paper, we provide a method to construct finitely many CG cuts for K which separate all the irrational parts of F. By separately removing these problematic pieces of F, we show that our inexact lifting of b is correct on the relevant parts of F, thereby proving the lemma.

Conclusion

In the paper, we develop new techniques to analyze the behavior of cutting planes over non-polyhedral convex sets. These prove useful for establishing polyhedrality results for the CG closure, and may likely be helpful for analyzing other cutting plane families in the future. Many of the techniques we use here however are non-constructive, as they rely on limit and compactness type arguments. We therefore leave as an open problem to give an alternative constructive proof for the polyhedrality of the CG closure.

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Nominations for Society Prizes Sought

The Society awards four prizes, now annually, at the INFORMS annual meeting. We seek nominations and applications for each of them, due by **June 30, 2012**. Details for each of the prizes, including eligibility rules and past winners, can be found by following the links from `http://www.informs.org/Community/Optimization-Society/Prizes`.

Each of the four awards includes a cash amount of US$ 1,000 and a citation certificate. The award winners will be invited to give a presentation in a special session sponsored by the Optimization Society during the INFORMS annual meeting in Phoenix, AZ in October 2012 (the winners will be responsible for their own travel expenses to the meeting).

The **Khachiyan Prize** is awarded for outstanding life-time contributions to the field of optimization by an individual or team. The topic of the contribution must belong to the field of optimization in its broadest sense. Recipients of the INFORMS John von Neumann Theory Prize or the MPS/SIAM Dantzig Prize in prior years are not eligible for the Khachiyan Prize. The prize committee for the Khachiyan Prize is as follows:

- Kurt Anstreicher (Chair)
  kurt-anstreicher@uiowa.edu
- Egon Balas
- Claude Lemaréchal
- Éva Tardos

Nominations and applications for the Khachiyan Prize should be made via email to the prize-committee chair. Please direct any inquiries to the prize-committee chair.

The **Farkas Prize** is awarded for outstanding contributions by a mid-career researcher to the field of optimization, over the course of their career. Such contributions could include papers (published or submitted and accepted), books, monographs, and software. The awardee will be within 25 years of their terminal degree as of January 1 of the year of the award. The prize may be awarded at most once in their lifetime to any person. The prize committee for the Farkas Prize is as follows:

- Monique Laurent
- David Shmoys (Chair)
  shmoys@cs.cornell.edu
- Laurence Wolsey
- Yinyu Ye

Nominations and applications for the Farkas Prize should be made via email to the prize-committee chair. Please direct any inquiries to the prize-committee chair.

The **Prize for Young Researchers** is awarded to one or more young researcher(s) for an outstanding paper in optimization that is submitted to and accepted, or published in a refereed professional journal. The paper must be published in, or submitted to and accepted by, a refereed professional journal within the four calendar years preceding the year of the award. All authors must have been awarded their terminal degree within eight calendar years preceding the year of award. The prize committee for the Prize for Young Researchers is as follows:

- Shabbir Ahmed
- Alper Atamtürk
- Endre Boros (Chair)
  Endre.Boros@rutcor.rutgers.edu
- Bob Vanderbei

Nominations and applications for the Prize for Young Researchers should be made via email to the prize-committee chair. Please direct any inquiries to the prize-committee chair.

The **Student Paper Prize** is awarded to one or more student(s) for an outstanding paper in optimization that is submitted to and received or published in a refereed professional journal within three calendar years preceding the year of the award. Every nominee/applicant must be a student on the first of January of the year of the award. Any co-author(s) not nominated for the award should send a letter indicating that the majority of the nominated work was performed by the nominee(s). The prize committee for the Student Paper Prize is as follows:

- Simge Küçükyavuz
- Katya Scheinberg (Chair)
  katyas@lehigh.edu
- Renata Sotirov

Nominations and applications for the Student Paper Prize should be made via email to the prize-committee chair. Please direct any inquiries to the prize-committee chair.

Nominations of Candidates for Society Officers Sought

We would like to thank three Society Vice-Chairs who will be completing their two-year terms at the conclusion of the INFORMS meeting: Oleg Prokopyev, Frank Curtis, and Huseyin Topaloglu. We are currently seeking nominations of candidates for the following positions:

- Vice-Chair for Global Optimization
- Vice-Chair for Nonlinear Programming
- Vice-Chair for Stochastic Programming

Self nominations for all of these positions are encouraged.

According to Society Bylaws, "The main responsibility of the Vice Chairs will be to help INFORMS Local Organizing committees identify cluster chairs and/or session chairs for the annual meetings. In general, the Vice Chairs shall serve as the point of contact with their sub-disciplines." Vice Chairs shall serve two-year terms.

Please send your nominations or self-nominations to Jim Luedtke (jrluedt1@wisc.edu), including contact information for the nominee, by **June 30, 2012**. Online elections will begin in mid-August, with new officers taking up their duties at the conclusion of the 2012 INFORMS annual meeting.

The Sun Shined Brightly upon the OS Conference at Coral Gables

The 2012 edition of the INFORMS Optimization Society Conference was held, February 24-26, at Coral Gables, Florida. The conference was superbly organized by the University of Miami's School of Business Administration. Many thanks are due to Anuj Mehrotra, Mike Trick, Ed Baker, Hari Natarajan and Tallys Yunes. There were 105 regular talks, 8 poster presentations, 5 plenary and featured talks, and one human pyramid. Trivia questions: Whose hat graces the top of the pyramid? Who has more experience participating in human pyramids, Cole Smith or Jeff Linderoth? The biennial conference of the OS has become a wonderful mid-winter tradition for keeping current with the latest trends in optimization — and doing so in a warm climate.

Human Pyramid at INFORMS OS Conference 2012