More Equitable Congestion-Mitigation Policies for Multimodal Transportation Networks*

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Background

- Congestion-Mitigation Policies
  - Congestion pricing
  - Tradable credit schemes (Verhoef et al., 1997; Yang and Wang, 2011)
    - Government distributes credits to travelers
    - Credits are charged for using transportation services
    - Credits can be traded among all the travelers
  - Equity is critical for both policies
Background (Cont’d)

- Despite the fact that successful implementations exist, congestion pricing remains very tough to sell.
- Much of the public opposition centers on the perceived inequalities:
  - Congestion pricing harms the poor who may have to pay more due to their inflexible schedules or switch to less desirable routes, departure times or transportation modes.
Background (Cont’d)

- We attempt to design more equitable pricing/credit schemes to alleviate congestion or improve social benefit in multimodal urban transportation networks
- Existing pricing models are not able to capture the distributional effects of pricing schemes on different income groups and thus do not offer meaningful discussions on the income-based equity
Objective

• Design more equitable pricing/credit schemes by
  – Directly taking into account the effects of income on choices of trip generation, mode and route
  – Explicitly capturing the distributional impacts of pricing/credit schemes across different income and geographic groups

• A pricing/credit scheme is deemed to be more equitable if it leads to a more uniform distribution of wealth across population
Basic Considerations

- A general multimodal transportation network
  - Three types of facilities, i.e., transit services, high-occupancy/toll (HOT) and regular lanes
- Multiple user groups with different incomes and different preferences among four modes:
  - No travel, transit, drive alone (SOV), and car pool (HOV)
  - The number of travelers between each OD pair is fixed
Choices of Mode and Route

- Nested Logit model

\[
p_{k}^{w,m,g} = \frac{\exp\left(\frac{v_{k}^{w,m,g}}{\theta_{w,m,g}}\right)}{\sum_{j \in K^{w,m}} \exp\left(\frac{v_{j}^{w,m,g}}{\theta_{w,m,g}}\right)} \cdot \frac{\exp(\tilde{v}_{w,m,g})}{\sum_{m' \in M} \exp(\tilde{v}_{w,m,g})}
\]

\[
\tilde{v}_{w,m,g} = \ln\left(\sum_{j \in K^{w,m}} \exp\left(\frac{v_{j}^{w,m,g}}{\theta_{w,m,g}}\right)\right)^{\theta_{w,m,g}}
\]
Utility Function

- Linear-in-income

\[ v = \beta_0 + \beta_1 T + \beta_2 (y_0 - \tau) \]

The above conventional specification with constant marginal utility of income may lead to an underestimate of the regressivity of a pricing scheme (e.g., Franklin, 2006; Bureau and Glachant, 2008)
Nonlinear Utility Function

- **Generalized Leonief**
  \[ \nu = \beta_0 + \beta_1 T + \beta_2 \sqrt{T} + \beta_3 y + \beta_4 \sqrt{y} + \beta_5 \sqrt{T} \sqrt{y} \]

- **Translog**
  \[ \nu = \beta_0 + \beta_1 \ln T + \beta_2 \ln^2 T + \beta_3 \ln y + \beta_4 \ln^2 y + \beta_5 \ln T \ln y \]

**Pricing:** \( y = y_0 - \tau \)
**Credit:** \( y = y_0 + p(q - \tau) \)
Tolled User Equilibrium

- VI Formulation

\((f^*, d^*) \in \Phi\) is in user equilibrium if

\[
\sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K} \left( -\nu_{k}^{w,m,g} + \theta_{w,m,g,\ln f_{k}^{w,m,g}} \right) \cdot \left( f_{k}^{w,m,g} - f_{k}^{w,m,g^*} \right) + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \left( 1 - \theta_{w,m,g,\ln d_{w,m,g}} \right) \ln d_{w,m,g} \cdot \left( d_{w,m,g} - d_{w,m,g^*} \right) \geq 0,
\]

\forall (f, d) \in \Phi
User Equilibrium under Credit Scheme

• VI Formulation

\((f^*, d^*, p^*) \in \Phi\) is in equilibrium if

\[
\sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^w,m} \left( -v_k^{w,m,g} + \theta_k^{w,m} \ln f_k^{w,m,g^*} \right) \cdot (f_k^{w,m,g} - f_k^{w,m,g^*}) \\
+ \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \left( 1 - \theta_k^{w,m} \right) \ln d_k^{w,m,g^*} \cdot (d_k^{w,m,g} - d_k^{w,m,g^*}) \\
+ \left( Q - \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^w,m} \sum_{l \in L} \Delta_{l,k}^{k_l,m} f_k^{w,m,g^*} \right) \cdot (p - p^*) \\
\geq 0, \\
\forall (f, d, p) \in \Phi
\]
Welfare Measure

- Equivalent Income
  - The income level that allows the individual to experience under the no-toll scenario the same level of utility as the original income does under the tolling scenario

\[ e^{w,g}(\tau) = \arg\{z : u^{w,g}(z, 0) = u^{w,g}(y^g, \tau)\} \]
Welfare Measure (Cont’d)

- Equivalent Income

\[ y_0 \rightarrow u_0 \text{ No toll} \]
Welfare Measure (Cont’d)

- Equivalent Income

Income

\[ y_0 \]

Utility

\[ u_0 \]

\[ u_1 \]

No toll

With toll
Welfare Measure (Cont’d)

- Equivalent Income

\[ y_0 \rightarrow u_0 \]

**No toll**

**With toll**

\[ u_1 \]
Welfare Measure (Cont’d)

- Equivalent Income

Equivalent Income

Income

Utility

No toll

With toll

\(-\mathcal{EV}\)

\(u_0\)

\(u_1\)
Welfare Measure (Cont’d)

- Equivalent income is a measure of how wealthy a traveler feels under the pricing/credit scheme.
- Due to the existence of the random error term in the utility function, the equivalent income is also random for each individual traveler.

\[ e^{w,g}(\tau) = \arg\{z: u^{w,g}(z, 0) = u^{w,g}(y^g, \tau)\} \]

- Expected equivalent income
  - Dagsvik and Karlstrom (2005)
Welfare Measure (Cont’d)

\[ E(e^{w,g}(\tau)) \]

\[ = \sum_{m^0 \in M} \sum_{k^0 \in K^{w,m^0}} \int_0^{p_{k^0}^{w,m^0,g}(\tau)} \left( \sum_{k \in K^{w,m^0}} \exp \left( \frac{h_k^{w,m^0,g}(z,\tau)}{\theta^{m^0}} \right) \right)^{\theta^{m^0} - 1} \cdot \exp \left( \frac{v_{k^0}^{w,m^0,g}(y_g,0)}{\theta^{m^0}} \right) \]

\[ \sum_{m \in M} \left( \sum_{k \in K^{w,m}} \exp \left( \frac{h_k^{w,m,g}(z,\tau)}{\theta^{m}} \right) \right)^{\theta^{m}} \]  

\[ v_{k^0}^{w,m^0,g}(y_g,\tau) = v_{k^0}^{w,m^0,g}(p_{k^0}^{w,m^0,g}(\tau),0) \]

\[ h_k^{w,m^0,g}(z,\tau) = \max(v_k^{w,m^0,g}(y_g,\tau),v_k^{w,m^0,g}(z,0)) \]
Equity Measure

- Gini coefficient
  - Calculated based on expected equivalent income with 0 being complete equality and 1 being complete inequality

\[
GN(\tau) = \frac{1}{2 \cdot \left( \sum_{g \in G} \sum_{w \in W} D^{w,g} \right)^2 \cdot E(e(\tau))}
\cdot \sum_{g_1, g_2 \in G} \sum_{w_1, w_2 \in W} (D^{w_1, g_1} \cdot D^{w_2, g_2} \cdot |E(e^{w,g}(\tau)) - E(e^{w,g}(\tau))|)
\]

- A more equitable pricing/credit scheme will lead to a smaller value of the Gini coefficient
Model Formulation

• Objective
  – Efficiency: maximize the sum of the total expected equivalent income (user benefit) and the toll revenue (producer benefit)
  – Equity: minimize the Gini coefficient

\[
\max_{\tau,d,f} \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1 - \alpha) \cdot \frac{GN(\tau)}{GN(0)}
\]
Design Decisions

- Congestion Pricing Scheme
  - Where to charge?
  - How much to charge?

The feasible toll set $\Psi$ can be defined as:

$$\tau_l^m \geq 0, \quad \forall l \in L, m \in \{S, H\}$$
$$\tau_l^S = \tau_l^H, \quad \forall l \in L_{RT}$$
$$\tau_l^R = 0, \quad \forall l \in L$$

$$\sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^w} f_{k, m, g} \sum_{l \in L} \Delta_{k, l} \tau_l^m \geq 0$$
Design Decisions (Cont’d)

- Tradable Credit Scheme
  - How to distribute the credits?
  - Where to charge the credits?
  - How many credits to charge?

The feasible credit scheme set $\Psi$ can be defined as:

$$
\tau^m_l \geq 0, \quad \forall m \in \{S, H, T\}, l \in L
$$

$$
\tau^R_l = 0, \quad \forall l \in L
$$

$$
q^{w,g} \geq 0, \quad \forall w \in W, g \in G
$$

$$
\sum_{w \in W} \sum_{g \in G} q^{w,g} D^{w,g} = Q
$$
Design Model for Pricing

\[
\max_{\tau, d, f} \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1 - \alpha) \cdot \frac{GN(\tau)}{GN(0)}
\]

s.t.

\[
\sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K_w^m} \left( -v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g} \right) \cdot (f_k^{w,m,g} - f_k^{w,m,g*}) \\
+ \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \left( 1 - \theta^{w,m,g} \right) \ln d^{w,m,g} \cdot (d^{w,m,g} - d^{w,m,g*}) \geq 0,
\]

\[\forall (f, d) \in \Phi\]

\[(f^*, d^*) \in \Phi, \quad \tau \in \Psi\]
Design Model for Credit

\[
\begin{align*}
\max_{d,f,p,q,\tau} & \quad \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1 - \alpha) \cdot \frac{GN(\tau)}{GN(0)} \\
\text{s.t.} & \quad \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} (-v^w_{k,m,g} + \theta^w_m \ln f^w_{k,m,g} \cdot (f^w_{k,m,g} - f^w_{k,m,g}^*) \\
& \quad + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} (1 - \theta^w_m) \ln d^w_{m,g} (d^w_{m,g} - d^w_{m,g}^*) \\
& \quad + \left(Q - \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} \sum_{l \in L} \Delta_{l,k;m} f^w_{k,m,g}^* \right) \cdot (p - p^*) \geq 0, \\
(f^*, d^*, p^*) & \in \Phi \\
(q, \tau) & \in \Psi
\end{align*}
\]
Solution Algorithm

- Mathematical programs with equilibrium constraints (MPEC), a class of problems difficult to solve
- Compounding the difficulty is that the computation of the expected equivalent income involves numerical integrations
- Derivative-free algorithms
  - Compass search algorithm
  - SID-PSM algorithm (Custódio and Vicente, 2007; Custódio et al., 2010)
Numerical Example

- **Seattle Regional Freeway Network**
  - Four income groups ($20,000; $40,000; $70,000; $120,000)
  - Translog utility function (Franklin, 2006)

\[

v_{k}^{w,m,g} = \beta_{0}^{R} \log y_{g} + \beta_{1} \ln \left( y_{g} - c_{k}^{w,m} \right) + \beta_{2} \ln T_{k}^{w,m} + \beta_{3} \ln^2 T_{k}^{w,m}

\]
Travel Demand

Production

Attraction
Optimal Pricing Schemes
Benefit Distribution

More equitable

Most efficient
## Optimal Pricing Schemes

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More efficient

More equitable
Optimal Credit Schemes
Benefit Distribution

Most efficient
### Credit Charging Schemes

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Comparison

![Graph showing comparison of Gini Coefficient vs Net Benefit (million USD)]
Summary

- We developed a modeling framework that explicitly captures the distributional effects of pricing/credit schemes on different income and geographic groups

- The framework can be used to generate more equitable yet efficient pricing/credit schemes for multimodal transportation networks
Optimal Pricing Schemes (Cont’d)
Observations

• Low-income travelers suffer the most when efficiency is maximized

• When equity is given enough weight, low-income and high-income travelers both benefit more than mid-income travelers
  – Low-income travelers: transit subsidy
  – High-income travelers: reduction in travel time

• Better equity is achieved via heavier transit subsidy
Observations

• Everybody is better off under the most efficient credit scheme.
  – Low-income travelers: selling extra credits
  – High-income travelers: reduction in travel time

• Better equity is achieved by increasing the number of credits charged at each link.
  – More credits charged \(\rightarrow\) more demand for credits \(\rightarrow\) higher credit price \(\rightarrow\) more subsidies