Determining the Number of Slots to Submit to a Market Mechanism at a Single Airport

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Research Question

Given various economic concerns, and airport-specific conditions, how many arrival slots should be made available in a given time period?

Outline:
1. Background
2. Models
3. Calibration
4. Slot value determination
5. Sensitivity analysis
6. Conclusions
Background Information

• **U.S. slot control**
  - High Density Rule (HDR) enacted 1969 for ORD, DCA, EWR, JFK, and LGA
  - FAA NPRM released in 2006 concerning redistribution of slots at LGA
  - New York Aviation Rulemaking Committee 2007

• **Motivation**
  - Cannot simply use VMC or IMC capacity
  - Higher value times-of-day
  - Cascading queuing delays

• **LGA used as case study**
  - Because of its significance as a slot-controlled airport
  - *Methodology is not limited to application at LGA*
Literature

• Primarily focused on how to allocate slots
  – Grether, et al. (1979,1989)
  – NEXTOR
    • Le et al. (2004)
    • Ball et al. (2006)
  – Brueckner (2002)

• International interest
  – DotEcon (2001)

Auctions
Congestion pricing
Modeling Framework

• Two formulations
• Common elements
  – Integer-valued linear optimization problems
  – Posed as network flow problems
    • Objects in network are flights
    • Exits from network are landings and cancellations
  – Model cancellation behavior
• Different elements
  – Objective functions
  – Policy trade-offs
Assumptions

• Day broken into discrete periods (e.g. hour, quarter hour)
• Number of slots to offer in each hour has upper and lower bounds, $D_{\text{min}}$ and $D_{\text{max}}$
  – Driven by operational limits and policy constraints
• Costs normalized to one unit delay cost
• Cancellations
  – Modeled as exits from network
  – Several capacitated arcs, each with increasing cost
• Delays
  – Modeled as horizontal arcs in flow diagrams
  – Flights can be delayed up to $U$ time periods
Capacity scenarios

- Work undertaken by Liu, et al. (2005) to cluster arrival capacity (AAR) profiles

Results shown for LGA, 2003
Base Formulation

- Constraints: Flow balance
- Objective function: Maximize total net benefit, as difference between total value of slots offered and cost of delays and cancellations
Parametric Formulation

- Same network structure as base model
- Constraints: Flow balance, **cap on average delay per flight and overall cancellation percentage**
- Objective function: Maximize total value of slots offered; no longer requires pecuniary equivocation between delays and cancellations
Mathematical Properties

• Base constraint matrix is TU under limited conditions
  – Very small problem sizes (not useful)
  – Even-valued $b$ vector

• Parametric constraint matrix is not TU under any conditions

• However, …
  – The base formulation solves with LP relaxation
  – The parametric formulation solves quickly
Parameter Calibration

• Need method to estimate several parameters from historical data
  – \( U \) - maximum delay length
  – \( N \) - number of cancellation arcs
  – \( P \) - cancellation arc capacity
  – \( \{\lambda_i\} \) - cancellation cost vector = \( \alpha[1, 2, 4, 7, 11, \ldots] \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Additional Constraints</th>
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<tbody>
<tr>
<td>( U )</td>
<td>4</td>
<td>20</td>
<td>Multiple of 4</td>
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<tr>
<td>( N )</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>2</td>
<td>6</td>
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<tr>
<td>( \alpha )</td>
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<td>10</td>
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Parameter Calibration Procedure

- Define search space by bounding parameters.
- For each unique combination of parameters:
  - Apply same network as Base model with one capacity scenario (historical realization) and historical demand.
  - Objective: minimize total cost, as weighted sum of cancellations and delays.
  - Conduct analysis for many days (1 year).
  - Quantify relationship between observed and predicted data for all days using several metrics.
- Choose “best” set of parameters based on metric criteria.

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Sample Calibration Results

ORD, 2005

$U = 1$ hour

$\lambda = \begin{bmatrix} 4 & 8 \end{bmatrix}$

$P = 6$ flights/arc

$R^2 = 0.774$

Slope = 1.320

Intercept = 1.9%
Determining Slot Valuation

- Acquire proprietary data
  - Best to have data from many carriers
  - Carriers obviously reluctant to release such data

- Inference from ticket pricing data
  - High cost of data access
  - Confounding issues in ticket pricing

- Infer from auction results
  - True values revealed
  - Phased in over several years
Value Inference from Auctions

• Goals
  – Determine true slot valuations
  – Spread slot removal over several years
  – Combines value inference, slot removal, and distribution

• General process
  – Wait several years for prices to stabilize
  – Use model to determine number of slots to offer in each time period
  – Remove slots from circulation over several years
  – Update values and slots to be removed each year until process stabilizes
Value Inference Example

Parameters: Delay: 15 minute/flight, 3.0% cancellations

Real results, generated by Parametric model

Fictitious, but realistic

Average of red values

Relative slot value

Number of slots

Hour

6  8  10  12  14  16  18  20  22

0  4  8  12  16  20  24  28  32  36  40

Current round values $\pi_t^{10}$

Overall average $\pi_t^J$

Already auctioned

To be removed

Remaining pool
Sensitivity Analysis

• Tested effects of variations in slot values $V_t$

• Used Parametric formulation
  – Data from LGA, 2003
  – Slot valuations from Year 10 of auction example
  – Cancellations: maximum of 9 per hour permitted
  – Delay: 15 minutes/flight
  – Cancellations: 3.0%
Model Sensitivity to Slot Valuation

Average delay (minutes/flight) vs. Number of slots to offer and Relative slot value

Monotonically increasing slot values
Summary of Results

• Presented two possible formulations to determine number of arrival slots to offer
• Addressed model calibration
• Discussed various methods to estimate slot valuation
  – Proposed procedure that integrates valuation, removal, and allocation
• Examined model sensitivity to slot values