Stochastic Network Models for Hospital Inpatient Flow Management

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Outline

• Part 1: Empirical observations
• Part 2: Stochastic network models
• Part 3: Two-time-scale framework
• Part 4: Managerial insights & future research
Empirical observation at NUH

- Average queue length curve over 78 weeks
  - # of patients who are waiting for inpatient beds from the emergency department (ED)
    - Period 1: Jan 2008 to Jun 2009
  - Two types of variations
    - Time-of-day
    - Day-of-week
Average queue length curve (547 days)

- Time-of-day variation
Waiting time statistics: Period 1

Average waiting time

Fraction of patients who wait at least 6 hours
Bed-request rate and discharge distribution
Early discharge campaign: 2nd half of 2009

- Discharge time distribution
  - Period 1: Jan 08 to Jun 09 (13% discharge by noon)
  - Period 2: 2010 (27% discharge by noon)
Period 2 performance

- Changing operating environment
Israel hospital (Armony et al., 2011)
**Motivation**

- Can we *build a model* and *find methods* to predict the average queue length curve?
- If so, how can we use it to make relevant decisions?
Part 2: Stochastic network models

- Time-varying queues
  - Massey (1981), non-stationary queues
  - Whitt (1991)
  - Massey, Mandelbaum and Reiman (1998)
  - $M_t/GI/N$ framework
$M_t/GI/N$ queues fail to capture

- Simulation results from an $M_{peri}/\text{lognormal}/N$ system

**avg waiting time**

**avg queue length**
A new stochastic network model

- Multi-server pools serving multi-class customers
New features

- Endogenous service times
- Allocation delays
- Overflow trigger times
Endogenous service times

Service time = Discharge time – Admission time
= LOS + Dis hour – Adm hour
Length-of-stay (LOS) = number of nights in hospital

- LOS distribution
  - Average is ~ 5 days
  - Depends on admission source and medical specialty
Checking the service time model

(a) Empirical

(b) Simulation output
An alternative iid service time model

- Directly generate service times from the empirical distributions
  - Discharge distribution does not match
$M_t/GI/N$ queues fail to capture

- Simulation results from an $M_{peri}/\text{lognormal}/N$ system

**Average waiting time**

**Average queue length**

![Graph showing average waiting time and queue length over time.](image-url)
Allocation delays

- Getting a bed is a process
  - Pre-allocation delay
    - Bed management unit searches/negotiates for beds
  - Post-allocation delay
    - Delays in ED discharge
    - Delays in transportation
    - Delays in ward admission

- Must model allocation delays