Strategic customers in queueing systems: Bridging observable and unobservable models

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Part I: Introduction
The basic queueing models with strategic customers
Queueing problems
Queueing problems

- **Performance evaluation problems**
  How does a given system perform?
  (no one makes decisions)
Queueing problems

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  How should we design a system? (choice of parameters)
  (the constructor makes decisions once)
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- **Dynamic optimization problems**
  How should we administrate a system? (dynamic control)
  (the administrator makes decisions)
- **Strategic behavior problems**
  How do the agents behave in a system? What can we do to induce a desirable behavior?
  (each agent makes his own decision)
Mathematical tools

Performance evaluation (no one makes decisions)

Stochastic processes (Markov, semi-Markov)

Static optimization (design) (the constructor makes decisions once)

Stochastic Processes + Nonlinear Programming

Dynamic optimization (control) (the administrator makes decisions)

Stochastic Dynamic Programming (Markov Decision Processes)

Strategic behavior (each customer makes his own decision)

Stochastic Processes + Game Theory

Strategic customers in queueing systems
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  (each customer makes his own decision)
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Strategic queueing problems

To join or balk?
To stay or renege?
To buy priority or not?
How often to retry for service?
Which queue to join?
Strategic queueing problems

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Typical questions

Strategic behavior problem:
At a certain system, the arriving customers follow a given joining strategy, e.g., join with prob. \( q \) (unobservable), join if the queue length \( \leq n \) (observable).

Consider a tagged customer. What is the best response of the tagged customer? Are there equilibrium strategies? Is a Follow-The-Crowd or Avoid-The-Crowd situation? Are the equilibrium strategies socially optimal? What level of information should be provided?
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The basic observable model (Naor (1969))

- Poisson(\(\lambda\)) arrival process.
- \(\text{Exp}(\mu)\) service times.
- 1 server that serves customers one by one.
- Infinite waiting space.
- FCFS queueing discipline.
- \(\rho = \frac{\lambda}{\mu}\): the utilization rate.
- \(R\): customer's benefit from completed service.
- \(C\): waiting cost per time unit for a customer (it is paid even when he is in service).
- Upon arrival, a customer inspects the queue length and decides whether to join or balk.
The basic observable model (Naor (1969))

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  9. Upon arrival, a customer inspect the queue length and decides whether to join or balk.
The basic observable model - Equilibrium I

A customer that observes $n$ customers in the system prefers to join if his expected net benefit is non-negative:

$$R - C_n + 1 \mu \geq 0.$$
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$$R - C \frac{n + 1}{\mu} \geq 0.$$
The basic observable model - Equilibrium II

The individual’s optimizing strategy for a customer that sees $n$ customers upon arrival is the threshold strategy that prescribes to join if $n + 1 \leq n_{e}$ with $n_{e} = \lfloor \mu R_{c} \rfloor$ (Naor’s threshold).

This is the unique equilibrium strategy, but also a dominant strategy.
The basic observable model - Equilibrium II

Theorem

The individual’s optimizing strategy for a customer that sees $n$ customers upon arrival is the threshold strategy that prescribes to join if $n + 1 \leq n_e$ with

$$n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor \text{ (Naor’s threshold).}$$
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The individual’s optimizing strategy for a customer that sees \( n \) customers upon arrival is the **threshold** strategy that prescribes to join if \( n + 1 \leq n_e \) with

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  n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor \quad \text{(Naor’s threshold)}.
\]

This is the unique equilibrium strategy, but also a dominant strategy.
The basic observable model - Social opt. I

Social benefit per time unit, under a threshold strategy $n$:

$$S^{\text{soc}}(n) = \lambda R_1 - \rho n_1 - \rho n_{n+1} - C [\rho_1 - \rho - (n+1) \rho_{n+1}]$$
The basic observable model - Social opt. I

- Social benefit per time unit, under a threshold strategy $n$:

$$S_{soc}^{(obs)}(n) = \lambda R \frac{1 - \rho^n}{1 - \rho^{n+1}} - C \left[ \frac{\rho}{1 - \rho} - \frac{(n + 1)\rho^{n+1}}{1 - \rho^{n+1}} \right].$$
The basic observable model - Social opt. II

Theorem

\[ S(\text{obs}) \leq S(\text{soc}) \]

Its unique maximum is attained for

\[ n_{\text{soc}} = \lfloor x_{\text{soc}} \rfloor \]

where \( x_{\text{soc}} \) is the unique solution to

\[ x(1 - \rho) - \rho (1 - \rho x)(1 - \rho^2) = \mu R \]

Moreover:

\[ n_{\text{soc}} \leq n_e \]

Individual optimization leads to longer queues than are socially desired.

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Strategic customers in queueing systems
The basic observable model - Social opt. II

Theorem

\[ S_{soc}^{(obs)}(n) \text{ is unimodal.} \]
The basic observable model - Social opt. II

**Theorem**

$S_{soc}^{(obs)}(n)$ is unimodal.

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$$n_{soc} = \lceil x_{soc} \rceil$$

where $x_{soc}$ is the unique solution to

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Moreover:

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Individual optimization leads to longer queues than are socially desired.
The basic observable model - Profit max. I

Profit of the administrator when he uses a fee

\[ p = R - C_n \mu \]

to induce a threshold strategy \( n \):

\[ S(\text{obs}) \text{prof}(n) = \lambda_1 - \rho n^{1 - \rho} n^{1 + 1} (R - C_n \mu). \]

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Strategic customers in queueing systems
Profit of the administrator when he uses a fee \( p = R - \frac{Cn}{\mu} \) to induce a threshold strategy \( n \):

\[
S^{(obs)}_{prof}(n) = \lambda \frac{1 - \rho^n}{1 - \rho^{n+1}} \left( R - \frac{Cn}{\mu} \right).
\]
The basic observable model - Profit max. II

Theorem

The unique profit-optimizing threshold $n_{\text{prof}}$ that maximizes $S(\text{obs})_{\text{prof}}(n)_{\text{prof}}$ is given by

$$n_{\text{prof}} = \left\lfloor x_{\text{prof}} \right\rfloor$$

where $x_{\text{prof}}$ is the unique solution to

$$x_{\text{prof}} + (1 - \rho x_{\text{prof}} - 1)(1 - \rho x_{\text{prof}} + 1)\rho x_{\text{prof}} - 1(1 - \rho) = \mu R.$$
The basic observable model - Profit max. II

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Moreover

$$n_{\text{prof}} \leq n_{\text{soc}} \leq n_e.$$
The basic unobservable model (Edelson and Hildebrand (1975))
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- Unobservable $M/M/1$ queue.
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- **Unobservable M/M/1 queue.**
  1. Same dynamics and operational parameters as in Naor’s model \((\lambda, \mu \text{ and } \rho)\).
  2. Same reward-cost structure \((R \text{ and } C)\).
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- **Unobservable M/M/1 queue.**
  1. Same dynamics and operational parameters as in Naor’s model ($\lambda$, $\mu$ and $\rho$).
  2. Same reward-cost structure ($R$ and $C$).
  3. Upon arrival, a customer decides whether to join or balk without observing the queue length.
The basic unobservable model - Equilibrium I

Suppose the customers follow a strategy $q$. The system behaves as an $M/M/1$ queue with arrival rate $\lambda q$ and service rate $\mu$.

Mean sojourn time of a customer: $\frac{1}{\mu} - \frac{\lambda q}{\mu}$.

A tagged customer prefers to join, prefers to balk or he is indifferent, if his expected net benefit $R - C$ is $>0$, $<0$ or $=0$.

If $R - C > 0$, then the best response is to join.

If $R - C < 0$, then the best response is to balk.

If $R - C = 0$, then any strategy is best response.
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The basic unobservable model - Equilibrium II

Theorem
There always exist a unique equilibrium strategy.

Case
Equil. prob. \( q \)
\[ R \leq C \mu \]
\[ C \mu < R < C \mu - \frac{\lambda}{R} \mu \]
\[ R \geq C \mu - \frac{\lambda}{R} \mu \]

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The basic unobservable model - Social, prof. opt.

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Social benefit function when customers follow a strategy $q$ (coincides with the profit function):

$$S_{soc}(q) = \lambda q R - \frac{C}{\mu - \lambda}.$$ 

Theorem $S_{soc}(q)$ is strictly concave, so there exists a unique socially optimal strategy $q_{soc}$.

Case

Social opt. prob. $q_{soc}$

- $R \leq C\mu$
- $C\mu < R < C\mu (\mu - \lambda)^2 \mu - \sqrt{\mu C/R} \lambda$
- $R \geq C\mu (\mu - \lambda)^2$

Moreover $q_{soc} \leq q_e$.

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The basic unobservable model - Social, prof. opt.

- Social benefit function when customers follow a strategy $q$ (coincides with the profit function):

$$S_{soc}^{(un)}(q) = \lambda q R - C\lambda/(\mu - \lambda).$$
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Moreover \( q_{soc} \leq q_e \).
Observable vs. unobservable I


Let $\lambda^{(\text{obs})}$ and $\lambda^{(\text{un})}$ be the equilibrium arrival rates in the observable and unobservable cases. Then, there exists a unique critical value $\lambda^*$ such that $\lambda^{(\text{un})} > \lambda^{(\text{obs})}$, for $\lambda < \lambda^*$, while $\lambda^{(\text{un})} < \lambda^{(\text{obs})}$, for $\lambda > \lambda^*$.

For low arrival rates, it is better to conceal information from the customers to increase the throughput.
Observable vs. unobservable I

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For low arrival rates, it is better to conceal information from the customers to increase the throughput.

Let $\lambda_e^{(obs)}$ and $\lambda_e^{(un)}$ be the equilibrium arrival rates in the observable and unobservable cases.

Then, there exists a unique critical value $\lambda^*$ such that

$$\lambda_e^{(un)} > \lambda_e^{(obs)}, \text{ for } \lambda < \lambda^*,$$

while

$$\lambda_e^{(un)} < \lambda_e^{(obs)}, \text{ for } \lambda > \lambda^*.$$

Let $\lambda_e^{(\text{obs})}$ and $\lambda_e^{(\text{un})}$ be the equilibrium arrival rates in the observable and unobservable cases.

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Observable vs. unobservable II

For any parameters of the model

\[ \max q \Rightarrow S (\text{un}) < \max n \Rightarrow S (\text{soc}) \]

The social planner prefers to reveal the queue length to the customers.

For any parameters of the model

$$\max_q S_{soc}^{(un)}(q) < \max_n S_{soc}^{(obs)}(n).$$

For any parameters of the model

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Observable vs. unobservable III

If $R \leq 2C\mu$, then
$$\max q_S(\text{un prof})(q) < \max n_S(\text{obs prof})(n),$$

The profit maximizer prefers to reveal the queue length to the customers.

If $R > 2C\mu$, then there exist $\lambda_{\text{prof}}$ such that
$$\max q_S(\text{un prof})(q) > \max n_S(\text{obs prof})(n),$$
for $\lambda < \lambda_{\text{prof}}$,

$$\max q_S(\text{un prof})(q) < \max n_S(\text{obs prof})(n),$$
for $\lambda > \lambda_{\text{prof}}$.

The profit maximizer prefers to conceal the queue length for low values of $\lambda$ and to reveal it for high values.

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Strategic customers in queueing systems
Observable vs. unobservable III

- If \( R \leq \frac{2C}{\mu} \), then

\[
\max_q S_{prof}^{(un)}(q) < \max_n S_{prof}^{(obs)}(n).
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The profit maximizer prefers to reveal the queue length to the customers.

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\[
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\]

and

\[
\max_q S_{prof}^{(un)}(q) < \max_n S_{prof}^{(obs)}(n), \quad \text{for} \quad \lambda > \lambda_{prof}.
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The profit maximizer prefers to conceal the queue length for low values of \( \lambda \) and to reveal it for high values.
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The profit maximizer prefers to conceal the queue length for low values of $\lambda$ and to reveal it for high values.
Structure of the talk

Models with imperfect observation structure. The customers observe imperfectly the queue length.

Models with delayed observation structure. The customers observe the queue length with delay.

Models with mixed observation structure. Only a fraction of the customers observe the queue length.

Models with partial observation structure. The state of the queue is 2-dimensional. The customers observe only one dimension of the state.

Other information structures, extensions, conclusions, bibliography.

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Structure of the talk

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Part II: Strategic customers in models with imperfect information structure
Models with imperfect information structure

Upon arrival, a customer decides whether to join or balk based on an 'imperfect' observation of the queue length. 'Imperfect' observation means that the customer gets some information about the queue length but not its exact value.
Models with imperfect information structure

- $M/M/1$ queue with same dynamics and operational parameters as in Naor’s model ($\lambda$, $\mu$ and $\rho$).
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- Same reward-cost structure ($R$ and $C$).
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An introduction to imperfect information structure

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Strategic customers in queueing systems

A model with imperfect information structure


The waiting space of the system is partitioned in compartments of fixed capacity for customers. Upon arrival, a customer gets informed about the number of the compartment in which he will enter (N case) or the position within his compartment (P case).

Suppose that a tagged customer arrives at a system with n present customers. In the N case, he gets informed about \( \lfloor \frac{n}{a} \rfloor + 1 \). In the P case, he gets informed about \( (n \mod a) + 1 \).
A model with imperfect information structure

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- The waiting space of the system is partitioned in compartments of fixed capacity for \( a \) customers.
A model with imperfect information structure


- The waiting space of the system is partitioned in compartments of fixed capacity for $a$ customers.

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A model with imperfect information structure


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- Suppose that a tagged customer arrives at a system with $n$ present customers.

- In the $N$ case, he gets informed about $\lfloor n/a \rfloor + 1$. 
A model with imperfect information structure


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- Suppose that a tagged customer arrives at a system with \( n \) present customers.

- In the \( N \) case, he gets informed about \( \lfloor n/a \rfloor + 1 \).

- In the \( P \) case, he gets informed about \( (n \mod a) + 1 \).
Example

Suppose \( a = 10 \). In the \( N \)-case, the customer gets informed that his position belongs to the set \( \{1, 2, \ldots, 10\} \) (first compartment) or to the set \( \{11, 12, \ldots, 20\} \) (second compartment) etc. In the \( P \)-case, the customer gets informed that his position belongs to the set \( \{1, 11, 21, \ldots\} \) (first position) or to the set \( \{2, 12, 22, \ldots\} \) (second position) or to the set \( \{3, 13, 23, \ldots\} \) (third compartment) etc.
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In the $P$-case, the customer gets informed that his position belongs to the set

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Extreme cases

$n$: number of present customers.

$N$ case - information: compartment number $\lfloor n/a \rfloor + 1$.

The customer gets a rough estimate of his position:

- He knows if it will belong to $\{1, 2, \ldots, a\}$
- or to $\{a+1, a+2, \ldots, 2a\}$
- or to $\{2a+1, 2a+2, \ldots, 3a\}$
- and so on.

$P$ case - information: compartment position $(n \mod a) + 1$.

- $a = 1$: $N$ case = Observable $M/M/1$ (Naor).
- $P$ case = Unobservable $M/M/1$ (E&H).
- $a \to \infty$: $N$ case = Unobservable $M/M/1$ (E&H).
- $P$ case = Observable $M/M/1$ (Naor).
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- $n$: number of present customers.

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Strategic customers in queueing systems
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- $a \to \infty$: $N$ case = Unobservable $M/M/1$ (E&H).
  $P$ case = Observable $M/M/1$ (Naor).
$N$ case - Equilibrium

An individual’s optimizing strategy for a customer that gets informed will be placed in the compartment $i$ is the threshold strategy that prescribes to join if $i \leq N_e$ with $N_e = \lfloor x_N \rfloor$, with $x_N = \begin{cases} \frac{R_a C}{a - 1} & \text{if } \rho \neq 1, \\ \frac{R_a C}{2} & \text{if } \rho = 1. \end{cases}$ This is the unique equilibrium strategy within the set of pure strategies.

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Strategic customers in queueing systems
An individual’s optimizing strategy for a customer that gets informed that will be placed in the compartment $i$ is the threshold strategy that prescribes to join if $i \leq i_e^N$ with $i_e^N = \lfloor x_e^N \rfloor$, with

$$x_e^N = \begin{cases} \frac{R\mu}{aC} + \frac{1}{1-\rho^a} - \frac{1}{a(1-\rho)} & \text{if } \rho \neq 1, \\ \frac{R\mu}{aC} + \frac{a-1}{2a} & \text{if } \rho = 1. \end{cases}$$
An individual’s optimizing strategy for a customer that gets informed that will be placed in the compartment $i$ is the threshold strategy that prescribes to join if $i \leq i_e^N$ with

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\frac{R\mu}{aC} + \frac{a-1}{2a} & \text{if } \rho = 1. \end{cases}$$

This is the unique equilibrium strategy within the set of pure strategies.
The social benefit function $S_{soc}(i)$ is unimodal. Its unique maximum is attained for $i_{soc} = \lfloor x_{soc} \rfloor$ where $x_{soc}$ is the unique solution of $g(x) = x_{soc}$, with

$$g(x) = \begin{cases} \frac{(x-a+1)(1-\rho a)}{a(1-\rho)(1-\rho a)} + 1 & \text{if } \rho \neq 1 \\ \frac{x^2 - (a^2 - 2a)}{2} & \text{if } \rho = 1 \end{cases}$$

Moreover: $i_{soc} \leq i_{ne}$.

Individual optimization leads to longer queues than are socially desired.
Theorem

The social benefit function $S_{soc}^N(i)$ is unimodal.

Moreover:

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Individual optimization leads to longer queues than are socially desired.
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$$i_{soc}^N = \lfloor x_{soc}^N \rfloor$$

where $x_{soc}^N$ is the unique solution of

$$g(x) = x_e^N$$

with

$$g(x) = \begin{cases} 
\frac{(xa+1)(1-\rho^a) - a(1-\rho^{xa+1})}{a(1-\rho)(1-\rho^a)} + \frac{1}{1-\rho^a} - \frac{1}{a(1-\rho)} & \text{if } \rho \neq 1 \\
\frac{a}{2}x^2 - \frac{a-2}{2}x & \text{if } \rho = 1.
\end{cases}$$

Moreover:

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Theorem

The social benefit function $S_{soc}^N(i)$ is unimodal. Its unique maximum is attained for

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where $x_{soc}^N$ is the unique solution of $g(x) = x_e^N$, with

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\frac{a}{2}x^2 - \frac{a-2}{2}x & \text{if } \rho = 1.
\end{cases}$$

Moreover:

$$i_{soc}^N \leq i_e^N.$$

Individual optimization leads to longer queues than are socially desired.
$N$ case - Profit maximization

The unique profit-optimizing threshold $i_{\text{prof}}$ that maximizes $S_{\text{prof}}(i)$ is given by

$$i_{\text{prof}} = \lfloor x_{\text{prof}} \rfloor$$

where $x_{\text{prof}}$ is the unique solution of the equation $h(x) = x \in [1, \infty]$ with

$$h(x) = \begin{cases} x + (x - 1)(x^a + 1), & \text{if } \rho \neq 1 \\ x + (x - 1)(x^a + 1), & \text{if } \rho = 1 \end{cases}$$

Moreover, $i_{\text{prof}} \leq i_{\text{soc}} \leq i_{\text{ne}}$.

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Strategic customers in queueing systems
N case - Profit maximization

**Theorem**

The unique profit-optimizing threshold $i_{prof}^N$ that maximizes $S_{prof}^N(i)$ is given by

$$i_{prof}^N = \lfloor x_{prof}^N \rfloor$$

where $x_{prof}^N$ is the unique solution of the equation $h(x) = x_{e}^N$ in $[1, \infty]$ with

$$h(x) = \begin{cases} 
  x + \frac{(1-\rho^{xa-a})(1-\rho^{xa+1})}{\rho^{xa-a}(1-\rho)(1-\rho^a)}, & \text{if } \rho \neq 1 \\
  x + (x - 1)(xa + 1), & \text{if } \rho = 1.
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Strategic customers in queueing systems
The unique profit-optimizing threshold $i_{prof}^N$ that maximizes $S_{prof}^N(i)$ is given by

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$$h(x) = \begin{cases} x + \frac{(1-\rho^{xa-a})(1-\rho^{xa+1})}{\rho^{xa-a}(1-\rho)(1-\rho^a)}, & \text{if } \rho \neq 1 \\ x + (x - 1)(xa + 1), & \text{if } \rho = 1. \end{cases}$$

Moreover,

$$i_{prof}^N \leq i_{soc}^N \leq i_e^N.$$
The best response of a customer against any strategy of the others is always a (mixed) threshold strategy. The equilibrium strategies are of (mixed) threshold type. The equilibrium strategy is unique, if we exclude some very special values of the parameters ($R$ being an integer multiple of $C$). The equilibrium, social optimizing and profit maximizing thresholds can be computed in closed form.
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The equilibrium strategy is unique, if we exclude some very special values of the parameters ($R$ being an integer multiple of $\frac{C}{\mu}$).
$P$ case - Results

- The best response of a customer against any strategy of the others is always a (mixed) threshold strategy.
- The equilibrium strategies are of (mixed) threshold type.
- The equilibrium strategy is unique, if we exclude some very special values of the parameters ($R$ being an integer multiple of $\frac{C}{\mu}$).
- The equilibrium, social optimizing and profit maximizing thresholds can be computed in closed form.
Numerical results I
Numerical results I

Figure 1: Optimal thresholds with respect to $R - N$ case
Numerical results II

Scenario: $\lambda = 0.7$, $\mu = 1$, $a = 4$, $C = 1$. The three thresholds are all increasing ladder functions of $R$. The individual optimal threshold increases more rapidly than the other optimal thresholds. Its increase is almost linear in $R$. The other two thresholds increase almost logarithmically in $R$. Antonis Economou, aeconom@math.uoa.gr
Numerical results II

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Strategic customers in queueing systems
Numerical results II

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Strategic customers in queueing systems
Numerical results II

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- The individual optimal threshold increases more rapidly than the other optimal thresholds. Its increase is almost linear in $R$.
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Numerical results III
Figure 2: Optimal social benefit and administrator’s profit with respect to $a - N$ case
Numerical results IV

Scenario: $\lambda = 0.9$, $\mu = 1$, $R = 25$, $C = 1$.

For $a = 1$ which corresponds to full information, the optimal social benefit is high, while the administrator's profit attains its minimum value. For small values of $a$ the difference of the two functions is positive, whereas for greater values of $a$ the two functions coincide. There is a value of $a$ ($a = 7$), such that the administrator's profit is maximized and then it decreases. This is in some sense the 'ideal' compartment size for the administrator.

Take-away message: The administrator can improve its profit by an adequate selection of the compartment size.
Scenario: $\lambda = 0.9$, $\mu = 1$, $R = 25$, $C = 1$. 
Numerical results IV

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For small values of \( a \) the difference of the two functions is positive, whereas for greater values of \( a \) the two functions coincide.

There is a value of \( a \) (\( a = 7 \)), such that the administrator’s profit is maximized and then it decreases. This is in some sense the ‘ideal’ compartment size for the administrator.

Take-away message: The administrator can improve its profit by an adequate selection of the compartment size.
Other models with imperfect information structure


The nonnegative integers are partitioned into intervals and a customer is informed about the interval that contains the queue length at the time of his arrival.


The customers get informed whether the queue size is smaller than some $L$ or not.
Other models with imperfect information structure

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Part III:
Strategic customers in models with delayed information structure
Models with delayed information structure

M/M/1 queue with same dynamics and operational parameters as in Naor's model \( (\lambda, \mu, \rho) \).

Same reward-cost structure \( (R, C) \).

Upon arrival, a customer decides whether to join or balk without observing the queue length. Later, the customer gets informed about the queue length.
Models with delayed information structure

- \( M/M/1 \) queue with same dynamics and operational parameters as in Naor’s model (\( \lambda, \mu \) and \( \rho \)).
Models with delayed information structure

- $M/M/1$ queue with same dynamics and operational parameters as in Naor’s model ($\lambda$, $\mu$ and $\rho$).
- Same reward-cost structure ($R$ and $C$).
Models with delayed information structure

- $M/M/1$ queue with same dynamics and operational parameters as in Naor’s model ($\lambda$, $\mu$ and $\rho$).
- Same reward-cost structure ($R$ and $C'$).
- Upon arrival, a customer decides whether to join or balk without observing the queue length.
Models with delayed information structure

- $M/M/1$ queue with same dynamics and operational parameters as in Naor’s model ($\lambda$, $\mu$ and $\rho$).
- Same reward-cost structure ($R$ and $C$).
- Upon arrival, a customer decides whether to join or balk without observing the queue length.
- Later, the customer gets informed about the queue length.
A model with delayed information structure

Burnetas, A., Economou, A. and Vasiliadis, G. (2015) Strategic balking behavior in a queueing system with delayed observations. M/M/1 queue with same dynamics and operational parameters as in Naor's model ($\lambda$, $\mu$ and $\rho$). Same reward-cost structure ($R$ and $C$).

The administrator of the system announces to all customers their positions in the system, every $\text{Exp}(\theta)$ time units. Upon arrival, each customer decides whether to join or balk, without observing the system. Joining customers may decide to renege at any later time.
A model with delayed information structure

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A model with delayed information structure


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A model with delayed information structure

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Extreme cases

The system is initially unobservable, but it becomes observable for any given customer after an $\text{Exp}(\theta)$ announcement time.

$\theta \to 0$: Delayed model = Unobservable $\text{M/M/1}$ (E&H).

$\theta \to \infty$: Delayed model = Observable $\text{M/M/1}$ (Naor).

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Extreme cases

- The system is initially unobservable, but it becomes observable for any given customer after an $\text{Exp}(\theta)$ announcement time.
- $\theta \to 0$: Delayed model = Unobservable $M/M/1$ (E&H).
- $\theta \to \infty$: Delayed model = Observable $M/M/1$ (Naor).
Because of the exponentiality assumptions, the customers may renege only at announcement instants. Because of the FCFS discipline and the full observation, they may renege only after the first announcement. A customer stays after the first announcement, if his position $n$ at the system is such that $R - C n < 0$. The best strategy of a customer taking into account the reaction of the others is to stay if his position $n$ at the first announcement is such that $n \leq n_e$, with $n_e = \lfloor \mu R / C \rfloor$ (Naor's threshold).

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Strategic customers in queueing systems
Because of the exponentiality assumptions, the customers may renege only at announcement instants.
Reneging

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Reneging

- Because of the exponentiality assumptions, the customers may renege only at announcement instants.
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- A customer stays after the first announcement, if his position $n$ at the system is such that $R - C\frac{n}{\mu} > 0$. 

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The best strategy of a customer taking into account the reaction of the others is to stay if his position $n$ at the first announcement is such that

$$n \leq n_e,$$

with

$$n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor \text{ (Naor’s threshold)}.$$
The system is unobservable at arrival instants. The strategic behavior of a customer regarding joining/balking is specified by a joining probability \( q^* \).

Suppose that all customers use a reneging threshold \( n^* \) and joining probability \( q^* \).

Under this \((n^*, q^*)\) strategy the number of customers in the system is a CTMC with diagram:

\[
\begin{array}{cccc}
0 & 1 & 2 & n^* - 1 \\
\lambda q^* & & & \\
& \lambda q^* & & \\
& & \ddots & \\
& & & \mu + \theta \\
& & & \mu \\
\end{array}
\]
**Balking**

- The system is unobservable at arrival instants.
Balking

- The system is unobservable at arrival instants.
- The strategic behavior of a customer regarding joining/balking is specified by a joining probability $q_\ast$. 

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Strategic customers in queueing systems
Balking

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Balking

- The system is unobservable at arrival instants.
- The strategic behavior of a customer regarding joining/balking is specified by a joining probability $q_\ast$.
- Suppose that all customers use a reneging threshold $n_\ast$ and joining probability $q_\ast$.
- Under this $(n_\ast, q_\ast)$ strategy the number of customers in the system is a CTMC with diagram

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Strategic customers in queueing systems
Balking

- The system is unobservable at arrival instants.
- The strategic behavior of a customer regarding joining/balking is specified by a joining probability $q_*$. 
- Suppose that all customers use a reneging threshold $n_*$ and joining probability $q_*$. 
- Under this $(n_*, q_*)$ strategy the number of customers in the system is a CTMC with diagram

```
0 ----> 1 ----> 2 ----> ... ----> n*_1 ----> n*_2 ----> n*_3 ----> ...
|      |      |      |       |      |      |      |
| μ    | μ      | μ     | n*_1  | μ      | μ      | μ      |

λq*  λq*  λq*  λq*  λq*  λq*  λq*
```

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Strategic customers in queueing systems
Stationary distrib. of the number of customers

\begin{align*}
\pi_n &= \pi_n(n^*, q^*, \rho^*) = \\
&\begin{cases} 
B^* \rho_{n^*} & \text{if } 0 \leq n \leq n^* - 1, \\
B^* \rho_{n^*} \rho_{n^* - n^* + 1} & \text{if } n \geq n^*,
\end{cases}
\end{align*}

where

\begin{align*}
\rho^*_1 &= \lambda q^* \mu, \\
\rho^*_2 &= \lambda q^* + \mu + \theta - \sqrt{(\lambda q^* + \mu + \theta)^2 - 4 \lambda q^* \mu^2}, \\
\rho^* &= (1 - \rho^*_1)(1 - \rho^*_2) ^{-1}.
\end{align*}

Antonis Economou, aeconom@math.uoa.gr
Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n^*, q^*, \rho) = \begin{cases} B^* \rho_n^1 & \text{if } 0 \leq n < n^* - 1, \\ B^* \rho_n^1 \rho_n - n^* \rho_n^2 & \text{if } n \geq n^* \end{cases} \]

where

\[ \rho_1^* = \lambda q^* \mu, \quad \rho_2^* = \lambda q^* + \mu + \theta - \sqrt{\left(\lambda q^* + \mu + \theta\right)^2 - 4 \lambda q^* \mu^2} \mu \]

and

\[ B^* = \left(1 - \rho_1^*\right)\left(1 - \rho_2^* - \rho_n + 1\right)^{-1} \rho_n^1 + \rho_n^2. \]
Stationary distribution of the number of customers:

Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n^*, q^*, 1) = \begin{cases} 
B^* \rho \pi_n^* 1 & \text{if } 0 \leq n \leq n^* - 1, \\
B^* \rho \pi_n^* 1 \rho n - n^* 1 & \text{if } n \geq n^*, 
\end{cases} \]

where

\[ \rho^*_1 = \lambda q^* \mu, \quad \rho^*_2 = \lambda q^* + \mu + \theta - \sqrt{(\lambda q^* + \mu + \theta)^2 - 4 \lambda q^* \mu} \]

and

\[ B^* = (1 - \rho^*_1)(1 - \rho^*_2) \]

Antonis Economou, aeconom@math.uoa.gr
Stationary distrib. of the number of customers

Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n_*, q_*) = \begin{cases} 
B_* \rho_{1*}^n & \text{if } 0 \leq n \leq n_* - 1, \\
B_* \rho_{1*} n_* \rho_{2*}^{n-n_*} & \text{if } n \geq n_, 
\end{cases} \]
Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n_*, q_*) = \begin{cases} 
B_* \rho_1^n & \text{if } 0 \leq n \leq n_* - 1, \\
B_* \rho_1^n \rho_2^{n-n_*} & \text{if } n \geq n_*,
\end{cases} \]

where

\[ \rho_1 = \frac{\lambda q_*}{\mu}, \quad \rho_2 = \frac{\lambda q_* + \mu + \theta - \sqrt{(\lambda q_* + \mu + \theta)^2 - 4\lambda q_* \mu}}{2\mu}. \]
Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n_*, q_*) = \begin{cases} 
B_* \rho_{*1}^n & \text{if } 0 \leq n \leq n_* - 1, \\
B_* \rho_{*1} \rho_{*2}^{n-n_*} & \text{if } n \geq n_*, 
\end{cases} \]

where

\[ \rho_{*1} = \frac{\lambda q_*}{\mu}, \quad \rho_{*2} = \frac{\lambda q_* + \mu + \theta - \sqrt{(\lambda q_* + \mu + \theta)^2 - 4\lambda q_* \mu}}{2\mu} \]

and

\[ B_* = \frac{(1 - \rho_{*1})(1 - \rho_{*2})}{1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}^{n_*} \rho_{*2}}. \]
Stationary mean number of customers

Proposition (continued)

The mean stationary number of customers in system is

\[ E(n^*, q^*, N) = (1 - \rho^* \cdot 2) \left( (n^* - 1) \rho_n^* + 1 \right) \]

\[ (1 - \rho^* 1) \left[ 1 - \rho^* 2 - \rho n^* + 1 + \rho n^* \rho \right] \]

\[ + (1 - \rho^* 1) \left[ n^* \rho n^* - (n - 1) \rho_n^* + 1 + \rho n^* \right] \]

\[ (1 - \rho^* 2) \left( 1 - \rho^* 2 - \rho n^* + 1 + \rho n^* \rho \right) \]

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Strategic customers in queueing systems
Stationary mean number of customers

Proposition (continued)

The mean stationary number of customers in system is

\[ E_{(n_*, q_*)}(N) = \frac{(1 - \rho_{*2})[(n_* - 1)\rho_{*1}^{n_*+1} - n_*\rho_{*1}^{n_*} + \rho_{*1}]}{(1 - \rho_{*1})[1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}\rho_{*2}]} + \frac{(1 - \rho_{*1})[n_*\rho_{*1}^{n_*} - (n_* - 1)\rho_{*1}^{n_*}\rho_{*2}]}{(1 - \rho_{*2})[1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}\rho_{*2}]} \].

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Strategic customers in queueing systems
Conditional expected net benefit

Proposition

Suppose that the customers follow an \((n^*, q^*)\) strategy.

Consider a tagged customer that finds \(n\) customers upon arrival (but he does not know about it).

The conditional expected net benefit of the tagged, if he decides to join is

\[
U(n | n^*) = \begin{cases} 
R - C(n + 1) & \text{if } n < n^*, \\
(R - Cn^* + C\theta) \frac{\mu}{\mu + \theta} n - n^* + 1 - C\theta & \text{if } n \geq n^*.
\end{cases}
\]

\(U(n | n^*)\) does not depend on \(\lambda\) nor on \(q^*\).

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Proposition

\[ U(n|n^*) = \begin{cases} R - C(n+1) \mu & \text{if } n < n^*, \\ R - Cn^* \mu + C\theta n - n^* + 1 - C\theta & \text{if } n \geq n^*. \end{cases} \]

\( U(n|n^*) \) does not depend on \( \lambda \) nor on \( q^* \).
Proposition

Suppose that the customers follow an \((n^*, q^*)\) strategy.
Proposition

Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged customer that finds \(n\) customers upon arrival.
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Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged customer that finds \(n\) customers upon arrival (but he does not know about it).
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Proposition

Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged customer that finds \(n\) customers upon arrival (but he does not know about it).

The conditional expected net benefit of the tagged, if he decides to join is

\[
U(n|n_*) = \begin{cases} 
R - \frac{C(n+1)}{\mu} & \text{if } n < n_*, \\
(R - \frac{Cn_*}{\mu} + \frac{C}{\theta}) \left(\frac{\mu}{\mu+\theta}\right)^{n-n_*+1} - \frac{C}{\theta} & \text{if } n \geq n_*.
\end{cases}
\]
Proposition

Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged customer that finds \(n\) customers upon arrival (but he does not know about it).

The conditional expected net benefit of the tagged, if he decides to join is

\[
U(n|n_*) = \begin{cases} 
R - \frac{C(n+1)}{\mu} & \text{if } n < n_*, \\
\left( R - \frac{Cn_*}{\mu} + \frac{C}{\theta} \right) \left( \frac{\mu}{\mu+\theta} \right)^{n-n_*+1} - \frac{C}{\theta} & \text{if } n \geq n_*. 
\end{cases}
\]

\(U(n|n_*)\) does not depend on \(\lambda\) nor on \(q_*\).
Unconditional expected net benefit

Theorem

Unconditional net benefit of a customer that decides to join given than the others follow a strategy \((n^*, q^*)\):

\[
U(n^*, q^*) = B^* (R - C \mu (n^* - 1))^{1 - \rho n^*} + B^* C \mu (n^* - 1)^{1 - \rho n^* + 1} \rho n^* + \rho^* \mu (1 - \rho^* 2) + B^* (R - C n^* \mu + C \theta) \mu (n^*)^{1 - \rho n^* + 2} - B^* C \theta (n^* - 1)^{1 - \rho n^* + 1} \rho n^* + \rho^* 2 - B^* C \theta (n^* - 1)^{1 - \rho n^* + 1} \rho n^* + \rho^* 2.
\]

\(U(n^*, q^*)\) is decreasing in \(q^*\) for any fixed \(n^*\).
Unconditional expected net benefit

Theorem

Unconditional net benefit of a customer that decides to join given than the others follow a strategy \((n^*, q^*)\):

\[
U(n^*, q^*) = B^* \left( R - C \mu \right) \frac{1}{1 - \rho n^*} - B^* C \mu \left( n^* - 1 \right) \rho n^* + 1 - B^* C \theta \rho n^*^2 - B^* C \theta (1 - \rho^*^2).
\]

\(U(n^*, q^*)\) is decreasing in \(q^*\) for any fixed \(n^*\).
Unconditional expected net benefit

**Theorem**

Unconditional net benefit of a customer that decides to join given than the others follow a strategy \((n_*, q_*)\):

\[
U(n_*, q_*) = B* (R - C \mu)^{1-\rho n_*} + B* (R - C n_* \mu + C \theta)^{1-\rho n_*}.
\]

\(U(n_*, q_*)\) is decreasing in \(q_*\) for any fixed \(n_*\).
Theorem

Unconditional net benefit of a customer that decides to join given than the others follow a strategy \((n_*, q_*)\):

\[
U(n_*, q_*) = B_* \left( R - \frac{C}{\mu} \right) \frac{1 - \rho_{1*}^{n_*}}{1 - \rho_{1*}} \\
- B_* \frac{C}{\mu} \frac{(n_* - 1)\rho_{1*}^{n_*+1} - n_*\rho_{1*}^{n_*} + \rho_{1*}}{(1 - \rho_{1*})^2} \\
+ B_* \left( R - \frac{Cn_*}{\mu} + \frac{C}{\theta} \right) \frac{\mu\rho_{1*}^{n_*}}{\mu + \theta - \mu\rho_{2*}} \\
- B_* \frac{C}{\theta} \frac{\rho_{1*}^{n_*}}{1 - \rho_{2*}}.
\]
Unconditional expected net benefit

Theorem

Unconditional net benefit of a customer that decides to join given than the others follow a strategy \((n_*, q_*)\):

\[
U(n_*, q_*) = B_* \left( R - \frac{C}{\mu} \right) \frac{1 - \rho_{*1}^{n_*}}{1 - \rho_{*1}} - B_* \frac{C}{\mu} \frac{(n_* - 1)\rho_{*1}^{n_*+1} - n_*\rho_{*1}^{n_*} + \rho_{*1}}{(1 - \rho_{*1})^2} + B_* \left( R - \frac{Cn_*}{\mu} + \frac{C}{\theta} \right) \frac{\mu\rho_{*1}^{n_*}}{\mu + \theta - \mu\rho_{*2}} - B_* \frac{C}{\theta} \frac{\rho_{*1}^{n_*}}{1 - \rho_{*2}}.
\]

\(U(n_*, q_*)\) is decreasing in \(q_*\) for any fixed \(n_*\).
Equilibrium strategies

Theorem

Let \( n_e = \lfloor \mu R \cdot C \rfloor \).

Case I: \( U(n_e, 0) \leq 0 \). The unique equilibrium is \((n_e, 0)\).

Case II: \( U(n_e, 1) < 0 < U(n_e, 0) \). The unique equilibrium is \((n_e, q_e)\), where \( q_e \) is the unique solution of the equation \( U(n_e, q) = 0 \) in \((0, 1)\), with respect to \( q \).

Case III: \( U(n_e, 1) \geq 0 \). The unique equilibrium is \((n_e, 1)\).
Equilibrium strategies

Theorem

Let \( n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor \).
Equilibrium strategies

Theorem

Let \( n_e = \lfloor \frac{\mu R}{C} \rfloor \).

Case I: \( \mathcal{U}(n_e, 0) \leq 0 \).
Theorem

Let $n_e = \lfloor \frac{\mu R}{C} \rfloor$.

Case I: $U(n_e, 0) \leq 0$. The unique equilibrium is $(n_e, 0)$.
Equilibrium strategies

Theorem

Let $n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor$.

Case I: $\mathcal{U}(n_e, 0) \leq 0$. The unique equilibrium is $(n_e, 0)$.

Case II: $\mathcal{U}(n_e, 1) < 0 < \mathcal{U}(n_e, 0)$.
Equilibrium strategies

**Theorem**

Let $n_e = \lfloor \frac{\mu R}{C} \rfloor$.

Case I: $U(n_e, 0) \leq 0$. The unique equilibrium is $(n_e, 0)$.

Case II: $U(n_e, 1) < 0 < U(n_e, 0)$. The unique equilibrium is $(n_e, q_e)$, where $q_e$ is the unique solution of the equation

$$U(n_e, q) = 0$$

in $(0, 1)$, with respect to $q$. 

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Strategic customers in queueing systems
Equilibrium strategies

**Theorem**

Let \( n_e = \lfloor \frac{\mu R}{C} \rfloor \).

**Case I:** \( U(n_e, 0) \leq 0 \). The unique equilibrium is \((n_e, 0)\).

**Case II:** \( U(n_e, 1) < 0 < U(n_e, 0) \). The unique equilibrium is \((n_e, q_e)\), where \( q_e \) is the unique solution of the equation

\[
U(n_e, q) = 0
\]

in \((0, 1)\), with respect to \( q \).

**Case III:** \( U(n_e, 1) \geq 0 \).
Equilibrium strategies

Theorem

Let \( n_e = \left\lfloor \frac{\mu R}{C} \right\rfloor \).

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Case III: \( U(n_e, 1) \geq 0 \). The unique equilibrium is \((n_e, 1)\).
Results

The equilibrium joining probability is an increasing function of $\theta$.

The equilibrium throughput is a unimodal function of $\theta$.

There exists an 'ideal' announcement rate that maximizes the equilibrium throughput.
• Effect of $\theta$ on the equilibrium:
Results

- **Effect of $\theta$ on the equilibrium:**
  The equilibrium joining probability is an increasing function of $\theta$. 

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Results

- Effect of $\theta$ on the equilibrium:
The equilibrium joining probability is an increasing function of $\theta$.

- Effect of $\theta$ on the equilibrium throughput:
Results

- **Effect of $\theta$ on the equilibrium:**
The equilibrium joining probability is an increasing function of $\theta$.

- **Effect of $\theta$ on the equilibrium throughput:**
The equilibrium throughput is a unimodal function of $\theta$. 

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Strategic customers in queueing systems
Effect of $\theta$ on the equilibrium:
The equilibrium joining probability is an increasing function of $\theta$.

Effect of $\theta$ on the equilibrium throughput:
The equilibrium throughput is a unimodal function of $\theta$. There exists an ‘ideal’ announcement rate that maximizes the equilibrium throughput.
Other models with delayed information structure
Other models with delayed information structure

Other models with delayed information structure


Other models with delayed information structure


Part IV:
Strategic customers in models with mixed observation structure
Models with mixed observation structure

The customers are heterogeneous regarding information and possibly also regarding the rewards, costs. There are customers that may observe the system and then decide whether to join or balk. There are also customers that cannot observe the system before making their decisions.
Models with mixed observation structure

- $M/M/1$ queue with known dynamics and operational parameters ($\lambda$, $\mu$ and $\rho$).

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Strategic customers in queueing systems
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A model with mixed observation structure
A model with mixed observation structure

A model with mixed observation structure

- $M/M/1$ queue $(\lambda, \mu, \rho)$ with 2 types of customers:
A model with mixed observation structure


- $M/M/1$ queue ($\lambda$, $\mu$, $\rho$) with 2 types of customers:
  - Observing customers that see the queue length before deciding whether to join or balk.

- $M/M/1$ queue ($\lambda$, $\mu$, $\rho$) with 2 types of customers:
  - Observing customers that see the queue length before deciding whether to join or balk.
  - Uninformed customers.
A model with mixed observation structure


- $M/M/1$ queue ($\lambda$, $\mu$, $\rho$) with 2 types of customers:
- Observing customers that see the queue length before deciding whether to join or balk.
- Uninformed customers.
- Each arriving customer is observing with probability $p_o$ or uninformed with probability $p_u$ ($p_o + p_u = 1$).
A model with mixed observation structure


- $M/M/1$ queue ($\lambda$, $\mu$, $\rho$) with 2 types of customers:
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- Upon arrival, each customer decides to join or balk.
A model with mixed observation structure


- $M/M/1$ queue ($\lambda$, $\mu$, $\rho$) with 2 types of customers:
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- Upon arrival, each customer decides to join or balk.

- $R_o, R_u$: Service rewards for $o$-cust, $u$-cust.
A model with mixed observation structure

- $M/M/1$ queue ($\lambda$, $\mu$, $\rho$) with 2 types of customers:
  - Observing customers that see the queue length before deciding whether to join or balk.
  - Uninformed customers.
- Each arriving customer is observing with probability $p_o$ or uninformed with probability $p_u$ ($p_o + p_u = 1$).
- Upon arrival, each customer decides to join or balk.
- $R_o, R_u$: Service rewards for $o$-cust, $u$-cust.
- $C_o, C_u$: Waiting costs per time unit.
Extreme cases
Extreme cases

- $p_0 = 0$: Mixed model = Unobservable $M/M/1$ (E& H).
Extreme cases

- \( p_0 = 0 \): Mixed model = Unobservable \( M/M/1 \) (E& H).
- \( p_0 = 1 \): Mixed model = Observable \( M/M/1 \) (Naor).
Observing customers

An o-customer joins, if his position in the system (including him) is such that $R_o - C_o \geq \mu$. The best strategy of a customer against any strategy of the others is to join, if his position given that he joins is such that $n \leq n_e$, with $n_e = \lfloor \mu R_o C_o \rfloor$ (Naor's threshold).
Observing customers

- An o-customer joins, if his position $n$ at the system (including him) is such that $R_o - C_o \frac{n}{\mu} \geq 0$. 

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Strategic customers in queueing systems
Observing customers

- An o-customer joins, if his position $n$ at the system (including him) is such that $R_o - C_o \frac{n}{\mu} \geq 0$.
- The best strategy of a customer against any strategy of the others is to join, if his position $n$ given that he joins is such that
  $$n \leq n_e,$$
  with
  $$n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor$$ (Naor’s threshold).
Uninformed customers

The system is unobservable for uninformed customers. The strategic behavior of a $u$-customer regarding joining/balking is specified by a joining probability $q^\ast$. Suppose that all $o$-customers follow an $n^\ast$-threshold policy and the $u$-customers use a joining probability $q^\ast$. Under this $(n^\ast,q^\ast)$ strategy the number of customers in the system is a CTMC with diagram:

$$
\begin{array}{cccc}
0 & 1 & 2 & \ldots \\
\lambda^\ast_1 & \lambda^\ast_1 & \lambda^\ast_2 & \ldots \\
\mu & \mu & \mu & \ldots \\
\end{array}
$$

where 

$$
\lambda^\ast_1 = \lambda p_o + \lambda p_u q^\ast, \\
\lambda^\ast_2 = \lambda p_u q^\ast.
$$

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Strategic customers in queueing systems
The system is unobservable for uninformed customers.
Uninformed customers

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Strategic customers in queueing systems
Uninformed customers

- The system is unobservable for uninformed customers.
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Uninformed customers

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- The strategic behavior of a $u$-customer regarding joining/balking is specified by a joining probability $q_*$. 
- Suppose that all $o$-customers follow an $n_*$-threshold policy and the $u$-customers use a joining probability $q_*$. 
- Under this $(n_*, q_*)$ strategy the number of customers in the system is a CTMC with diagram

\[ \begin{array}{cccccc}
0 & 1 & 2 & n_* - 1 & n_* & n_* + 1 & n_* + 2 & \cdots \\
\lambda_1 & \mu & \lambda_2 & \lambda_2 & \lambda_2 & \cdots & \mu \\
\end{array} \]
The system is unobservable for uninformed customers.

The strategic behavior of a $u$-customer regarding joining/balking is specified by a joining probability $q_\ast$.

Suppose that all $o$-customers follow an $n_\ast$-threshold policy and the $u$-customers use a joining probability $q_\ast$.

Under this $(n_\ast, q_\ast)$ strategy the number of customers in the system is a CTMC with diagram

$$
\begin{align*}
&0 \quad 1 \quad 2 \quad \cdots \quad n_\ast - 1 \quad n_\ast \quad n_\ast + 1 \quad n_\ast + 2 \quad n_\ast + 3 \quad \cdots \\
&\lambda_{\ast 1} \quad \lambda_{\ast 1} \quad \lambda_{\ast 1} \quad \lambda_{\ast 2} \quad \lambda_{\ast 2} \quad \lambda_{\ast 2} \quad \lambda_{\ast 2} \\
&\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu
\end{align*}
$$

where $\lambda_{\ast 1} = \lambda p_o + \lambda p_u q_\ast$, $\lambda_{\ast 2} = \lambda p_u q_\ast$. 

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Stationary distrib. of the number of customers

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n^*,q^*) = \begin{cases} B^* \rho_{n^*1} & \text{if } 0 \leq n \leq n^* - 1, \\ B^* \rho_{n^*1} \rho_{n^*2} & \text{if } n \geq n^* , \end{cases} \]

where \( \rho^*_1 = \lambda^*_1 \mu, \rho^*_2 = \lambda^*_2 \mu \) and \( B^* = (1 - \rho^*_1)(1 - \rho^*_2) \).
Proposition

Stationary distribution of the number of customers:

\[
\pi_n = \pi_n(n^\ast, q^\ast) =
\begin{cases} 
B^\ast \rho_n^\ast 1 & \text{if } 0 \leq n \leq n^\ast - 1, \\
B^\ast \rho_n^\ast 1 \rho_{n^\ast - n}^\ast 2 & \text{if } n \geq n^\ast, 
\end{cases}
\]

where

\[
\rho_{n_1}^1 = \lambda_{n_1} \mu, \quad \rho_{n_2}^2 = \lambda_{n_2} \mu
\]

and

\[
B^\ast = \left(1 - \rho_{n_1}^1 \right) \left(1 - \rho_{n_2}^2 \right)^{-1}.
\]
Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n^*, q^*) = \begin{cases} B^* \rho_n^* 1 & \text{if } 0 \leq n \leq n^* - 1, \\ B^* \rho_n^* 1 \rho_n - n^* 2 & \text{if } n \geq n^*. \end{cases} \]

where \( \rho_1^* = \lambda_1^* \mu \) and \( \rho_2^* = \lambda_2^* \mu \) and 
\[ B^* = (1 - \rho_1^*) (1 - \rho_2^*) \frac{1}{1 - \rho_2^* - \rho_n^* + 1} + \rho_n^* 1 \rho_2^*. \]
Stationary distrib. of the number of customers

Proposition

Stationary distribution of the number of customers:

\[
\pi_n = \pi_n(n_*, q_*) = \begin{cases} 
B\rho_1^n & \text{if } 0 \leq n \leq n_* - 1, \\
B\rho_1^{n_*} \rho_2^{n-n_*} & \text{if } n \geq n_*,
\end{cases}
\]

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Stationary distrib. of the number of customers

Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n_*, q_*) = \begin{cases} 
B_\ast \rho_{\ast 1}^n & \text{if } 0 \leq n \leq n_\ast - 1, \\
B_\ast \rho_{\ast 1} \rho_{\ast 2}^{n-n_\ast} & \text{if } n \geq n_\ast,
\end{cases} \]

where

\[ \rho_{\ast 1} = \frac{\lambda_{\ast 1}}{\mu}, \quad \rho_{\ast 2} = \frac{\lambda_{\ast 2}}{\mu}. \]
Proposition

Stationary distribution of the number of customers:

\[ \pi_n = \pi_n(n_*, q_*) = \begin{cases} 
B_* \rho_{*1}^n & \text{if } 0 \leq n \leq n_* - 1, \\
B_* \rho_{*1} \rho_{*2}^{n-n_*} & \text{if } n \geq n_*,
\end{cases} \]

where

\[ \rho_{*1} = \frac{\lambda_{*1}}{\mu}, \quad \rho_{*2} = \frac{\lambda_{*2}}{\mu} \]

and

\[ B_* = \frac{(1 - \rho_{*1})(1 - \rho_{*2})}{1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}^{n_*} \rho_{*2}}. \]
Stationary mean number of customers

The mean stationary number of customers in the system is

\[ E(n^*,q^*) = (1 - \rho^*) \left[ \frac{(n^* - 1)\rho n^* + 1 - n^*\rho n^*\rho^*}{1 - \rho^*} \right]. \]
Stationary mean number of customers

Proposition (continued)

The mean stationary number of customers in the system is

\[ E(n^*, q^*) (N) = (1 - \rho^* 2) \left[ (n^* - 1) \rho^* + 1 \right] \]

\[ + (1 - \rho^* 1) \left[ n^* \rho^* 1 - (n^* - 1) \rho^* 1 \right] \]

\[ + (1 - \rho^* 2) \left[ 1 - \rho^* 2 - \rho^* n^* + 1 \right] \]

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Stationary mean number of customers

Proposition (continued)

The mean stationary number of customers in the system is

\[ E(n^*,q^*) \times (N) = (1 - \rho^*2)(n^* - 1)\rho n^* + 1 - n^*\rho n^*\rho + (1 - \rho^*1)(1 - \rho^*2 - \rho n^*\rho + 1 + \rho n^*\rho^*2). \]
The mean stationary number of customers in the system is

\[ E_{(n^*, q^*)}(N) = \frac{(1 - \rho_2)[(n^* - 1)\rho_1^{n^*+1} - n^*\rho_1^{n^*} + \rho_1]}{(1 - \rho_1)[1 - \rho_2 - \rho_1^{n^*+1} + \rho_1^{n^*}\rho_2]} + \frac{(1 - \rho_1)[n^*\rho_1^{n^*} - (n^* - 1)\rho_1^{n^*}\rho_2]}{(1 - \rho_2)[1 - \rho_2 - \rho_1^{n^*+1} + \rho_1^{n^*}\rho_2]} \]
Proposition (continued)

The mean stationary number of customers in the system is

\[
E(n_*,q_*)(N) = \frac{(1 - \rho_{*2})[(n_* - 1)\rho_{*1}^{n_*+1} - n_*\rho_{*1} + \rho_{*1}]}{(1 - \rho_{*1})[1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}\rho_{*2}]} + \frac{(1 - \rho_{*1})[n_*\rho_{*1}^{n_*} - (n_* - 1)\rho_{*1}\rho_{*2}]}{(1 - \rho_{*2})[1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}\rho_{*2}]}.
\]
Expected net benefit

Suppose that the customers follow an $(n^\ast,q^\ast)$ strategy. Consider a tagged $u^\ast$-customer upon arrival. His expected net benefit, if he decides to join, is

$$U(n^\ast,q^\ast) = R_u - C_u E(n^\ast,q^\ast)(N) + \mu.$$ 

$U(n^\ast,q^\ast)$ is a decreasing function of $q^\ast$ for any fixed $n^\ast$ (a coupling argument shows that $N$ is stochastically increasing in $q^\ast$).
Expected net benefit

Proposition
Expected net benefit

**Proposition**

Suppose that the customers follow an \((n_*, q_*)\) strategy.
Proposition

Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged \(u\)-customer upon arrival.
Expected net benefit

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Proposition

Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged \(u\)-customer upon arrival. His expected net benefit, if he decides to join is

\[
U(n_*, q_*) = R_u - C_u \frac{E(n_*, q_*)(N) + 1}{\mu}.
\]
Expected net benefit

Proposition

Suppose that the customers follow an \((n_*, q_*)\) strategy. Consider a tagged \(u\)-customer upon arrival. His expected net benefit, if he decides to join is

\[
\mathcal{U}(n_*, q_*) = R_u - C_u \frac{E(n_*, q_*)(N) + 1}{\mu}.
\]

- \(\mathcal{U}(n_*, q_*)\) is a decreasing function of \(q_*\) for any fixed \(n_*\) (a coupling argument shows that \(N\) is stochastically increasing in \(q_*\).
Equilibrium strategies

Theorem

Let \( n = \lfloor \mu R \rfloor \), \( E_0 = E(n, 0)_{\mathcal{N}} + 1 \) and \( E_1 = E(n, 1)_{\mathcal{N}} + 1 \).

Case I: \( E_0 \geq \mu R_{\mathcal{U}} \).

The unique equilibrium is \((n, 0)\).

Case II: \( E_0 < \mu R_{\mathcal{U}} < E_1 \).

The unique equilibrium is \((n, q_e)\), where \( q_e \) is the unique solution of the equation \( E(n, q)_{\mathcal{N}} + 1 = \mu R_{\mathcal{U}} \) in \((0, 1)\), with respect to \( q \).

Case III: \( E_1 \leq \mu R_{\mathcal{U}} \).

The unique equilibrium is \((n, 1)\).
Equilibrium strategies

Theorem

Let $n_e = \lfloor \mu R o C \rfloor$, $E_0 = E(n_e, 0)(N) + 1$, and $E_1 = E(n_e, 1)(N) + 1$.

**Case I:** $E_0 \geq \mu R u C$. The unique equilibrium is $(n_e, 0)$.

**Case II:** $E_0 < \mu R u C < E_1$. The unique equilibrium is $(n_e, q_e)$, where $q_e$ is the unique solution of the equation $E(n_e, q_e)(N) + 1 = \mu R u C$ in $(0, 1)$, with respect to $q_e$.

**Case III:** $E_1 \leq \mu R u C$. The unique equilibrium is $(n_e, 1)$.
Equilibrium strategies

Theorem

Let \( n_e = \left\lfloor \frac{\mu R_o}{C_o} \right\rfloor \),
Equilibrium strategies

**Theorem**

Let $n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor$, $E_0 = E_{(n_e,0)}(N) + 1$ and $E_1 = E_{(n_e,1)}(N) + 1$. 
Equilibrium strategies

Theorem

Let \( n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor \),
\[ E_0 = E_{(n_e,0)}(N) + 1 \] and \[ E_1 = E_{(n_e,1)}(N) + 1. \]

Case I: \( E_0 \geq \frac{\mu R_u}{C_u} \).

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Theorem

Let \( n_e = \lfloor \frac{\mu R_0}{C_o} \rfloor \),
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**Case I:** \( E_0 \geq \frac{\mu R_u}{C_u} \). The unique equilibrium is \((n_e, 0)\).
Theorem

Let $n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor$, $E_0 = E(n_e,0)(N) + 1$ and $E_1 = E(n_e,1)(N) + 1$.

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Strategic customers in queueing systems
Equilibrium strategies

Theorem

Let \( n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor \),
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Case II: \( E_0 < \frac{\mu R_u}{C_u} < E_1 \). The unique equilibrium is \((n_e,q_e)\), where \( q_e \) is the unique solution of the equation

\[
E_{(n_e,q)}(N) + 1 = \frac{\mu R_u}{C_u}
\]

in \((0,1)\), with respect to \( q \).
Equilibrium strategies

**Theorem**

Let \( n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor \),
\[
E_0 = E(n_e,0)(N) + 1 \quad \text{and} \quad E_1 = E(n_e,1)(N) + 1.
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E(n_e, q)(N) + 1 = \frac{\mu R_u}{C_u}
\]
in \((0, 1)\), with respect to \( q \).

**Case III:** \( E_1 \leq \frac{\mu R_u}{C_u} \).
Equilibrium strategies

Theorem

Let \( n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor \),
\( E_0 = E_{(n_e,0)}(N) + 1 \) and \( E_1 = E_{(n_e,1)}(N) + 1 \).

Case I: \( E_0 \geq \frac{\mu R_u}{C_u} \). The unique equilibrium is \((n_e, 0)\).

Case II: \( E_0 < \frac{\mu R_u}{C_u} < E_1 \). The unique equilibrium is \((n_e, q_e)\), where \( q_e \) is the unique solution of the equation

\[
E_{(n_e,q)}(N) + 1 = \frac{\mu R_u}{C_u}
\]

in \((0, 1)\), with respect to \( q \).

Case III: \( E_1 \leq \frac{\mu R_u}{C_u} \). The unique equilibrium is \((n_e, 1)\).
Social benefit per time unit

The social benefit per time unit under a strategy \((n^*, q^*)\) is

\[
S(n^*, q^*) = n^* - \sum_{n=0}^{n_e} \pi_n(n^*, q^*) \lambda_p o(R_o - C_o + 1) + \sum_{n=0}^{\infty} \pi_n(n^*, q^*) \lambda_p u q^* (R_u - C_u + 1) \mu .
\]

It is too complicated to reduce it in closed form and to maximize. For each \(n^* = 0, 1, 2, ..., n_e\) we find \(q^*\) that maximizes \(S(n^*, q^*)\) and then choose the one that gives the overall maximum, namely \((n_{soc}, q_{soc})\).

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The social benefit per time unit under a strategy $(n_*, q_*)$ is
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\[
S(n_*, q_*) = \sum_{n=0}^{n_*-1} \pi_n(n_*, q_*) \lambda p_o \left( R_o - C_o \frac{n + 1}{\mu} \right) \\
+ \sum_{n=0}^{\infty} \pi_n(n_*, q_*) \lambda p_u q_* \left( R_u - C_u \frac{n + 1}{\mu} \right).
\]
The social benefit per time unit under a strategy \((n_*, q_*)\) is

\[
S(n_*, q_*) = \sum_{n=0}^{n_*-1} \pi_n(n_*, q_*) \lambda p_o \left( R_o - C_o \frac{n+1}{\mu} \right) \\
+ \sum_{n=0}^{\infty} \pi_n(n_*, q_*) \lambda p_u q_* \left( R_u - C_u \frac{n+1}{\mu} \right).
\]

It is too complicated to reduce it in closed form and to maximize.
Social benefit per time unit

- The social benefit per time unit under a strategy 
  \((n_*, q_*)\) is

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+ \sum_{n=0}^{\infty} \pi_n(n_*, q_*) \lambda p_u q_* \left( R_u - C_u \frac{n+1}{\mu} \right).
\]

- It is too complicated to reduce it in closed form and to maximize.

- For each \(n_* = 0, 1, 2, \ldots, n_e\) we find \(q_*\) that maximizes \(S(n_*, q_*)\) and then choose the one that gives the overall maximum, namely \((n_{soc}, q_{soc})\).
Conclusions

Effect of $p_o$ on the social benefit:

The optimal social benefit per time unit seems to be an increasing or unimodal function of $p_o$.

There exists a somehow 'ideal' fraction of observing customers for the society. In many cases, it is strictly between 0 and 1.

Effect of $p_o$ on the price of anarchy (PoA), defined as $\text{PoA} = \frac{S(n_{soc}, q_{soc})}{S(n_e, q_e)}$:

In most cases, PoA is a convex smooth function of $p_o$.

Again this shows the existence of an 'ideal' fraction of observing customers for the society.

But there are cases where the graph of PoA shows peculiar behavior with very abrupt changes.
Conclusions

- Effect of $p_o$ on the social benefit:
Conclusions

- **Effect of $p_o$ on the social benefit:**
  The optimal social benefit per time unit seems to be an increasing or unimodal function of $p_o$. 

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Conclusions

- **Effect of $p_o$ on the social benefit:**
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Conclusions

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  In many cases, it is strictly between 0 and 1.
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- **Effect of $p_o$ on the social benefit:**
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  But there are cases where the graph of PoA shows peculiar behavior with very abrupt changes.
Other models with mixed information structure
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Other models with mixed information structure

Part V:
Strategic customers in models with partial information structure
Models with partial information structure

- **M/M/1 queue** with the same dynamics and operational parameters ($\lambda$, $\mu$, and $\rho$) but with some additional characteristic.
  - Additional characteristic regarding the server: setup times, vacations, random environment that influences the arrival/service rates, etc.
  - **Same reward-cost structure** ($R$ and $C$).

Upon arrival, a customer decides whether to join or balk.

There are various informational cases:

1. Observe both the queue length and the server's status.
2. Observe only the queue length.
3. Observe only the server's status.
4. Observe nothing.
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A model with partial information structure

Burnetas, A. and Economou, A. (2007) Equilibrium customer strategies in a single server Markovian queue with setup times. Queueing Systems. M/M/1 queue with setup times. When the server becomes idle, he deactivated immediately. When a new customer arrives at an empty system, a setup process starts. The setup times are Exp(θ) random variables.
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The model

\[ N(t) \]: Number of customers in the system.

\[ I(t) \]: State of the server.

\{ \[ N(t), I(t) \] \} is a CTMC with transition diagram:

\[ 0 \rightarrow 0 \]

\[ 0 \rightarrow 1 \]

\[ 0 \rightarrow 2 \]

\[ 0 \rightarrow n \]

\[ 0 \rightarrow n + 1 \]

\[ \lambda \rightarrow \lambda \] \ldots

\[ \mu \rightarrow \mu \] \ldots

\[ \theta \rightarrow \theta \] \ldots

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Strategic customers in queueing systems
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![CTMC Transition Diagram]

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Strategic customers in queueing systems
Informational cases

- Fully observable case: Customers observe both $N(t)$ and $I(t)$.
- Almost observable case: Customers observe $N(t)$ but not $I(t)$.
- Almost unobservable case: Customers observe $I(t)$ but not $N(t)$.
- Fully unobservable case: Customers do not observe $N(t)$ nor $I(t)$. 

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Numerical results I
Figure 3: Equilibrium social benefit for various information levels with respect to $R$. 

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Numerical results II

The difference in the equilibrium social benefits is small between the fully and almost observable case, when $\theta$ is high. But it may be large for low values of $\theta$. There are quite significant differences between the observable and the unobservable cases. The lowest optimal social benefit corresponds to the fully case. For low values of $R$ the optimal social benefit under ao may surpass the optimal social benefit under fo.
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For low values of $R$ the optimal social benefit under almost observable may surpass the optimal social benefit under fully observable.
Part VI: Final remarks
Conclusions

Controlling the information provided to the customers in various ways can improve the equilibrium social benefit. This indirect influence of the customers can be less disturbing for them than imposing admission fees etc. The throughput of the system can be also controlled by tuning the information provided to the customers.
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Bibliography I


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Bibliography II

B. Some papers treating intermediate strategic situations between observable and unobservable queues for non-Markovian models.


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Questions?