Discussion on

Risk Analytics

Markov Lecture by David Yao

Risk and Central Counterparties

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INFORMS – Philadelphia
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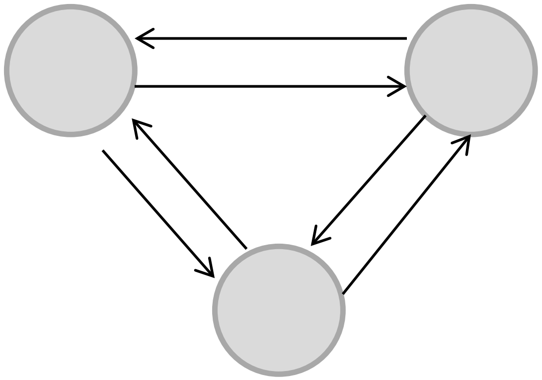
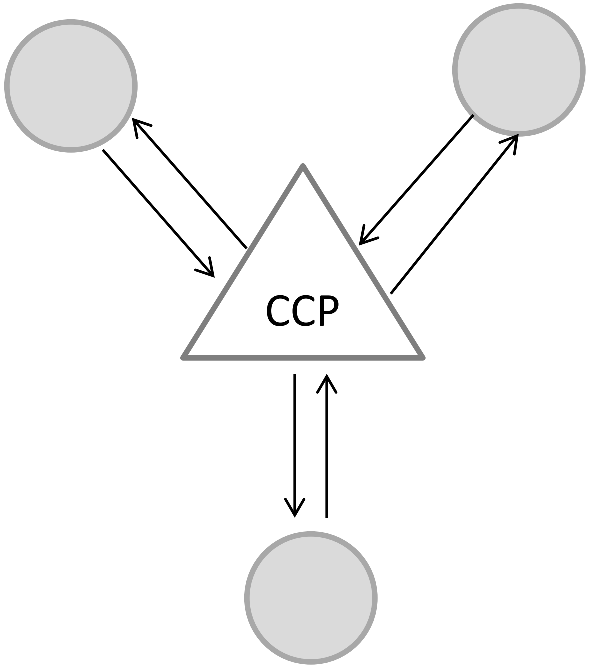
OTC vs CCP

Bilateral over-the-counter market

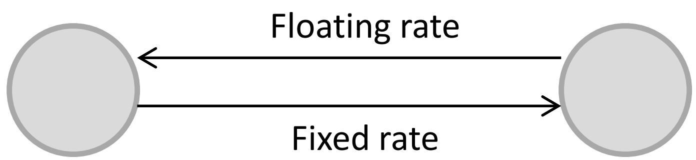


Centrally cleared market

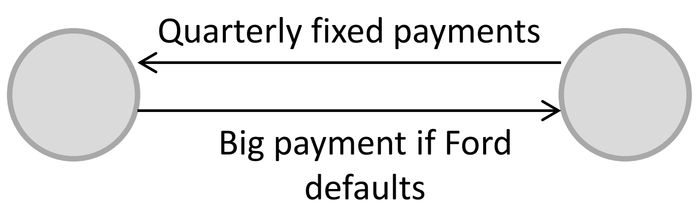


Swaps: Two Examples

- Interest rate swap



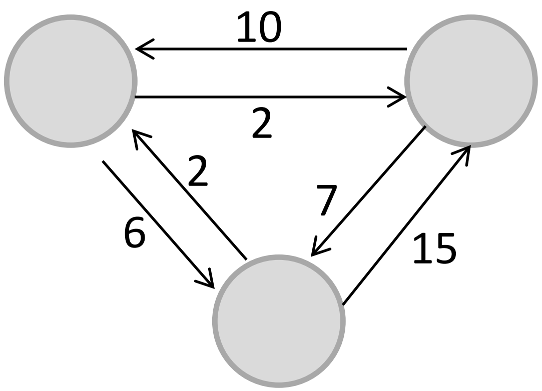
- Credit default swap



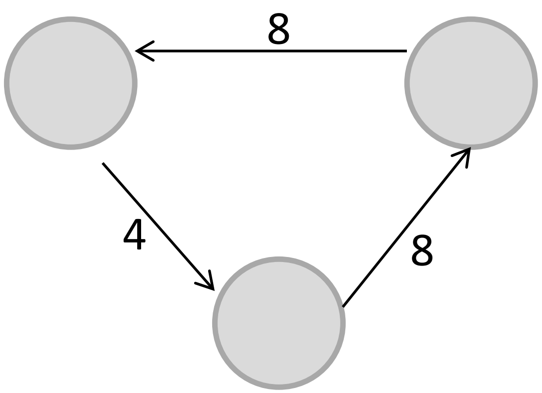
- As of Dec 2013, total market value outstanding of $18 trillion, down from pre-crisis peak

Netting Reduces Total Counterparty Risk

Over-the-counter market



Bilateral netting



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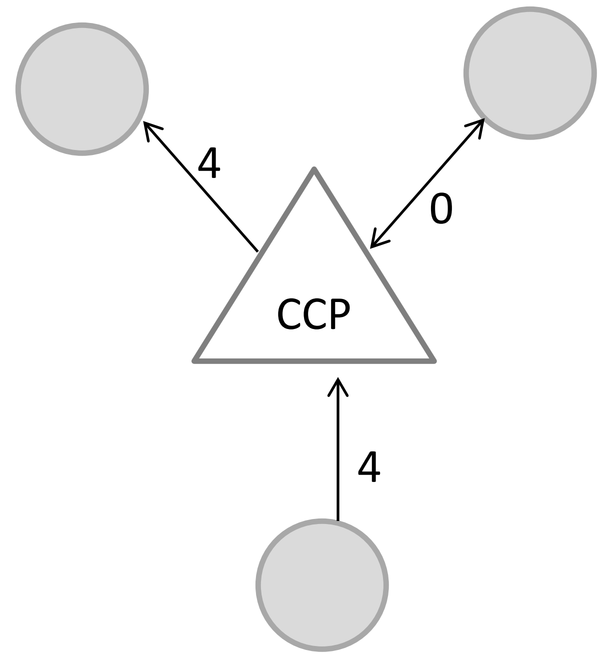
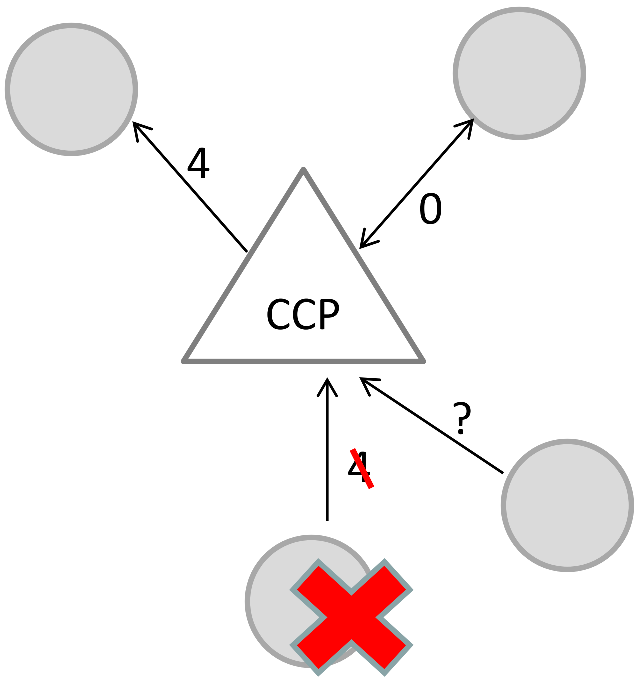


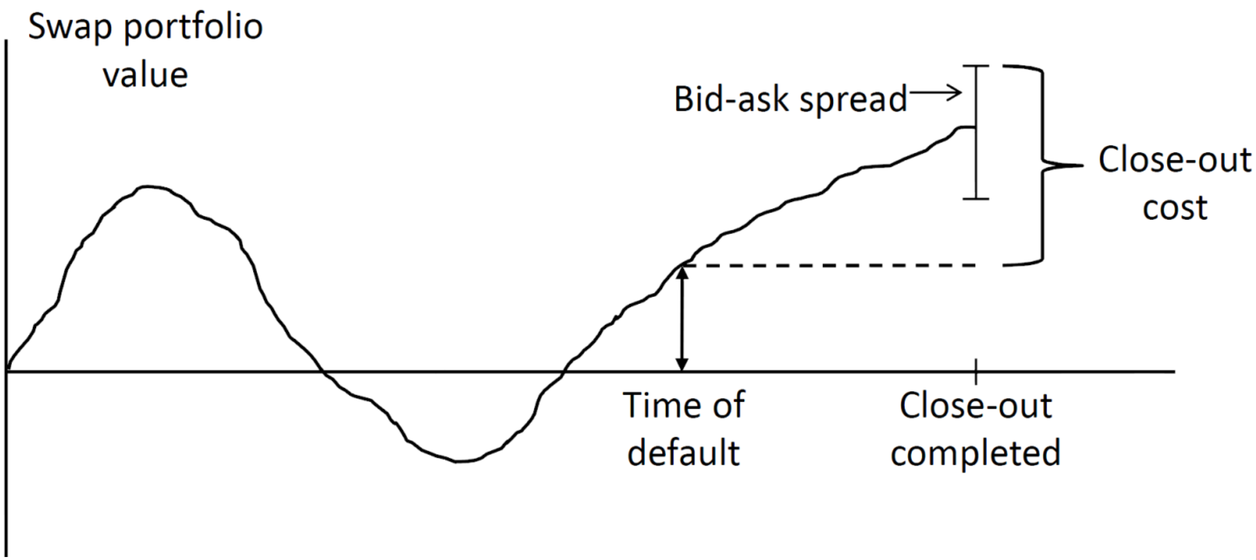
The CCP always has a matched book and zero net exposure, in theory

But What Happens If A Clearing Member Fails?

- If a clearing member fails, the CCP needs to restore a matched book but may incur a loss in doing so
- The failure of a CCP could cascade to failures of other clearing members
- CCPs are a potential source of systemic risk



Margin Protects the CCP Against Default Risk



- CCP holds margin from each clearing member to absorb potential losses over a liquidation period of 5-10 days

- This is “initial” margin as opposed to variation margin

- Clearing members also contribute to a default fund to cover larger losses

The CCP's Default Waterfall

Defaulting Member's Margin
Defaulting Member's Default Fund Contribution
CCP's Capital
Surviving Member's Default Fund Contributions
Assessment Rights on Surviving Members

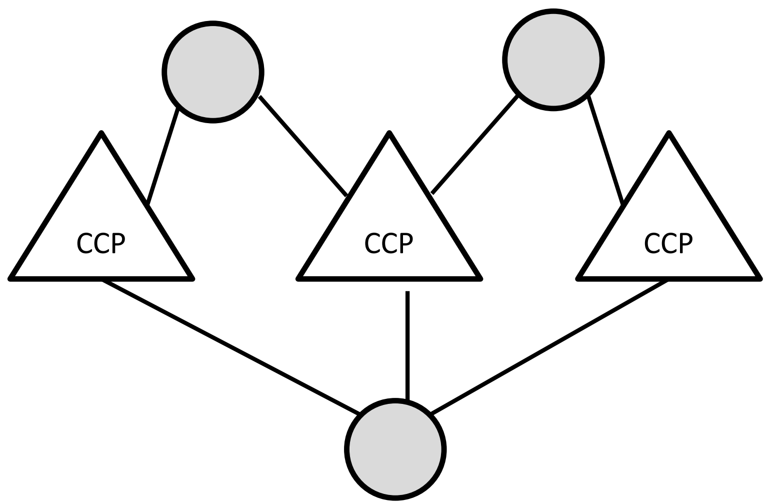
Total resources should be sufficient to cover losses from the failure of the two members with the largest positions

Risk Modeling Questions

- How much margin should the CCP collect to cover losses with 99% probability?
- How should losses beyond this level be allocated among the CCP and surviving members? In particular, how should the default auction be designed?
- When is central clearing “systemically safer” than bilateral clearing?
- Do current and proposed rules create incentives for dealers to clear through CCPs rather than bilaterally?

Risks From Overlapping Membership

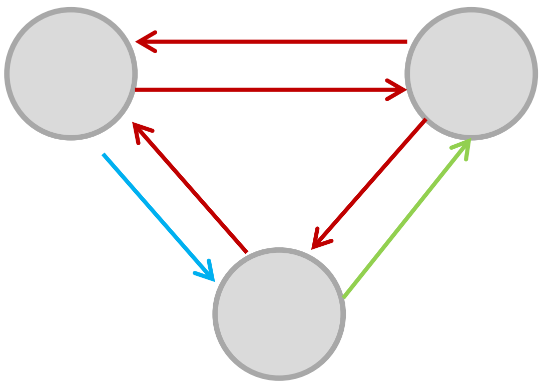
- The same dealers are members of many CCPs
- If a dealer defaults at one CCP, it defaults at all CCPs
- CCPs need to compensate for “hidden” positions at other CCPs in setting margin requirements

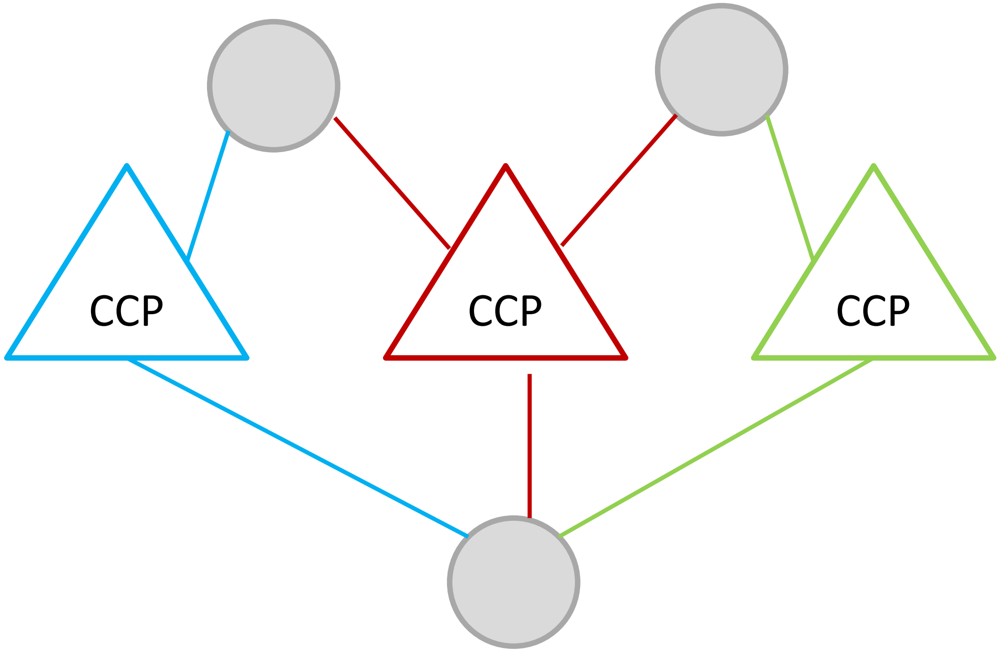


- See Kai Yuan’s talk tomorrow, joint work with Ciamac Moallemi

Netting Tradeoff With Multiple Types of Swaps

- Bilateral netting may be more effective





- See Duffie and Zhu (2011), Cont and Kokholm (2014)

Costs and Incentives

- Trading through a CCP: Each clearing member
  - posts margin to the CCP, tied to the risk of its trades
  - contributes to the CCP's default fund
  - Incurs a capital charge through its exposure to the CCP

- Trading bilaterally: Under proposed rules, each party
  - posts margin to the other
  - Incurs a capital charge through its exposure to the other party

In current work with Samim Ghamami, Federal Reserve, we compare these costs and calibrate the model to confidential exposure data

Perspective of a Single Dealer

- Dealer has a fixed set of trades *V*
- Each trade (*j*,*k*) has a counterparty *j* and a type *k* (interest rate swap, CDS, etc.)
- Dealer will decide which trades to clear through a CCP and which to clear bilaterally
- Each CCP clears one type of swap

- So, dealer partitions all trades into portfolios: one portfolio for each counterparty, and one portfolio for each CCP
- Total cost

$$\sum_{\text{counterparties } j} a_j \sigma_j + \sum_{\text{types } k} b_k \nu_k$$

where σ_j and ν_k are the portfolio standard deviations

Standard Deviation of a Subset

Given a set of random variables $\{X_1, X_2, \ldots, X_N\}$, let $V = \{1, 2 \ldots, N\}$. For $S \subseteq V$ define

$$\sigma(S) = \text{Standard Deviation}\left(\sum_{i \in S} X_i\right).$$

Let Σ be the covariance matrix of $(X_1, X_2, \ldots, X_N)$, with entries $\Sigma_{ii} = \sigma_i^2$ and $\Sigma_{ij} = \sigma_{ij}$, $j \neq i$.

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Proposition. Suppose Σ satisfies the following two conditions:

(i) $\sigma_{ij} \leq 0$, for all distinct $i, j = 1, \ldots, N$;

(ii) $\sigma_i^2 \geq -2 \sum_{j \neq i} \sigma_{ij}$, for all $i = 1, \ldots, N$.

Then $\sigma : 2^V \mapsto \mathbb{R}$ is increasin and submodular,

$$\sigma(A \cup B) + \sigma(A \cap B) \leq \sigma(A) + \sigma(B), \quad A, B \subseteq V$$

Properties of the Cost Function

- Under these conditions, each portfolio standard deviation (as a function of the set of included trades) satisfies
  - $\sigma(\emptyset) = 0$
  - $\sigma(\cdot)$ is increasing
  - $\sigma(\cdot)$ is submodular

- This defines a *polymatroid rank function*, in the sense of Edmonds (1973), which I first learned about from David through

  Shanthikumar and Yao (1992) “Multiclass Queueing Systems: Polymatroid Structure and Optimal Scheduling Control,” *Operations Research.*

- These properties are clearly preserved by positive linear combinations

The Dealer's Optimization Problem

- We can write the dealer's problem of minimizing

$$\sum_{\text{counterparties } j} a_j \sigma_j + \sum_{\text{types } k} b_k \nu_k$$

As

$$\min_{S \subseteq V} \rho_1(S) + \rho_2(V - S)$$

where ρ_1 and ρ_2 are polymatroid rank functions

This is a *polymatroid intersection problem.* Edmonds (1973) showed that the solution coincides with

$$\{\max \sum_i x_i : x \in P(\rho_1) \cap P(\rho_2)\}$$

where $P(\rho)$ is the polymatroid

$$P(\rho) = \{x \in \mathbb{R}^{|V|} : x(A) \leq \rho(A), \forall A \subseteq V\}$$

Universal Bases

- The dealer's optimal cost

$$\{\max \sum_i x_i : x \in P(\rho_1) \cap P(\rho_2)\}$$

has a representation as (Nakamura 1988, Fujishige and Nagano 2009)

$$\sum_i (y_i^* \wedge z_i^*), \quad y^* \in B(\rho_1), z^* \in B(\rho_2)$$

where $B(\rho)$ is the base of $P(\rho)$,

$$B(\rho) = \{x \in P(\rho) : x(V) = \rho(V)\}$$

- This is a separable decomposition into a sum of costs over trades
- The cost per trade is the minimum of costs of trading bilaterally or through a CCP

Consequences

- This connection opens up a wide set of tools to characterize (and calculate) the optimal solution
- In particular, using the idea of universal bases developed by Nakamura (1988) and Fujishige and Nagano (2009)
  - Results characterize how this optimal cost decomposition varies parametrically with cost coefficients

- Current work: stitching together the optimization problem of multiple dealers trading with each other

Summary

- The transformation of the OTC derivatives market to a centrally cleared market raises many interesting research problems
  - Including problems of stochastic modeling and optimization

- Tomorrow: **TA06 Systemic Risk session, 8:00-9:30**
  - Talks on CCPs by Kai Yuan and Agostino Capponi

- Congratulations and thanks, David, for this year’s Markov Lecture