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Background information

More than a century after the seminal work of Louis Bachelier, the quantitative approach to financial markets has become omnipresent.

Nowadays, many investment outfits specialize in research, development, and implementation of systematic trading strategies, while other active asset managers have added quantitative strategies to their business lines. Individual clients may also now delegate management of their portfolios to robo-advisors.

These are just a few manifestations of the changing landscape, and we believe that quantitative portfolio management will become ever more important because of the discipline offered by the scientific approach and full automation of the investment process.

The QuantAwards competition offers students a unique opportunity to showcase their creativity and their understanding of this highly timely subject.

QuantAwards

The Quantitative Finance Competition

Private Equity Target Selection Using Artificial Intelligence

August 2022

I. Introduction

Private Equity (PE) is known as the superior form of capitalism (Swensen, 2017). Aggressive use of leverage providing tax advantages, high-powered management incentives, freedom from restrictive public company regulations, and superior risk management as well as focus on cash flow margin improvement result in continued above-average performance. Historical data comparing private equity returns to regional listed Indices show clear evidence for the out-performance of this investment type (Bain&Company, 2022).

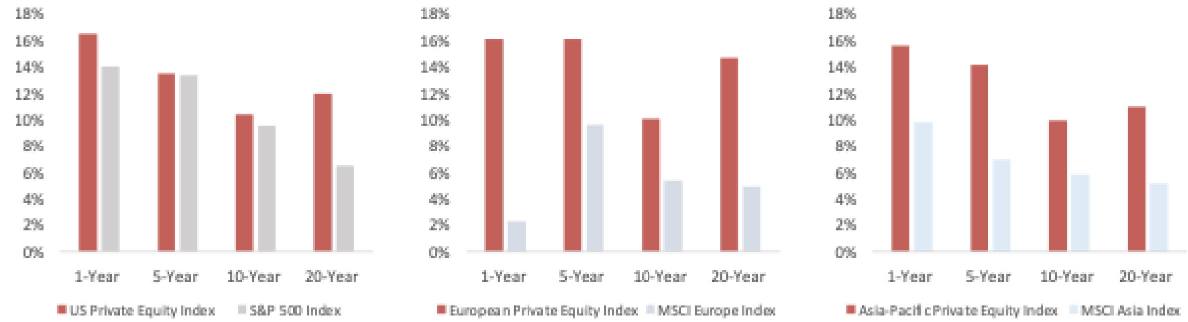

Figure 1: Private Equity Performance versus regional Index (Bain&Company, 2022) (CambrigeAssociates, 2018)

However, this sovereignty of private equity comes with significantly higher default risk, volatility and liquidity concerns (McStay, 2020) which shifts additional importance to the management team and the selection of the target. Harris et al. (2014) have studied this persistent out-performance of equity investments by examining nearly 1,400 private equity and venture capital funds since 1984 across 25 vintages each. They found that each dollar invested in the average fund returned at least 20% more than a dollar invested in the S&P 500 and that the a better-than-average manager is likely to out-perform his or her next fund. Table 1 shows this evidence of persistence in performance and shows that a manager is likely to outperform his or her current fund if the previous one outperformed too. This indicates that the skills of the general partner of a private equity firm, who is responsible for the investment strategy, plays a crucial role in its success. But what are the determinants for a successful private equity investment and how important are the skills of the general partner (GP) really?

This working paper is an excerpt from my Master's degree, in which I developed an artificial intelligence (AI) that is able to select and predict private equity targets. In the following, I briefly present the relevance of the topic for the industry, the theoretical framework, the data examined and the results of the AI.

		Current Fund			
		1st Q	**2nd Q**	**3rd Q**	**4th Q**
Previous Fund	**1st Q**	43%	25%	23%	10%
	2nd Q	28%	31%	28%	14%
	3rd Q	21%	27%	33%	18%
	4th Q	13%	26%	26%	35%

Table 1: Evidence of Persistence in Performance (Harris et al., 2014) (McStay, 2020)

II. Literature

The investigation of relevant criteria is part of the heterogeneous field of investigation of value generation in private equity and takes two different approaches. Predicting future takeover targets based on their financial ratios is relevant from an investment strategy perspective in order to achieve the usual "bid premium" of 10-50% (Bloomberg, 2022), while the other approach investigates why certain leveraged buy-outs (LBOs) are more successful than others by analysing which company characteristics act as relevant determinants. However, both literature reviews, when considered separately, come to similar conclusions regarding the factors that influence the selection of an LBO target. An in-depth search of the current literature was conducted throughout the original paper, while only a conclusion is presented here. A table with the relevant results can be found in the appendix.

The investigation shows clear evidence that key financial indicators such as good operating margins, high free cash flow (FCF), stable income, and company growth are decisive factors, and is confirmed by several studies such like Palepu (1986), Jansen (1987), and Kranz and Gustafsson (2020). In addition, undervaluation of a company increases the likelihood of becoming a takeover target, as shown by Bermann (2006) and Opler and Titman (1993). Contradictory results emerge in the consideration of business risk. We know from the Capital Structure Theory that a low business risk is crucial for an effective capital structure (Modigliani and Miller, 1958). However, several studies such as Le Nadant and Perdreau (2006) show that higher business risk is more likely to lead to acquisitions, which can be explained by the superiority of the PE investor , whose skills allow for a better handling of increased risk, as he makes decisions based on a broader knowledge that runs counter to the public market and thus exceeds the efficient market hypothesis (EMH). It is precisely this effect that needs to be investigated, which is what the method selected here is capable of doing.

While some studies classify the tax rate and the financial visibility of a company before acquisition as a relevant factor, the insignificant results of Krantz and Gustafsson (2020) reflect the assumptions of the theoretical framework in private equity. Their hypothesis, that the tax level of a company has an impact of its acquisition is seen as a weak argument, since taxation of a company is highly dependent on the sector in which the company operates. Moreover, the tax rate is closely intertwined with the jurisdiction of a company and can be actively influenced by the investors after acquisition. We know that PE style investing based on capital structure theory takes advantage of tax benefits, but this has no implications for the pre-acquisition tax rate, only for the potential factors that influence the subsequent tax rate, such as industry, capital expenditures, and country.

Another weakness of the literature is that - with the exception of a few studies with very small samples such as Brar et al. (2009) - an potential undervaluation is only calculated on the basis of the ratios reported and is not considered in relation to other companies or the market surrounding. This study attempts to overcome this weakness, by implementing industry averages to consider all factors relative to the company's environment. Furthermore, recent research also shows that an exclusive consideration of public-to-private (P2P) deals has the advantage that more data is available, which on the other hand leads to unavoidable distortions in the data sample (Acharya et al.,2009). In this paper, therefore, a wider range of LBO deals will be considered.

Overall, the investigation of the literature shows, that the different approaches up to date have in common that they try to identify the relevant financial and non-financial criteria of a potential acquisition candidate and apply these findings by means of a statistical model to identify LBO targets. Thus, they use simple or semi-simple statistical methods to see which numerical variables are able to scientifically explain the selection of a specific company without paying attention to the ability of the manager in charge, the GP. We know, that research and theory of PE investing show that the qualitative capabilities of the general partners play a central role in target identification, and that some identified financial criteria can be explained by the underlying theory (Harris, 2014). However, statistical models have difficulties to successfully apply qualitative factors (Hedges, 2014) and even if they are powerful in describing correlations, description cannot be the only research goal. In summary: Research has successfully mapped the superiority of private equity investing and addressed the importance of management and the takeover target. However, until now these two findings have not been considered and studied together. In addition to the extension of the factors considered, a clear gap was identified in trying to successfully address investor capabilities and identify clear non-financial criteria and design predictions for LBO candidates using more powerful computer models. The aim of this scientific work therefore is, in addition to broadening the criterion studied, to break the limitation of statistical methods and to investigate whether we can learn, better understand and even apply the superior selection of PE investors.

III. Hypothesis

Based on the theoretical framework of PE investing and the literature review conducted, the research hypothesis of this paper is, that the superior skill of private equity firms has a significant influence on the success of investments and that determinants should therefore subsequently be considered in relation the respective market in order to get a reflected picture of the company’s situation. Furthermore, analogous to the theories stated in the Variables section, subsidiary hypotheses are put forward that state:

- Certain firm characteristics, such as potential for a stable cash flow or efficiency margins, must be met for a successful LBO investment.
- The most relevant determinant for the selection of a target is the relative under- performance compared to the company’s peers.
- Private equity investors prefer cheaper companies that can be brought back to the top of the respective industry on the basis of the theoretical framework.
- The conditions in the company’s respective market have a significant influence on its suitability as an LBO target.
- The country in which the company is based also determines whether a company should be considered as a target.

IV. Methodology

Data Sample

For the research, a total of 1,420 global private equity deals between 2018 and 2021 were extracted using advanced search in Bureau van Dijk's Zephyr M&A database. Financial data from reporting in the year prior to the acquisition was used and matched with reporting from 67,512 companies that were not involved in an acquisition. In total, the underlying data sample contains companies from over 60 countries and 24 industries.

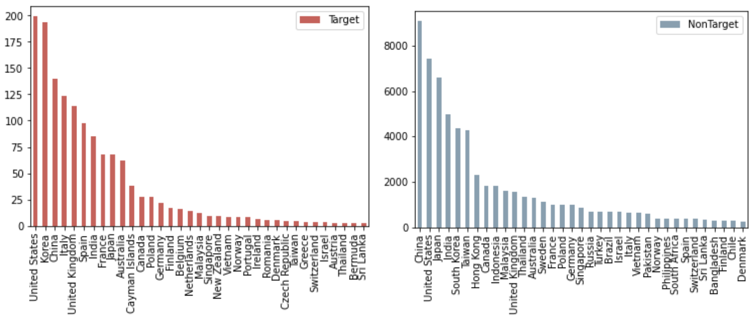

Figure 2: Target Distribution by Country

Based on the literature review, the theoretical framework for private equity investments and interviews with industry experts, a total of 21 financial and non-financial determinants were calculated and related to the respective industry average for the year using data from Capital IQ. The selected variables can be classified by defining five theories: Firm Characteristics Theory, Under-performance Theory, Undervaluation Theory, Industry/Market Conditions Theory, and Country-level Characteristics Theory.

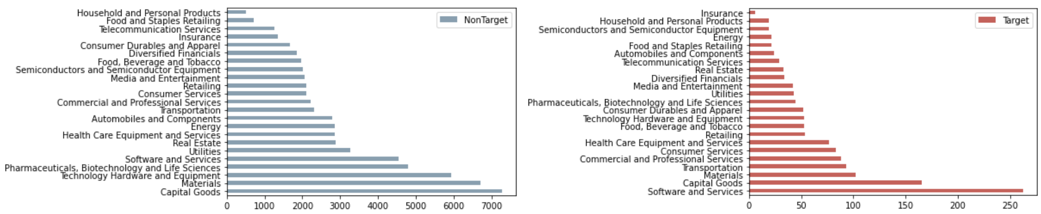

Figure 3: Target Distribution by Industry

Variables

As outlined before, the literature identified financial criteria that allow clear conclusions to be drawn about the suitability of a company as an LBO target, which is also confirmed by the theoretical framework of PE investing. This forms the basis for the Firm Characteristics Theory, which states that specific financial ratios and operational characteristics must be given in order to support the motivation of PE investors.

Firm Characteristics Theory

Based on the theoretical framework of private equity investing, there are specific characteristics in the financial health of a company that must be met.

- The growth of a company's turnover is an indicator of its health, the quality of its management and the conditions in the market. Sales_GR describes the growth of sales in the year before and of the deal.
- Margins are indicators of the efficiency of a company. EBITDA_M and EBIT_M are calculated as a percentage of total sales.
- Conclusions on the efficiency and growth of a company can also be drawn from net assets and depreciation amortisation. NA_M and DA_M are also considered relative to total sales.
- Free cash flow is important because the high leverage of an LBO requires available capital to repay interest. It is also an indicator of the efficiency and management of the company. FCF_M is, as already described, IS calculated by NI_Y1 + D&A

An extension of the previous theory is the under-performance theory, which states that these hard financial criteria must be considered relative to their environment. Previous studies such as Weir et al. (2005) and Brar et al. (2009) recommend that a company should be considered relative to their environment, but lack the inclusion of industry values of the respective companies. However, as a central element of the PE investment process, the comparable companies analysis is a fundamental component of the company's value creation strategy and serves as an indication of possible scope for

Under Performance Theory

Operational metrics should always be considered relative to the companies' peer group to identify room for potential improvement operational improvement. The theory suggests that PE investors actively seek out under performing companies based on their superior capabilities, which are then re positioned and improved over the long-term time horizon (McStay 2020).

- The dummy variable D_VARIABLE indicates whether the performance of a company is below or above the industry average. 1 is under-performance, 0 is over-performance.
- Furthermore, all firm characteristics variables should be considered with their total distance from the sector average to see how large a potential headroom is. These are indicated by R_VARIABLE and calculated by VARIABLE_COMPANY – INDUSTRY_AVERAGE. Negative values therefore describe an under-performance and positive values an over-performance.

The undervaluation theory states that PE investors are interested in acquiring companies that appear cheap relative to their peer group. This is based on the assumption that the alternative investment approach of LBOs is able to optimise and increase the value of punished or poorly managed companies (McStay, 2020). Ideal targets are therefore companies that are undervalued compared to their peers, and optimally show a relatively good performance. This assumption is based on the idea that PE is the "repair shop of capitalism" and that the core competencies of private equity firms - governance and operational engineering, incentives and financial structuring - create value (Meyer, 2014). In addition, the multiple expansion is considered the strongest force in the value expansion of the total return (Street, 2014). This theory is considered by means of the EV/EBITDA multiple relative to the industry. Here, too, a dummy variable and the total distant from the average are used, while an 1 in the dummy variable means that the target company is undervalued compared to its respective market.

Undervaluation Theory

As "repair shop of capitalism", private equity pursues an alternative investment approach that makes it possible to operationally and structurally improve companies that have been punished by the market. The target selection of PE firms therefore focuses on companies that are undervalued relative to their market.

The industry/market conditions theory states that certain markets and industries are more attractive to PE investors than others. On the one hand, this is based on the fact that certain industries are so-called "trend industries", such as tech or software, and on the other hand, that industries have characteristics that make an investment more attractive, as suggested by Akhigbe and Madura (1999) and Abdesselam et al. (2008). These can be, for example, industries with higher margins, lower capital expenditure, or stable customers.

Industry/Market Conditions Theory

The industry in which a company operates is central component of the investment performance. This is based on the fact that trend industries promise more growth and that operating in certain industries offers more attractive cost-return ratios.

The fifth theory is based mainly on the literature reviewed and not on the theoretical framework. While the long-term horizon offers PE investors more freedom from regulations in changing jurisdictions and tax environments, the existing literature like Halpern et al. (1999), Kosedag and Lane (2002), and Le Nadant and Perdreau (2006) shows that the business environment can indeed be a determining factor in the selection of the target company. These findings are examined and tested by means of this theory.

Country-level Characteristics Theory

The country of domicile of the company prior to the acquisition provides an indication of the factors relevant to the company's performance.

Neural Network

Based on these formulated theories, a machine learning algorithm was developed that tries to recognize patterns in the data set used on the basis of the historical date in order to draw conclusions about the investment behavior and the determinants of the target selection. The neural network attempts to explain the variation in the dependent variable TARGET using the characteristics stated above. The variable TARGET is therefore formulated as a dummy variable that takes the value 1 if the company was the target of an LBO transaction and 0 if this is not the case.

Since the neural network can only operate with numerical data, the variables county and industry must first be transformed. For this purpose, a special numerical code is assigned to each country or economic sector, which has the disadvantage of making it difficult to interpret the weighting of these determinants later on. To improve the learning effect of the neural network, the Synthetic Minority Oversampling Technique (SMOTE) was applied to obtain a balanced dataset. As described by Chawla et al. (2002), SMOTE works by selecting examples of observations from a class that are close to each other in the feature space, drawing a line between the examples in the feature space, and drawing a new sample at a point along that line. The new over sampled set now has a total of 92,518 observations, of which 50% are targets and 50% are non-targets. Finally, before implementation, the data set used was standardised and divided into a training and a test part. The training sample makes up the larger part with 70% of the observations, i.e. 64,762 companies.

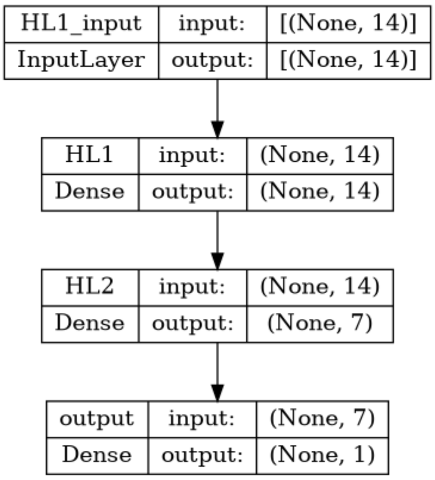

Figure 4: Structure of the Neural Network

After an analysis of the correlations and the VIF values of the respective variables, the architecture and the chosen properties of the neural network used were determined on the basis of these results and the methodology presented in the original research paper. For the development of the network the Python programming language and the open source software libraries Keras and Tensorflow by Chollet et al. (2015) and Abadi et al. (2015) were used, which provides an interface for artificial neural networks. A four-layer network with two hidden layers was implemented, as this took into account the complexity of the data to a sufficient extent without being too complex in order to avoid over-fitting. For the two hidden layers the activation function TanH was chosen, which provides the best processing for input values between -1 and 1, which is the case for the majority of the used input variables. The output layer uses the activation function Sigmoid, which is ideal for 0/1 classifications (Goodfellow et al., 2017). Furthermore, a supervised learning process was applied, which learns the weighting of the data itself, in order to also obtain information about the interpretation of the input variables. For training the network, the best batch size was determined using GridSearchCV by Pedregosa et al. (2011).

V. Results

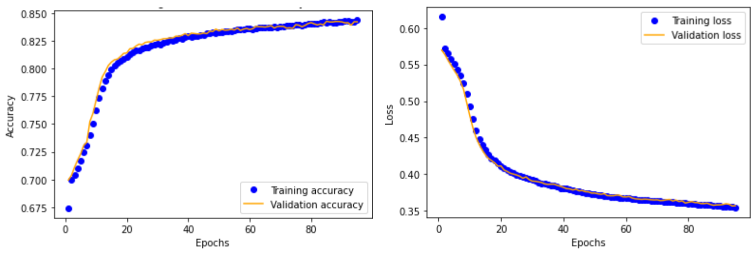

Figure 5: Training and Validation Accuracy and Loss

Figure 5 shows the learning process of the network and the accuracy and loss of the individual epochs. In total, the network was trained for 93 epochs before the integrated early stopping mechanism interrupted the process, to avoid over-fitting. For the training process we obtain a maximum accuracy of 84.1%, with a minimum loss of 34.1%. The model created was then tested against the test sample, which comprised 27,756 random observations of the total data sample. The model was able to correctly classify 83% of all targets and 85% of all non-targets, resulting in an overall training accuracy of 84% and an F1 score of 0.84. The consistency of the target and non-target classification shows, that clear overall differences in the pool of variables can be identified by the network.

	Non-Target	Target
Non-Target	11,860	2,051
Target	2,319	11,526

	precision	recall	f1-score	support
0	0.84	0.85	0.84	13,911
1	0.85	0.83	0.84	13,845
accuracy			0.84	27,756
macro avg	0.84	0.84	0.84	27,756
weighted avg	0.84	0.84	0.84	27,756

Figure 6: Classification and Test Results of the Neural Network

The interpretation of the processing of the input variables was carried out using Shapley Additive Explanations (SHAP), which allows us to deduce how strongly the respective variables influence the decision-making and in which direction the weights operate. As shown in Figure 6, D_Sales_GR and R_Sales_GR have by far the greatest influence on the classification problem, and can thus be considered the most important determinants in the selection. However, the relative performance of sales growth cannot be viewed separately from the other factors. The relative and separately considered efficiency of the company measured as EBITDA, net income and FCF margin is also decisive. The question of whether a company is undervalued or not (D_EVEBITDA) is also in the middle of the relevant determinants, but the extent to which it is undervalued (R_EVEBITDA), if this is the case, is classified as less relevant. The least important determinants are considered to be Industry, DA_M and Country, which have small influence on the overall decision-making process. It should also be noted that the values with the four highest contributions are all relative values related to the industry of the respective company, indicating that a target should always be considered relative to its environment.

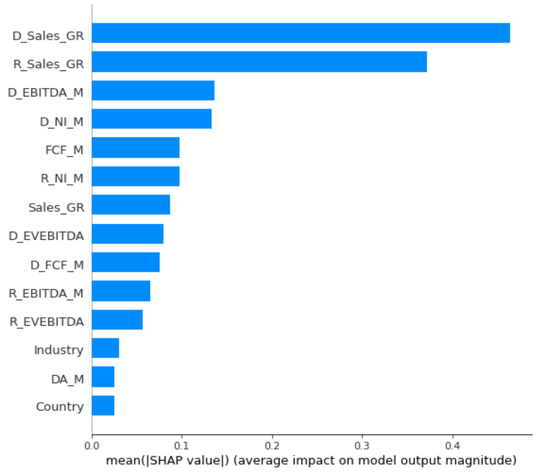

Figure 7: Shapley Values – Average Impact on Model Output Magnitude

In addition to the respective average contributions, SHAP also provides us with an impression of the strength of the respective determinants in the specific decision-making process. The force diagram in Figure 8 makes it easy to see where the "initial value" lies in relation to the "base value". At the beginning, the network assumes the output value 0, i.e. it assumes that the company is a non-target, which is represented as the "initial value". The "base value" is the average output value of the whole network after the training process. Since we have the same number of targets and non-targets, this is 0.5. The force diagram now shows the direction in which the respective determinants act in the classification of the companies. The red features have a positive influence, while the blue values have a negative. We recall that the higher value, 1, means a target classification. Of the 10 most important variables shown here, R_NI_M, D_EBITDA and D_Sales_GR have an increasing effect on output, i.e. they have a positive impact on target selection when their respective values increase. All others are considered to work in the direction of the non-target classification if the input parameters increase.

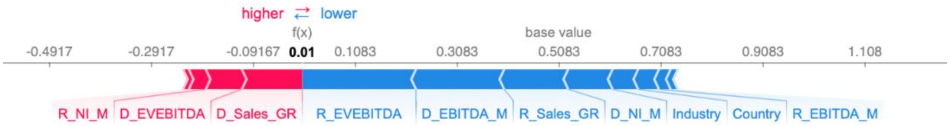

Figure 8: SHAP Force Diagram

In a further step, the waterfall diagram is displayed Figure 9, which also lets us see the amplitude and the type of impact of a characteristic. In addition, we can see the order of importance of the characteristics and the values that each characteristic occupies in the sample. Analogous to the force diagram, the determinants D_Sales_GR, D_EBITDA, and R_NI_M also have a positive effect on the classification. The output values show that under-performance in sales growth increase the probability of being selected as a target by 12%. In addition, undervaluation is shown to also increase the probability of a takeover by 7%, and a marginal increase in relative net profit margin over-performance increases the probability by 3%. We can observe that PE investors therefore prefer to invest in under-performing and undervalued companies. However, if we look at the average relevance of the two values in Figure 7 we see that they only play a subordinate role and only become relevant in connection with other determinants.

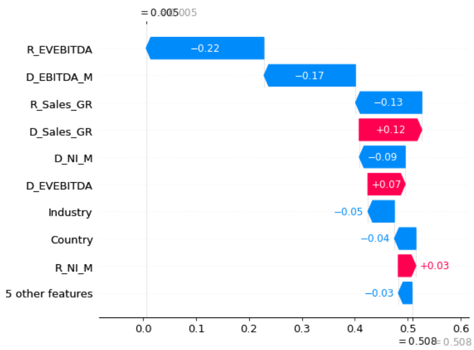

Figure 9: SHAP Waterfall Plot

We can see, that an increase in R_EVEBITDA leads to a decrease in model output, which means that, each marginal increase in overvaluation relative to its market, the probability of the company being selected as a target drops by 22%. Similarly, a marginal increase in R_Sales_GR, i.e. out-performance, has a negative 13% impact on selection as a target. From the coefficients of the dummy variables D_EBITDA_M and D_Sales_GR it can be seen, that a pure out-performance in EBITDA margin and sales growth reduces the likelihood of an LBO takeover for a company. The five other features not shown here are Sales_GR, DA_M, FCF_M, D_FCF_M, and R_EBITDA_M. It should also be noted that the SHAP waterfall coefficients of Industry and County are not interpretable here, as both were transmitted to the network as numeric code without ranking.

VI. Conclusion

The evaluation of the Shapley coefficients shows which criteria are the most relevant for the selection as a target and how they affect the classification. Analogous to previous studies, it becomes clear that the respective company characteristics play a role in the selection of PE targets. It turns out that by far the most important company characteristics are net profit generation efficiency and sales growth, while the other factors can be classified as less relevant. The dominant classification of sales growth is exactly what is expected from the investors' perspective. Sales is considered as a representative indicator of the overall development of the company and is very resistant to manipulation (Wahlmann, 2022). This hedging of the GP against possible pre-acquisition changes in the accounting is also reflected in the low relevance of depreciation and amortisation, which is strongly dependent on which accounting standard is used. For this reason, it is also difficult to compare this figure with companies in other countries.

However, the overall view of the effect of the respective variables shows that the firm characteristic parameters only play a minor role and have a moderate influence on the overall classification, thus should not be considered separately from the relative determinants in individual selection. This leads to the relevance of the relative under-performance theory, which can be confirmed by the results presented. It turns out that both the dummy and the overall distance to the industry average are the most important factors, which clearly confirms the hypothesis and addresses the identified gap in the literature. The undervaluation theory can also be confirmed on the basis of the network, which shows that in addition to lower efficiency and higher growth, undervaluation in the market is also relevant for the selection. However, the positive coefficient of the EBITDA multiple also shows that the question of whether a company is undervalued or not plays a minor role. This could be explained by the fact that the purchase price and thus the acquisition multiple are usually public, and an improvement in the company's key figures on exit must almost automatically lead to an increase in the multiple. In addition, only a relatively short period of four years, which was also influenced by the Covid pandemic, was used for the training of the network. This period is marked by a sharp fall in global markets and a subsequent bull market that has driven valuations up enormously, making overvaluation at the time of purchase less relevant. The last two hypotheses, that the industry and the country of origin have an influence on target selection, can only be partially confirmed. Although it can be seen that these contribute to the overall picture and support the classification decision, it can also be seen that both tend to be classified as less relevant. As already indicated above, this can be explained with the fact that the origin of a company has no real influence on its attractiveness as a target, since the associated determinants such as jurisdiction and taxation can be changed by the PE investor over the long-term investment horizon. The fact that the company's industry plays only a minor role could be connected to the actuality that targets are always considered relative to their market, which means that trend measures play a subordinate role, since profit maximisation and the exit multiple are always considered in comparison to industry.

In conclusion, the superior competence of private equity firms has a significant impact on the success of investments and specific firm characteristics are decisive for selection as a target. As based on the theoretical framework, PE investors prefer under-performing and undervalued companies relative to their industry, but valuation is considered less important. The last two hypotheses, that company's respective market and country have an influence on suitability as a target, are rejected, as these determinants were rated as least important in the classification network.

Overall, these results provide further evidence on the determinants of LBO targets and, therefore, are relevant for academics as well as investors in private equity to increase the chances for a successful buyout. Furthermore, these results may also be of interest to financial analysts and fund managers, as the significant bid premium of LBO takeovers, as well as the ability to predict future takeovers, may be the basis for a successful investment strategy. The findings contribute to the superiority of PE investors and indicate that the basis for a successful investment strategy is the capabilities of general partners and their target selection. While the results presented here are encouraging, they open up a new field for further research. The research project is considered as a light scratching of the surface, as there is a lot of room for improvement in both the variables used and the data sample to investigate. While the selection of determinants used here tries to represent the whole picture as broadly as possible, and at the same time to reconcile this with data availability, a much more fragmented consideration of the data can significantly increase the precision of the model. Possible attributes of this investigation could be: characteristics of market position, more accurate cash flow measurement, longer history, determinants about management and financial reporting systems, and operational improvement and exit opportunities. Despite these limitations, the method used shows that neural networks are much better suited to the problem studied than previous studies, which means huge potential for the future. In a further step, we are currently investigating how accurately the developed AI is able to screen the current market for possible draws and predict LBO targets.

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Appendices

Paper	Period	Sample	Identified Variables
Schwartz (1982)	1968-1977	186 Companies from the US	Asset Growth, Sales Growth
Palepu (1986)	n/a	n/a	Size, Industry Market/book values Price/earnings values Inefficient management
Jensen (1987)	n/a	n/a	Free Cash Flow
Lehn and Poulsen (1989)	1980-1987	263 US going private transactions	Cash flow/Equity Growth Tax/Equity
Ippolito and James (1992)	1980-1987	US Private Equity Targets	Growth, Takeover Threat Tax/Equity Book/Market
Opler and Titman (1993)	1980-1984 & 1985-1990	180 US companies	Management Cost of financial distress Cash flows Market-to-book-value-ratios
Kieschnick (1998)		US P2P Transactions	Tax/Equity, Size
Halpern et al. (1999)	1981-1986	US Private Equity Targets	Prior acquisition interest Tax expenditure Stock performance Management ownership
Akhigbe and Madura (1999)	1980-1996	245 large acquisition	Industry
Kosedag and Lane (2002)	1980-1996	21 US reLBO firms	Tax/ Equity Threath
Weir et al. (2005) Weir* et al. (2005)	1998-2000	1212 (a) / 116 (b) UK P2P Transactions	CEO shareholder Institutional shareholder Perceived undervaluation
Le Nadant and Perdreau (2006)	1996-2002	175 French LBO targets (mainly private held)	Debt Liquidity, Size Business risk, Profitability Tax levels
Aslan and Kumar (2007)	1996-2006	157 PE deals (P2P) in UK and Ireland	Growth, Liquidity Return on assets, Profit margin Market-to-book ratio Leverage ratios
Mehran and Peristiani (2008)	1990-2007	150 US companies	Analyst coverage/Financial visibility Investor Interest Cost of Debt / Equity Number of institutional investors Market-to-book-ratio Debt-to-equity ratio Free Cash Flow, Stock turnover, Stock volatility
Abdesselam et al. (2008)	1996-2004	3495 French LBO deals	Industry, Profitability
Acharya et al. (2009)	1996-2005	95 LBO deals in Western Europe	Stability Sustainability in growth
Brar et al. (2009)	1992-2003	896 MA deals	Size, Liquidity Undervaluation, Inefficient management Leverage/financial distress Barrier to Entry, Momentum
Bharath and Dittmar (2010)	1980-2004	P2P US transactions	Analyst coverage, Stock Turnover Free Cash Flow, Liquidity Institutional ownership
Billett et al. (2010)	1980-2006	407 US Targets	CIC Covenants Free Cash Flow
Krantz and Gustafsson (2020)	2000-2018	294 European P2P LBO targets	Cash Flow, Growth Tax benefits, Undervaluation Financial visibility

Appendix 1: Literature Overview

Theory	Variable
Firm Characteristics Theory	Sales_GR EBITDA_M EBIT_M NI_M DA_M NA_M FCF_M
Under performance Theory *& Firm Characteristics Theory*	R_Sales_GR D_Sales_GR R_EBITDA_M D_EBITDA_M R_EBIT_M D_EBIT_M R_NI_M D_NI_M R_FCF_M D_FCF_M
Undervaluation Theory	R_EVEBITDA D_EVEBITDA
Industry/Market Conditions Theory	Industry Affiliation
Country-level Characteristics Theory	Country

Appendix 2: Overview of all Variables used and the Underlying Theories

	Sales_GR	R_Sales_GR	D_Sales_GR	R_EBITDA_M	D_EBITDA_M	R_NI_M	D_NI_M	DA_M	FCF_M	D_FCF_M	R_EVEBITDA	D_EVEBITDA
Sales_GR	1.000000	0.750275	-0.466561	0.070731	-0.061412	0.073343	-0.059447	-0.024896	0.010661	-0.027279	-0.003241	0.013622
R_Sales_GR	0.750275	1.000000	-0.671492	0.079756	-0.081533	0.081807	-0.110790	-0.024536	-0.009841	0.005486	-0.012663	0.048197
D_Sales_GR	-0.466561	-0.671492	1.000000	-0.086369	0.099425	-0.082133	0.133983	0.035302	0.004576	0.030403	-0.006718	-0.016475
R_EBITDA_M	0.070731	0.079756	-0.086369	1.000000	-0.640502	0.550841	-0.502442	0.056552	0.102971	-0.252357	0.087766	-0.079445
D_EBITDA_M	-0.061412	-0.081533	0.099425	-0.640502	1.000000	-0.291335	0.592140	-0.143509	-0.047289	0.233404	-0.017486	-0.091252
R_NI_M	0.073343	0.081807	-0.082133	0.550841	-0.291335	1.000000	-0.407878	-0.251540	0.177307	-0.186898	0.068981	-0.033752
D_NI_M	-0.059447	-0.110790	0.133983	-0.502442	0.592140	-0.407878	1.000000	0.076539	-0.058331	0.234059	-0.011966	-0.129130
DA_M	-0.024896	-0.024536	0.035302	0.056552	-0.143509	-0.251540	0.076539	1.000000	-0.007434	-0.080316	-0.012017	-0.078159
FCF_M	0.010661	-0.009841	0.004576	0.102971	-0.047289	0.177307	-0.058331	-0.007434	1.000000	-0.251682	-0.000843	-0.008520
D_FCF_M	-0.027279	0.005486	0.030403	-0.252357	0.233404	-0.186898	0.234059	-0.080316	-0.251682	1.000000	-0.018362	0.010755
R_EVEBITDA	-0.003241	-0.012663	-0.006718	0.087766	-0.017486	0.068981	-0.011966	-0.012017	-0.000843	-0.018362	1.000000	-0.343314
D_EVEBITDA	0.013622	0.048197	-0.016475	-0.079445	-0.091252	-0.033752	-0.129130	-0.078159	-0.008520	0.010755	-0.343314	1.000000

Appendix 3: Correlation Matrix of the Variables

Extreme co-movement between the US equity market and geopolitical risks: an analysis from the extreme value perspective

August 30, 2022

Abstract

The paper aims to delve into the extreme co-movement between geopolitical events (incl. warfare, terrorism attacks, etc.) and the equity market (proxied by the S&P 500 Index), by applying extreme value theories (the BM-GEV-ACF-TQCC framework in short). The following results are evidenced: 1) the extrema of GPR and equity market return have "fat" tails and follow the Type II Fréchet distributions from GEV; 2) The tail and volatility indices of the ACF model scoherently jump with the occurrence of geopolitical conflicts and they could act as indicators for identifying the start of extreme states; 3) The extreme co-movement of geopolitical risks and the equity market exists, but it is not prominent. GPR accounts for one of the systemic and tail risk factors.

Key words: Geopolitical risks; Equity market; Tail risk contagions; Extreme value theory; Autoregressive conditional Fréchet; Tail quotient correlation coefficient

1. Introduction

Human history has been a chronicle of conquests where empires rise and fall. Even though wars are not alien to human beings, the advancements of modern technologies have been intensely adding to the deterrence and destructiveness of current-day warfare. The tightly knitted international relations also mount up to the intricacy of the impact of such conflicts. The geopolitical risks (GPR) also take on a broader horizon, from the conventional nationalist or colonial conquests to independence and human rights struggles, to racial and religious conflicts, and to proxy wars represented by superpowers.

Not striking to received logics, geopolitical conflicts, notably wars, bring about catastrophic blows to infrastructure and established trade orders, thus disrupting the value chain and further alienating countries. (Salisu, Lasisi, & Tchankam, 2022) revealed that stock markets in advanced economies suffer adversely from threats of GPR. In terms of fundamental firm valuation, Goldman Sachs pointed out that valuations on Chinese equities are negatively correlated with GPR with 1 standard deviation upward move in its US-China Relations Barometer translated into a 3.2% cutback in the price-to-earnings ratios. (Sipahutar, 2022)

Whereas, without getting too deep into the discussion of morality concerning reaping from wars, geopolitical hostilities are not always regarded as negative events from the perspective of investors. For example, for certain countries (Indonesia, Austria, Sri Lanka, etc.), the estimated effects of wars on their national stock markets lie in the positive range. (Leigh, Wolfers, & Zitzewitz, 2003) It is also found that the equity market tends to act positively in the longer term following the direct downfalls as a consequence of the conflicts in topic. During the course of the WWII (1939-1945), the Dow Jones Industrial Average (DJIA) index appreciated for approximately 50% (+7% YoY growth). (Calson, 2020) The trajectories of the equity market following the Vietnam War and the Gulf War also illustrate such a viewpoint. (WolfReport, 2022) The relationship between geopolitical conflicts and market outcomes is intricate and manifold.

Various studies have been conducted to understand the spillover effects of GPR with the performances of different asset classes. Based on a GARCH-MIDAS model, positive spillover effects along with GPR are observed in the energy, agriculture and livestock commodity markets and negative effects in precious metal and industrial metal commodity markets. (Gong & Xu, 2022) Employing the wavelet coherence analysis, the green bonds, Swiss franc, precious metal and real estate are also proved to be resilient to GPR fluctuations. (Będowska-Sójka, Demir, & Zaremba, 2022) Featuring a GED-GARCH(1,1) model, the hedging power of gold, specifically, in face of various risk forms is validated. (Chiang, 2022) Relevant analyses of connectedness for different countries have also been performed. (Fernandez, 2007; Singh & Roca, 2022)

In history, financial crises arise alongside disruptive events such as pandemics, wars, or major market failures. The 2007-2008 financial crisis has been a recent and pertinent opportunity for market participants and academia to reflect on the causal factors to the crisis. The hindsight could be conducive to strengthening the market resilience faced with such events in the future and avoiding dire consequences that were previously witnessed. The Gaussian copula, a statistical tool used to manage the risk of the collateralized debt obligations (CDOs) that triggered the flare-up of the crisis, has been under serious reproach for its essential flaw to overlook the occurrence and the magnitude of extreme events. (Salmon, 2012; Watts & Sam,

2016) To effectively understand and cope with the extreme events, the extreme value theory (EVT), born in the 19th century, has regained its popularity and importance, especially amid the financial turmoil. Capital requirements for financial institutions, such as the Basel guidelines for banks and the Solvency II Directive for insurers, have their theoretical base in the EVT.

The aforementioned studies majorly focus on the connectedness of relevant indices during their normal states and do not cover the aspect of the extreme dynamics. In the gloom of Russian invasion in Ukraine and rising tension across the Taiwan Strait, it would be helpful in understanding the dynamics of extreme states and formulating effective hedging strategies to alleviate the impact from possible major crashes. Inspired by the BM-GEV-ACF-TQCC framework put together by (Lin & Zhang, 2022) to study the extreme co-movement between pandemics and crude oil prices from an EVT perspective, this study employees such a paradigm, attempting to unveil the interrelatedness between GRP and the US equity market under the extreme circumstances.

This paper is organized as follows. Section 2 presents the data acquired for this analysis and the preliminary data cleansing conducted. Section 3 explains the methodology (BM, GEV, ACF and TQCC) of this study. Section 4 summarizes the results from the study and related analysis and observations. Section 5 includes further discussion and reflection on this topic.

2. Data

This study revolves around 2 datasets: the geopolitical risk (GPR) index and the S&P 500 index. Originally, both datasets are of daily frequency, spanning the period from January 01, 1985 to July 31, 2022. For modelling extreme behaviour, the monthly maxima method is utilized for each variable. Table [1] offers a presenation of the definitions of the variables in topic and their domains of attraction, and Table [2] presents the descriptive statistics of the three variables.

(Caldara & Iacoviello, 2022) constructed the Geopolitical Risk index, a news-based measure of adverse geopolitical events and related risks. They also maintain a daily update of this index and it is downloadable from their site. There are two components of the index, the geopolitical threats (GPT) and the geopolitical acts (GPA). The GPA component serves as the subject in this paper since it is more specific for actual occurrence of geopolitical escalations, including categories of beginning of wars, escalation of wars, and terror attacks.

For the equity marketplace, the S&P 500 index acts as the proxy for the US equity market and its daily close price is used as the price source and is converted to logarithm returns for further analysis.

Table 1: *Description of the monthly block maxima variables and data*

Variable	Definition	Domain of attraction
GPA	The monthly block maxima of the geopolitical acts (GPA) component index of the GPR index constructed and maintained by (Caldara & Iacoviello, 2022). The original data of GPA are devided by 100 to improve the comparability with the other two varibales.	Type II GEV (Fréchet)
SPX_up	The monthly block maxima of the upward movement of the S&P 500 index (logarithm return * 100)	Type II GEV (Fréchet)
SPX_down	The monthly block maxima of the downward movement of the S&P 500 index (- logarithm return * 100)	Type II GEV (Fréchet)

Figure [1] presents the evoluation of the monthly block maxima of the three extreme variables from January 1985 to July 2022. We can observe that the peaks of the GPA index correspond to the occurrence of geopolitical conflicts and that some volatile periods between the GPA index and S&P 500 index are matching, but not always.

3. Methodology

3.1. Static modelling of monthly block maxima — generalized extreme value (GEV) distributions

In extreme value analysis, there are mainly two different ways to identity extreme values, the block maxima (BM) approach and the peaks-over-threshold (POT) approach. The BM approach divides the observation period into non-overlapping, continuous and equal intervals and collects the maximum entries of each interval. (Gumbel, 1958) Maxima from these blocks (intervals) can be fitted into a generalized extreme value (GEV) distribution. The POT approach selects the observations that exceed a certain high threshold. A generalized Pareto distribution (GPD) is usually used to approximate the observations selected with the POT approach. (Pickands III, 1975)

Here we get our maxima samples by selecting the monthly block extrema and therefore the BM-GEV approach is applicable here. Let us denote $M_n = \max(X_1, ..., X_n)$ with X_i being independent and identically distributed (iid). If there exist sequences of constants $\{a_n > 0\}$ and $\{b_n\}$ and a non-degenerate distribution

Figure 1: *Evolution of monthly block maxima of SPX_up, SPX_down and GPA from 1985 to 2022*

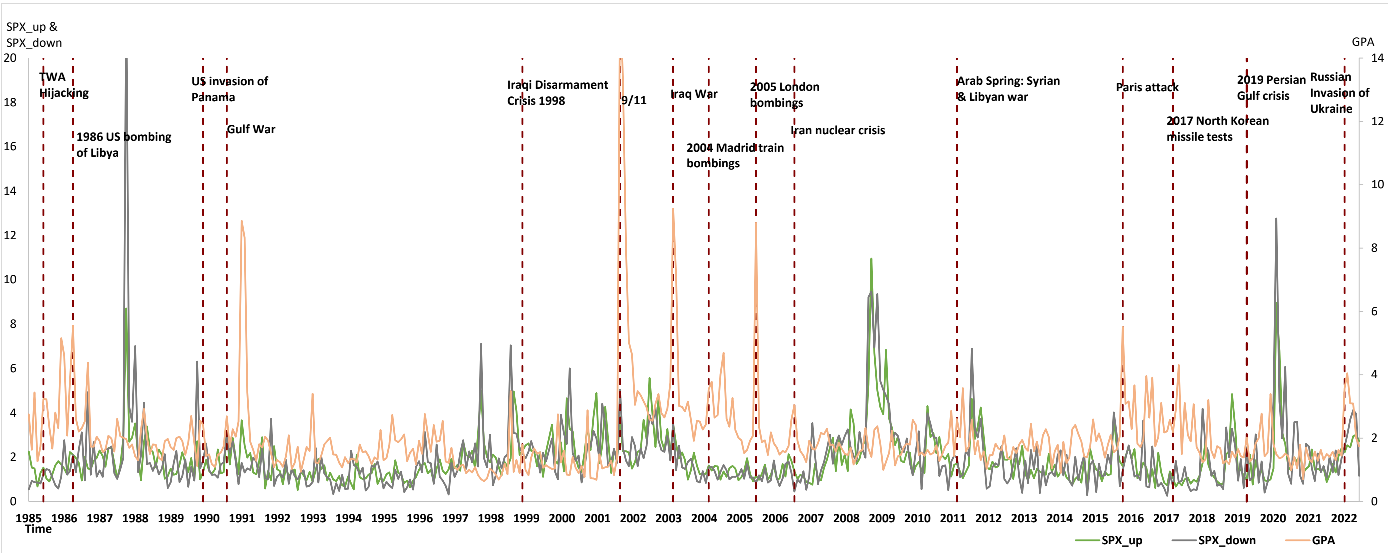


Table 2: *Descriptive statistics of the monthly block maxima variables*

Variable	No. of obs.	mean	std. dev.	kurtosis	skewness	min	max
GPA	451	2.0372	1.387509	41.12925	5.095496	0.6447	16.2743
SPX_up	451	1.9072	1.158307	17.81721	3.04191	0.5205	10.9572
SPX_down	451	1.9753	1.712401	56.85043	5.592084	0.2582	22.8997

function G such that

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{M_n - a_n}{b_n} < x\right) = \lim_{n\to\infty} F^n(a_n x + b_n) = G(x) \tag{1}$$

Then $G(x)$ belongs to the GEV family

$$G(x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} \tag{2}$$

where $1+\xi(\frac{x-\mu}{\sigma}) > 0$. ξ is the shape index (a.k.a tail index, extreme value index), μ the location index and σ the scale index. (Gnedenko, 1943) The GEV distributions have three subtypes corresponding to different tail features (Mises, 1936; Hosking, Wallis, & Wood, 1985):
Type I GEV: Gumbel distribution ($\xi = 0$)

$$G(x) = \exp\left\{\exp\left[-\left(\frac{x-b}{a}\right)\right]\right\}, \qquad -\infty < x < +\infty, a > 0, -\infty < b < +\infty \tag{3}$$

Type II GEV: Fréchet distribution ($\xi > 0$)

$$G(x) = \exp\left\{-\left(\frac{x-b}{a}\right)^{-\alpha}\right\}, \qquad x > b, a > 0, \alpha > 0, a > 0, -\infty < b < +\infty \tag{4}$$

Type III GEV: Weibull distribution ($\xi < 0$)

$$G(x) = \exp\left\{\left[-\left(-\frac{x-b}{a}\right)^{\alpha}\right\}, \qquad -\infty < x < +\infty, a > 0, -\infty < b < +\infty \tag{5}$$

In this study, to further analyze the GEV parameters, especially the tail index that determines the maximum domain of attraction (MDA), the logarithm maximum likelihood estimation (MLE) method is used to fit our data.

The result of our parameter estimation is presented in Table [3].

3.2. Dynamic modelling of monthly block extrema — Autoregressive Conditional Fréchet model (ACF)

As is presented in Table [3], the fitting of GEV of the three block extrema variables shows that all the variables are in the MDA of Fréchet distributions. Note that for the fitting of GEV, it is assumed that all the extreme parameters are static. This static fashion of modelling, however, does not offer a robust description of the processes and it is worthwhile to explore the dynamics of the extreme parameters. (Zhao, Zhang, & Chen, 2018) proposed an Autoregressive Conditional Fréchet (ACF) model to disentangle the dependencies between $\{Q_t\}$. This work adopts the ACF(1,1) model and it is specified as below:

$$Q_t = \mu + \sigma_t Y_t^{\frac{1}{\alpha_t}} \tag{6}$$

$$\log \sigma_t = \beta_0 + \beta_1 \log \sigma_{t-1} - \beta_2 \exp(-\beta_3 Q_{t-1}) \tag{7}$$

$$\log \alpha_t = \gamma_0 + \gamma_1 \log \alpha_{t-1} + \gamma_2 \exp(-\gamma_3 Q_{t-1}) \tag{8}$$

where$\{Y_t\}$ is a sequence of iid unit Fréchet random variable. $\{Q_t\}$ is the sequence in topic (in this study, the three block maxima variables). μ is the Fréchet location parameter, $\{\sigma_t\}$ the dynamic scale parameter sequence, and $\{\alpha_t\}$ the dynamic shape parameter sequence. $0 \leq \beta_1 \neq \gamma_1, \beta_2 > 0, \beta_3 > 0, \gamma_2 > 0$ and $\gamma_3 > 0$.

For the parameter estimation of the ACF model ($\theta = (\beta_0, \beta_1, \beta_2, \beta_3, \gamma_0, \gamma_1, \gamma_2, \gamma_3, \mu)$), we can use the conditional maximum likelihood estimation (cMLE). (Zhao et al., 2018) derived the following log-likelihood function with observations $\{Q_t\}_{t-1}^n$:

$$L_{n(\theta)} = \sum_{t=1}^{n} l_t(\theta) \tag{9}$$

$$\begin{aligned} &= \sum_{t=1}^{n} [\log \alpha_t + \alpha_t \log \sigma_t - (\alpha_t + 1) \log(Q_t - \mu) \\ &\quad - \sigma_t^{\alpha_t} (Q_t - \mu)^{-\alpha_t}] \end{aligned} \tag{10}$$

where $\{\sigma_t, \alpha_t\}_{t=1}^n$ can be obtained recursively through the Formulae [7] and [8] with an initial value $\{\sigma_1, \alpha_1\}$. The recovered values of unit Fréchet random variables $\{Y_t\}$ from Equation [6] could be used to study extreme co-movement. (Zhang, 2008)

In (Zhao et al., 2018), it is also pointed out that even though the true value of the initial value (σ_1, α_1) of a hidden process (σ_t, α_t), denoted as (σ_1^*, α_1^*), is not known, the impact of (σ_1, α_1) on preceding (σ_t, α_t) decays exponentially as t increases, with $0 \leq \beta_1, \gamma_1 < 1$ from the ACF(1,1) model. It is proved that the consistency and asymptotic normality of the estimation, as well as the asymptotic distribution, do not depend on the initial value (σ_1, α_1). Intuitively and also as is suggested in their work, the estimated $(\widehat{\sigma}, \widehat{\alpha})$ from the static GEV fitting is used as the initial value (σ_1, α_1).

3.3. Extreme co-movement — tail quotient correlation coefficient (TQCC)

Tail risk contagion has been a hot topic for academia since it is found to occrur with volatility clustering and could massively increase the magnitude of fluctuations during extreme states. Tail dependence (Embrechts, McNeil, & Straumann, 2002) of two random variables X and Y, with marginal distributions F_1 and F_2 respectively, is defined as

$$\lambda = \lim_{\alpha \to 1^-} \mathbb{P}[Y > F_2^{-1}(\alpha) | X > F_1^{-1}(\alpha)] \tag{11}$$

with its limit $\lambda \in [0, 1]$. Note that F^{-1} is the quantile function of F (i.e. $F^{-1}(\alpha) = \inf\{x | F(x) \geq \alpha)\}$).

X and Y are deemed to be asymptotically dependent in the upper tail if $\lambda \in (0, 1]$ or asymptotically independent in the upper tail if $\lambda = 0$.

(Zhang, 2008) proposed a sample-based alternative to model tail dependence that outperforms Pearson's linear correlation or Gumbel copula. Following this effort, (Zhang, Zhang, & Cui, 2017) further formalized the idea by proposing the concept of tail quotient correlation coefficient (TQCC) as follows.

If $\{X_i, Y_i\}_{i=1}^n$ is a random sample of unit Fréchet random variables (X,Y), the tail quotient correlation coefficient (TQCC) is then defined as

$$q_{u_n} = \frac{\max\limits_{1 \leq i \leq n} \{\frac{\max(X_i, u_n)}{\max(Y_i, u_n)}\} + \max\limits_{1 \leq i \leq n} \{\frac{\max(Y_i, u_n)}{\max(X_i, u_n)}\} - 2}{\max\limits_{1 \leq i \leq n} \{\frac{\max(X_i, u_n)}{\max(Y_i, u_n)}\} \times \max\limits_{1 \leq i \leq n} \{\frac{\max(Y_i, u_n)}{\max(X_i, u_n)}\} - 1} \tag{12}$$

where u_n is varying thresholds that tend to infinity.

To calculate the TQCC, I followed the procedure and the threshold of 97.5% presented in (Zhang, 2021).
for $k = 1 : 1000$,

- Simulate two sequences of unit Fréchet random variables $\{A_t\}$ and $\{B_t\}$;
- Sort $\{A_t\}$ and $\{B_t\}$, and denote them as $\{A_t^s\}$ and $\{B_t^s\}$;
- Set $X_t = A^s_{rank(Y_t^A)}$, $Y_t = B^s_{rank(Y_t^B)}$;
- Set $u_n = \min(P_{97.5}(\{X_t\}), P_{97.5}(\{Y_t\}))$, where $P_i(\cdot)$ denotes the i-percentile of a vector or a series.
- Compute $q_{(0.975,k)}$ using the Formula [12]

repeat until $k = 1000$, and set $q_{AB} = \frac{1}{1000} \sum\limits_{1 \leq k \leq 1000} q_{0.975,k}$

4. Results and Analysis

4.1. The static GEV fitting

Table [3] presents the results from the static GEV fitting of the three variables. The shape parameters (ξ) of all the variables are positive, indicating that they all follow certain Type II GEV (Fréchet) distributions and that they have "fatter" tails. The tail parameter of SPX_down is the highest amongst the three, showing that an extreme downward movement is more probable than the other two.

Table 3: *The GEV fittings for the monthly block maxima variables*

Variable	ξ (shape parameter)	μ (location parameter)	σ (scale parameter)
GPA	0.2290226 (0.03317374)	1.5146056 (0.02986104)	0.5713723 (0.02382386)
SPX_up	0.2421508 (0.03883130)	1.3885173 (0.03115824)	0.5844076 (0.02532777)
SPX_down	0.2608989 (0.03987525)	1.2913939 (0.03918002)	0.7314727 (0.03219391)

Inside the brackets are presented the standard errors of the estimations.

Table 4: *The estimated ACF(1,1) parameters*

Variable	γ_0	γ_1	γ_2	γ_3	β_0	β_1	β_2	β_3	μ
GPA	-0.064105	0.015632	1.739222	0.080362	0.419856	0.659353	0.751856	0.677522	-0.185247
	(0.886796)	(0.313199)	(0.857042)	(0.053473)	(0.075298)	(0.047522)	(0.102104)	(0.133147)	(0.173105)
			*		***	***	***	***	
SPX (up)	1.2189697	0.0011499	1.0956572	0.6289272	1.1993289	0.5246406	1.0068983	0.1433697	-0.9750196
	(0.3063568)	(0.1918525)	(0.2741579)	(0.3463841)	(0.3829128)	(0.0643114)	(0.3419549)	(0.0711408)	(0.3238699)
	***		***	.	**	***	**	*	**
SPX (down)	0.626287	0.300577	0.551901	0.373702	1.251867	0.532662	0.986380	0.097709	-1.168773
	(0.439383)	(0.367035)	(0.259385)	(0.310602)	(0.516476)	(0.096418)	(0.471039)	(0.062881)	(0.301705)
			*		*	***	*		***

Significance levels: 0 '***'; 0.001 '**'; 0.01 '*'; 0.05 '.'

The tail parameter of GPA is the lowest, indicating that the occurrence of actual geopolitical conflicts is less frequent than the extreme fluctuations of the equity market proxied by the S&P 500 index.

4.2. The ACF model estimation and extreme parameter dynamics

Table [4] provides the estimation results based of the cMLE method of the ACF(1,1) model. As is shown in the significance levels indiated, not all parameters reach ideal levels of significance.

Figures [2] and [3] are illustrations of the evolutions of the volatility index (σ) and the tail index (α) respectively. In term of GPA, we can observe the increases of the volatility index (σ) and decreses of the tail index (decreasing α, i.e. increase ξ) notably during the following periods: US bombing of Libya (1986); Gulf War (1990-1991); 9/11 Attack (2001); Iraq War (2003); London terrorism attack (2005); Paris terrorism attack (2015); and Russian invasion of Ukraine (2022).

Here we have two implications from the dynamic ACF analysis. First, the tail index could act as one of the identifiers of entering into extreme scenarios from normal states. Then, the extreme situations are coherently accompanied by "fatter"-tailed distributions and higher volatility.

In terms of the extreme behaviour of the S&P 500 index, from glances of the graphic illustrations, we can observe that the rise of σ and the drop of α of GPA during the specific events identified above are not always necessarily accompanied by the same movements for SPX_up and SPX_down. On the contrary, the equity index displays such behaviour when the GPA index is comparatively stable. More precise numeric analysis of co-movement will be prensented in the next subsection.

Table 5: *Full sample estimation of TQCC of the pairs between the three block extrema variables*

	GPA	SPX_up	SPX_down
GPA	-		
SPX_up	0.132124	-	
SPX_down	0.136427	0.300430	-

4.3. The TQCC estimation and spillover effects

According to the results of pair-wise TQCC calculation, SPX_up and SPX_down are tail connected at a 0.3/1.0 level. This is also an indication of volatility clustering where extreme upward movements are accompanied by extreme downward movements.

The TQCC of the SPX_up-GPA pair and the SPX_down-GPA pair are 0.132124 and 0.136427. Both are lower than that of the SPX_up-SPX_down, indicating that the extreme co-movement between GPA and the equity index is less prominent. Between the two, the TQCC of the SPX_down-GPA pair is slightly higher, showing that extremely high geopolitical risks are more correlated with extreme downward movement of the equity market.

Figure 2: *Evolution of the volatility index of SPX_up, SPX_down and GPA from 1985 to 2022*

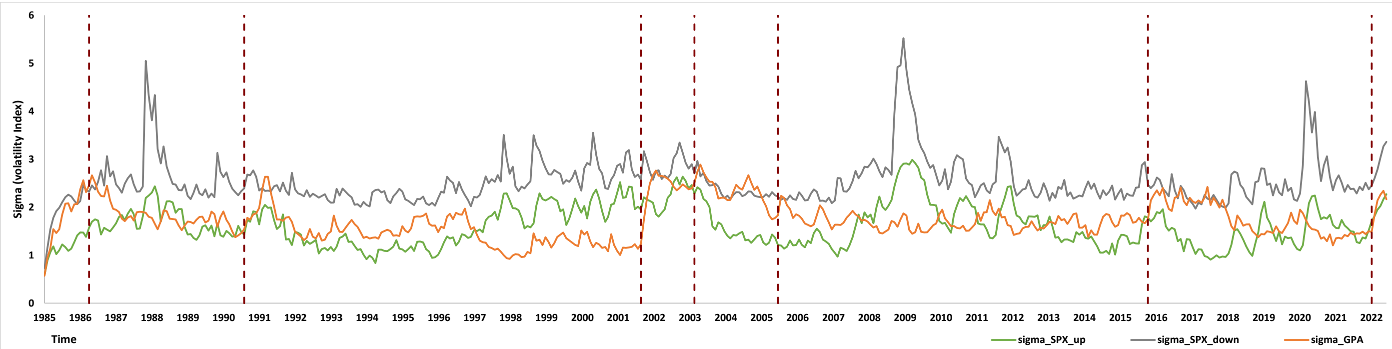
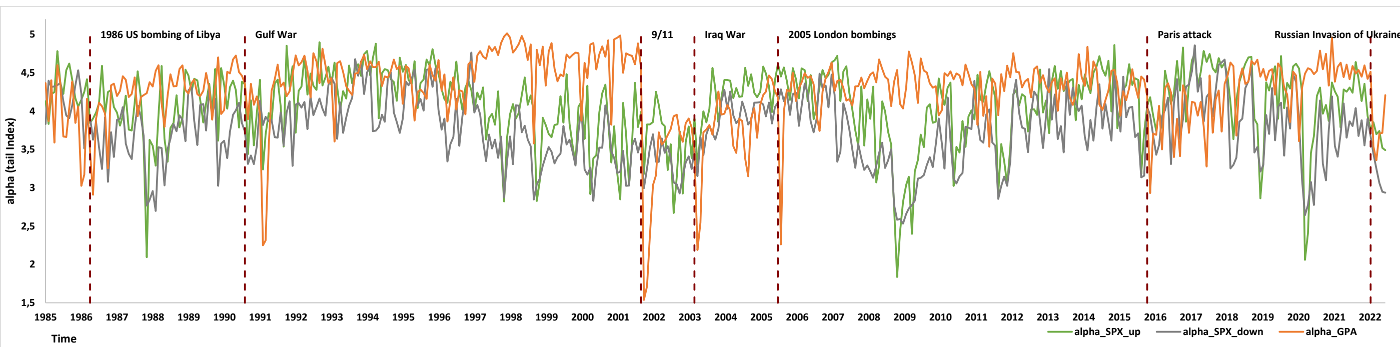


Figure 3: *Evolution of the tail index of SPX_up, SPX_down and GPA from 1985 to 2022*



5. Discussion

The extrem movements of the US equity market are partially contributed by the geopolitical risks. Proven to be one of the constituents of the systemic risks, the GPR accounts for part of the risk premium in asset pricing. For rational risk-averse investors, they are also tail risk-averse and the tail risk factors should be priced and compensated.

This paper took the chance to explore the dynamic relationship between the geopolitical risks and the US equity market in case of extreme events. The BM-GEV-ACF-TQCC framework could be applicable to further investigate the extreme linkages between such geopolitical risks with the equity markets in other parts of the world, or with other asset classes (Forex, bonds, commodity, etc.). Considering the TQCC analysis in this study is static, a time-varying one could be conducted to better understand the dynamics of the tail dependence structures, as (Lin & Zhang, 2022) did in terms of infectious diseases and crude oil price.

Faced with the comeback of geopolitical unrest, the results could be instructive to academia, asset managers, multinationals and also governments in understanding more in-depth the extreme co-movement of these elements and in establishing and improving infrastructure and strategies for more robust risk management.

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The R script for this project could be downloaded via this link (annonymous link and file).

Has Manipulation in the VIX Decreased?

August 30, 2022

Manipulation in the VIX settlement can cause significant losses to investors, concerning an open interest of more than a billion US-Dollars. Analysing high-frequency data, we present indications of VIX manipulation accelerating since 2017. Deviations from a fair value have an upward direction and average at around 6%. Specific effects accompany settlement days. The put/call ratio of underlying options surges by 10.9%. A time series decomposition demonstrates that this difference exceeds the day-specific variations of all other days by 80%. Data on open interest point towards leveraged funds, who systematically gather additional exposure in the seven days before settlement. All other players seem to reduce their VIX exposure before settlement.

JEL Codes: G12, G13, G14, G23, G24

Keywords: VIX, Market Manipulation, Expiration Day Effects, Volatility, Settlement Design, Term Structure

1 Introduction

Market manipulation is a criminal act, asymmetrically transferring capital to few undeserving beneficiaries at the expense of honest investors. In addition to direct costs, there are indirect costs as such conduct undermines the trust in financial instruments and impedes the intermediary function of financial markets (Allen and Gale, 1992). In 2011, the Financial Times pointed towards conspicuous settlement prices that cast doubt on the integrity of the VIX settlement procedure (Kaminska, 2011). Several lawsuits (still pending) examine a potential market manipulation in the VIX fixing, which is supported by empirical evidence (Griffin and Shams, 2018).[1] Since the open interest in VIX derivatives is in the order of 5 to 10 billion dollars, rigging the fixing price can incur substantial damage to holders of VIX derivatives.

In this paper, we examine indications of manipulation in the VIX. We are particularly interested in checking whether any potential rigging has slowed down since 2017, when Griffin and Shams (2018) drew attention to the vulnerabilities of the VIX. Using high-frequency data, we assess whether VIX settlement prices are in line with intraday trading and study to what extent settlement price outliers spill over into continuous trading. We further investigate what conditions are peculiar to settlement days in contrast to the other trading days. Considering open interest data, we point out how various trader groups behave differently in settlement weeks, aiming to reveal the actors behind a potential manipulation. Finally, we introduce difference-in-difference estimators between the VIX and realized volatility as an additional benchmark. For the use case of holding a VIX exposure beyond a short time horizon, we address the difficulty of being dependent on potentially manipulated settlement prices. We also suggest strategies for investors to mitigate the costs of possibly manipulated settlement prices.

We find indications of upward manipulation in the VIX index: The settlement price is on average 5.5% above the first trading quote thereafter. In addition, the settlement price lies 6.9% higher than the mid-point of intraday high and low VIX levels. Considering fixings outside the span of index values reached within the first 60 minutes of trading and extended by the last close, we observe such exceptions on more than 42% of the days with a tendency of strikingly high (as opposed to low) settlement prices. In the upward cases, the difference lies between 11% and 16% of the prevailing index level, on average. The downward cases are more moderate. These results are driven by a number of outliers, whereas most settlement prices remain within a reasonable range. We estimate that positive (negative) manipulation attempts take place on 13.4% (3.5%) of the days. Over time, the evidence for a rigged VIX index is most pronounced in the years 2017, 2018, and 2020, when it was well known by market participants how to influence the VIX in a cost-efficient manner.

Zooming in to the hours after settlement, we compare the VIX to its three-month adaption VIX3M which is analogously computed for a longer horizon. This comparison serves as a placebo test because there are no incentives for manipulating the VIX3M: Derivatives upon the VIX3N seem not sufficiently liquid and the options corresponding to the VIX3M are disjunct to those determining the VIX. The difference reaches 1.5% and dissolves over the first 90 minutes in the case of upward gaps.

On settlement days, the put/call ratio of the underlying S&P 500 options surges on average by 10.9%. The observation is in line with rigging the VIX index because the most cost-efficient manipulation is achieved by predominantly trading out-of-the-money (OTM) put options. A time series decomposition shows that the increased put/call ratio is particularly present on settlement days, exceeding the day-

[1]In contrast, Saha et al. (2019) find no evidence for manipulation of the VIX index.

specific level of all other days by almost 80%. A comparable increase is not present in the VIX3M placebo index.

From the Commodity Futures Trading Commission's (CFTC) open interest data[2], we extract that leveraged funds increase their exposure on average by 10% within the seven days before settlement. Additionally, we find that this increase is not driven by market participants with the eight largest open interest positions, but rather by entities with smaller exposures.

In the literature, to the best of our knowledge, only two papers focus on manipulation in the VIX. Griffin and Shams (2018) analyse intraday option prices and volumes around the settlement. They find that the volume of options traded increases abnormally in the settlement auction and only in the OTM options relevant to the VIX formula. Option volumes furthermore exhibit a distribution proportional to optimal cost-effectivity for manipulative purposes. While Griffin and Shams (2018) discuss and rule out a battery of alternative explanations, they admit that they cannot reject all legitimate explanations.

The second paper regresses VIX index levels on moving windows of the S&P 500 level and volatility (Saha et al., 2019). The findings suggest that the VIX and its settlement prices are in line with market fundamentals.

We contribute to this strand of the literature by considering the settlement prices themselves and their relation to VIX intraday index values based on high-frequency data ranging from 1990 to 2021. In contrast, Griffin and Shams (2018) concentrate on intraday option data, while Saha et al. (2019) only have end-of-day data at their disposal. In addition, we include the VIX adaption VIX3M as placebo index since its underlying options have different expirations and would thus not be affected by a manipulation of the main VIX index. Based on this complementary data set and additional approaches, we present novel indications of manipulation. While existing literature primarily discusses whether the fixing prices are manipulated or not, we add characterisations of potentially manipulated settlements such as their scale and directions (upwards or downwards). We are also the first to study the development over time and identify the accelerating trend of growing deviations. Furthermore, this paper is – to the best of our knowledge – the first to investigate open interest data of the CFTC regarding VIX manipulation, shedding light on the behaviour of different trader groups around the VIX settlement. Finally, we suggest strategies for investors to circumvent the costs of manipulation while keeping a VIX exposure.

Related literature outlines the dynamics of the VIX relative to the general market and derive pricing methods for the VIX.[3] We contribute to this strand of the literature by characterising the VIX' relation to the VIX3M as well as by explaining the VIX' movements. Various papers discuss the VIX as an instrument in portfolios (Doran, 2020; Hood and Malik, 2013). Our proposed strategies to circumvent potentially distorted settlement prices while keeping a VIX exposure add to this literature.

2 Data

The data used in this paper comprises daily index data, intraday index data on a minute basis, and open interest data collected once per week. The daily data consists of four data points for each time series per day, comprising the opening and closing quote as well as the high and low attained within the day. Cboe Exchange, Inc. provides such historical data going back to 1990 until 2021 for the VIX, VIX3M,

[2]The CFTC is a regulatory agency of the U.S. government to which all large market participants report their open interest held at the end of the day on every Tuesday. For instance, Ho and Lauwers (2017) provide background on the CFTC reports and consider the exposure of money managers in commodity futures as a predictor of commodity producers' stocks.

[3]Whaley (2000), Carr and Wu (2006), Bardgett et al. (2019), Fernandez-Perez et al. (2019), and Hülsbusch and Kraftschik (2018).

and other relevant U.S. indices. Besides, the exchange publishes volume statistics of S&P 500 (SPX) and VIX options as well as the settlement prices of the VIX and the settlement dates. Concerning futures, Cboe Exchange, Inc. supplies end-of-day data of VIX futures for the next six months' expirations.

Intraday data with one-minute granularity has been collected from Active Tick LLC and Barchart.com, Inc.[4] Including the years 2008 to 2021, there is one quote per minute of the VIX, VIX3M, and the S&P 500 indices, resulting in almost 1.5 million observations per index. The VIX3M is a variant of the VIX, which the Cboe computes by the same formula. However, the Cboe replaces the options serving as parameters by different expirations fitting the three-month time horizon.

The Commodity Futures Trading Commission (CFTC), a regulatory agency of the U.S. government, collects statistics from large market participants on their open interest held. Relevant market participants have a legal obligation to report to the CFTC every week. These reports comprise for any instrument – e.g., the VIX – the aggregated open interest (number of contracts) held in that instrument to Tuesday's record date (end of the day). Based on these reports, the CFTC publishes two sets of aggregated data divided by long and short exposure. First, it provides commitment data on the open interest held in the groups of leveraged funds, asset managers, dealers, other reportables and the open interest in sum, including market participants not obligated to report (non-reportables). Second, the CFTC supplies data on open interest concentration, namely the net open interest held by the largest eight and four market participants.

3 Are Settlement Prices in line with Intraday Trading?

This section aims to put the monthly VIX settlement prices into perspective by comparing them with the intraday VIX quotes. We will answer the central question whether the settlement prices are regularly on abnormal levels compared to continuous trading after settlement.

The VIX is particularly susceptible to manipulation due to its cash-settlement (Kumar and Seppi, 1992). As opposed to a settlement in kind, cash settlement facilitates making profits from manipulation. The VIX is an index based on a formula representing a weighted sum of prices of S&P 500 OTM options, to capture implied volatility more adequately. However, these OTM options are highly illiquid compared to in-the-money-options (Griffin and Shams, 2018; Saha et al., 2019). In contrast, investors trade the VIX derivatives heavily, comprising futures and options on the index. They are settled in cash based on the same formula that serves to calculate the continuous VIX quotes.

The Cboe determines option prices in a special auction taking place on the monthly settlement days in the morning before regular trading begins. During the auction, there is no order execution except the order book clearing at its end. These execution prices serve as parameters to the VIX formula, yielding the VIX derivatives' settlement price.

The natural approach to take advantage of this procedure is as follows: A manipulator enters a considerable position in VIX options or futures, regardless of whether this is long or short exposure. This transaction can be executed at low premiums to the market consensus of implied volatility as spreads are small in the liquid market for VIX derivatives. In the auction on the settlement day, the manipulator can drive the prices of the underlying illiquid OTM-S&P 500 options to an abnormal level by buying or selling them to move their prices in the direction favourable to the cash settlement he is going to receive on his VIX derivatives. The cost representing the unjustified premiums paid on S&P 500

[4]We use two data providers to cross-check the collected data.

options is negligible compared to the difference obtained in the cash settlement of the VIX derivatives. However, outside of the settlement auction, there is no direct incentive to manipulate option prices in advance to or after the settlement because solely the clearing price matters. As a result, we compare the settlement days to all other days to detect manipulative patterns.

3.1 Opening Quote

Settlement prices can be most directly compared to opening quotes. In the settlement auction, traders can provide orders from 8.30 a.m. until 9.15 a.m. (Eastern Time). In the following 15 minutes, the order books of options included in the VIX formula are frozen.[5] The Cboe executes SPX option orders at 9.30 a.m. These execution prices are the parameters to the VIX formula yielding the settlement price. Continuous trading begins directly after that.

To gauge whether there is a bias between the settlement price and opening quote, we consider their difference (settlement price minus opening quote). To facilitate interpretation, we divide this difference by the VIX opening quote. We assume that this difference should be relatively small, given that the amount of new information is limited due to the 15 minutes time difference. The average difference should also hover around zero in absence of manipulation, because positive and negative information events should be as likely.

The first column of Table 1 shows the empirical results. The settlement price was 5.5% higher than the quote not being subject to potential interventions, on average. The median difference (1.5%) is lower but with a high standard deviation (59.2%). This contrast can be traced back to the positive skew (0.7), pointing out that single days drive the higher mean. If manipulation is the reason for these differences, it implies that interventions are conducted to a substantial extent in isolated cases, rather than more frequent and more cautious rigging. These single occurrences can be observed in Figure 1. There are multiple differences exceeding one index point. However, there are also several substantial negative deviations.

To grasp how the difference between settlement price and open develops over time, we aggregate them for half-year periods. We employ the absolute value of the difference such that positive and negative differences do not cancel each other out. Figure 2 presents this development. We observe a growing trend starting in 2015, moving upwards within a difference of 2 to 6 index points per half-year. These findings are consistent with an increasing extent of manipulation conducted. The accelerating trend might be driven by imitators, who have learnt about the possibility to manipulate and subsequently taken part in the conduct.

3.2 Range of Daily High and Low

However, another explanation of this observation might be that in fact – for some reason apart from manipulation – the opening quotes are biased. To obtain further information on what happens after the settlement took place, a further natural reference are the highest and lowest index values after settlement. We use a scale that is defined by the low – fixing the point of 0% – and the high marking the point of 100%. In a second step, the settlement price is mapped accordingly to the scale $\frac{\text{Settlement Price}-\text{Low}}{\text{High}-\text{Low}}$ expressing the position of the settlement price on this scale as a percentage[6]. These high and low values

[5]Market participants can still submit orders in other options.

[6]relative to the low mapped to 0% and the high mapped to 100%

refer to continuous trading and exclude the settlement auction. For this reason, settlement prices may be below 0% or above 100 % on this scale if they are outside the range in which continuous trading takes place. However, such occurrences should be relatively infrequent as the continuous trading starts right after the fixing of the settlement price. Furthermore, by symmetry of information arrival and the law of large numbers, we expect the position of settlement prices on the scale to be around 50%.

The second column of Table 1 shows the empirical results. On average, the settlement prices are 6.9% above the mid-point of 50%. We test the realizations against the null hypothesis given this mid-point. Both a standard t-test (for the mean) and the Wilcoxon signed-rank test (for the median) indicate that realizations are higher than the mid-point. 13.4% of the settlement prices are above the intrady high, while 3.5% of the settlement prices are below the intraday low. These findings are consistent with manipulators rather biasing the VIX upwards.

Figure 3 depicts a histogram of the intraday position between high and low, highlighting that the cases above the span are – in addition to the higher frequency – also more pronounced in magnitude. This impression supports the hypothesis that manipulation only occurs on selective dates. Overall, our observations are remarkable because they reveal that settlement prices strongly differ from subsequent intraday trading levels.

3.3 Do Index Values normalize after Settlement?

It seems natural that a shock of settlement price manipulation continues to influence the index also after manipulation has ceased as the effect on supply and demand endures for a certain amount of time before the index arrives at an unbiased level again. A striking correction effect at the beginning of the continuous trading may indicate potential manipulation, in particular when comprising only minutes.

To study the intraday index values of the VIX on settlement days, we consider the deviation of the VIX in relation to the VIX3M in percent.[7] To aggregate these time series over the different days, we plot their means split by four categories. These categories are "positive gap", "negative gap" and "no gap" defined as above. Recall that for positive (negative) gaps the settlement price is above (below) both open and preceding close, and else there is no gap. For comparison purposes we add non-settlement days as a fourth category.

Figure 4 reports the results. Most notably, we see in the case of positive gaps that starting with strong first 30 minutes, the overall correction takes roughly 90 minutes. The plot also indicates that the scale of the correction is approximately 1.5%. There is a weaker effect for negative and no gaps, although also these categories arrive at an average level after 90 minutes. There is a systematic difference enduring throughout the whole day between the three categories of settlement days which the gaps' definition might (at least partly) explain. However, the non-settlement days serving for comparison are indeed almost constant around 0%.

4 What Settlement Day Effects do occur?

This section shows to what extent settlement days differ from the other days. Specifically, we review the external conditions, being possibly consequences of manipulation. We focus on the put/call ratio of SPX options, the fluctuation range of the VIX and VIX3M, and the open interest in VIX derivatives.

[7] More precisely, we use the centred and standardized difference of VIX and VIX3M. We consider the percentage deviation from the mean by dividing by the mean of their means and multiplying by their average standard deviation.

4.1 S&P 500 Options

Characteristically to the VIX formula, put options receive such a high weight that the call options' influence is negligible. Only out-of-the money (OTM) options are included, while all other options are ignored. Moreover, if two consecutive options have no bid at all, all deeper OTM options are discarded. Each individual option receives a weight proportional to $\frac{1}{K_i^2}$, implying the higher weight of put options: Put options are deeper OTM the lower the strike price is (implying a higher $\frac{1}{K_i^2}$). Excluding in-the-money (call) options removes the call options whose summand would otherwise be influential, as the low K_i, corresponding to a high $\frac{1}{K_i^2}$, are in-the-money.

This mechanism incentivizes manipulators to concentrate on put options. Such behaviour would leave footprints in the put/call ratio of these SPX options, representing the ratio of the volume traded in puts divided by the volume traded in calls. Therefore, manipulation – being conducted by primarily trading put options – would result ceteris paribus in a higher put/call ratio.

Panel A of Table 3 sketches the statistics of the put/call ratio of SPX options. It outlines descriptive statistics on non-settlement days in the first column and the same measures for settlement days in the second column. There is a higher put/call ratio on settlement days (mean: 1.94, median: 1.90) compared to the other days (mean: 1.74, median: 1.71). The difference of 0.19 is significant at the 1% level both under a t-test and the Wilcoxon signed-rank test.

Table 3's Panel B presents the results for an adjusted time series of the put/call ratio. We divide each day's put/call ratio by its moving two-week average (ten trading days) and subtract 1 for centring. This approach aims at including the market environment that prevailed in the two weeks before. We choose a length of two weeks since it comprises half of the four-week period between settlements. A shorter period would make the measure more volatile, while a sufficient distance to the previous settlement day is necessary to avoid a spillover (from the earlier settlement day). Thus, Panel B in Table 3 implies that under this transformation the difference found is 14% (mean) or 12% (median), both significant at the 1% level. These observations show that discrepancies of settlement days exceed 10%, which would be in line with manipulation.

We also employ a time series decomposition of the SPX options' put/call ratio, by decomposing daily put/call ratios additively into three components. Every period lasts four weeks (20 trading days) and settlement days are fixed as day 20.[8] The first component is a repetitive series of length 20, assuming the same value on any day n, $n \in \{1, ..., 20\}$. The second component constitutes a trend, deduced as the moving average of the preceding ten days. Finally, the third component is the residual, incorporating the remainder such that the put/call ratio equals the sum of the three components.

The time series decomposition confirms the relevance of the settlement day as the magnitude of the repetitive component on settlement days is approximately 80% higher than on non-settlement days. Moreover, this result implies that the difference of the SPX put/call ratio in Table 3 is exclusive to settlement days (i.e., isolating any other day instead of the settlement day does not yield similar results). Overall, we find a settlement day effect in the options which determine the VIX. This effect is consistent with manipulation as cost-efficient manipulation would entail a higher put/call ratio.

[8] If there are more than 20 trading days within a month, the first day(s) is (are) dropped since our procedure requires periods of equal length. The omitted days are of little relevance as the emphasis lies on the days before settlement days.

4.2 Index Value Fluctuation Range

Next, we consider the range in which the VIX index values move within the day. We define this fluctuation range for every day as the difference of the highest minus the lowest value. To reveal what effect is characteristic of the VIX settlement, we compare the results to the fluctuation range of the VIX3M. Table 4 provides the results. For the VIX, Panel A points out that the fluctuation range mean of 1.58 index points broadens to 1.92 on settlement days. This difference of 0.35 index points is significant at the 5% level under a t-test. The median difference (0.27) is significant at the 1% level using a Wilcoxon signed-rank test. Hence, the most extreme fluctuations occur on the settlement day, i.e., after the special opening auction. The VIX3M placebo index does not exhibit significant deviations (Panel B), which is again in line with manipulation in the VIX index.

5 Who drives the VIX Market on Settlement days?

In this section we try to reveal the market participants behind the potential market manipulation by analyzing CFDC open interest information. To analyse groups' commitment, we spotlight the week-on-week changes which Table 5 states. Given that this change from week to week reflects the chronological sequence of the commitment's development, it is more relevant than the absolute figures.

In sum, leveraged funds exhibit a conspicuous deviation on settlement days, being vastly stronger committed in the long direction. The other groups expose no special effect except other reportables. In connection with the notion emerging from the previous sections, the findings concerning leverage funds are consistent with with a preference for upward manipulation. Overall, the observations indicate that leveraged funds might be the manipulators, being mostly active in long positions. For this reason, we consider this group's long exposure in the following in greater detail. Leveraged funds are more substantially engaged in the long direction of VIX derivatives before settlement than the three weeks before. Furthermore, the funds enter into a relevant portion of this extra exposure in the seven days ahead of settlement. These observations further support the hypothesis that leveraged funds might bear responsibility for a suspected manipulation.

6 Conclusion

We find a vast amount of empirical findings that potential manipulation in the VIX has not slowed down since the financial press and academic papers (Griffin and Shams, 2018) started raising concerns in this regard. We show that settlement prices are often not in line with continuous trading. Upward deviations occur more often and are substantially more intense than downward distortions. Investors trade substantially more put options on settlement days, which is in line with cost-efficient VIX manipulation. Leveraged funds go into the settlement with 10% more exposure compared to the week before.

The Cboe Exchange, being responsible for the settlement design, failed to refute the massive criticism. A high share of Cboe's earnings attributable to the VIX may create an incentive against admitting weak spots in its settlement procedure. Until this issue is resolved, the main VIX index appears to be ill-suited for volatility hedging. The VIX3M seems a better alternative as the Cboe Exchange also issues derivatives on that index.

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Table Appendix

Table 1: VIX Settlement Prices

This table reports statistics on VIX settlement prices in comparison to intraday index quotes. The first column shows the difference between the settlement price and the opening quote as a percentage of the average VIX open on all days. In the second column, the position of the settlement price within the intraday trading range – spanning from the intraday low to the intraday high – is reported as a percentage of this span (defining the scale from 0%, the low, to 100%, the high).

As the quotes in continuous trading fix this scale, the settlement is excluded. As a result, it is possible that the intraday trading range does not contain the settlement price, resulting in values below 0% or above 100%. The number of these settlement days as a percentage of the total number of settlement days is shown in the last two rows. *, **, *** denote significance at the 10%, 5% and 1% level.

	Difference to Open	Intraday Position within High and Low
mean	5.5%	56.9%**
median	1.5%	50.9%***
standard deviation	59.2%	37.7%
skew	0.7	0.3
# values $\leq$ 0%		3.5%
# values $\geq$ 100%		13.4%

Table 2: Intraday Gaps around Settlement

This table reports on gaps between the settlement price and the last day's VIX close and intraday VIX values shortly after settlement. A gap is recorded if the settlement price is higher than the previous day's close but trades shortly after settlement below the settlement price again (positive direction) or vice versa (negative direction), having the character of a "reversal". For the exact point in time after settlement, different reference times are chosen and separately reported. Panel A reports the empirical probability that such a gap occurs. Panels B and C report descriptive statistics on the gaps after non-gap days have been deleted.

reference time	**9:30**	**9:35**	**9:40**	**9:45**	**10:00**	**10:15**	**10:30**
Panel A: Empirical Probability of Gaps (Settle Price not between Close and Open)							
Gap Probability	57.8%	42.0%	42.7%	43.9%	45.5%	49.8%	52.3%
thereof:							
positive	28.7%	22.0%	22.7%	22.3%	25.5%	26.2%	28.8%
negative	29.1%	20.0%	20.0%	21.6%	20.0%	23.6%	23.5%
Panel B: Width of Upward (positive) Gaps – in percent of last VIX Close							
mean	16.1%	13.0%	11.3%	11.8%	12.3%	13.9%	14.1%
median	1.9%	10.6%	9.9%	10.2%	10.6%	12.3%	12.1%
standard dev.	29.0%	12.8%	10.3%	10.4%	9.6%	10.3%	10.3%
skew	5.5	1.7	1.7	1.8	1.8	1.8	1.7
Panel C: Width of Downward (negative) Gaps – in Percent of last VIX Close							
mean	−9.3%	−9.5%	−9.6%	−9.8%	−9.5%	−8.9%	−9.5%
median	−9.0%	−9.4%	−9.2%	−9.4%	−9.0%	−9.2%	−9.0%
standard dev.	5.7%	5.4%	5.4%	6.0%	6.0%	5.7%	5.5%
skew	−0.3	−0.3	−0.3	−0.5	−0.8	−0.6	−0.4

Table 3: Underlying Options' Put/Call Ratio

This table shows descriptive statistics of the put/call ratio of the SPX options. The statistics are reported separately for settlement days and all other days. The VIX formula is based on a weighted sum of SPX options prices. As put options receive significantly higher weight, cost efficient manipulation will – ceteris paribus – result in a higher put/call ratio. Panel A shows a the descriptive statistics directly, whereas Panel B is computed from an adjusted and zero-centered time series ("excess multiple"). The adjustment is conducted by dividing each day's value by the ten day moving average. *, **, *** denote significance at the 10%, 5% and 1% level.

	other days	settle days	difference
Panel A: Put/Call Ratio			
mean	1.74	1.94	0.19***
median	1.71	1.90	0.19***
standard deviation	0.38	0.43	
skew	0.57	0.39	
Panel B: Excess Multiple of Put/Call Ratio			
mean	0.01	0.15	0.14***
median	−0.01	0.11	0.12***
standard deviation	0.21	0.27	
skew	0.71	0.76	

Table 4: Index Value Fluctuation Range

This table shows descriptive statistics of the intraday range in which index values fluctuate, defined as highest index value minus lowest index value of each day. In the second column, the monthly settlement days are isolated. The last column shows their absolute difference. The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. Characteristically, no derivatives are issued upon the three-month counterpart VIX3M of the VIX whose index values are calculated by the same formula. This fact makes them irrelevant to manipulators. Therefore, the VIX3M is a suitable comparison to validate whether the fluctuation range of the VIX is abnormally enlarged on settlement days. *, **, *** denote significance at the 10%, 5% and 1% level.

	other days	settle days	difference
Panel A: Fluctuation Range of VIX			
mean	1.58	1.92	0.35**
median	1.15	1.42	0.27***
standard deviation	1.65	1.88	
skew	5.97	3.91	
Panel B: Fluctuation Range of VIX3M			
mean	1.32	1.37	0.05
median	0.88	1.00	0.12
standard deviation	1.69	1.54	
skew	6.49	5.39	

Table 5: Commitment by Trader Group, Week-on-Week Change

This table shows the **week-on-week change in percent** of the net open interest in VIX derivatives (in 1000 USD * index value) held by the respective (disjunct) groups in Panels A to D. For the groups, reporting their exposure to the American Commodity Futures Trading Commission (CFTC), who publishes the aggregated data, is mandatory. The groups make up the majority of the open interest in VIX derivatives; smaller traders are not obliged to report. The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. *, **, *** denote significance at the 10%, 5% and 1% level.

	Long			**Short**		
	other days	settle days	difference	other days	settle days	difference
Panel A: Leveraged Funds (%)						
mean	6.0	16.0	10.1*	1.6	0.8	−0.7
median	−0.5	6.8	7.2***	1.0	1.0	−0.1
st. dev.	52.3	71.4		16.0	14.9	
Panel B: Asset Managers (%)						
mean	0.1	0.0	−0.1	0.0	0.0	0.0
median	0.0	0.0	0.0**	0.0	0.0	0.0
st. dev.	0.6	0.5		0.7	0.3	
Panel C: Dealer (%)						
mean	4.6	4.9	0.3	3.4	4.2	0.8
median	0.6	0.7	0.1	0.7	1.3	0.6
st. dev.	36.0	30.6		28.7	24.0	
Panel D: Other Reportables (%)						
mean	7.4	11.0	3.5	12.3	21.1	8.8
median	0.0	1.4	1.4	0.3	6.0	5.7*
st. dev.	56.6	59.0		140.5	139.6	

Figure Appendix

Figure 1: VIX Settlement Price Minus Open

The histogram shows the distribution of the difference between the VIX settlement price and the opening quote fixed minutes later in absolute index points (x-axis) and the corresponding frequency in percent (y-axis). The dotted line indicates the arithmetic mean.

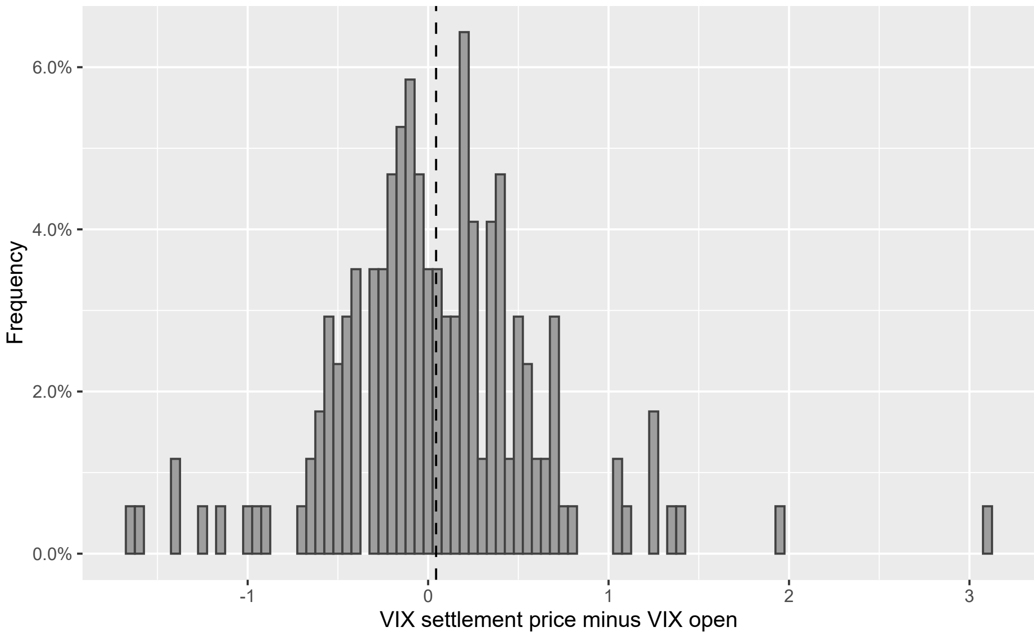


Figure 2: Difference of Settlement Price and Open

Summed up for each half year, this plot depicts the sum of the absolute values of the difference "VIX Settlement Price minus VIX Open" in index points. The dotted line represents the smoothed trend.

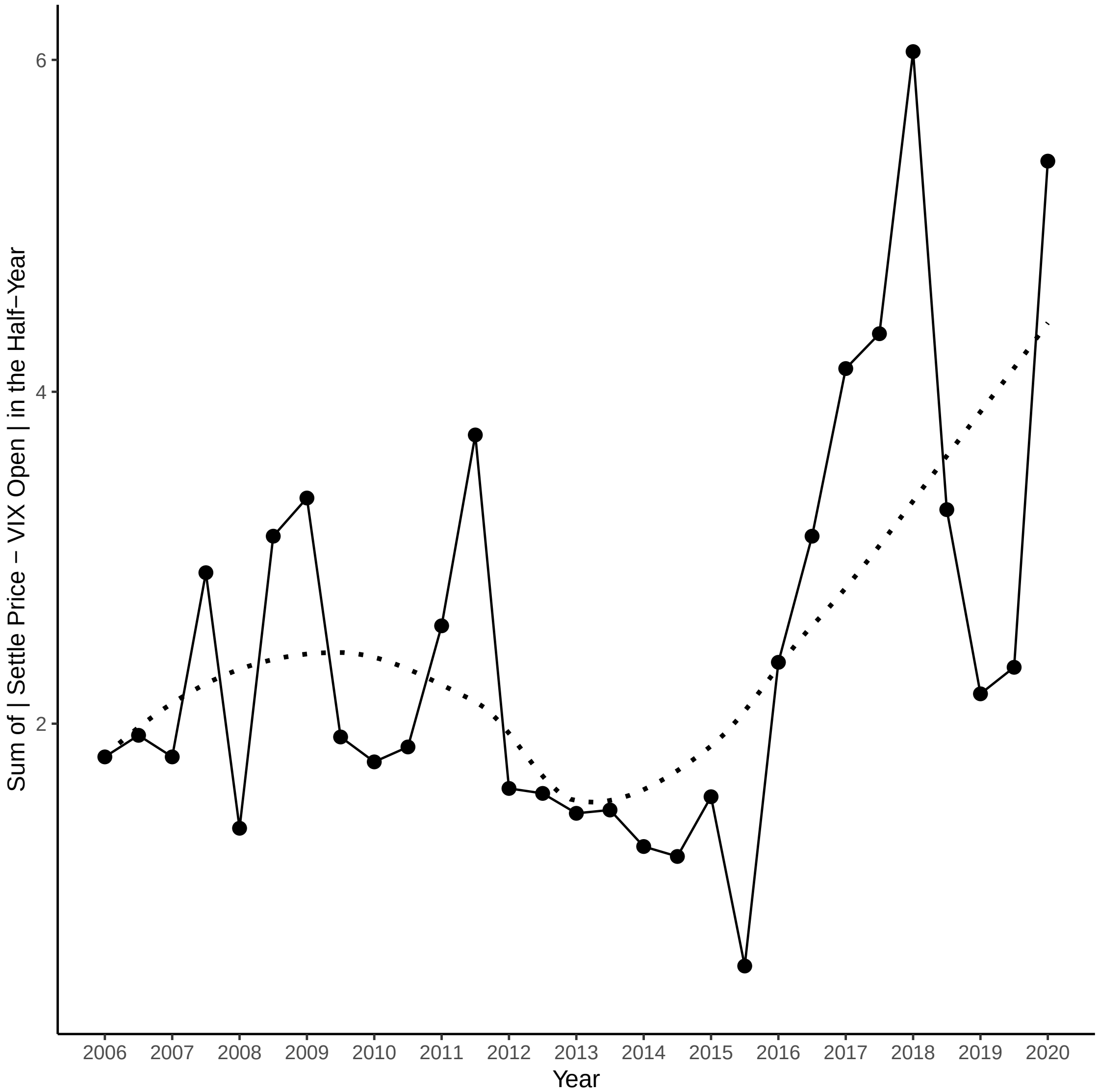


Figure 3: VIX Settlement Price in Fluctuation Range

The histogram depicts the distribution of the settlement price within the fluctuation range mapped to 0 to 1. For each settlement day, the settlement price position within the intraday fluctuation range is calculated, where the intraday low (excluding the settlement auction) serves as 0 and the high as 1. As the auction is excluded, values may be outside the interval [0; 1].

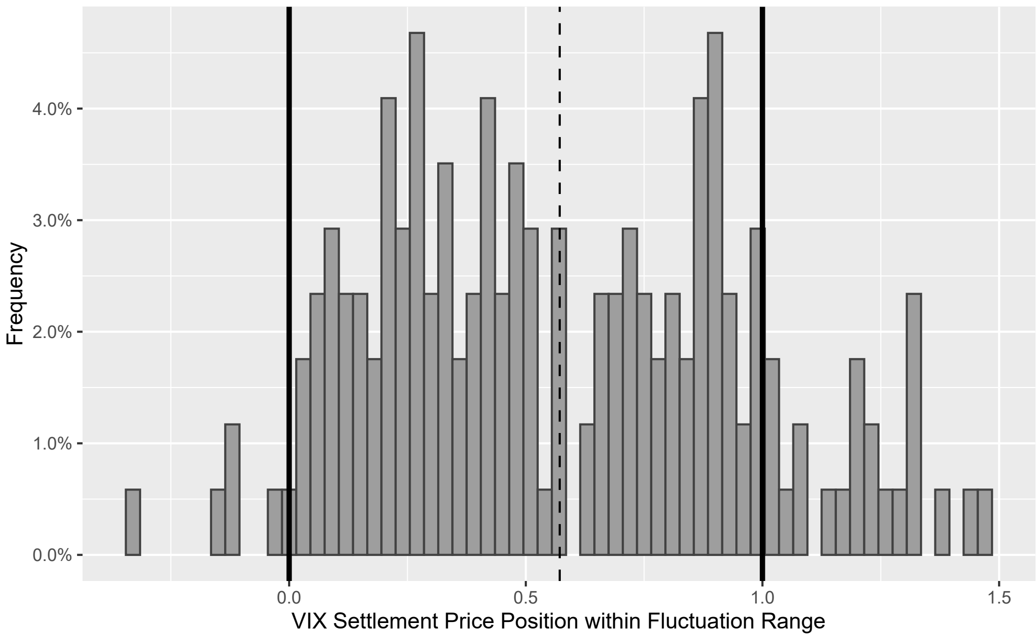


Figure 4: Intraday Difference of VIX and VIX3M

This figure presents the average development of the standardized difference of the VIX and VIX3M intraday in the following sense: A settlement day is a "positive gap"-day if both the VIX open and previous day's close are below the settlement price and it is a "negative gap"-day the settlement price is above. Other days, with a settlement price between VIX open and the previous day's close are "no gap"-days. For non-settle days there is no distinction.

Within these four categories, a variance-standardized difference (expressed in percent) of the VIX minus the VIX3M serves as basis for minutely means calculated from all index values after the n-th minute after market open at 9:30 a.m.

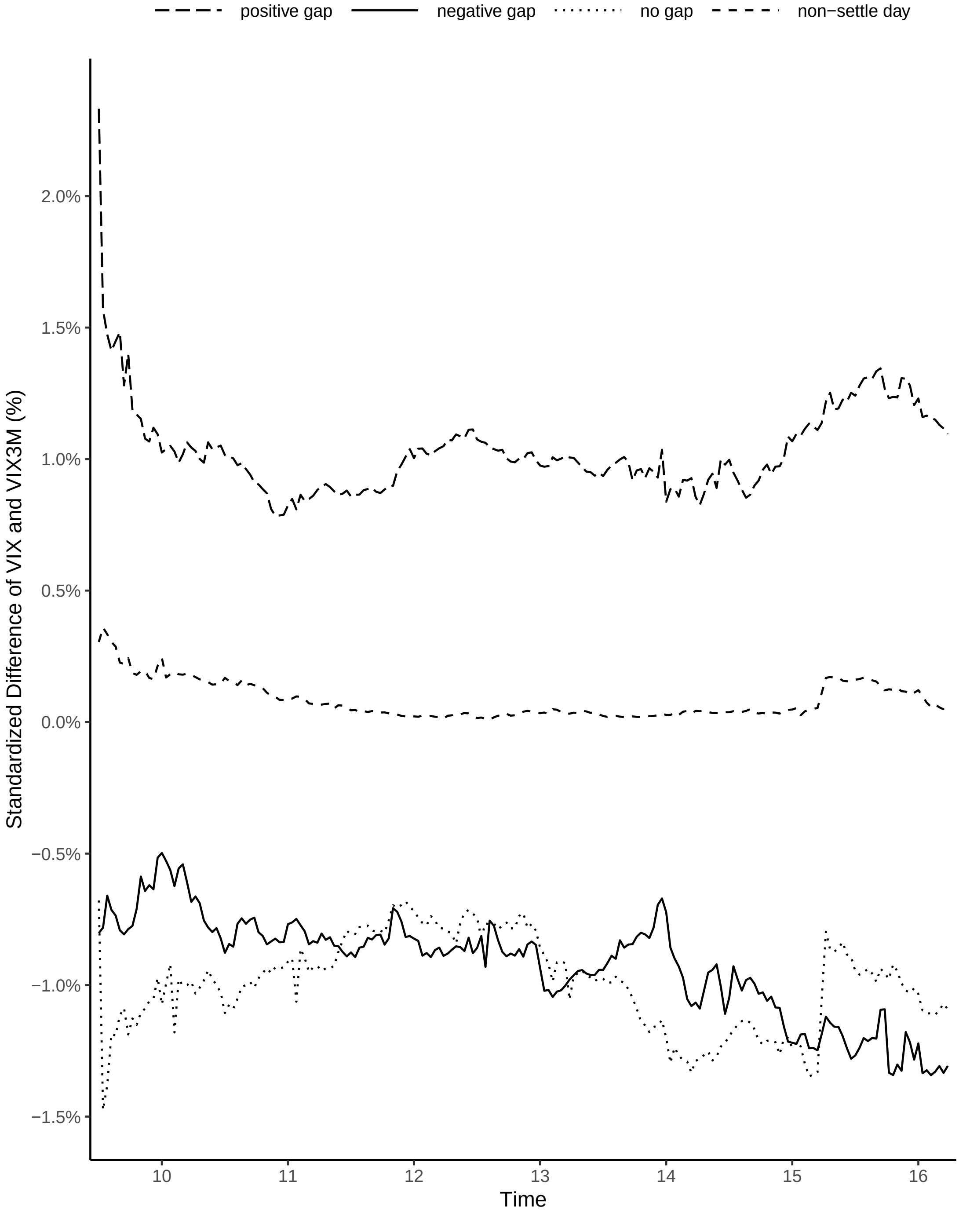
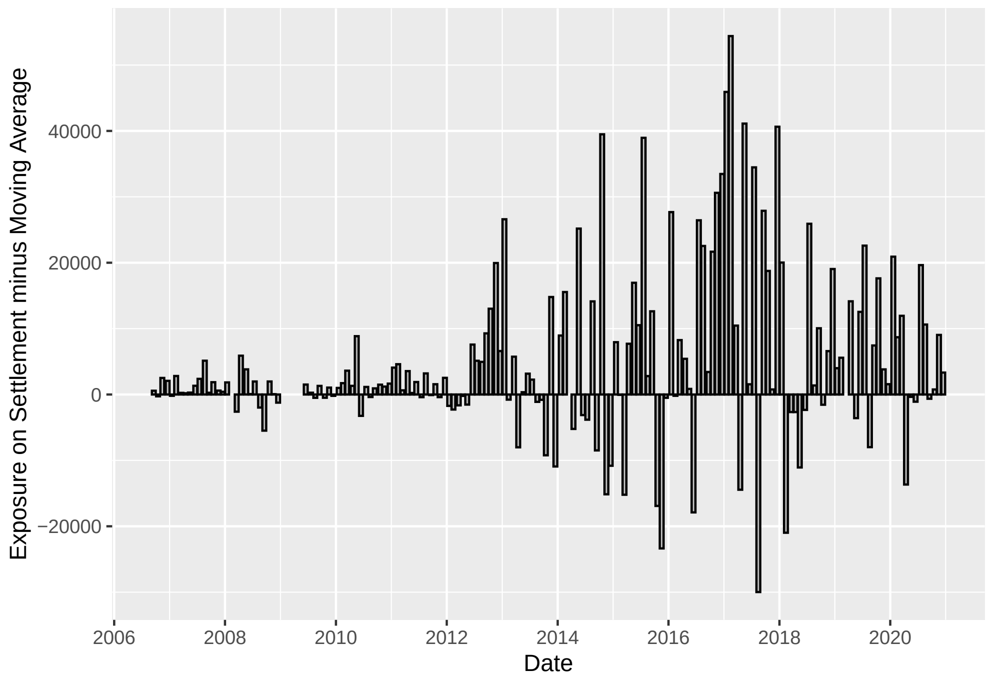


Figure 5: Commitment of Leveraged Funds

Panel A shows the long exposure to VIX derivatives of leveraged funds on (monthly) settlement days minus the three-week moving average. Panel B shows the difference between the long exposure's week-on-week change of settlement days minus the three-week moving average.

Panel A: Absolute Difference to the Average of the three Weeks before



Panel B: Relative Difference to the Percentage Change Average