Speed Harmonization using Optimal Control Algorithm under Mixed Traffic of Automated Vehicles and Human Driven Vehicles

Byungkyu Park\textsuperscript{1}, Ph.D., Seongah Hong\textsuperscript{1}, Andreas A. Malikopoulos\textsuperscript{2}, Ph.D. and Joyoung Lee\textsuperscript{3}, Ph.D.

1) Department of Civil and Environmental Engineering, University of Virginia; 2) Department of Mechanical Engineering, University of Delaware; 3) Department of Civil Engineering, New Jersey Institute of Technology

ABSTRACT

We address the problem of harmonizing the speed of an increasing number of vehicles on a highway in real time. We formulated the control problem and provided an analytical and closed-form solution that can be implemented in real time. The objective is to derive the optimal acceleration/deceleration of each vehicle that harmonizes the speed of an increasing number of vehicles at a speed reduction zone on the highway, under the hard safety constraint to avoid rear-end collision. The proposed speed harmonization algorithm was evaluated under various market penetrations of automated vehicles (AV). The simulation results showed that the travel time was improved by 4-28% and the fuel consumptions was improved by 6-21% for different market penetrations of AVs. This study demonstrated the feasibility of the control algorithm under mixed traffic and provided quantitative assessment in various aspects of mobility, fuel economy and safety compared to the state of the art SPD-HAR algorithms.

Introduction and Objectives

Advanced Variable Speed Limit (VSL) strategies use the Model Predictive Control (MPC) framework which incorporates dynamic traffic flow model. Although the effectiveness of the MPC method has been demonstrated by many studies, it has challenges including real-time implementation due to computational complexity and heuristic optimization approach, and requirement of model calibration and parameter estimation.

To address these challenges, we developed an analytical approach-based control strategies using Pontryagin’s Minimum Principle (PMP) with an objective of minimizing control efforts. Unlike the Model Predictive Control (MPC) method, the optimal control theory approach does not require model development and calibration as the control scheme is subject to the vehicle dynamics of associated parameters when optimization is solved. The merits of the optimal control algorithms are:

- The algorithm minimizes control efforts;
- The algorithm is computationally less expensive; and
- The solution guarantees true optimal strategies.

In this research, the simulation study was conducted to implement the optimal control algorithm under mixed traffic of Automated Vehicles (AVs) and human driven vehicles at a freeway corridor with a speed reduction zone. The measures of mobility and environmental sustainability were examined.

Development of Optimal Control Algorithm

Problem Formulation

Once a vehicle reaches the control zone, we seek to derive the optimal acceleration profile for each vehicle when the leading vehicle, indexed by \( i = 1 \), either accelerates or brakes between the interval \( \Delta t = t^t - t^d \) where \( t^t \) and \( t^d \) are the initial and final time of this event for vehicle \( i \). The problem can be formulated as follows:

\[
\min J = \min \frac{1}{2} \sum_{t, \xi} u_t^2 dt
\]

Subject to:

Vehicle dynamics

\[
x_t = v_t,
\]

\[
v_t = u_t,
\]

Initial conditions

\[
x_{t^d}(i) = x_{t^t}(i) = 0
\]

\[
v_{t^d}(i) = v_{t^t}(i) = v_t
\]

Final conditions

\[
x_{t^f}(i) = x_{t^f}(i) + \delta
\]

\[
v_{t^f}(i) = v_{t^f}(i) = v_t
\]

where \( x_t \) [m] is vehicle's position, \( v_t \) [m/s] is speed, \( u_t \) [m/s²] is acceleration, and \( \delta \) is the desired safe distance between the adjacent vehicles.

Analytical Solution

Pontryagin’s minimum principle was applied to find the analytical closed-form solution for Equation (1). We seek to find the control strategy to drive the system along an optimal trajectory. Using Hamiltonian function and the vehicle dynamics equations, the optimal speed and position for each vehicle such as:

\[
v_t(i) = \frac{1}{2} a t^2 + b t + c \quad \text{and} \quad x_t(i) = \frac{1}{6} a t^3 + \frac{1}{2} b t^2 + c t + d \quad \text{for vehicle } i
\]

To derive online optimal control policy for each vehicle, we need to update the constants at each time \( t \). Equation (2) and (3) along with the initial and final conditions defined in the optimization problem (1) can be used to form a system of four equations of the form \( T_p = q \).

\[
\begin{align*}
p_i & = \{T_i\}^T q_i, \quad \text{where } p_i \text{ is a vector containing the four unknown constants } a, b, c, \text{ and } d. 
\end{align*}
\]

Simulation Framework

- Advanced simulation environment which integrates a controller and a simulator using the Visual C# programming
- A hypothetical test-bed network consists of a 300-meter long speed reduction zone operated at the speed limit of 35 mph and a 300-meter long control zone located immediately upstream of the speed reduction zone
- The VISSIM model was calibrated by assessing the key parameters for the car following model. The maximum traffic flow rate was approximated as 1,800 veh/hr in reference to the Highway Capacity Manual (HCM) 2010.

Evaluations Results

- The base case scenario was developed to emulate the human driver behavior using the Wiedemann model in VISSIM to assess the net benefit of the control algorithm over the status without the control algorithm
- The performance of the optimal control algorithm improved as the market penetration of AVs increased in general
- Travel time was reduced by 4-28%, the per-vehicle fuel consumptions was reduced by 6-21%, and the vehicle throughput was increased by 0.5-6% compared to the base case

Conclusions and Future Research

- Optimal control algorithm yielded the environmentally optimized control policy for individual vehicle while maximizing the traffic throughput under the hard constraint of collision avoidance
- The future work includes (i) evaluations of the optimal control algorithm compared with the state-of-the-art existing speed harmonization algorithm; and (ii) implementation of the optimal control algorithm to various scenarios such as ramp metering, intersection control, etc.

Acknowledgment

This research was supported by the SMART Mobility Initiative of the Department of Energy. This research project was also supported by the Global Research Laboratory Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (2013K1A1A2A02078326). These supports are gratefully acknowledged.