Multi-Criteria Decision Making for Financial Managers – 26B
Professional Development Institute
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Naval Postgraduate School
Part 1: Identify and Structure objectives
Part 2: Build and use a quantitative preference model
Cost-Effectiveness Analysis

from OMB Circular A-4:

“Cost-effectiveness analysis can provide a rigorous way to identify options that achieve the most effective use of the resources available without requiring monetization of all of relevant benefits or costs. Generally, cost-effectiveness analysis is designed to compare a set of regulatory actions with the same primary outcome (e.g., an increase in the acres of wetlands protected) or multiple outcomes that can be integrated into a single numerical index (e.g., units of health improvement).”
Choose the Best Alternative

Choose the alternative that minimizes cost and maximizes effectiveness

How can I combine all my objectives into one measure of effectiveness?

What should I do if one alternative is the cheapest and another alternative is the most effective?
How to be a Mathematician!
What?
This lecture introduces the MOE, a model and a process for measuring effectiveness quantitatively, and comparing alternatives.
Under certain conditions, there exists a value function that represents the decision maker’s preferences using a single number.

\[ A_1 \rightarrow V(A_1) \rightarrow \text{MOE}(A_1) \]

\[ \text{MOE} = \text{Measure of Effectiveness} \]
Roles

V(A₁)

Preference Model

A₁

Decision Maker

Analyst

Stakeholder

Subject-Matter Expert (SME)
Terminology

- MCDM (multi-criteria decision making)
- MODM (multi-objective decision making)
- MAVT (multi-attribute value theory)
- MODA (multi-objective decision analysis)
- MAUT (multi-attribute utility theory)
- AHP (analytic hierarchy process)
Defining Effectiveness

• Measures the extent to which an alternative helps to achieve objectives
• Takes into account relative importance of each objective
• Does **not** consider cost
• Relies on the decision maker’s preferences

Different people may have different values... and therefore the same alternative has different levels of effectiveness for them
Additive Value Function

Data about a given alternative’s performance with respect to many attributes

\[ V(x_j) = V(x_{1,j}, x_{2,j}, \ldots, x_{n,j}) = w_1 v_1(x_{1,j}) + \ldots + w_n v_n(x_{n,j}) \]

Value function for one attribute

Overall value function

Trade-off weight (global)

Number of attributes

\[ V(A_1) \]

contribution of first attribute to Alternative \( j \)'s overall desirability

contribution of \( n^{th} \) attribute to Alternative \( j \)'s overall desirability
The MOE structure

\[ V(x_j) = V(x_{1,j}, x_{2,j}, \ldots, x_{n,j}) \]

\[ = w_1 v_1(x_{1,j}) + \ldots + w_n v_n(x_{n,j}) \]

\( x_{i,j} \)'s are the data about the attribute levels achieved by each alternative. The \( w_i \)'s and \( v_i(\cdot) \)'s are the parameters of the preference model. They describe preference. They do not change with alternative.
Screening criteria / requirements

• If you have absolute requirements, you must screen out unacceptable alternatives separately

• The model discussed in this class will **not** eliminate an unacceptable alternative automatically

• An additive model allows other attributes to compensate for unacceptable levels of other attributes
How?
Process for building MOE

STEP 1: Build an objectives hierarchy

STEP 2a: Construct a value function, \( v_i(x_i) \), for each attribute

STEP 2b: Only then, Assign weights, \( w_i' s \) for all objectives/attributes

STEPs 3-5: Collect data, calculate MOEs, and compare alternatives
Defining Effectiveness

What is important?

How much is enough?

How important is it?
How Much Is Enough?

• Each attribute is translated to a value scale between 0 and 1

• For each attribute, determine:
  • Not enough performance → value = 0
  • Good enough performance → value = 1
  • Is “more” or “less” better?
Value function

\[ v(x_{i,1}) \]

Translates attribute’s original units to [0, 1] scale so all attributes can be combined.

Attribute’s scale
e.g. “meters” or %
Marginal Value

- Numerical attributes
- Lowest possible level (minimum)
- Example - coffee
Marginal Value

1. Lowest possible level $\rightarrow$ value = 0
2. Highest possible level $\rightarrow$ value = 10
3. Assess marginal value of moving to the next possible level
4. Repeat step #3
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<th>$v_{\text{coffee}}$ (cups)</th>
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Marginal Value

$v_{\text{coffee}}$ (cups)

cups of coffee
Can My Value Function Look Like This?

Short answer. No! This is actually two different issues.
• \( v_i(x_i) \) is a value function over a **single attribute** \( i \)

• More is better
  - Decision maker prefers more of the attribute to less of the attribute
  - As \( x_i \) increases, \( v_i(x_i) \) increases

• Less is better
  - Decision maker prefers less of the attribute to more of the attribute
  - As \( x_i \) increases, \( v_i(x_i) \) decreases

• Value function can be scaled so that \( 0 \leq v_i(x_i) \leq 1 \)
Example: Field Radio Range

- Not enough range \( \leq 1 \text{ km} = x_i^L \)
- Enough range \( \geq 6 \text{ km} = x_i^H \)

Consider a radio with a range of 4 km

**Calculation:**

\[
\frac{\text{measure} - \text{not enough}}{\text{good enough} - \text{not enough}} = \frac{4-1}{6-1} = 0.6
\]

\[
\frac{\text{Range} - x_i^L}{x_i^H - x_i^L}
\]
Example of a value function for an attribute for which *more is better*. 

![Value function diagram](image-url)
Example of a value function for an attribute for which *less is better*.
Example: Field Radio Weight

- Too much weight $\geq 10 \text{ kg}$
- Light enough weight $\leq 0.5 \text{ kg}$

Consider a radio with a weight of 8 kg

Calculation

$$\frac{x_{i}^{H} - \text{Weight}}{x_{i}^{H} - x_{i}^{L}} = \frac{(10-8)}{(10-0.5)} = 0.21$$
Creating a single-attribute value function (Basic Steps)

1. Determine if the scale of the evaluation (attribute) measure is monotonically increasing or decreasing.

2. Define end points for the measure based on the decision maker’s preferences for the attribute.
   – Bad, worst, not enough, minimum performance
   – Good, best, enough, maximum useful performance

3. Fill in intermediate points – various methods.
Individual attribute value function
(example – radar range)

\[ v_R(x_i^-, 0) \]

\[ v_R(x_i^+, 1) \]
1. Assume a linear relationship
   Each (equal-size) increment is equally valuable
Assume a linear scale between low and high

Or ... Capture marginal value
Example of a value function for an attribute for which *more is better*.

Is a linear approximation good enough?
Is a linear approximation good enough?

Example of a value function for an attribute for which *more is better*.

![Value Scale vs Helicopter Payload](image.png)
Example of a value function for an attribute for which *less is better*.

Is a linear approximation good enough?
Is a linear approximation good enough?

Example of a value function for an attribute for which *less is better.*
Methods to fit intermediate points

1. Assume a linear relationship
   Each (equal-size) increment is equally valuable

2. Find a few (1, 3, maybe more) intermediate points and assume piecewise linear
Fitting a piecewise linear function: Direct assessment

- Given a set of values for $x_i$, the stakeholder(s) specify $v_i(x_i)$ test. For all other values of $x_i$, $v_i()$ is piecewise linear.
- Often the set of $x_i$, to be assessed corresponds to the values occurring in the set of available alternatives.
- The stakeholders must calibrate themselves (or the analyst must normalize) so that $v_i(x_i)$ are monotonic and between zero and one.
Direct assessment with Equal Measurement Intervals

\[ v_i(x_i) \]

\[ v_i \left( \frac{x_i^+ + x_i}{2} \right) \]

\[ x_i^- + 0.25 \times (x_i^+ - x_i^-) \]

\[ x_i^- + 0.75 \times (x_i^+ - x_i^-) \]
How do we measure . . .

- Radar range
- Cruising speed

These are **quantitative** measures

- Radar electronic countermeasures (Yes/No)
- Cognitive Load (Low, Medium, High)

These are **categorical** measures
Example of a categorical measure
What is important?

How much is enough?

How important is it?
How Important Is It?

• Assess weights that describe the relative importance of one attribute versus another attribute
• Represent the trade-offs a decision maker is willing to make between attributes
• Each weight is between 0 and 1 and sum of all the weights equals 1
• Preferences of the decision maker
Process for building MOE

STEP 1: Build an objectives hierarchy

STEP 2a: Construct a value function, $v_i(x_i)$, for each attribute

STEP 2b: **Only then**, Assign weights, $w_i's$ for all objectives/attributes

STEPs 3-5: Collect data, calculate MOEs, and compare alternatives
Weights and value functions are related

• Weights (preferences of one attribute relative to another) depend on the extent to which the single-attribute value functions differentiate among alternatives
• Value functions – elicit these first
• Weights are meaningful only with knowledge of functions

Warning: Often people feel they can provide meaningful weights and are mistaken...
Why We Don’t Always Do This

• Time
• Accessibility of decision maker
• Amount of information required
• Time
How do we get weights?

How much information on attribute importance can we get from the decision maker?

- None
- Rank order
  - Equal weighting
  - Invert and normalize

Tradeoffs

- Direct assessment
  - Pairwise comparison
- Swing Weights
Two Kinds of Weights

**Local Weights** – trade-off weights at one branching of an objectives hierarchy; they represent the relative importance of the objectives below this branch in contributing to the objective above them in the hierarchy.

**Global Weights** (also called *attribute weights*) – trade-off weights for each attribute; they represent the relative importance of the attributes to the overall (maximum potential) value of an alternative.
Local Weights

Effectiveness

Availability  Performance  Complexity

$0 \leq w_i \leq 1$

$\sum w_i = 1$

Interoper.  ECCM  Range

$0 \leq w_i \leq 1$

$\sum w_i = 1$

Cognitive Load  Ease-of-Use

$0 \leq w_i \leq 1$

$\sum w_i = 1$
Global Weights

Effectiveness

Availability

Performance

Complexity

Interoper.

ECCM

Range

Cognitive Load

Ease-of-Use

\[ 0 \leq w_i \leq 1 \]

\[ \sum w_i = 1 \]
How Do We Get Weights?

- How much information on attribute importance can we get from the decision maker?
- How comfortable is the decision maker with numbers?

Best  Worst

Swing Weights

Full Trade-offs

No Time  No Access

No Information

Full Trade-offs
Swing Weighting

• Step 1: Use established ordering of attributes
• Step 2: Imagine a system which is the worst in all attributes: *Benchmark*
• Step 3: Imagine other systems
  ➢ *Best* in one attribute
  ➢ *Worst* in all others
• Step 4: Compare and express preferences
### Radar Performance

#### Fictional Scenarios

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<th>Interop</th>
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**Weight**

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**Total Weighting**

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Pairwise Comparison (I)

Effectiveness

- $w_A$: Availability
- $w_P$: Performance
- $w_C$: Complexity

Relative Weights

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<td>$w_C$</td>
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Total: 230

1
Graphical Point Allocation

Effectiveness

- $w_A$ Availability
- $w_P$ Performance
- $w_C$ Complexity

Availability: 8/15
Performance: 4/15
Complexity: 3/15
• Some people just don’t think in numbers
• Scales

A

P

C

• Pie charts
• Consider pairs of attributes relative to each other attribute

• Example: “Performance is three times as important as complexity and twice as important as availability”

\[
\begin{align*}
w_P &= 3 \times w_C \\
wp &= 2 \times w_A \\
w_A + w_P + w_C &= 1
\end{align*}
\]

• Caution: Pairwise comparison seems easy but it often fails to account for the ranges of the attributes and may violate transitivity
### Weighting Schemes

#### Effectiveness

<table>
<thead>
<tr>
<th>Weighting Scheme (w)</th>
<th>Domain</th>
<th>Weight (w)</th>
<th>Rank</th>
<th>Value Coefficients</th>
<th>Reciprocal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_A ) Availability</td>
<td>1</td>
<td>1/1</td>
<td>6/6</td>
<td>6/11</td>
<td>.55</td>
</tr>
<tr>
<td>( w_P ) Performance</td>
<td>2</td>
<td>1/2</td>
<td>3/6</td>
<td>3/11</td>
<td>.27</td>
</tr>
<tr>
<td>( w_C ) Complexity</td>
<td>3</td>
<td>1/3</td>
<td>2/6</td>
<td>2/11</td>
<td>.18</td>
</tr>
</tbody>
</table>

### Rank Reciprocal

\[
\begin{align*}
\text{Sum} &= \frac{11}{6} \\
\text{Reciprocal} &= 1
\end{align*}
\]
Effectiveness

\[ \begin{align*}
  w_A & \quad \text{Availability} & 1 & \rightarrow & 3 & 3/6 & .5 \\
  w_P & \quad \text{Performance} & 2 & \rightarrow & 2 & 2/6 & .33 \\
  w_C & \quad \text{Complexity} & 3 & \rightarrow & 1 & 1/6 & .17 \\
\end{align*} \]


**Rank Sum**

\[ \begin{align*}
  6 & \rightarrow & 1 & \Rightarrow & 1 \\
\end{align*} \]
Weighting Schemes

Effectiveness

\[ w_A \quad \text{Availability} \]
\[ w_P \quad \text{Performance} \]
\[ w_C \quad \text{Complexity} \]

Direct Assessment

\[ \begin{array}{c}
.5 \\
.3 \\
.2 \\
1 \\
\end{array} \]
Weighting Schemes

<table>
<thead>
<tr>
<th>Effectiveness</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_A$ Availability</td>
<td>.33</td>
</tr>
<tr>
<td>$w_P$ Performance</td>
<td>.33</td>
</tr>
<tr>
<td>$w_C$ Complexity</td>
<td>.33</td>
</tr>
</tbody>
</table>

Total: 1
Key Takeaways

Weights

• Answer how important is each objective relative to the other objectives
• Should take into account the range of the value functions
• Can be assessed in multiple ways
• Find what works best with your decision maker
Assessing Trade-off Weights

- If decision maker will not say which objective is more important → equal weighting
- If decision maker orders objectives but cannot say how much more important → rank sum

<table>
<thead>
<tr>
<th>Objective</th>
<th>Order</th>
<th>Reverse order</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective a</td>
<td>1</td>
<td>4</td>
<td>4/10 = 0.4</td>
</tr>
<tr>
<td>Objective b</td>
<td>2</td>
<td>3</td>
<td>3/10 = 0.3</td>
</tr>
<tr>
<td>Objective c</td>
<td>3</td>
<td>2</td>
<td>2/10 = 0.2</td>
</tr>
<tr>
<td>Objective d</td>
<td>4</td>
<td>1</td>
<td>1/10 = 0.1</td>
</tr>
</tbody>
</table>

- If decision maker answers how much more important → direct assessment or swing weighting
Putting it all together

Let’s choose a new Tank!
Effectiveness of Alternatives

- Calculate measure of effectiveness (MOE) for each alternative
- Alternatives: Tank (A), Tank (B), etc...
- Attributes: Caliber, Mussel Velocity, Speed, Range, Height, Armor
- The same objectives hierarchy, value functions, and tradeoff weights are used for each alternative
Example: Main battle tank

Effectiveness

Firepower
- Caliber (mm)
- Muzzle Velocity (mps)

Mobility
- Speed (kph)
- Range (km)

Survivability
- Height (meters)
- Armor (mm)
Effectiveness
Main Battle Tank (A)

- Firepower: .45, .40, .60
- Mobility: .30, .67, .33
- Survivability: .25, .44, .56

Caliber, MV, Speed, Range, Height, Armor
Effectiveness

Main Battle Tank (A)

Firepower
- .45
- Caliber 0.500
- MV 0.929

Mobility
- .30
- Speed 0.500
- Range 0.560

Survivability
- .25
- Height 0.700
- Armor 0.800
Effectiveness
Main Battle Tank (A)

Firepower
- Caliber 0.500
- MV 0.929

Mobility
- Speed 0.500
- Range 0.560

Survivability
- Height 0.700
- Armor 0.800

V(Mobility) = 0.67 × 0.50 + 0.33 × 0.56 = 0.52
Effectiveness
Main Battle Tank (A)

Firepower
- .45
  - Caliber: 0.500
    - .40
    - .60
- .757

Moility
- .30
  - Speed: 0.500
    - .67
    - .33
- .520

Survivability
- .25
  - Range: 0.560
    - .44
    - .56
  - Height: 0.700
  - Armor: 0.800
- .756

Effectiveness
- .500
- .929
- .500
- .560
- .700
- .800
Effectiveness
Main Battle Tank (A)

0.686

0.45
Firepower

0.757

0.40 0.60
Caliber 0.500 0.929

0.30
Mobility

0.520

0.67 0.33
Speed 0.500 Range 0.560

0.25
Survivability

0.756

0.44 0.56
Height 0.700 Armor 0.800
Effectiveness
Main Battle Tank (A)

Firepower
- Caliber: 0.500
- MV: 0.929

Mobility
- Speed: 0.500
- Range: 0.560

Survivability
- Height: 0.700
- Armor: 0.800

Effectiveness: 82
Effectiveness
Main Battle Tank (A)

Firepower
- Caliber: 0.500
- MV: 0.929
- Effectiveness: 0.090

Mobility
- Speed: 0.500
- Range: 0.560
- Height: 0.700

Survivability
- Armor: 0.800
Effectiveness
Main Battle Tank (A)

Firepower
- Caliber: 0.500 \times 0.180 = 0.090
- MV: 0.929 \times 0.270 = 0.251
- Speed: 0.500 \times 0.201 = 0.101

Mobility
- Range: 0.560 \times 0.099 = 0.055
- Height: 0.700 \times 0.110 = 0.077

Survivability
- Armor: 0.800 \times 0.140 = 0.112

Total: 0.090 + 0.251 + 0.101 + 0.055 + 0.077 + 0.112 = 0.686
Measure of Effectiveness (MOE)

Effectiveness measures (attributes)
Preferences for individual attributes (value functions)
Relative importance of attributes (trade-off weights)

(additive) value function

\[ v(A) = w_1 v_1(A) + w_2 v_2(A) + \ldots \]

EFFECTIVENESS
Measures of effectiveness

- **Tank A**: MOE = 0.686
- **Tank B**: MOE = 0.561
- **Tank C**: MOE = 0.808
- **Tank D**: MOE = 0.770
- **Tank E**: MOE = 0.742
Measures of effectiveness

<table>
<thead>
<tr>
<th>Tank</th>
<th>MOE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.686</td>
</tr>
<tr>
<td>B</td>
<td>0.561</td>
</tr>
<tr>
<td>C</td>
<td>0.808</td>
</tr>
<tr>
<td>D</td>
<td>0.770</td>
</tr>
<tr>
<td>E</td>
<td>0.742</td>
</tr>
</tbody>
</table>

- **Firepower**
- **Mobility**
- **Survive**
Why?
Choose the Best Alternative

Choose the alternative that minimizes cost and maximizes effectiveness

Answering three questions to define effectiveness

Ask decision maker if they are willing to spend more money to gain extra capabilities of more effective alternative
Cost-Effectiveness Analysis (CEA)

Measures of Effectiveness (attributes)

Preferences for Individual Attributes (value functions)

Relative Importance of the Attributes (weights)

EFFECTIVENESS (MOE)

COST

COST- EFFECTIVENESS
CEA powered by MCDM
CEA powered by MCDM

![Diagram showing the relationship between Effectiveness and Cost, with MOE and C marked on the axes.](image-url)
CEA powered by MCDM
CEA powered by MCDM

\[ v(A) = w^1 v^1(A) + w^2 v^2(A) + w^3 v^3(A) + \ldots \]

Diagram showing the relationship between MOE and Cost with points plotted on a graph.
An alternative that dominates all other alternatives.
The tyranny of fixed requirements
Which efficient alternative is best?

That depends on what matters most to you.

- Only cost matters
- Only MOE matters
Benefits of Process

- Clarifies thinking
- Reveals and documents stakeholder preferences
- Easy to determine points of disagreement
- Supports sensitivity analysis
- Allows rapid re-evaluation
  - under a variety of scenarios, and
  - of new alternatives if they arise
- Justifies decisions
- Outlasts analyst
Key Takeaways

• The answers to three questions define effectiveness:
  • What is important? → objectives hierarchy
  • How much is enough? → value functions
  • How important is it? → trade-off weights

• Preferences of the stakeholders explicitly incorporated into analysis.

• Decision maker evaluates trade-off between effectiveness and cost – simplified by MOE.

• In many cases, the process is more important than the numbers.
Resources


Short courses

Defense Resources Management Institute

- Defense resources management (4 weeks)
- Multiple criteria decision making (2 weeks)
- Risk management (2 weeks)
- Data analytics for financial and resources management (2 Weeks)
- Human capital resources management (2 weeks)
- Performance management and budgeting (1 week)
- Budget preparation, execution and accountability (8 days)

http://www.nps.edu/DRMI

drburton@nps.edu
BACKUP
<table>
<thead>
<tr>
<th>G</th>
<th>O</th>
<th>A</th>
<th>M</th>
<th>P</th>
</tr>
</thead>
</table>

Maximize Effectiveness
Lessons to Learn

- Time Allocation
  - Goals } 1/3rd of your time
  - Objectives } 1/3rd of your time
  - Alternatives } 1/3rd of your time
- Analysts tend to spend too much time on Alternatives (especially data) and Models
- Not enough time spent defining Goals and Objectives
- Not enough time spent on Preferences
- Too much time spent in the details of the models
- “It is not about a number. The primary role of any planning process is to stimulate critical thinking.” - Douglas Burton
Lessons to Learn (2)

• Objectives, Objectives, Objectives
• Bias is everywhere (Limits to Rationality Lecture)
• “All models are wrong, but some are useful” – George Box
• Good analysis is hard to do
• Put the Right Person in charge (horsepower)
• Analytical Aperture
• Make sure that the Thinking Process utilized has an inherent ability to Change and be Changed
• Unintended Consequences
• “Perfect is the Enemy of Good Enough” – Voltaire
• You are not alone, so synergize
• Make time to Think
What would you do?
What would you do?
Decision Support for Defense Management

Analytical Decision Making
- Systems Analysis
- Cost Issues
- Effectiveness and Evaluation Issues

Integrating Case Studies: Drmecia

TEMPO Military Planning Game
- Economics / Environment
- Quantitative Analysis
- Management and Resources Management / Systems

Decision Support for Defense Management

Strategy / Policy Formulation
- Program Analysis and Implementation
- Program Execution and Control
“Major decisions should be made by choices among explicit, balanced, feasible alternatives”

“The Secretary should have an active analytic staff to provide him with relevant data and unbiased perspectives”

“Open and explicit analysis, available to all parties, must form the basis for major decisions”
Who are we?

Defense Resources Management Institute (DRMI)

• Sponsored by Secretary of Defense
  ➢ Department of Defense (DoD) Instruction 5010.35

• Established in 1965 at the Naval Postgraduate School (NPS)
  ➢ Dr. Charles Hitch, OSD comptroller under SecDef McNamara, used NPS faculty to teach analytical and business approaches to make the best use of defense resources
Course Goals

• To develop a broad-based analytical framework for defense decision makers
  – emphasizing the economic and efficient allocation of scarce defense resources to competing mission areas.

• To provide an environment for the comparative exchange of ideas related to the management of national security
What do we teach?

Resident Courses:

- **General**
  - International Defense Management Course (IDMC)—10 weeks; 2 per year
  - Defense Resources Management Course (DRMC)—4 weeks; 4 per year
  - Senior International Defense Management Course (SIDMC)—4 weeks; once per year

- **Specific**
  - Multi-Criteria Decision Making—2 weeks
  - Introduction to Budget Concepts—8 days
  - Risk Management—2 weeks
  - Performance Management & Budgeting—1 week
  - Human Capital RM—2 weeks

Non-Resident Events:

- Mobile Courses—1-2 weeks
- Workshops—3-5 days
- Seminars—3-5 days

- Tailored to country’s specific needs
- Conducted in appropriate language
- Opportunity to quickly build a large cohort
More information

- **Target audience:** Military 0-3 or above; Civilian GS-9 or above
- **NO** tuition charged for DoD military and civilians
- Sending agency responsible for travel and per diem
- **Website:** http://my.nps.edu/web/drmi/
• Mission Statement
  – Provide relevant and unique advanced education and research programs to increase the combat effectiveness of commissioned officers of the naval service to enhance the security of the United States.
  – In support of the foregoing and to sustain academic excellence, NPS will foster a program of relevant and meritorious thesis and research experiences for NPS students that informs the curricula, supports the needs of Navy and Department of Defense, and builds the intellectual capital of NPS faculty.
  – To support the core Navy mission, NPS’ programs are inherently joint, inter-agency, and international.
Calculating Weights (Theory)

• Developing a set of weights
  ➢ By convention, they sum to one (need to be normalized)

• Concept of indifference between alternatives
  ➢ Implied tradeoff between “swings” on attributes equals the ratio of the weights
  ➢ Can solve for weights mathematically
Thinking about Values and Weights

• Suppose performance has two attributes
  - **Interoperability** varies between 1 and 10 data links
  - **Range** varies between 150 and 500 km

• Consider two hypothetical choices
  - A: Range = 150, Interoperability = 10
  - B: Range = 500, Interoperability = 1

What amount of interoperability is the decision maker willing to give up to increase range (by 350 km)?
Thinking about Values and Weights

Interoperability

10 data links

4 data links

1 data links

Range

150

500
We established indifference between alternatives

\[ V(x) = w_R v_R(x_R) + w_I v_I(x_I) \]

\[ w_R v_R(150) + w_I v_I(10) = w_R v_R(500) + w_I v_I(4) \]
Value Functions

Value function for interoperability

Value function for range
Calculations

\[ w_R \nu_R (150) + w_I \nu_I (10) \]
\[ = w_R \nu_R (500) + w_I \nu_I (4) \]
\[ w_R * 0 + w_I * 1 = w_R * 1 + w_I \nu_I (4) \]
\[ w_R = w_I - w_I \nu_I (4) \]
\[ w_R + w_I = 1 \]
\[ \nu_I (4) = \frac{1}{3} \]

\[ w_I = \frac{3}{5} \text{ and } w_R = \frac{2}{5} \]
• Suppose performance has two attributes
  ➢ **Interoperability** varies between 1 and 10 data links
  ➢ **Range** varies between 400 and 500 km

• Consider two hypothetical choices
  ➢ A: Range = 400, Interoperability = 10
  ➢ B: Range = 500, Interoperability = 1

What amount of interoperability is the decision maker willing to give up to increase range (by 100 km)?
Does Tradeoff Change?

Interoperability

10 data links

8 data links

1 data links

Range

400

500
Value Functions

Value function for interoperability

Value function for range

Number of interoperable systems

Range (km)
Calculations

\[ w_R \nu_R (400) + w_I \nu_I (10) \]
\[ = w_R \nu_R (500) + w_I \nu_I (8) \]
\[ w_R \times 0 + w_I \times 1 = w_R \times 1 + w_I \nu_I (8) \]
\[ w_R = w_I - w_I \nu_I (8) \]
\[ w_R + w_I = 1 \]

\[ \nu_I (8) = \frac{7}{9} \]

\[ w_I = \frac{9}{11} \text{ and } w_R = \frac{2}{11} \]
Simplification

• Requires a lot of work
• Lot of time
  ➢ Function of the number of attributes
  ➢ Note: NOT the number of alternatives
• Heavy on the math / numbers
• How to make this faster, quicker, and yet still “right”
Weights and Measures

• Buying a car: which is more important?
  • Gas mileage
  • Engine power

• If you selected gas mileage, do you prefer
  • Car A: 30 mpg and 2.0 L engine
  • Car B: 29 mpg and 4.2 L engine

• If you selected engine power, do you prefer
  • Car C: 15 mpg and 4.2 L engine
  • Car D: 35 mpg and 4.1 L engine
Swing Weighting

- Imagine an alternative which is the worst (value=0) in all attributes
- Then imagine other alternatives which are the best (value=1) in ONE attribute, and the worst in all others
- Compare the desirability of these alternatives
Swing Weighting

- Gas mileage: 15 to 50 mpg
- Engine power: 1.8 to 6 L
- Trunk space: 15 to 90 ft³

<table>
<thead>
<tr>
<th>Hypothetical alternative</th>
<th>Gas mileage</th>
<th>Engine power</th>
<th>Trunk space</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst</td>
<td>15 mpg</td>
<td>1.8 L</td>
<td>15 ft³</td>
<td>0</td>
</tr>
<tr>
<td>“Alternative 1”</td>
<td>50 mpg</td>
<td>1.8 L</td>
<td>15 ft³</td>
<td>100</td>
</tr>
<tr>
<td>“Alternative 2”</td>
<td>15 mpg</td>
<td>6 L</td>
<td>15 ft³</td>
<td>60</td>
</tr>
<tr>
<td>“Alternative 3”</td>
<td>15 mpg</td>
<td>1.8 L</td>
<td>90 ft³</td>
<td>80</td>
</tr>
</tbody>
</table>

Weight

<table>
<thead>
<tr>
<th>Gas mileage</th>
<th>Engine power</th>
<th>Trunk space</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 / 240</td>
<td>60 / 240</td>
<td>80 / 240</td>
<td>240</td>
</tr>
</tbody>
</table>
## Sloat Radar

<table>
<thead>
<tr>
<th>Effectiveness</th>
<th>Rank reciprocal</th>
<th>Pairwise comparison</th>
<th>Direct assess.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_A = .5$ Availability</td>
<td>1/1</td>
<td>10</td>
<td>.5</td>
</tr>
<tr>
<td>$w_P = .4$ Performance</td>
<td>1/2</td>
<td>8</td>
<td>.4</td>
</tr>
<tr>
<td>$w_C = .1$ Complexity</td>
<td>1/3</td>
<td>5</td>
<td>.1</td>
</tr>
<tr>
<td></td>
<td>11/6</td>
<td>23</td>
<td>1</td>
</tr>
</tbody>
</table>
Graphical Point Allocation

Effectiveness

\[ W_F \] Fuel consumption  
\[ W_T \] Trunk space  
\[ W_P \] Engine power

8/15  
4/15  
3/15