

# CONDENSED MATTER THEORY: FROM MODELS TO FIRST PRINCIPLES

MARVIN L. COHEN

Department of Physics, University of  
California, and Materials Sciences Division,  
Lawrence Berkeley Laboratory  
Berkeley, CA

IT WAS SUGGESTED THAT I

“REVIEW CMT WITH EMPHASIS ON  
ELECTRONIC STRUCTURE AND THE  
DEVELOPMENT OF COMPUTATIONAL  
METHODS TO CALCULATE AND  
PREDICT PROPERTIES OF REAL  
MATERIALS AND GIVE MODERN  
EXAMPLES (MY OWN WORK)”

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ALL MY REJECTION  
LETTERS FROM PRL

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NANOSCIENCE AND  
SUPERCONDUCTIVITY

# PREHISTORY

EINSTEIN, DIRAC,  
SOMMERFELD-BETHE, FERMI

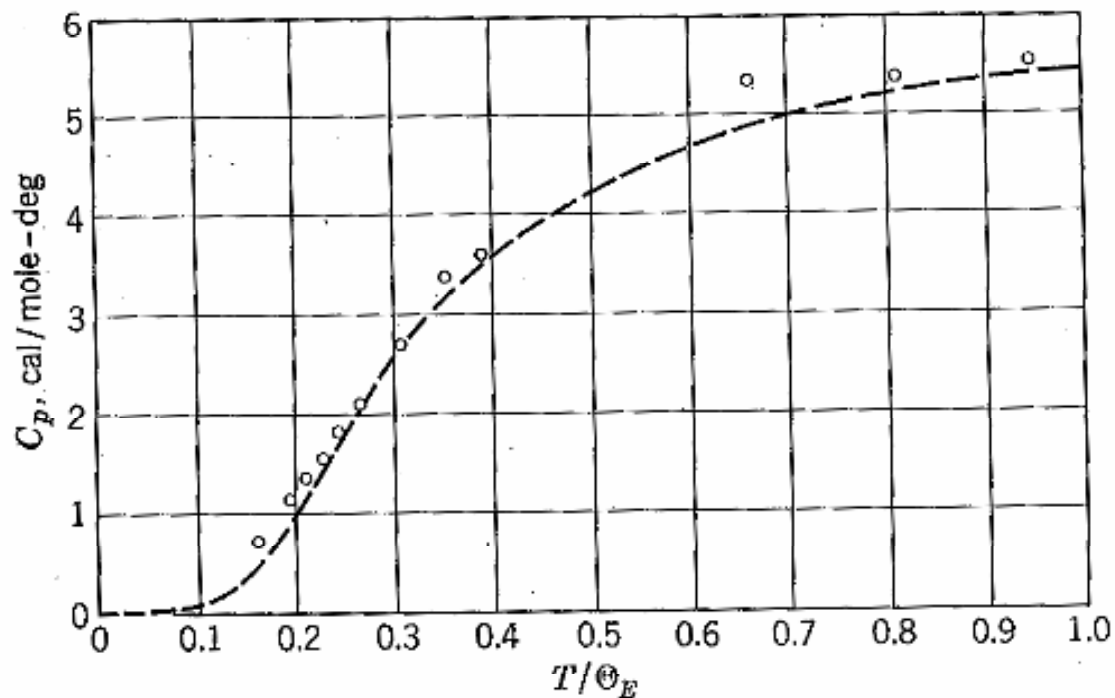
# EINSTEIN 1905

DETERMINATION OF MOLECULAR DIMENSIONS

BROWNIAN MOTION

SPECIAL RELATIVITY

PHOTOELECTRIC EFFECT



**Figure 3** Comparison of experimental values of the heat capacity of diamond with values calculated on the Einstein model, using the characteristic temperature  $\Theta_E = \hbar\omega/k_B = 1320^\circ\text{K}$ . [After A. Einstein, *Ann. Physik* **22**, 180 (1907).]

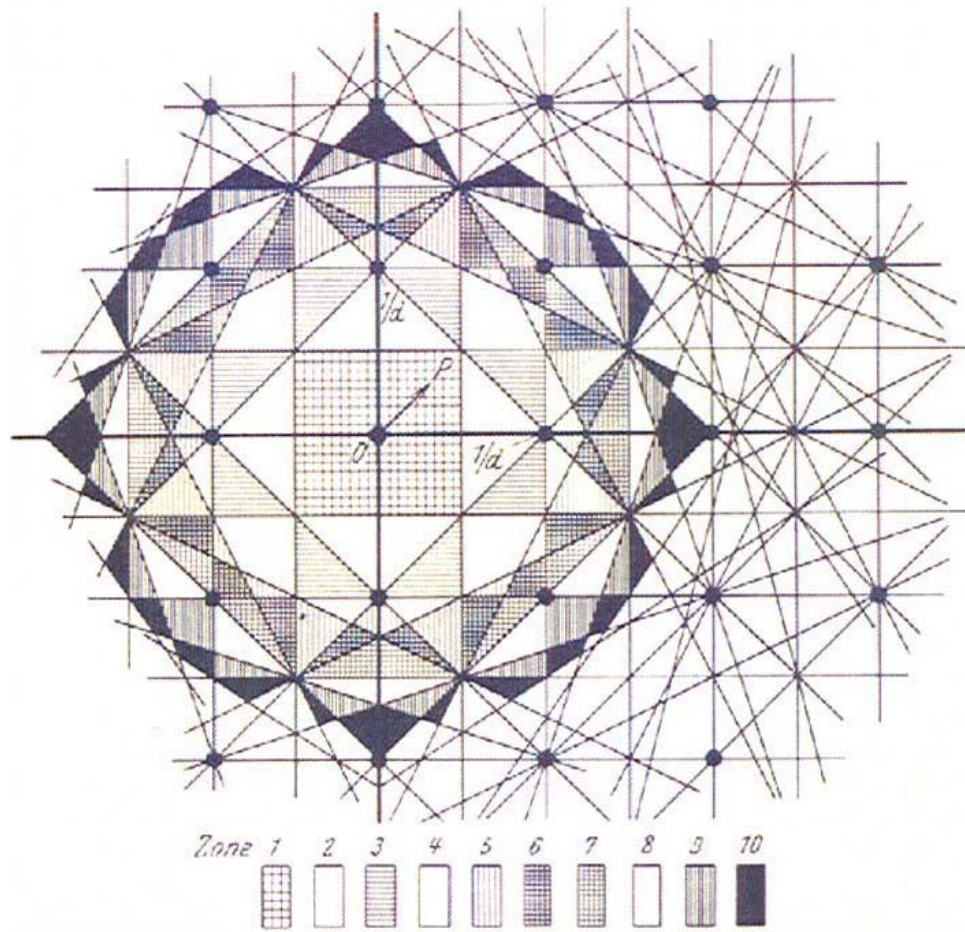
This figure is still in textbooks after a century.

Dirac (1929)

"The underlying physical laws necessary for a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble."

[Proc. Roy. Soc. (London) A123, 714]





BRILLOUIN ZONES  
SOMMERFELD AND BETHE 1933

# PSEUDOPOTENTIAL - FERMI 1934

95. - *Sopra lo spostamento per pressione delle righe, ecc.*

711

All'esterno della buca, dove  $V(r)$  si annulla, la  $u$  è dunque una funzione lineare di  $r$ . E siccome il valore di  $\psi$  lontano dalla buca

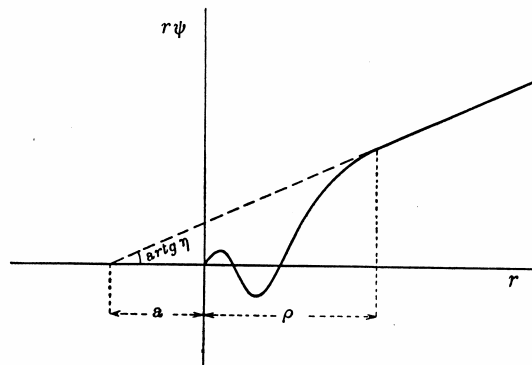


Fig. 1.

deve tendere approssimativamente al valore  $\bar{\psi}$ , si potrà porre, all'esterno della buca:

$$(11) \quad u(r) = (a + r) \bar{\psi}$$

dove  $a$  è una lunghezza, il cui significato è chiarito nella Fig. 1. In essa sono riportati in ascisse i valori di  $r$  e in ordinate quelli di  $u$ . La  $u$ , come risulta dalla (9), è nulla per  $r = 0$ , mentre, per  $r$  maggiore di  $\rho$  ha per grafico una retta. Prolunghiamo questa retta fino ad incontrare l'asse delle ascisse;  $a$  è la distanza del punto di intersezione dall'origine delle coordinate.

Tenendo conto di (9), (10) e (11) troviamo

$$(12) \quad \frac{8\pi^2 m}{h^2} \int V \psi d\tau = 4\pi \frac{8\pi^2 m}{h^2} \int V u r dr = 4\pi \int u' r dr \\ = 4\pi [u' r - u]_0^\infty = -4\pi a \bar{\psi}$$

e siccome nell'unità di volume sono contenute  $n$  buche di potenziale, ricaviamo infine

$$(13) \quad \frac{8\pi^2 m}{h^2} \Sigma V_i \bar{\psi} = -4\pi a n \bar{\psi}$$

Con ciò la (8) diventa

$$(14) \quad \Delta \bar{\psi} + \frac{8\pi^2 m}{h^2} (W_0 - U) \bar{\psi} = 0$$

dove si è posto

$$(15) \quad W_0 = W + \frac{h^2 a n}{2\pi m}$$

# DENSITY FUNCTIONAL THEORY - DIRAC 1930

1940-1960

HERRING-SLATER-PHILLIPS

1957--PRL, BCS, but no  
accurate/detailed Si  $E(k)$

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accurate/detailed Si  $E(k)$

BUT BY...

1965--14 accurate semiconductor  $E(K)$ 's-EPM

1970--optical structure of semiconductors solved

1980's--structural properties, superc., surf., high P

1990-2008 --complex materials, nanostructures,  
and a variety of properties

“I'm often asked whether doing physics research using computers is ‘mindless research.’ My answer is that I can do ‘mindless research’ without a computer.”

*M. L. Cohen (1970)*

***For calculating materials properties: “If given the choice between the computers of today together with the physical concepts of the 1970's—or—the computers of the 1970's along with current concepts, I'd choose the latter.”***

***J. R. Chelikowsky (2000)***

# CONCEPTUAL BASIS

ONE CAN ARGUE FOR TWO MODELS OR  
“MENTAL PICTURES” OF A SOLID:

“INTERACTING ATOMS”

and

“ELEMENTARY EXCITATIONS”  
MODELS



# INTERACTING ATOMS MODEL

A solid is a collection of strongly  
interacting atoms.

The particles are electrons and nuclei  
interacting via EM interactions.

(associated with reductionism)

# ELEMENTARY EXCITATION MODEL

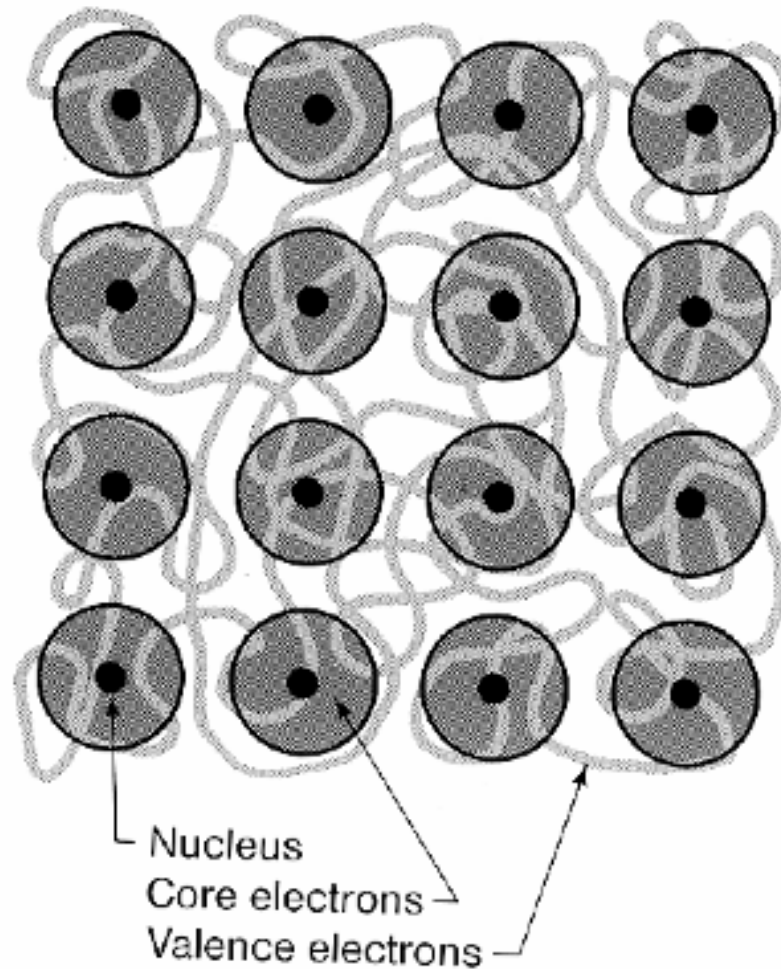
particles are mainly:  
quasiparticles and collective excitations  
[probe-response]  
(emergent behavior)

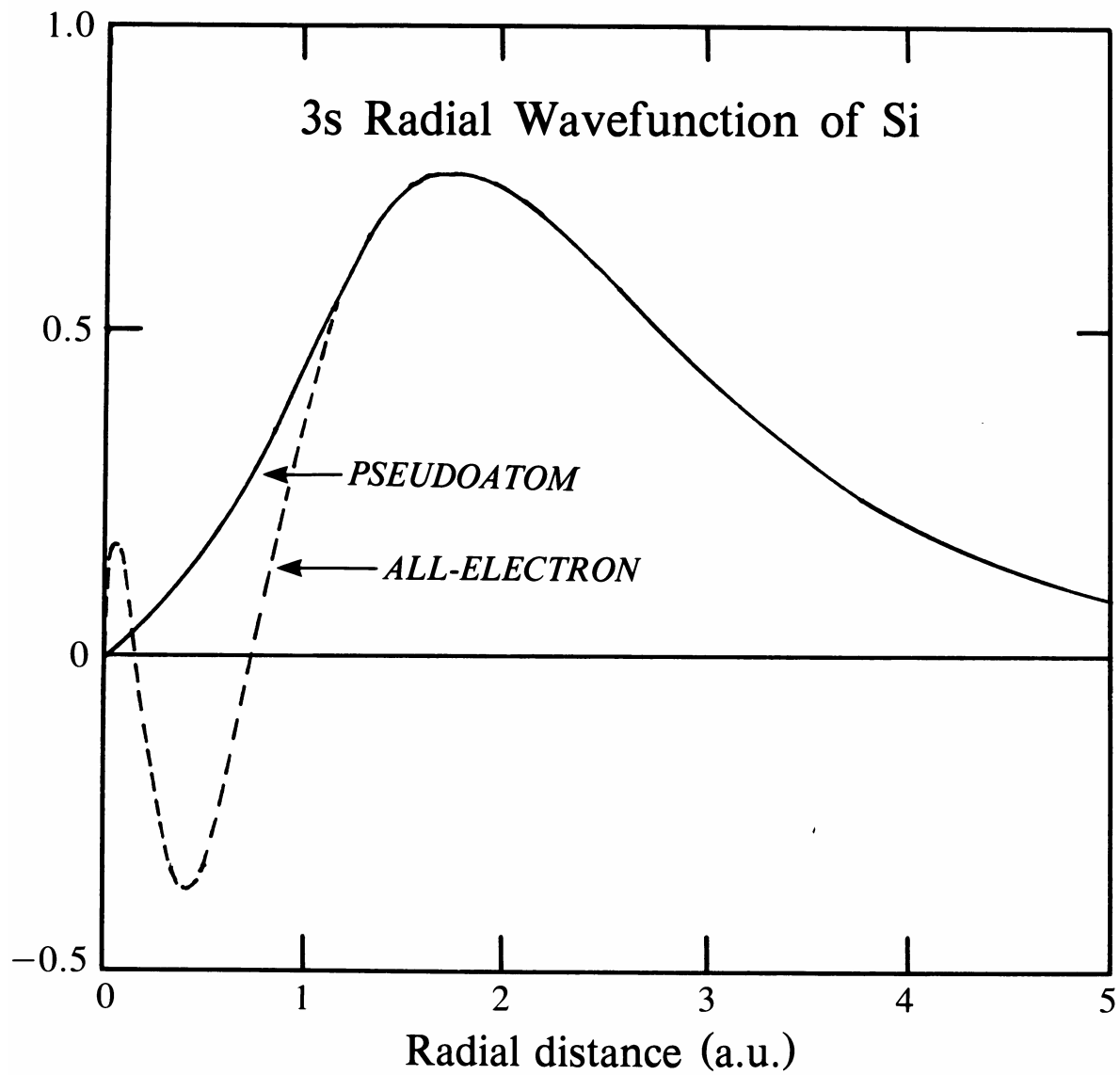
quasiparticles: quasielectrons (like polarons), holes,  
superconducting quasiparticles,...

collective excitations: phonons, plasmons, magnons,...

# Standard Model

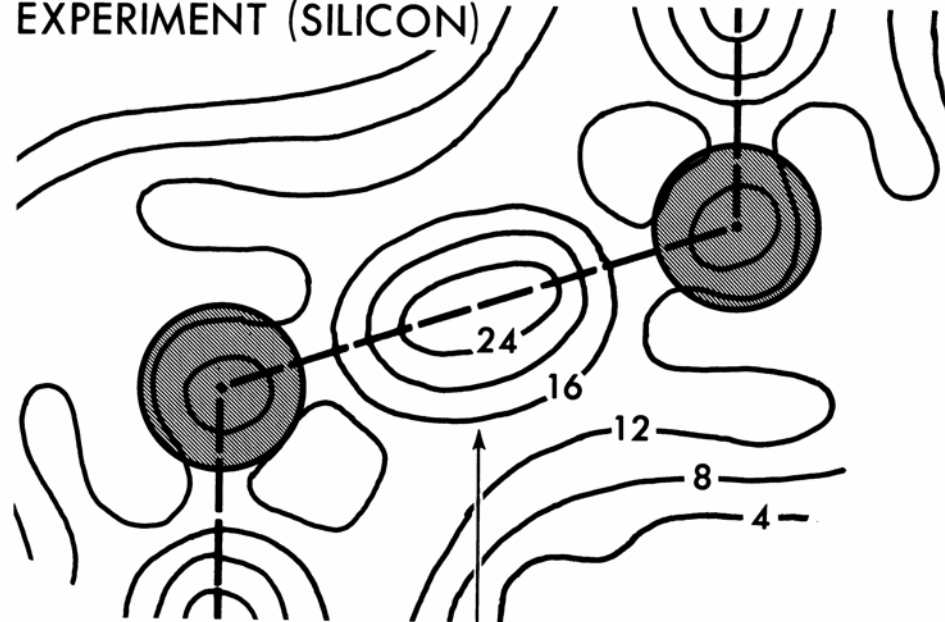
## Plane Wave Pseudopotential Method [PWPM]





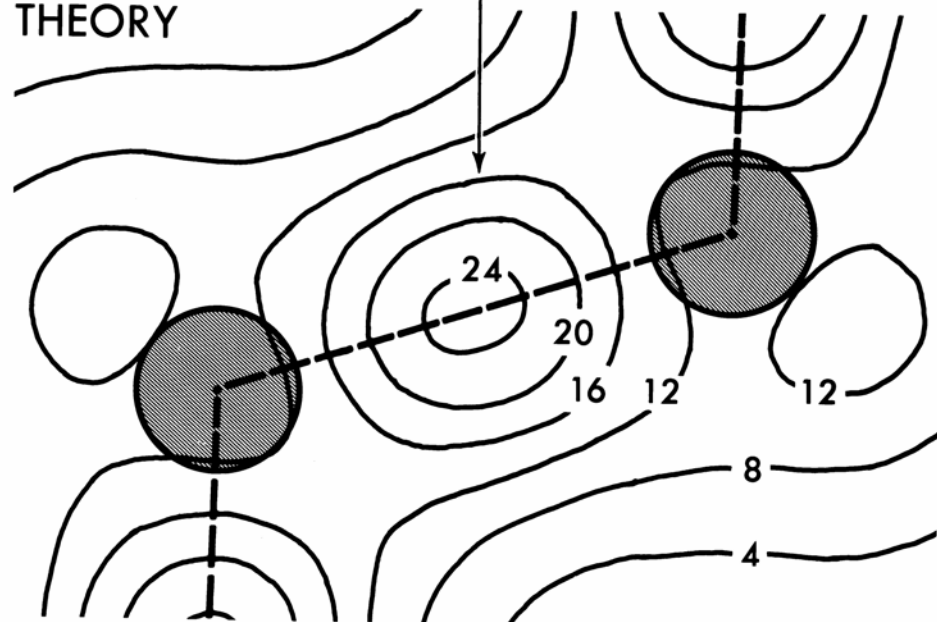
# Charge Density of Si

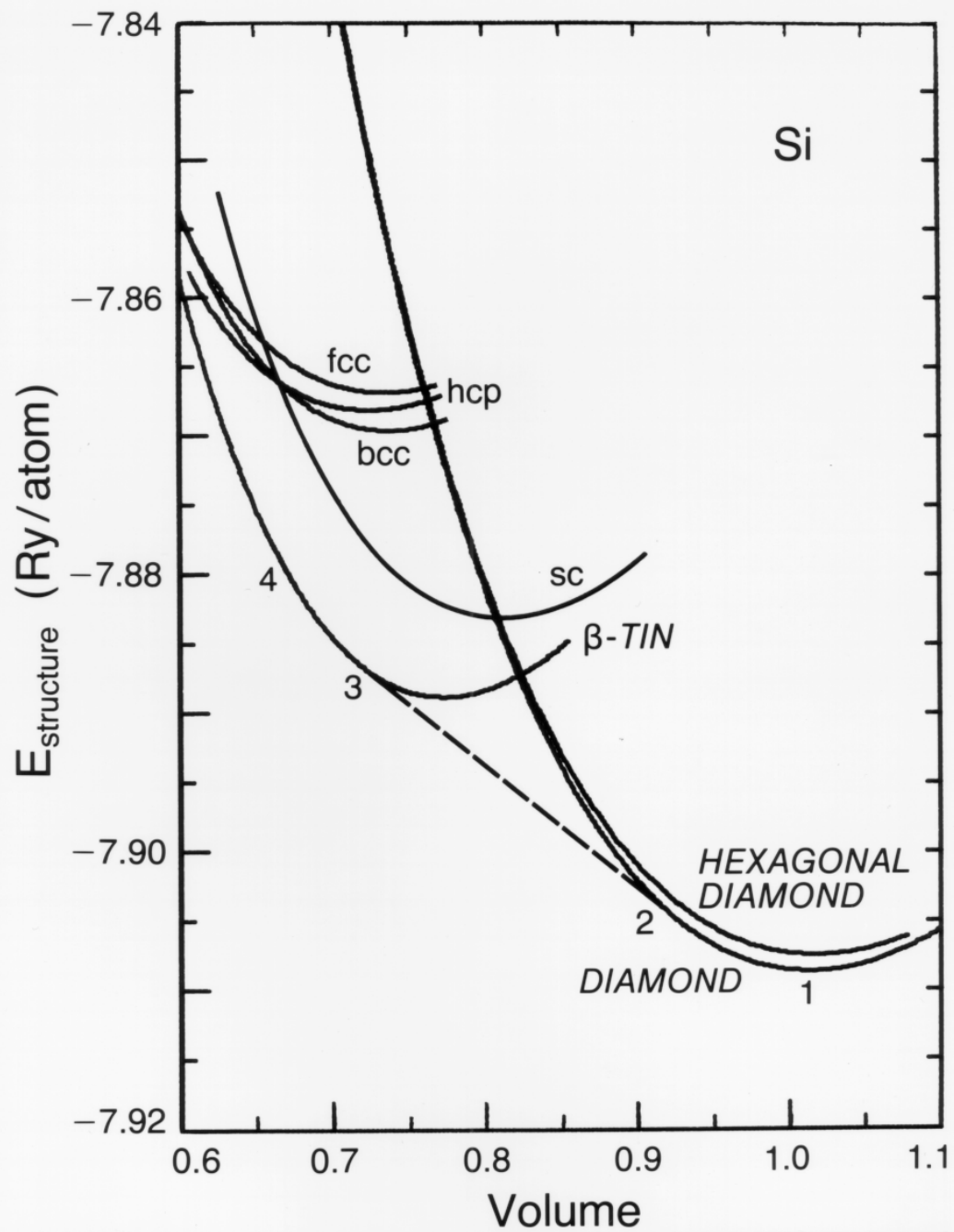
EXPERIMENT (SILICON)



BOND

THEORY





## Static Structural Properties

	lattice constant (Angstroms)	bulk modulus (GPa)
Si		
calc.	5.45	98
expt.	5.43	99
% diff.	0.4%	-1%
Ge		
calc.	5.66	73
expt.	5.65	77
% diff.	0.1%	-5%
C		
calc.	3.60	441
expt.	3.57	443
% diff.	0.8%	-1%

# **Plane Wave Pseudopotential Method**

**(Standard Model of Solids)**

**For a broad class of solids, clusters, and molecules, this method describes ground-state and excited-state properties such as:**

**electronic structure**

**crystal structure and structural  
transitions**

**structural and mechanical properties**

**vibrational properties**

**electron-lattice interactions**

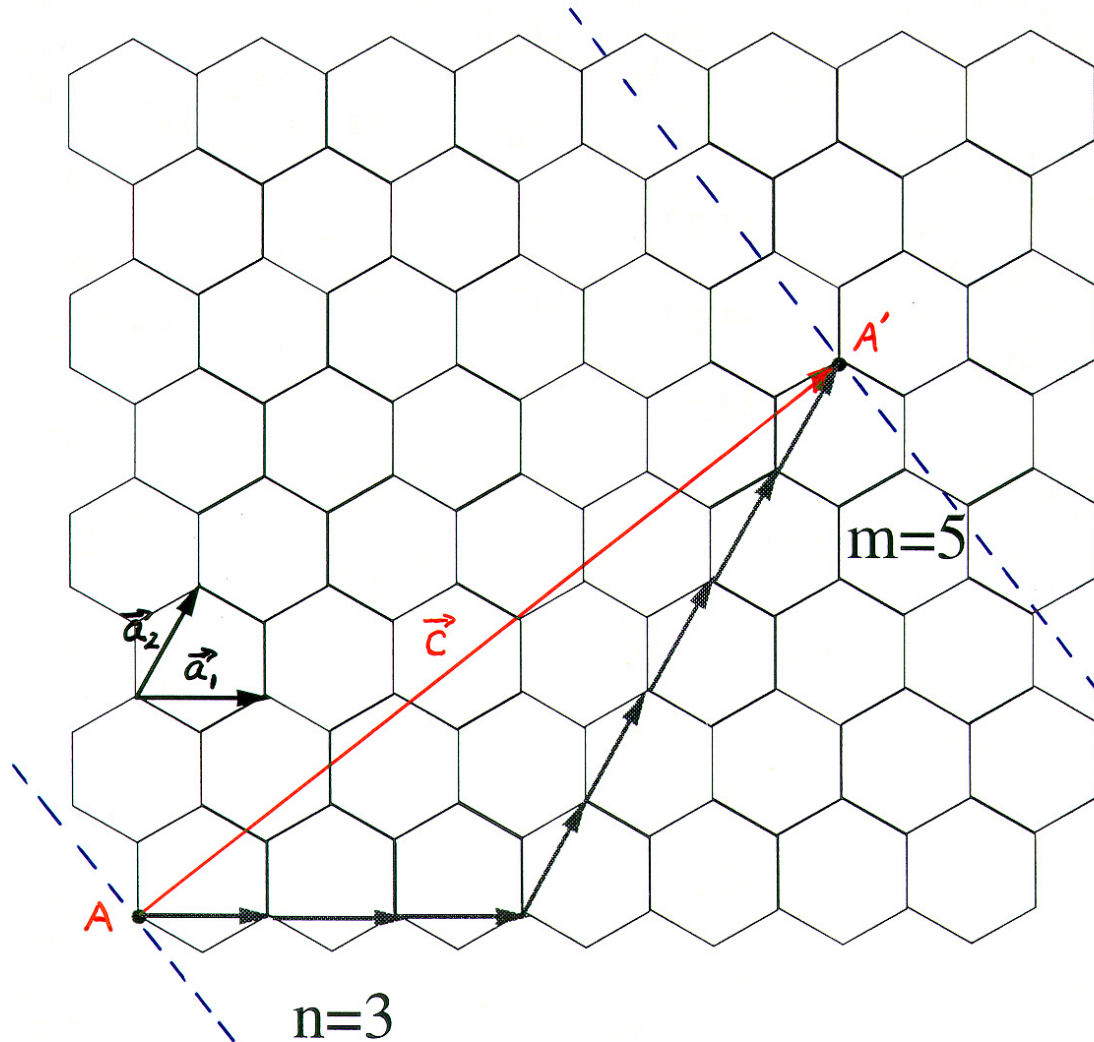
**superconductivity**

**optical properties**



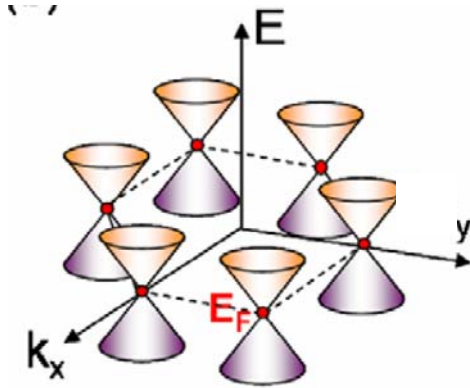
CONFINEMENT  
REDUCED DIMENSIONALITY  
SYMMETRY

Nanotubes are indexed by the  
circumferential periodicity.

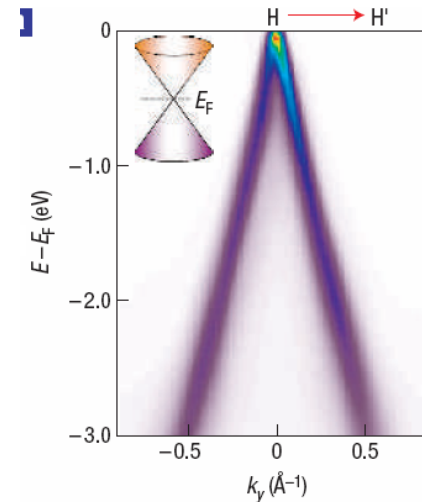


The (3,5) nanotube.

# 2-D graphene as physical realization of (2+1)D QED



Single particle energy dispersion



ARPES, S. Y. Zhou *et al*,  
Nature Phys.**2**, 595 (2006)

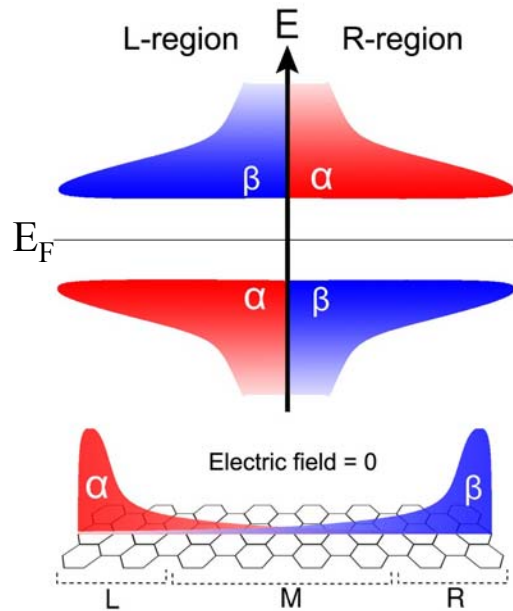
Massless Dirac equation with  $c^* \sim c/300 \sim 10^6 \text{m/s}$

Quantum Hall effect in graphene observed

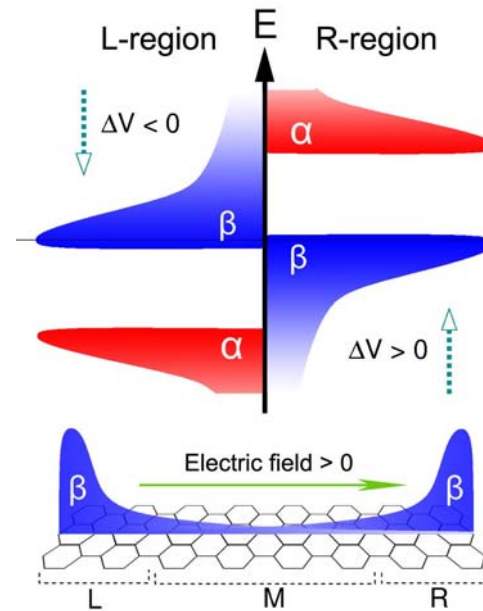
Electric field induced half-metallic states in graphene nanoribbons

# Effect of Transverse Electric Field on Edge States

no field

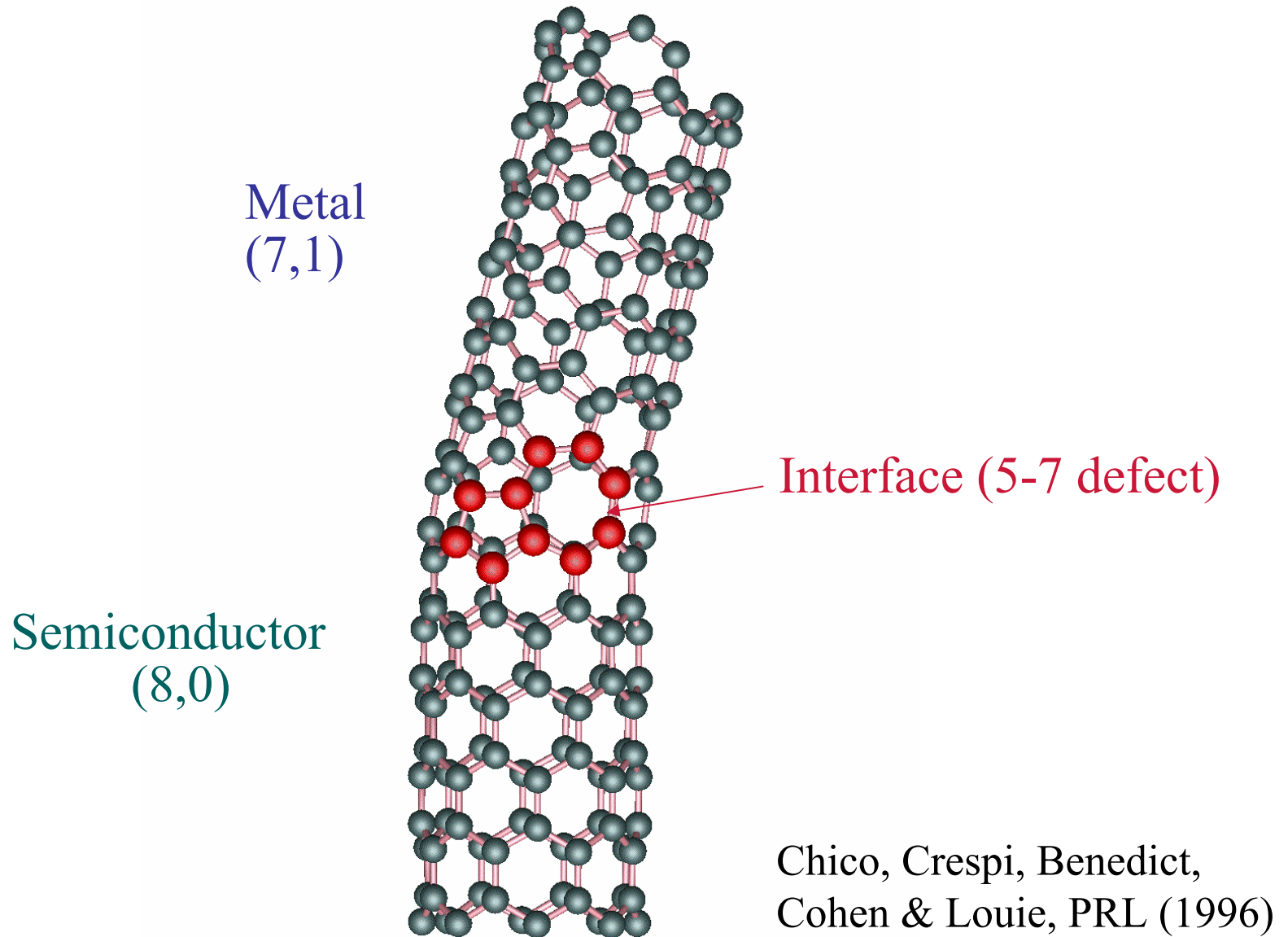


with field



- Spin polarization of carriers is 100%.
- It is tunable and reversible!
- Electric field is more effective on wider nanoribbons ( $E_c \sim 1/w$ )

# $(8,0)/(7,1)$ Nanotube Schottky Barrier

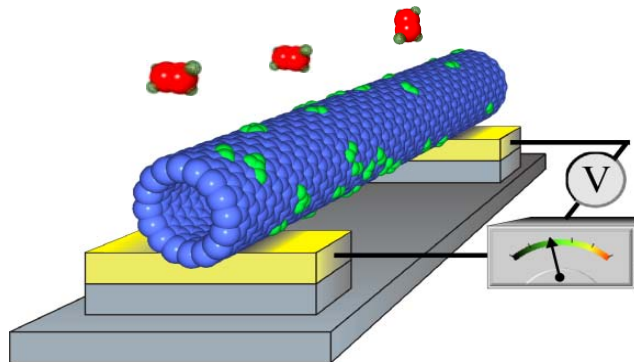
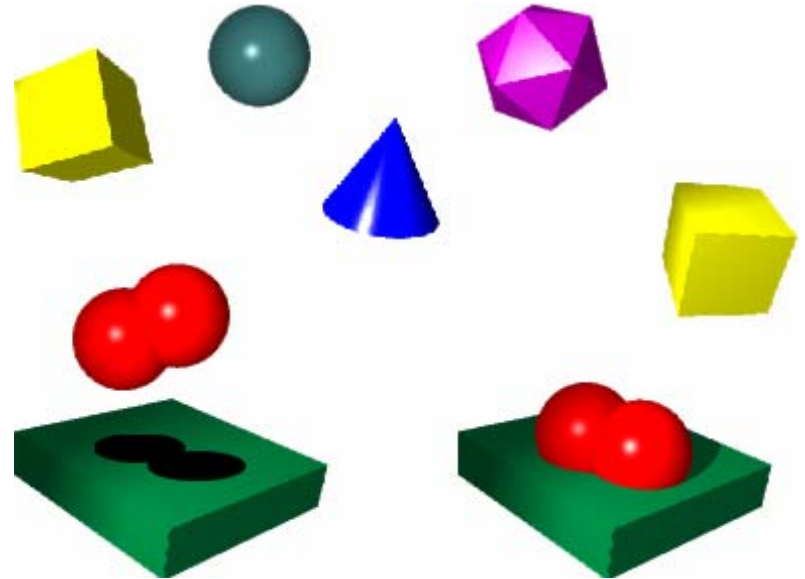


# Sensor Concept: Sensing Specific Analytes

## Architecture:

integration of three layers

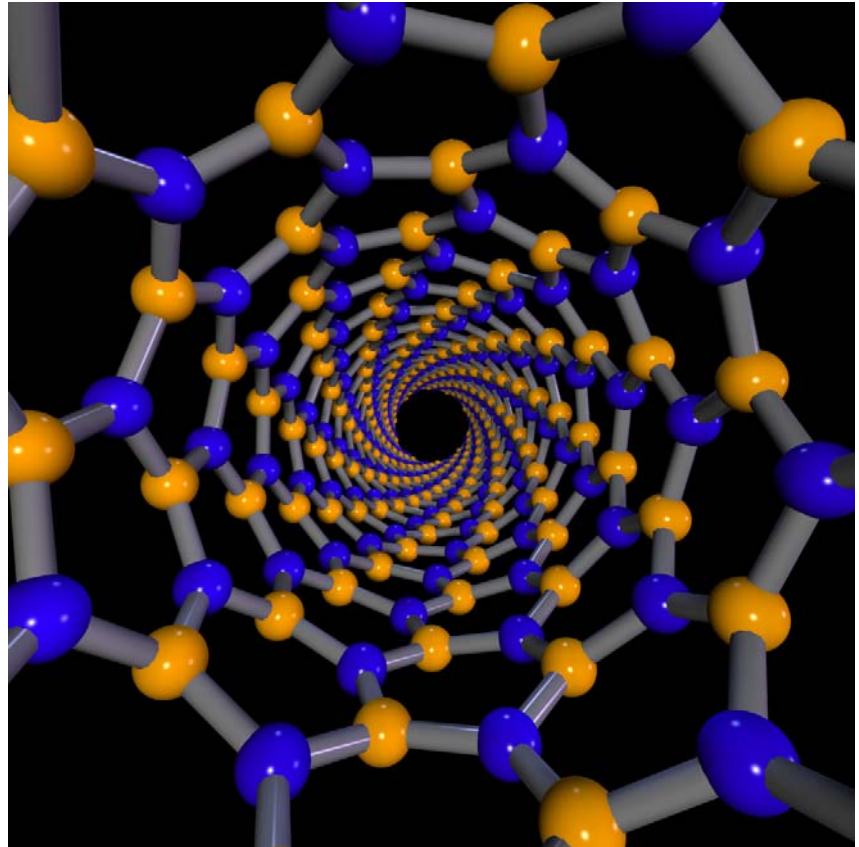
1. Recognition layers or recognition molecules to achieve analyte specificity
2. NTFET as transducer
3. Si CMOS architecture



# Boron Nitride Nanotubes

Predicted by theory  
Semiconductors  
Electronic  
properties  
independent of tube  
chirality

Zettl, Cohen, Louie, et al., *Science*  
(1995)



# CLASSES OF SUPERCONDUCTORS

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BCS : conventional metals, C60, some organics, doped semiconductors, MgB<sub>2</sub>,...

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“BCS” EXOTIC: copper oxides, heavy fermion metals, some organics,...



CAN BCS THEORY PREDICT  $T_c$ ?

$$T_c \propto T_D \underbrace{e^{-\frac{1}{NV}}}_{\substack{\sim 0.3 \\ 300K}} \sim 11K$$

$$\text{IF "NV"} \rightarrow 0.03 \quad T_c \rightarrow 10^{-12}$$

---

NEED TO KNOW "NV" **VERY** ACCURATELY  
TO PREDICT  $T_c$

---

DOPED SEMICONDUCTOR

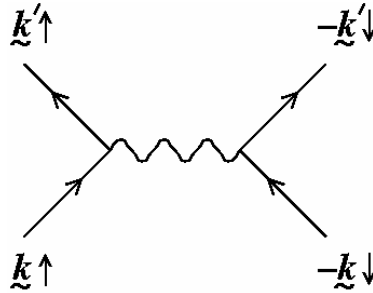


**FIRST SUPERCONDUCTING OXIDE**

(COHEN 1963; SCHOOLEY ET AL 1964)

# Superconductivity in the Eliashberg Formalism

BCS Theory



Electron pairing  
via phonon exchange

Main ingredient: momentum- and frequency-dependent Eliashberg function

$$\alpha^2 F(\vec{k}, \vec{k}', \omega) \equiv N(\epsilon_F) \sum_j \left| g_{\vec{k}\vec{k}'}^j \right|^2 \delta(\omega - \omega_{j\vec{q}})$$

where  $N(\epsilon_F)$  = density of states per spin at Fermi level

$g_{\vec{k}\vec{k}'}$  = electron-phonon matrix element

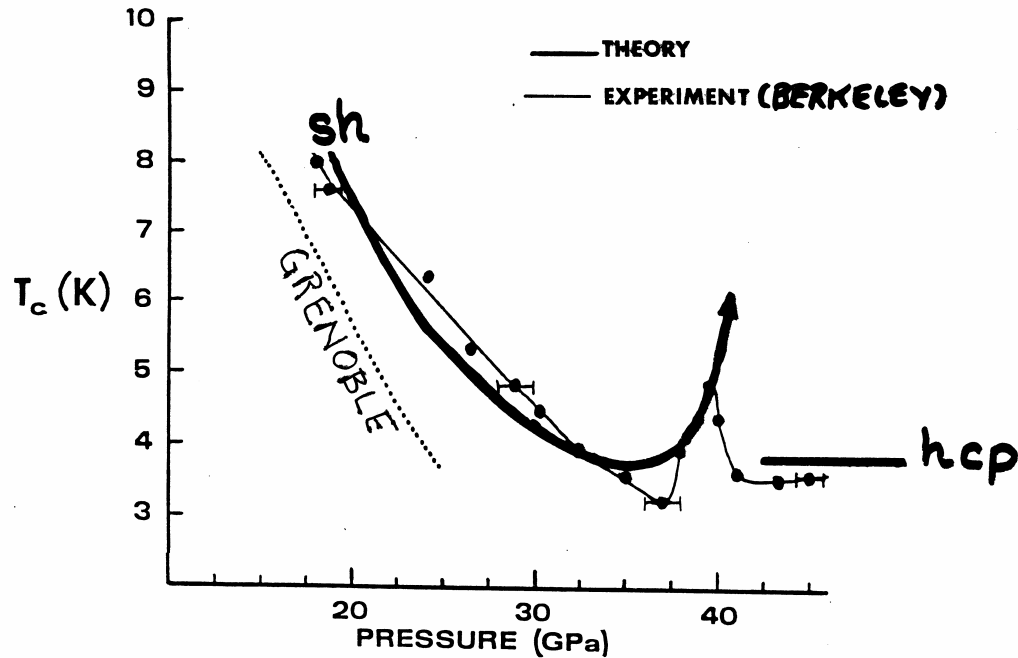
$\omega_{j\vec{q}}$  = frequency of phonon in  $j$ th branch with  $\vec{q} = \vec{k} - \vec{k}'$

Equivalently:

$$\lambda(\vec{k}, \vec{k}', n) \equiv \int_0^\infty d\omega \alpha^2 F(\vec{k}, \vec{k}', \omega) \frac{2\omega}{\omega^2 + (2n\pi T)^2}$$

$$\lambda = \langle \lambda(\vec{k}, \vec{k}', 0) \rangle$$

# sh and hcp SILICON $T_c$ (PRESSURE)



THEORY

CHANG, DASROGNA & COHEN

EXP.

GRENoble: MIGNOT, CHOUTEAU & MARTINEZ

BERKELEY: ERSKINE & YU

# Transition Temperature and Isotope Effect

	harmonic		anharmonic		experiment
	isotropic	anisotropic	isotropic	anisotropic	
$T_c$	28 K	55 K	19 K	39 K	39 K
$\alpha_B$	0.42	0.46	0.25	0.32	0.26, 0.30
$\alpha_{Mg}$	0.04	0.02	0.05	0.03	0.02
$\lambda$	0.73		0.61		0.58, 0.62
$\omega_{ph}$	62.7 meV		75.9 meV		75.9, 76.9

$$\mu^*(\omega_c) = 0.12.$$

Anharmonicity --> small  $\alpha_B$

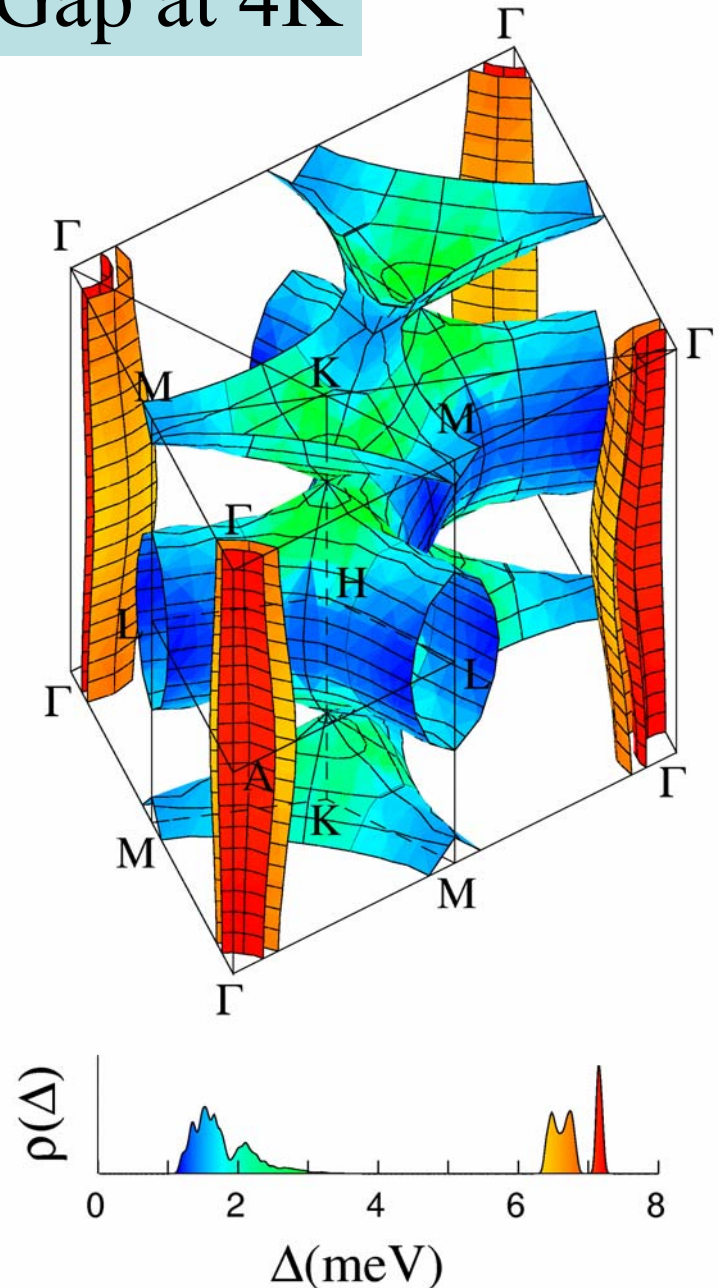
$\lambda$  : averaged electron-phonon coupling.

$\omega_{ph}$  : frequency of the in-plane B-B stretching modes ( $E_{2g}$ ) at  $\Gamma$ .

$$\text{For } 0.10 \leq \mu^*(\omega_c) \leq 0.14, 41 \text{ K} \geq T_c \geq 37 \text{ K}$$

# Superconducting Gap at 4K

- $\Delta(\mathbf{k})$  on Fermi surface at  $T=4$  K
- Large gap on cylindrical  $\sigma$ -sheets
- 2 dominant sets of gap values



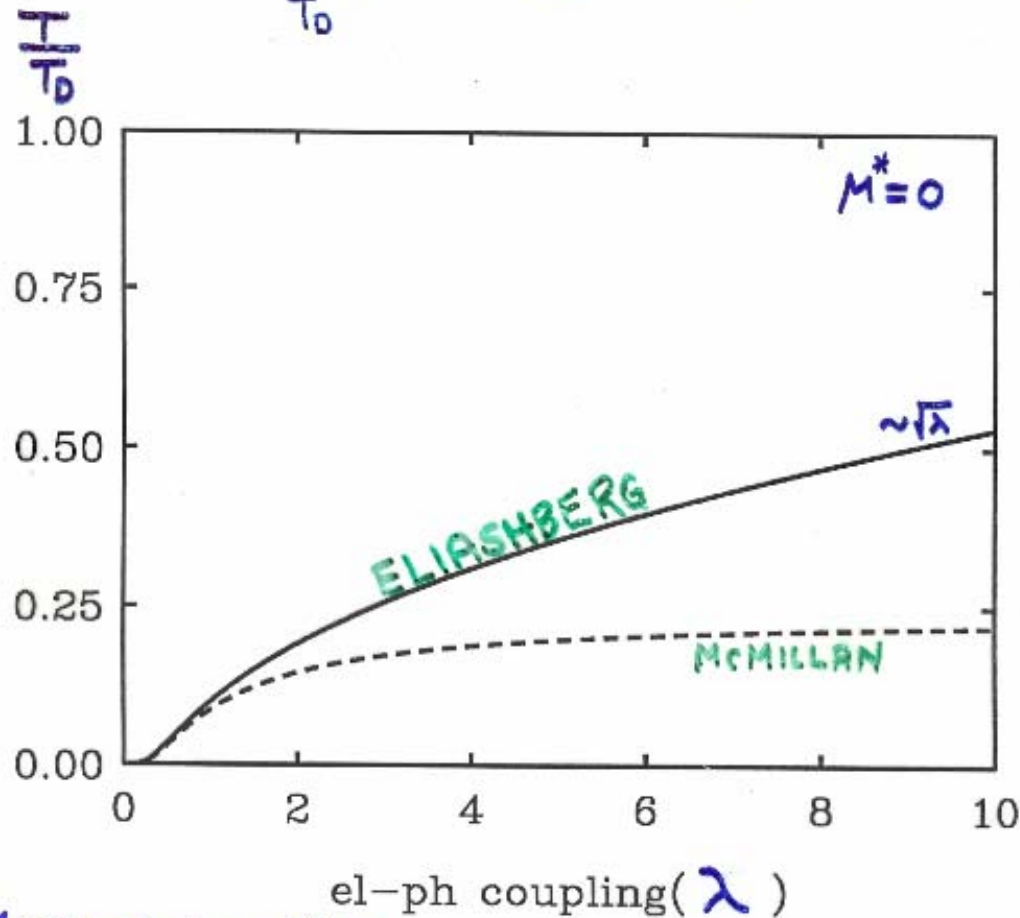
# RAISING $T_c$

WE TRIED TO USE THEORY TO SUGGEST  
HOW TO INCREASE THE TRANSITION  
TEMPERATURE OF MAGNESIUM DIBORIDE  
SIGNIFICANTLY BUT FAILED!

THIS RESULT IS CONSISTENT WITH  
EXPERIMENTS UP TO NOW.

McMILLAN 1968

$$\frac{T_c}{T_0} \approx 0.69 e^{-\frac{1}{\lambda^* - \mu^*}}$$



Kresin-Barbee-Cohen

$$\frac{T_c}{\langle \omega \rangle} = 0.26 (e^{\lambda} - 1)^{-1}$$

## ELECTRON-PHONON COUPLING

$$\lambda \langle \omega^2 \rangle = \sum_i \frac{\eta_i}{M_i}$$

SO  $\lambda$  CAN BE VIEWED AS THE RATIO OF AN  
ELECTRONIC SPRING CONSTANT  $\eta$  AND A LATTICE  
SPRING CONSTANT



# Superconductivity in diamond

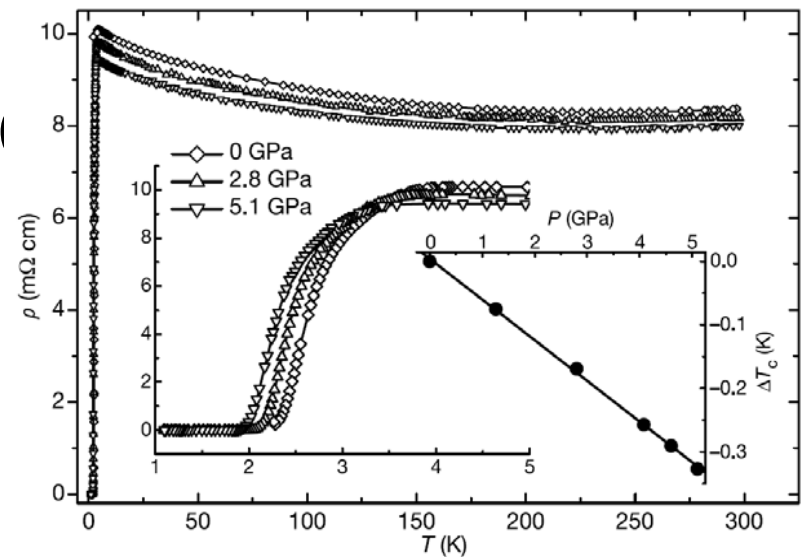
**E. A. Ekimov<sup>1</sup>, V. A. Sidorov<sup>1</sup>, E. D. Bauer<sup>2</sup>, N. N. Mel'nik<sup>3</sup>, N. J. Curro<sup>2</sup>,  
J. D. Thompson<sup>2</sup> & S. M. Stishov<sup>1</sup>**

<sup>1</sup>*Vereshchagin Institute for High Pressure Physics, Russian Academy of Sciences,  
142190 Troitsk, Moscow region, Russia*

<sup>2</sup>*Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

<sup>3</sup>*Lebedev Physics Institute, Russian Academy of Sciences, 117924 Moscow, Russia*

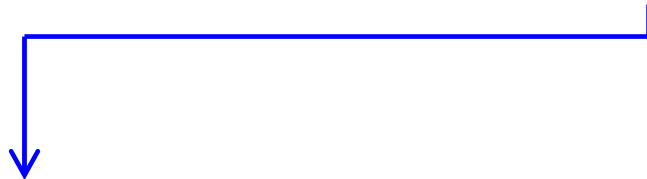
NATURE | VOL 428 | 1 APRIL 2004 | [www.nature.com/nature](http://www.nature.com/nature)



# Bloch to Wannier Representation

$$\underbrace{g(\mathbf{k}, \mathbf{q})}_{\text{Bloch}} = \sum_{\mathbf{R}_e, \mathbf{R}_p} e^{i\mathbf{k} \cdot \mathbf{R}_e} e^{i\mathbf{q} \cdot \mathbf{R}_p} u_{\mathbf{q}} U_{\mathbf{k}+\mathbf{q}} \underbrace{g(\mathbf{R}_e, \mathbf{R}_p)}_{\text{Wannier}} U_{\mathbf{k}}^\dagger$$

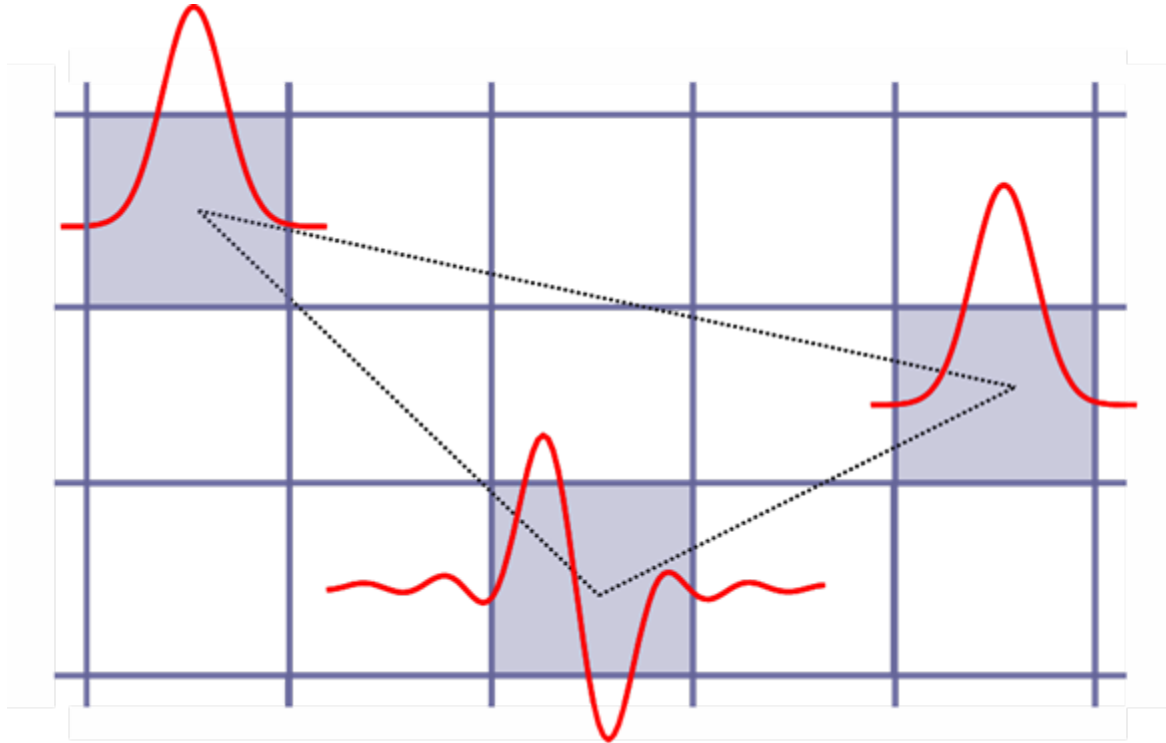
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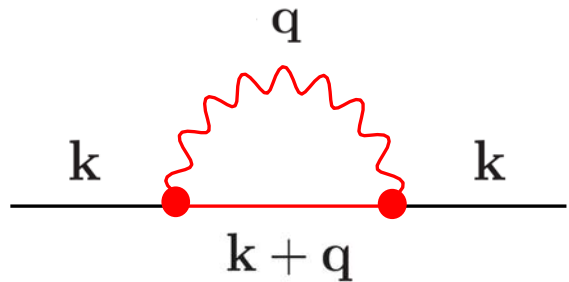
$$\langle m\mathbf{0}_e | \Delta_{\kappa\alpha, \mathbf{R}_p} V(\mathbf{r}) | n\mathbf{R}_e \rangle$$

# Wannier Representation

$$\langle m\mathbf{0}_e | \quad \Delta_{\kappa\alpha, \mathbf{R}_p} V(\mathbf{r}) \quad | n\mathbf{R}_e \rangle$$



# Electron Self-energy



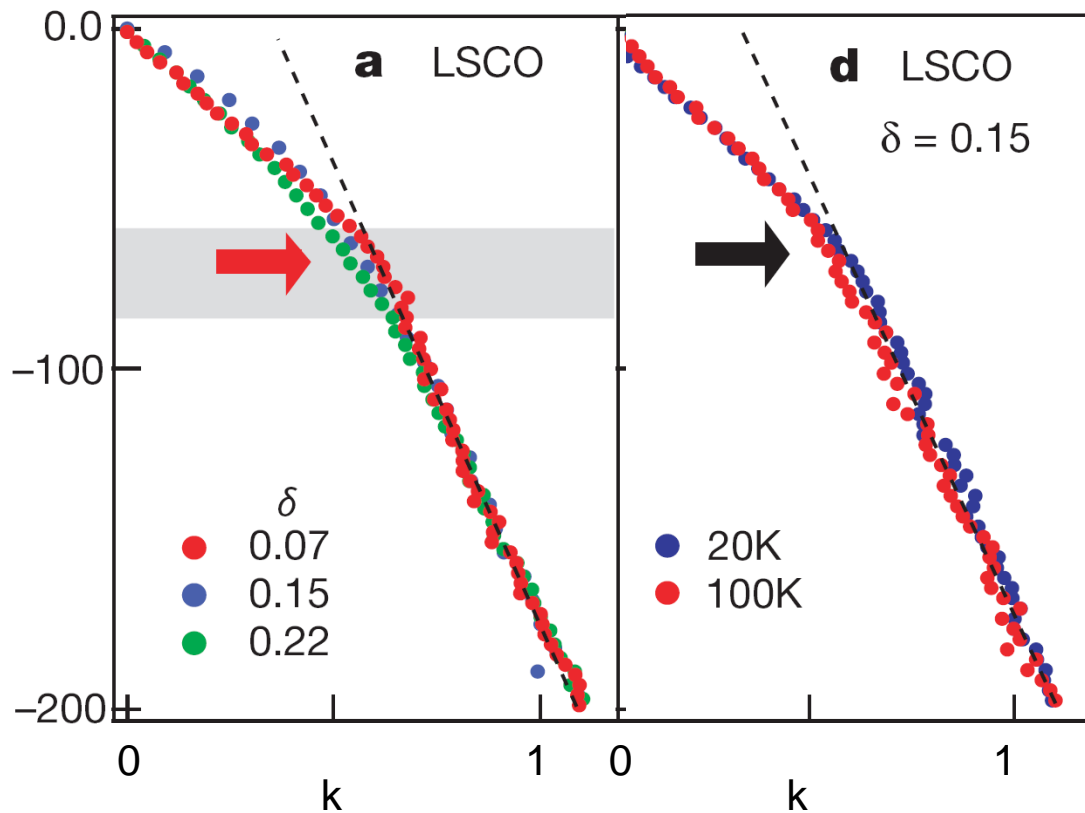
A Feynman diagram representing the electron self-energy. It consists of a horizontal black line representing an electron propagator. Two red dots are placed on this line. A red wavy line, representing a phonon, connects these two dots. The momentum of the incoming electron is labeled  $k$  above the left dot. The momentum of the outgoing electron is labeled  $k$  above the right dot. The momentum of the phonon is labeled  $q$  above the wavy line. The momentum of the internal electron line is labeled  $k + q$  below the line between the two dots.

$$\Sigma = i \int \frac{d^2}{(2\pi)^4} |g(1, 2)|^2 D(1 - 2) G(2)$$

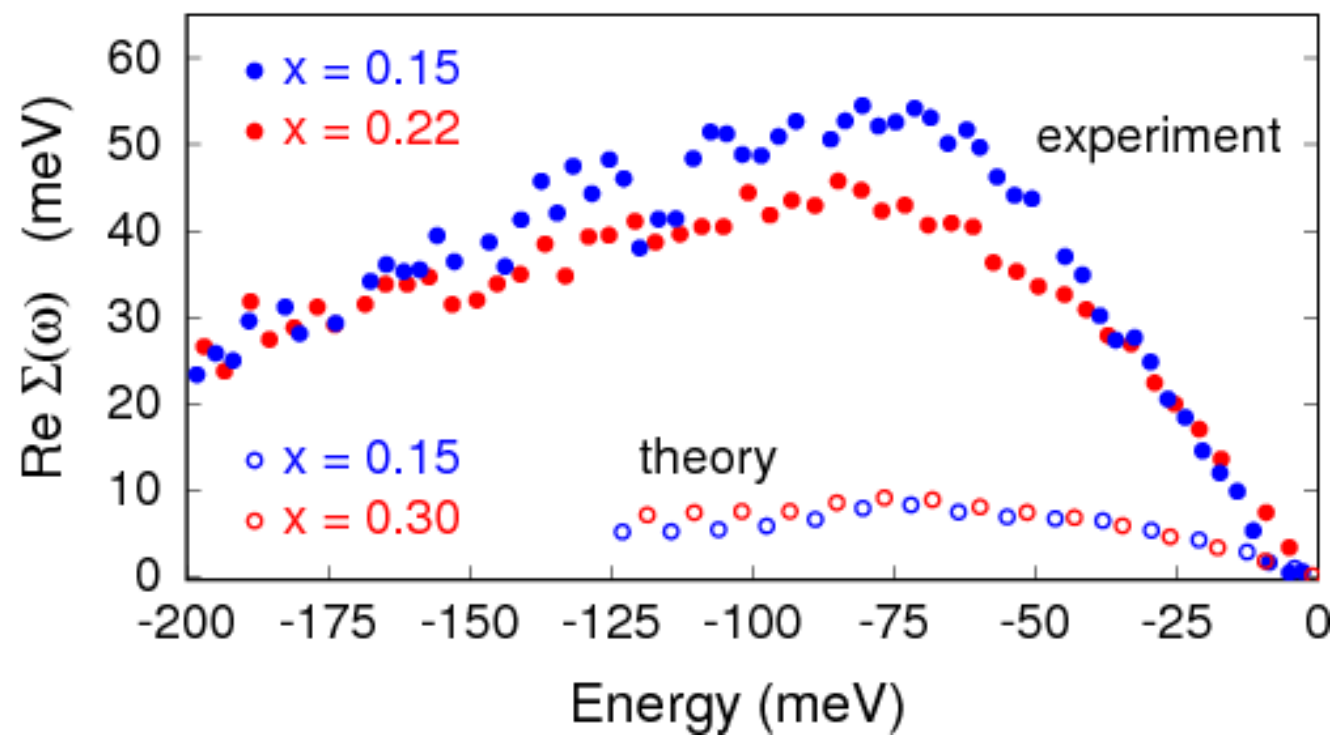
YIELDS A MASS ENHANCEMENT AND ASSOCIATED “KINK” AT THE FERMI SURFACE.”KINKS” HAVE BEEN OBSERVED IN ARPES DATA AND INTERPRETETED AS SIGNATURES OF STRONG ELECTRON-PHONON COUPLING.

# Electron-Phonon Interaction in the Photoemission Spectrum of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ from First Principles

“Kink” (for example, Lanzara et al, Nature) 2001



By measuring the change in slope, the electron-phonon coupling is estimated



## CONCLUSION

BASED ON THE WANNIER  
FORMALISM FOR CALCULATING  
ELECTRON-PHONON SELF-  
ENERGIES, THE COUPLING IS  $1/7$   
OF WHAT IS NEEDED TO  
REPRODUCE THE OBSERVED  
ARPES “KINKS”



# EINSTEIN'S VIEW

Title of 1905 photoelectric effect paper:

"Concerning the generation and transformation of light from a heuristic point of view"

heuristic = model (an emergent view)

# EMERGENCE

HENRI BERGSON (1859-1941)

“all we sense are images”

# EINSTEIN'S VIEW

Title of 1905 photoelectric effect paper:

"Concerning the generation and transformation of light from a heuristic point of view"

heuristic = model (an emergent view)

$a(\text{reductionism}) + b(\text{emergence})$

WHERE BOTH  $a$  AND  $b$  WERE FUNCTIONS OF TIME

Standard Model =  
“interacting atoms” model +  
“elementary excitations” model  
(reductionism + emergent behavior)

Standard Model =  
“interacting atoms” model +  
“elementary excitations” model  
(reductionism + emergent behavior)

Theorists can explain and predict ground and excited state properties of many condensed matter systems, but experimentalists still make the decisions on “what’s right”. They also make the major new discoveries (for now).

HAPPY 50TH BIRTHDAY TO  
PHYSICAL REVIEW  
LETTERS AND MANY  
THANKS TO THE PEOPLE  
WHO HAVE MADE IT SUCH  
A SUCCESS!

END