Modeling and Reality in Early Twentieth-Century Physics

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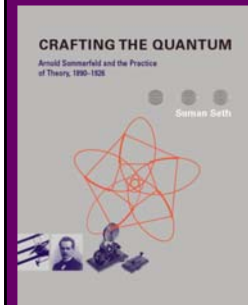


1. Background: The "Intellectual Landscape" for Modeling as Method

2. Change Over Time: Arnold Sommerfeld

3. Teasers: Payoffs for looking at Sommerfeld.

Ludwig Boltzmann, "On the Methods of Theoretical Physics," (1892)

Compare the almost ethereally structured and crystal clear though colourless theory of elasticity in Kirchhoff's lectures with the crudely realistic account in Vol. 3 of Thomson's *Mathematical and Physical Papers* which concerns not ideally elastic bodies but steel, rubber, glue; or with the often childlike naiveté of Maxwell's language, who in the midst of formulae mentions a really effective method for removing fat stains.

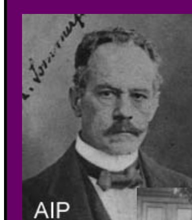
Ernst Mach, "The Economical Nature of Physical Inquiry," (1882)

When a geometer wishes to understand the form of a curve, he first resolves it into small rectilinear elements. In doing this, however, he is fully aware that these elements are only provisional and arbitrary devices for comprehending in parts what he cannot comprehend as a whole. When the law of the curve is found he no longer thinks of the elements. Similarly, it would not become physical science to see in its self- created, changeable, economical tools, molecules and atoms, realities behind phenomena…The atom must remain a tool for representing phenomena, like the functions of mathematics. Gradually, however, as the intellect, by contact with its subject-matter, grows in discipline, physical science will give up its mosaic play with stones and will seek out the boundaries and forms of the bed in which the living stream of phenomena flows. The goal which it has set itself is the simplest and most economical abstract expression of facts.

Max Planck on Models

[1887] "I have the intention to extend a little further in several successive treatments the series of conclusions that can be drawn from the Carnot-Clausius principle *in and of itself*, i.e. without reference to certain conceptions of the essence of molecular motions, merely taking as a basis the principle of the conservation of energy."

[1943] "What interested me most in physics were the great general laws that possess meaning for all natural processes, independent of the characteristics of the bodies that took part in the processes.... Hence the two laws of thermodynamics captivated me to a particular degree."



Arnold Sommerfeld (1868-1951)


ATOMBAU
UND
SPEKTRALLINIEN
von
Arnold Sommerfeld

Wolfgang Pauli (1900-1958), Nobel Lecture, 1945. Two paths to the Exclusion Principle

At that time there were two approaches to the difficult problems connected with the quantum of action. One was an effort to bring abstract order to the new ideas by looking for a key to translate classical mechanics and electrodynamics into quantum language which would form a logical generalization of these. This was the direction which was taken by Bohr's Correspondence Principle. Sommerfeld, however, preferred, in view of the difficulties which blocked the use of the concepts of kinematical models, a direct interpretation, as independent of models as possible, of the laws of spectra in terms of integral numbers, following, as Kepler once did in his investigation of the planetary system, an inner feeling for harmony. Both methods, which did not appear to me irreconcilable, influenced me.

Pauli: Scylla and Charybdis (Letter to Kramers, July 1925)

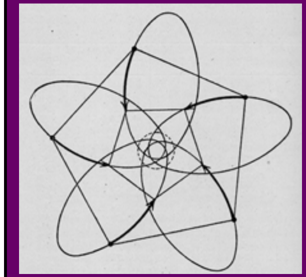
"I feel at the moment a little less lonely as around half a year ago, when I found myself (spiritually as well as spatially) pretty much alone between the Scylla of the number mysticism of the Munich school and the Charybdis of the reactionary Copenhagen putsch propagated by you with the excesses of a zealot!"

Arnold Sommerfeld, "Theoretisches über die Beugung der Röntgenstrahlen," *Zeitschrift für Mathematik und Physik* 46 (1901).

… the most fruitful path for theoretical physics appears to be this: to lay down as specific and determinate [*specielle und bestimmte*] hypotheses as possible, to develop their consequences exactly, and to compare these with experiment: if this shows no contradiction with experience, good, then our hypotheses were valid and may be retained until later; if, however, an opposition emerges, then all the better; then our hypothesis is displayed as invalid and we have gained a definitive, if also negative, finding.

The Solvay Congress, 1911

Sommerfeld's *Ellipsenverein (1918)*



For my feeling, the artful interlocking of the electronic paths in our 'Ellipsenverein' is nothing unnatural; I see much more a sign therein for the high harmony of motion that must rule within the atom.

A Number Mystery

Runge's Rule: Let *a* be the Lorentz separation

$\Delta\nu = q / r * a$

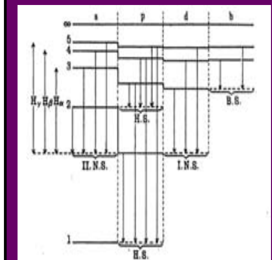
Sommerfeld's use of the combination principle

$q / r = q_1 / r_1 - q_2 / r_2 = (q_1 r_2 - q_2 r_1) / r_1 r_2$

Re-Write:

$r = r_1 r_2$

Take home: The "Runge Denominator" is made up of terms from the initial and final levels of an electron's "jump."



For the main series (H. S.) and the second subsidiary series (II. N. S.), transitions are between *s* and *p* orbitals.
For doublets in these, r = 3, for triplets, r = 2.

For the first Subsidiary series (I. N. S.) transitions are between d and p orbitals.
For doublets, r = 15
For triplets, r =6.

	s	*p*	*d*	*b*
Einfachlinien	1	1	1	1
Triplettlinien	1	2	3	(4)
Dublettlinien	1	3	5	(7)

Modellmässigkeiten vs *Gesetzmässigkeiten* (Sommerfeld, 1919)

…at the moment we are at a loss with the *modellmässigen* (model-based) meaning of the line-multiplicities of the non-hydrogenic elements, in spite of repeated efforts from various sides. All the more valuable are all the lawful aspects [*Gesetzmässigkeiten*] that present themselves empirically for the line-multiplicities, above all when they are of such a radical and simple kind as those here at hand.

Sommerfeld, Innsbruck Lecture, September 1924

…I would like to say a few words about the soundness of our *Modellvorstellungen* [conceptions of models]. The difficulties that obtrude ever more clearly into atomic physics appear to me to lie less in an immoderate application of the quantum theory and much more in a perhaps immoderate belief in the reality of our *Modellvorstellungen*. Certainly the Hydrogen model functions correctly in all cases (except perhaps in strong magnetic fields) and certainly Bohr's explanation of the chemical systematics of the periodic system is sound in general terms. But the phenomena are much simpler than one would expect according to the model... The atomic model would then be more of a calculational scheme [*Rechenschema*] than a state of reality [*Zustandsrealität*].

From Modellmässigkeiten to Gesetzmässigkeiten: 1919-1922.

- Wolfgang Pauli: In Munich from 1918 to July, 1921.
- Werner Heisenberg: In Munich from the Fall of 1920 to mid-1922.

Pauli to Sommerfeld 6 December, 1924

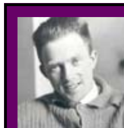
I found it particularly beautiful in the presentation of the complex structure that you have left all *modellmässig* considerations to one side. The model-idea now finds itself in a difficult, fundamental [*prinzipiellen*] crisis, which I believe will end with a further radical sharpening of the opposition between classical and quantum theory. One now has the impression with all models, that we speak there a language that is not sufficiently adequate for the simplicity and beauty of the quantum world. For that reason I found it so beautiful that your presentation of the complex structure is completely free of all model-prejudices.

Oskar Klein (1894-1977) on Sommerfeld's Style

During Sommerfeld's lecture I was sitting at the side of Bohr. Sommerfeld had a few numbers for these anomalous levels. The first was so and the second was so—they were something like 1, 3, and so. And then Sommerfeld said the next must be 5, or something like that. Then Bohr smiled and said to me, "I don't believe that.

Payoff 1: Pauli Origins of the Exclusion Principle

1. Timing
2. Pauli's own claims.
3. Style: Pauli's paper is phenomenological.
4. The path:
   a. Pauli's debts to Stoner, via Sommerfeld.
   b. Sommerfeld on Stoner: Stoner's scheme "possesses more of an arithmetical than a geometric-mechanical character, makes no assumptions about the symmetry of orbital arrangements and uses not only a part, but the entirety of Röntgen-spectroscopic facts."
5. The meaning: Zweideutigkeit: "The doublet structure of the alkali spectrum, as well as the violation of Larmor's theorem comes about through a peculiar, classically non-describable kind of *Zweideutigkeit* of the quantum-theoretical characteristics of the light-electron."



Payoff II: Heisenberg: The roots of the Observability Criterion in Quantum Mechanics

Route 1: Relativity: "To my astonishment, Einstein…thought that every theory in fact contains unobservable quantities. And when I objected that in this I had merely been applying the type of philosophy that he, too, had made the basis of his special theory of relativity, he answered, simply: 'Perhaps I did use such philosophy earlier, and also wrote it, but it is nonsense all the same.'

Encounters and Observations with Albert Einstein (1974)

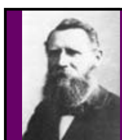
Route 2: Sommerfeld's phenomenology in quantum theory.

Payoff 3: Re-thinking the "Crisis" of the Older Quantum Theory

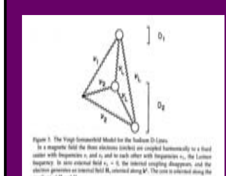
NOT, When was the crisis, what was its cause etc.

BUT rather: *What do you have to think* in order for there to be a crisis in quantum theory in 1923/24?

OR EVEN BETTER: *How do you have to work* in order for there to be a crisis in quantum theory in 1923/24?



Woldemar Voigt's (1850-1919) 'Coupling Theory,' as re-worked by Sommerfeld



Source: Cassidy, David C. "Heisenberg's First Core Model of the Atom: The Formation of a Professional Style." *Historical Studies in the Physical Sciences* 10 (1979): 187-224

Voigt's theory provides "the adequate expression of the facts in the language of oscillation-theory"

Sommerfeld's aim: "to show how these equations could be translated from an oscillations-theoretical to a quantum-theoretical language.

Sommerfeld, 1922: "Quantentheoretische Umdeutung der Voigtschen Theorie des anomalen Zeemaneffektes vom D-Linientypus."

Heisenberg's first paper: "The Quantum Theory of Line Structure and the Anomalous Zeeman effect." (1922)

"[Describing Sommerfeld's approach] A further extension of the empirical materials can thus be effected if one translates Voigt's theory of the D-Line splitting or the simplification of this by Sommerfeld into the language of the quantum theory."

Pauli on *Zweideutigkeit*, 1923

$$g = \frac{3}{2} + \frac{1}{2} \cdot \frac{r(r-1) - k(k-1)}{j(j-1)}$$

- "…each momentum will act not through a single number, but through a *pair* of numbers. The momenta appear in a certain sense to be *zweideutig*…One sees further that this *Zweideutigkeit* also includes *k…"*
- (*r* is the angular momentum of the core, *k* of the valence electron, *j* of the atom as a whole.)

Pauli on *Zweideutigkeit*, 1924

The doublet structure of the Alkalis is, in essence, a property of the valence electron alone. This classically non-describable *Zweideutigkeit* expresses itself first through the fact that a different quantum number is responsible for the size of the relativistic correction as for the central force between the valence electron and the core and second, through the fact that a different quantum number is responsible for the magnetic moment than for the moment of momentum.

Bohr and Coster, 1922

$$W = Rh\frac{(N-\gamma)^2}{n^2} + Rh\frac{(N-\delta)^4}{n^4}\left(\frac{2\pi e^2}{hc}\right)^2\left(\frac{n}{k}-\frac{3}{4}\right)+\ldots$$

γ is the "total screening number", dependent on *n* and k_1.

δ Is a measure of the relativistic correction, depending on *n* and k_2

Sommerfeld and Zweideutigkeit *Atombau* (4th Ed., 1924)

"*k* splits apart into two quantum numbers *k1* and *k2*... With regard to the n_k electronic orbits *k1* plays the role of the azimuthal quantum number *k*; on the other hand, with respect to the quantitative representation of the doublets the number *k2* plays this role. Under these circumstances, which are not yet fully explained, the Bohr-Coster labeling seems more correct than our previous one..."

Sommerfeld and Zweideutigkeit II *Atombau* (4th Ed., 1924)

The remarkable ambiguity [*Zweideutigkeit*] upon which we strike here clearly depends on the still completely unsolved question of why it is that the L-shell is divided into three and not two, the M-shell into five and not three etc. [...]The relativistic representation of terms takes L_{21} and L_{11} for the elliptical orbital type 2_1, but the theory of the periodic system ascribes L_{22} and L_{21} to the orbital type 2_2. The meaning of the L_{21} level is therefore contradictory. This is true of other shells, namely for all terms for which k_1 and k_2 are different. We are forced to conclude from this that our model-based conception [*Modellvorstellung*] with regard to types of orbits is too narrow and that the real circumstances are not yet satisfied.