Anderson Localization in the Seventies and Beyond

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Importance of Anderson’s 1958 paper on Absence of Diffusion in Certain Random Lattices was little understood for the first ten years. The paper was addressed to the problems of spin diffusion on a lattice, and to electrical conduction in a semiconductor impurity band.

23 citations in Science Citation Index to end of 1964, with 3 by Mott. 8 more to end of 1967, with 4 of those by Mott.

These citations do not suggest the excitement which might lead to a new subfield in physics and a Nobel Prize. However, many of the names of those who cited the paper are well known to me for other reasons.

Explosion of interest and controversy started in 1968. International Conference on Amorphous and Liquid Semiconductors, 1969, in Cambridge was a forum for this controversy.

My own involvement with the subject started just after this, when John Ziman asked for my opinion of his and his colleagues’ criticism of the Anderson theory, and of Anderson’s response.
Anderson’s original paper discussed the problem of noninteracting electrons moving on a fixed regular lattice, hopping from one site to a neighboring site with a fixed matrix element $V_{ij} = -V$, while the site energy $\epsilon_i$ is a static random variable spread about some mean value $\epsilon_0$, over some width $W$.

Anderson used expansion in ratio $V/W$ to produce a correct, but incomplete argument, using the hopping matrix $V$ as a perturbation on the localized states. This argument showed convincingly that, for sufficiently small values of $V/W$, states remained localized.

The tricky part of the argument is that distant regions of the system must always contain regions arbitrarily close to the energy of the state under consideration.

This leads to the small denominator problem, which was solved in the 1983 work of Fröhlich, Spencer.

Various people, including Mott and Twose, had shown that all states were localized in one-dimensional systems.
Over first ten years there were a number of developments that brought Anderson’s paper into the main stream.

Work of Anderson, Kondo and others on localized magnetic moments in conductors raised some of the same issues in a different context.

Walter Kohn had argued that the difference between metals and insulators was that the electron wave functions were exponentially localized in insulators, but extended in metals.

Ovshinsky, encouraged by several eminent physicists, was arguing for the technological importance of amorphous semiconductors.

Mott and his colleagues were analysing the electrical properties of highly disordered and amorphous materials.

Domb, Fisher, Pokrovsky, Kadanoff, and others were making a general study of continuous phase transitions, such as the critical point of water vapor and magnetic Curie points.
Surprise in 1968-9 was that exact solution of Anderson model for Cauchy distribution of density of site energies gave, even in three dimensions, a smooth analytic answer, with no sign of a transition.

The reason given by Anderson and others was that the density of states can be obtained from the average of a one-particle Green’s function, but conduction properties depend on the average of a two-particle Green’s function, such as the current-current correlation formula of Kubo.

Note however that many of us, including me, continued to teach our students that the oscillatory spin-spin correlation function in a metal (Ruderman-Kittel oscillations) was exponentially damped by disorder, until we became aware of the Zyuzin and Spivak work of 1986. Our mistake rested on the same confusion about averaging.

MacKinnon’s numerical work of 1984 showed that the two-dimensional Anderson model with a Cauchy distribution of site energies shows localization features similar to other distributions, despite the smooth Lorentzian shape of the density of states.
Figure 1. The renormalised decay length of the transmission coefficient $\Lambda = \lambda_M / M$ of strips of width $M$ as a function of $\xi / M$. $\xi$ is the scaling parameter, which is chosen to fit all data onto one curve. + signifies Cauchy disorder and × rectangular. The inset shows $\log \xi$ versus $\gamma / V(+)$ and $W / V(\times)$ respectively.
There are sharp distinctions between localized and extended states. Electrical conductivity $\sigma$ of a low temperature Fermi gas of electrons tends to a finite limit as $T \to 0$ if states round Fermi energy are extended, but goes to zero exponentially for localized states near the Fermi energy.

Mott argued that this conductivity is not simple activated conduction over barriers of a given height, which gives $\ln \sigma \propto 1/T$, but is variable range hopping with $\ln \sigma \propto 1/T^{1/(d+1)}$.

This $T^{-1/4}$ law was confirmed experimentally, but Efros and Shklovskii showed that a Coulomb gap develops at the Fermi surface, changing the exponent from $-1/4$ to $-1/2$. At lower temperatures, experiments showed the crossover to $-1/2$.

The mathematically literate told us that the spectrum is continuous for the energy region where states are extended, but discrete where the states are localized. In the limit of a system of an infinite system size the eigenstates are still distinct in the energy range of localized states.

Some of us found this hard to comprehend. When an average over disorder is taken, this distinction is lost, because the spectrum is smoothed.
The most important theoretical development of the 1970s in my view was the development of a workable scaling theory by the Gang of Four (Abrahams, Anderson, Licciardello and Ramakrishnan 1979).

This is based in part on the idea that the strength of the coupling between neighboring parts of the electronic system is closely related to the electrical conductivity of a block (Licciardello and Thouless 1975).

This idea was combined with a detailed treatment of weak disorder.
FIG. 1. Plot of $\beta(g)$ vs $\ln g$ for $d > 2$, $d = 2$, $d < 2$. $g(L)$ is the normalized “local conductance.” The approximation $\beta = s \ln(g/g_c)$ is shown for $g > 2$ as the solid-circled line; this unphysical behavior necessary for a conductance jump in $d = 2$ is shown dashed.
It was found that in one dimension, as larger and larger systems were considered, the systems crossed over from a traditional Ohmic behavior to an exponentially localized situation (Abrahams, Anderson, Fisher and Thouless 1979), as earlier theory had shown, and experiments were beginning to indicate.

In three dimensions, weakly disordered systems scaled to the metallic state, but more strongly disordered systems scaled to the insulating and exponentially localized state.

Smooth transition between localized and extended states, as the localization length tends to zero, which is confirmed by simulations, contradicts Mott’s minimum metallic conductivity and Ioffe-Regel criterion that mean free path should not be less than wavelength.

In two dimensions it was predicted (in the absence of spin-dependent forces) that arbitrarily weak disorder would eventually scale to the localized state, but that the crossover should be logarithmically slow.
What do we know about interacting systems? Real charge carriers have a Coulomb interaction with one another and interactions mediated by the lattice.

We know this can be important, but experimental systems can be designed to minimize these effects.

In optical systems and in cold atom systems the strength of interactions can be controlled. You will hear about recent work this afternoon.

Mott showed that, even without disorder, attractive force between electrons and positive ions can lead to an insulator-metal transition, the Mott transition. For most solids both disorder and interaction are present, so real transitions are likely to be Mott-Anderson transitions.

There are many theories of interacting disordered systems, but few command wide support.

Simulations have shown attractive interactions delocalizing one dimensional fermion systems.
Experiments (Sarachik et al) have shown a metal-insulator transition at low temperatures in two-dimensional systems, contrary to expectations of scaling theory.

At the mean field level, we can argue that repulsion between bosons forces localized Bose condensates to expand through the system to form extended, superfluid condensates. Similar arguments would suggest that attraction would cause collapse.
The **Quantum Hall effect** taught us a lot of unfamiliar things about localization.

At low temperatures and high magnetic fields very flat plateaus of the Hall conductance, as function of gate voltage or of magnetic field are accompanied by very low values of longitudinal conductance.

Insensitivity of Hall conductance to field strength implies that when Fermi energy is right for plateau, there are only localized states or edge states at that energy.

Electric field can induce edge currents, or can cause localized states to drift along equipotentials.
I wish to thank numerous people who have guided and stimulated my thoughts on this topic, particularly Phil Anderson, Don Licciardello, Sir Nevill Mott, and numerous experimentalists who have patiently explained what they were doing and why I should care.

I wish to thank Gloria Lubkin and Dan Kleppner for setting up and helping me to organize this Forum.