Landau and theory of quantum liquids

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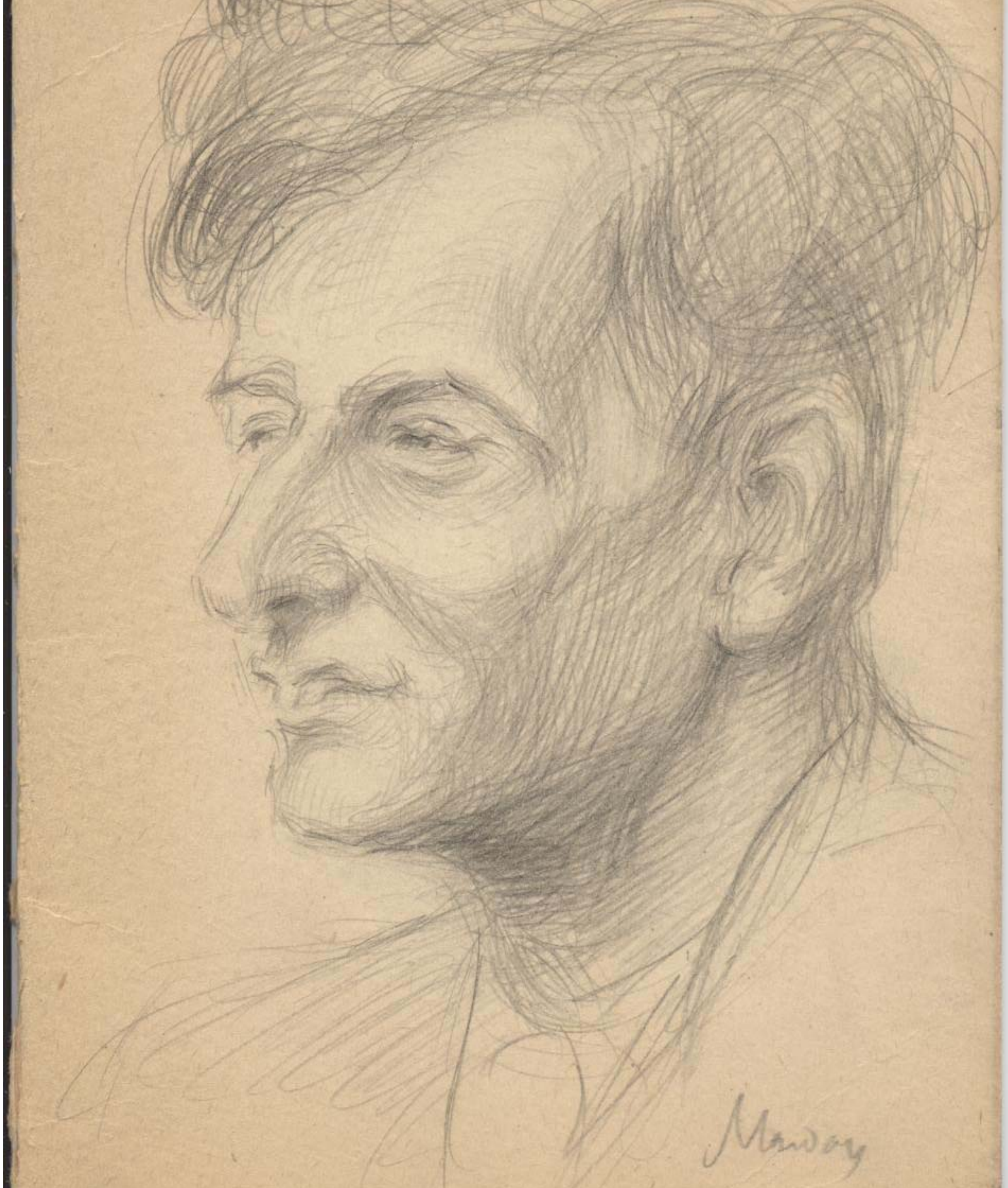
L. D. Landau

Life sketch
by Galkina,
~1961



From letter by Kapitza to Molotov of April 6, 1939

Comrade Molotov,

Recently, during work on liquid helium, at temperatures near absolute zero, I have been able to discover a number of new phenomena which can clear up one of the most puzzling areas in modern physics. I propose to publish some of this work during the next few months. But to do so I need the aid of a theoretician. We had at the Soviet Union one who thoroughly understood the area of theory that I need, namely Landau, but he has been under arrest a year now.

I had much hoped that he would be released, especially as I must say frankly that I cannot believe that he is a traitor.

…

Part I

Theory of superfluidity

Landau's theory of superfluidity

1. Observable properties of a macroscopic body at low temperature can be described in the terms of elementary excitations.

2. Liquid helium is consisted of a "mixture" of a superfluid liquid without viscosity and a normal one:

$$\rho = \rho_{\text{s}} + \rho_{\text{n}} \text{ , } \mathbf{j} = \rho_{\text{s}} \mathbf{v}_{\text{s}} + \rho_{\text{n}} \mathbf{v}_{\text{n}}$$

ρ_{s} is not related to the condensate density,

$$\rho_{\text{s}} \neq m n_c ; \rho_{\text{n}} = -\int p^2 \frac{\partial n_0}{\partial \varepsilon} d\tau_{\mathbf{p}}$$

$$\rho_{\text{n}} \to 0 \text{ at } T \to 0$$

Explanation of the absence of dissipation

Creation of excitations is impossible

if velocity of flow V stisfied

"Landau condition":

$$V < \min \frac{\varepsilon(p)}{p}$$

Potentiality of superfluid flow

Wave function of moving superfluid

$$\Psi(\mathbf{r}_\alpha) = \Psi_0 \exp\left\{\sum \chi(\mathbf{r}_\alpha)\right\} \qquad (1)$$

$$\mathbf{v}_\mathrm{s} = \frac{\hbar}{m}\,\mathrm{grad}\,\chi$$

$$\mathrm{curl}\,\mathbf{v}_\mathrm{s} = 0$$

Wave function (1) was used by Feynman in his theory of vortex lines

Energy spectrum of excitations, 1941

$$p \to 0,\ \varepsilon(p) = cp \text{ - phonons}$$

$$S(T) \propto T^3$$

Experiment:

$$S(T) \propto e^{-\Delta/T},\ \Delta \cong 8-9\,\mathrm{K}$$

Second branch with gap - "rotons"

Initial version (1941):

$$\varepsilon_{\mathrm{r}}(p) = \Delta + p^2/2\mu,\ \mu \cong (7-8)m_{\mathrm{He}}$$

Energy spectrum of excitations, 1947

Measurements of ρ_n gave

$$\left\langle \mathrm{p}^2 \right\rangle = const \equiv p_0^2$$

$$p_0 / \hbar \cong 2 \times 10^8 \text{ cm}^{-1}$$

$$\varepsilon_r(p) = \Delta + \frac{\left(p - p_0\right)^2}{2\mu}, \ \mu \cong 0.8 m_{\mathrm{He}}$$

Single curve of spectrum

“These considerations…lead to a spectrum consisting of a single curve; after a linear initial part, the function passes through a maximum, $\varepsilon\left(p\right)$ then has a minimum and increases again.”

Palevsky, Otnes and Larsson, 1958

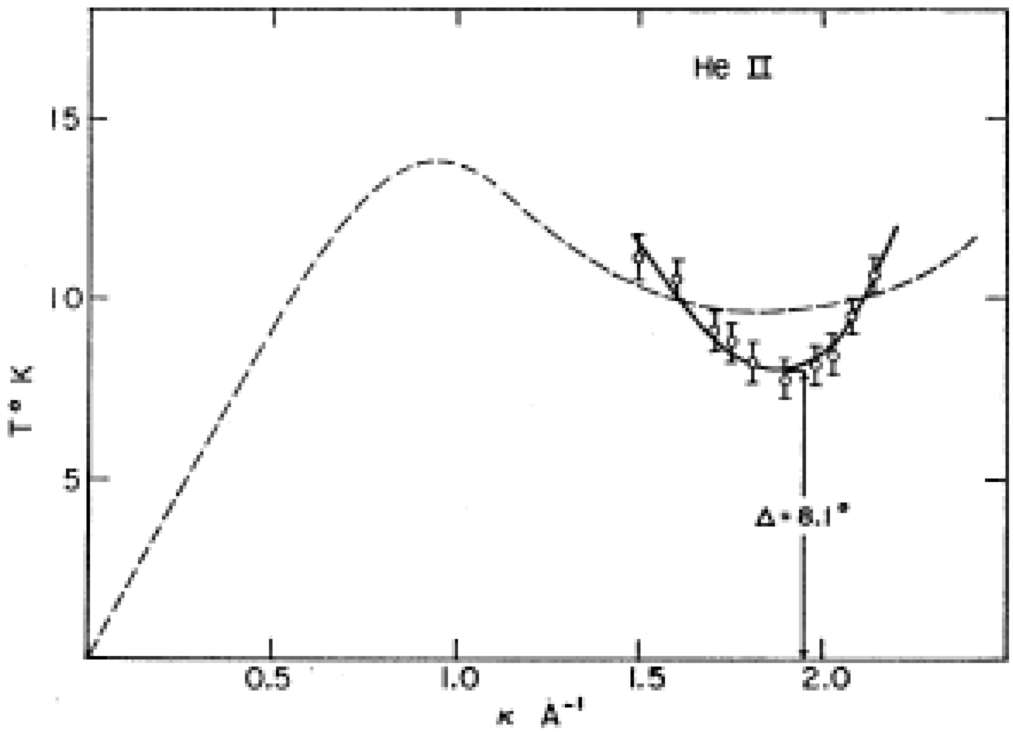


FIG. 8. Dispersion curve for excitations in He II. The temperature of the sample was kept between 1.4 and 1.5°K. The dashed line is the Landau-Feynman dispersion curve given in reference 6.

"Old" roton exists also!

Bound state of two rotons.

Dispersion law :

$$\varepsilon_2(p) = \Delta_2 + \frac{p^2}{2\mu_2},$$

$$\Delta_2 \leq 2\Delta, l = 2$$

Greytak et al., 1970.

Relation to Bogoliubov paper

It is useful to note that N. N. Bogoliubov has succeeded recently, by an ingenious application of second quantization, in determining the general form of the energy spectrum of a Bose-Einstein gas with a weak interaction between the particles. As it should be, the “elementary excitations” appear automatically, and their energy $\mathcal{E}$ as a function of the momentum p is presented by a single curve, which has a linear initial part. Although the model of such a gas does not have any direct bearing on the actual helium *II*, it shows the manner in which the quantum-mechanical mathematical formalism leads, in fact, from a macroscopical body to an energy spectrum with the indicated properties.

L. Landau, 1948; [Phys. Rev. **75**, 884 (1949)]

Bose-Einstein condensation in liquid, 1951.

Long-range behavior of one-body density matrix

$$\rho(\mathbf{r}',\mathbf{r}) = \int \Psi^*(\mathbf{r}',q)\Psi(\mathbf{r},q)dq \rightarrow \rho_\infty = const$$

"Thus this property of the density matrix is equivalent to the statement that in a superfluid liquid…a finite number of particles have zero momentum. However, … we must emphasize that these particles cannot be identified with the "superfluid part" of the liquid."

Part II

Theory of Fermi liquid

Basic conception of the theory of normal Fermi liquid with strong interaction

At $T<<E_E$ only a small fraction of particles ~(T/E_F) are active. These particles interacts with a “background” of the rest particles and with other active particles by means of the back-ground.

Elementary excitations

Energy of elementary excitation

$$\varepsilon(p) = v_F \left| p - p_F \right| = \left(p_F / m^* \right) \left| p - p_F \right|$$

$$p > p_F \text{ – particles}$$

$$p < p_F \text{ – holes}$$

$$\frac{N}{V} = 2 \times \frac{4\pi p_F^3}{3(2\pi\hbar)^3} \quad (!)$$

Volume of the Fermi-sphere is the same as in an ideal gas in spite of strong interaction.

Landau comment

“I did not like this assumption myself and tried
to change. However, I discovered that it is impossible.”

Entropy

$$S = -\int \left(n \ln n + (1-n)\ln(1-n) \right) d\tau_{\mathbf{p}}$$

$$E = E[n] \neq n\varepsilon$$

$$\delta \left[S - \beta E + \mu \int n \, d\tau_{\mathbf{p}} \right] = 0$$

Interaction of excitations

Interaction results in
dependence of
energy of excitation on
distribution of excitations:

$$\delta\varepsilon(\mathbf{p}) = \int f(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p}')d\tau_{\mathbf{p}'}$$

$$\frac{1}{m^*} = \frac{1}{m} - \frac{p_F}{(2\pi\hbar)^3}\int f\cos\vartheta\, do_{\mathbf{p}'}$$

Physical meaning of the *f*-function

"The quantity *f* is nothing else but the scattering amplitude of two excitations on angle 0."

Prediction of “zero sound”

$$\text{Dispersion law of zero sound}:$$

$$\omega = c_0 k$$

BUT

$$c_0 = c_0\left[f\right] \neq \left(\frac{\partial p}{\partial \rho}\right)$$

Clouds on the horizon I

$$f_{\uparrow\uparrow,\uparrow\uparrow}(\mathbf{p},\mathbf{p}) \neq 0$$

as it must be for the

forward scattering amplitude

of identical fermions.

(Pomeranchuk question).

Clouds on the horizon II

Scattering amplitude for

$$\mathbf{p}_1, \mathbf{p}_2 \to \mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2 - \mathbf{k}$$

$$\varepsilon_1, \varepsilon_2 \to \varepsilon_1 + \omega, \varepsilon_2 - \omega$$

at $\omega, k \to 0$

depends on ratio k / ω

(Migdal)

The final step in developing of the theory

The function *f* is the forward scattering amplitude for $k/\omega \to 0.$

The assumption (!) was proved using a gauge invariance identity. (Never published!)

A different proof – Luttinger (1960).