Bose and His Four-Page Masterpiece: Was It Just Luck?

By: Advait Iyer, September 2021

Of all the scientists whose names are associated with quantum statistics, Satyendra Nath Bose is perhaps the least well known, despite there being elementary particles and states of matter named after him. In fact, he is often confused with the botanist Jagdish Chandra Bose in his own country, India; and mistaken for a German by many others. Hence, while a comprehensive overview of Bose's life is beyond the scope of this paper, it focuses on his 'magnum opus': a 'clean' derivation of Planc's Law based entirely on quantum theory, which lay the foundation for quantum statistics. More specifically, it addresses some notions about the serendipity of his contribution, and tries to answer the question: Did Bose just get lucky?

Previous attempts to derive Planck's Law: Planck, Debye, Einstein

Quantum theory was born in December 1900, when Max Planck presented a heuristically derived theoretical formula for the observed trends in black body radiation (Planck, 1900). He pictured black body radiation as being produced by tiny 'resonators', and established the relation between incident radiation's energy density and a resonator's average energy using classical electromagnetic theory. Then, he made an ad hoc assumption that these resonators could only emit fixed 'quanta' of energy, as opposed to continuous streams of energy. Finally,, using adjusted Maxwell-Boltzman statistics, he arrived at the following equation:

$$p(v,\ T) \ = \ (8\pi v^2/c^3) \ \cdot \ [hv/(exp(hv/kT) \ - \ 1)]$$

— (1)

Surprisingly, this formula was consistent with experimental data. Yet, there was no real basis for Planck's assumption other than it yielding the desired results. As he himself remarked "...a theoretical interpretation had to be found at any price, however high it might be" (Planck to Wood, 1932, Mehra and Rechenberg, 1982).

In 1905, Einstein, based on the conclusions of his newly-formulated light-quantum hypothesis, argued that Planck's relation based on the classical electromagnetic theory could not be used, since it was not consistent with his assumption about energy quanta. Hence, in 1910, Debye sought to provide a derivation of Planck's law that avoided this pitfall. His method eschewed Planck's relation, but still relied on classical theory to determine the energy stored inside a radiant cavity by counting the standing modes of electromagnetic radiation within it. He also used the same statistics as Planck. (Debye, 1910)

This was followed by Einstein's own derivation in 1916, which didn't refer to either resonators or standing waves. Rather, he arrived at Planck's law by drawing on his own light-quantum hypothesis combined with Bohr's atomic theory, also referring to Wien's law, Maxwell-Boltzman distributions, and Bohr's correspondence principle. Einstein concluded that "…the simplicity of the hypothesis…persuaded me to regard it as very probable that all this constitutes the fundamental outline of the future theoretical derivation." (Einstein, 1916)

Bose's Magnum Opus: A rigorous theoretical derivation of Planck's Law

Satyendra Nath Bose, then a reader at Dacca University in East Bengal (modern Bangladesh), was not satisfied with any of these derivations. He felt that Planck and Debye's derivations were logically flawed since they were essentially based on classical electromagnetic theory, which was deemed to be fundamentally incompatible with the basic assumptions of quantum theory. Additionally, while Bose believed that Einstein's derivation was "remarkably elegant", he disagreed with his use of Wien's law and the correspondence principle: the former was based on classical theory, while the latter was a heuristic principle assuming that the behaviour of systems described by both classical and quantum theory agreed asymptotically. (Bose, 1924)

This motivated him to submit his own papers on the subject to *Philosophical Magazine*, which were rejected after a delay of more than six months. Undeterred, Bose sent his papers to Albert Einstein in June 1924, requesting him to translate and publish them in *Zeitschrift für Physik* (Masters, 2013). Einstein was rather impressed with Bose's work, remarking to Ehrenfest in a 1924 letter, "the Indian Bose has given a beautiful derivation of Planck's law". He got both papers published with his own comments lauding the first one's significance, calling it an "important step forward". Einstein's delight was, in large part, due to the fact that Bose had reconciled his light-quantum hypothesis with Planck's law, which he had himself been unable to do (Ghose, 1994). However, he was not as pleased with Bose's second paper, as we shall discuss later.

The first paper, titled 'Planck's law and the light quantum hypothesis', started by stating that all previous derivations of Planck's law were logically discordant, for the reasons mentioned earlier, and hence, unacceptable. Then, he went on to give an overview of his derivation, which, he explained, used only Einstein's light-quantum hypothesis (which was gaining increasing acceptance post the discovery of the Compton effect in 1923) and statistical mechanics "in the form adjusted by Planck to the needs of the quantum theory". (Bose, 1924)

Drawing on Einstein's light-quantum hypothesis, he denoted black-body radiation as being composed of N light-quanta (or photons) enclosed in a given volume. Now, each light-quantum is located in a 6-dimensional phase space, with three space coordinates and corresponding momentum coordinates. Bose divided this phase space into cells of volume h^3, with each cell denoting one quantum state. Next, based on Compton's discovery, he took the momentum of a light-quantum with frequency v to be $h\nu/c$in the direction of its forward motion. He then demonstrated that the phase space of such a light-quantum would be constrained to a cylindrical surface. Using this result, he showed that the total number of quantum states of radiation was equal to $8\pi\nu^2 V d\nu/c^3$, the coefficient in Planck's equation (1). But in order to obtain this value, his paper took into account "the polarization", a classical concept. Interestingly, Bose always averred that he had provided an quantum theory explanation based on the light-quantum's spin, but, as he told his doctoral student Ghose, "the old man crossed it out"! Einstein probably did this because he found the idea too radical; light-quanta having spin was unheard of in 1924 (Ghose, 1994). Unfortunately, Bose didn't maintain a copy of his original paper, and it is not present in the Einstein archives, so the only written account of this is in CV Raman and S Bhagavantam's 1931 paper, where it is mentioned that Bose "envisaged the possibility of the quantum possessing...also an intrinsic spin angular momentum $\pm\ h/2\pi$" in his paper.

Moving on, Bose needed to calculate "the thermodynamic probability of a macroscopically defined state" as Planck had done before him. In order to do so, he used Planck's adjusted statistical mechanics mentioned earlier. Since each of the $8\pi\nu^2 V d\nu/c^3$cells in his phase space denoted a quantum state, the macrostate resulting from the distribution of N light-quanta amongst these cells was defined only by the number of quanta in each cell: the permutations of quanta in a cell did not create a new macrostate! This implied that light-quanta were indistinguishable; essentially different from the distinguishable particles described by Maxwell-Boltzman statistics! Then, after achieving this remarkable result, all Bose had to do was continue with the process adopted by Planck and Debye to complete his rigorous theoretical derivation of Planck's law. (Bose, 1924)

Significance of Bose's Derivation and Further Developments

Bose's paper has been termed as the "fourth and last of the revolutionary papers of the old quantum theory" by Abraham Pais due to the important problems it resolved and the developments it spurred. Firstly, due to the implicit treatment of photons as being indistinguishable, it solved Gibb's paradox, which had indicated the inapplicability of Maxwell-Boltzman statistics to such particles. Next, when Einstein extended Bose's theory to material particles through three papers in 1924-25, he concluded that it satisfied Nerst's theorem, a pivotal result that led to the prediction of Bose-Einstein Condensate, a new state of matter that was experimentally confirmed in 1995 (Masters, 2013). Additionally, post the development of Fermi-Dirac statistics for particles that obeyed Pauli's exclusion principle, Dirac associated Bose-Einstein and Fermi-Dirac statistics with the symmetry of their wave functions, formally distinguishing between these branches of quantum statistics. Schrodinger's discovery of wave mechanics was also inspired by these developments in quantum statistics, with him remarking "wave mechanics was born in statistics" (Moore, 1989, p. 188)

Did Bose get lucky?

However, despite having played a crucial role in inspiring these exciting developments in quantum theory and wave mechanics, Bose's breakthrough was termed as a "shot in the dark" by Abraham Pais, with even Max Delbrück suggesting he got lucky, and didn't understand the importance of his own work. Bose's lack of justifications for his assumptions were the major cause of such accusations, with the most notable one coming from Delbrück, regarding Bose's implicit assumption about the indistinguishability of photons. (Pais, 1982 p. 428, Delbrück, 1980)

Delbrück believed that a large part of Bose's success was down to a simple statistical mistake when determining the number of ways in which photons were to be distributed. As photons do not behave like other familiar (distinguishable) particles, this 'error' helped Bose accurately describe their thermodynamics. Yet, Bose's student, Ghose, disputes the claim of Bose's fortuitousness, arguing that although Bose didn't explicitly state the term 'indistinguishable' in conjunction with photons, he mentioned using statistical mechanics similar to Planck's, which had already been shown to treat photons as being indistinguishable by Natanson and Ehrenfest more than a decade ago. Ghose adds that most people misconstrue Bose's method as involving the calculation of the number of ways in which individual photons could be distributed among $8\pi v^2 V dv/c^3$ cells. Instead, Bose had sought to compute the thermodynamic probability of a macrostate defined by the number of photons in each cell, as explained earlier, for which his counting method is perfectly justified.

Somaditya Banerjee sheds some more light on Bose's unconventional derivations. As he explains, Bose didn't have the same scientific background as his European peers, given the relative isolation of the colonial Indian scientific community (Banerjee, 2020). Additionally, during his time, theoretical physics and mathematics in India hadn't diverged as much as in Europe—Bose himself was a mathematician by education. Taking these two factors into consideration, it is unsurprising that he did not have rigid pre-conceptions about the distinguishability of light quanta as physicists in Europe did, permitting him to come up with a consistent theoretical derivation of Planc's Law which would not have occurred to them.

The next issue of Bose not understanding his work's importance can be countered by recounting his persistence in trying to get it published: he did not hesitate to contact even Einstein, and request publication in a journal as well-known as *Zeitschrift für Physik*. Addressing this issue, Bose, in an interview with Ramaseshan, mentions that his emotion upon cleanly deriving Planck's law was more akin to relief than elation or pride: the previous derivations had caused him 'ceaseless pain'. He felt this unassuming attitude towards his discovery led many people (including Einstein) to believe he "did not understand what he was saying" (Ramaseshan, 2000). Nevertheless, it is likely that Bose did not understand the physical implications of his statistical method as well as Einstein did, recognising the indistinguishability implied by Bose's counting and generalising it to particles that were not massless.

It is also worth noting that there was perceptible haste in Einstein's translation of Bose's paper, with Delbrück even suspecting that he rewrote some portions of the original manuscript, with citations missing (explaining why references to others' ideas he adopted are absent) and Bose's initials incorrectly mentioned. Taking all of this into consideration, it would certainly be harsh to attribute such a crucial contribution by Bose to sheer luck, or to claim that he was ignorant of his result's significance.

Bose's Exit from the International Science Scene

Sadly, Bose departed from the forefront of theoretical physics research as quickly as he had arrived. His unsuccessful time in Europe is cited as a major reason behind this. Blanpied attributes Bose's lack of success to him being unprepared for the rapid pace of scientific developments, coming off the back of working on a single problem for years.

Bose's disagreement with Einstein also played a role in his exit. Einstein was critical of Bose's second paper, largely due to his idea of a purely statistical interpretation of quantum physics at the cost of the causal processes that Einstein espoused (Ghose, 1974). Bose (whose notions were ultimately proved to be right by Dirac in 1927) was hurt by this dismissal of his ideas by a man who he considered to be his mentor, not publishing a single theoretical physics related paper for the next 12 years!

Post his return to India, Bose was promoted to the post of professor at Dacca University, and primarily engaged himself with teaching his students, alongside studying chemistry and crystallography. He was elected a Fellow of the Royal Society in 1958, and also served as the President of the Indian Physical Society, National Institute of Science and Indian Statistical Institute. In recognition of his contributions to quantum physics, the S.N. Bose National Centre for Basic Sciences was established by the Government of India (Masters, 2013).

Satyendra Nath Bose may not have won any of the seven Nobel Prizes in Physics awarded to research related to the boson, Bose-Einstein Statistics, and Bose-Einstein Condensate, but he has been forever immortalised by their names.

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