Why is the gas converted into stars?
What sets the mass distribution and the formation rate of stars?
Gravitational Instability

Instability of linear density perturbations of a uniform, isothermal, static gas, extending to infinity (Jeans 1902):

\[ \lambda > \lambda_J = \left( \frac{\pi \sigma^2_{th}}{G \rho_0} \right)^{1/2} \quad \Rightarrow \quad M_J = \frac{4}{3} \pi \left( \frac{\lambda_J}{2} \right)^3 \rho_0 = 24 M_{\text{sun}} \left( \frac{n}{200 \, \text{cm}^{-3}} \right)^{-1/2} \left( \frac{T}{10 \, \text{K}} \right)^{3/2} \]

The cold interstellar medium has a complex hierarchical structure:

\[ n \approx 2 \times 10^3 \, \text{cm}^{-3} \left( \frac{l}{1 \, \text{pc}} \right)^{-1}, \quad T \approx 10 \text{K} \]

So clouds of 10 pc size have \( n \sim 200 \, \text{cm}^{-3} \), \( M_{\text{cl}} \sim 10^4 \, M_{\text{sun}} \), and \( M_J \sim 24 \, M_{\text{sun}} \).

**Prediction 1:**
The characteristic stellar mass in these molecular clouds is \( \sim 24 \, M_{\text{sun}} \)
At what rate is the gas converted into stars?

Without pressure support, a uniform sphere collapses in a free-fall time:

\[ \tau_{\text{ff}} = \left( \frac{3 \pi}{32 G \rho} \right)^{1/2} = 2.3 \times 10^6 \text{ yr} \left( \frac{n}{200 \text{ cm}^{-3}} \right)^{-1/2} \]

(roughly a sound crossing time of the Jeans length).

Prediction 2:
Molecular clouds are converted into stars in two million years.

Both predictions from the linear gravitational instability are quite wrong....
Large range of stellar masses: 0.01 - 100 $M_{\text{Sun}}$

Characteristic stellar mass: 0.2 $M_{\text{Sun}}$
1. Broad range of masses, characteristic mass $M_{ch} \ll M_J$

2. Gas conversion into stars $\sim 2\%$ per free-fall time

Why are the predictions from the gravitational instability so wrong?
The cold ISM is highly turbulent, \( Re = \frac{UL}{\nu} \sim 10^8 \).

The turbulence is supersonic, \( M_s \sim 30 \).

--> Highly non-linear velocity and density.
Turbulence Solution to Star Formation

1) Mass range of stars:
Stellar masses are set by turbulence, not by self-gravity ($M>J$ is possible).
Density peaks that become stars are pieces of postshock gas.
Their size scales with the thickness of the postshock gas, set by shock jump conditions and velocity scaling.

MHD shocks: $\lambda = l / \mathcal{M}_A(l), \quad \rho(l) = \rho_0 \mathcal{M}_A \Rightarrow M \sim \lambda^3 \rho \sim l^3 \rho_0 / \mathcal{M}_A^2$

Velocity scaling: $\mathcal{M}_A(l) \sim u(l) \sim l^{\zeta_2/2} \Rightarrow M \sim l^{3-\zeta_2} \sim l^2$

$\Rightarrow M_{\text{max}} / M_{\text{min}} = \left( L_0 / l_1 \right)^{3-\zeta_2} = \mathcal{M}_A,0^{-2+6/\zeta_2} = \mathcal{M}_A,0^4$

$\mathcal{M}_A,0 = 10 \Rightarrow M_{\text{max}} / M_{\text{min}} = 10^4$
2) Characteristic stellar mass:

Bonnor-Ebert mass: isothermal sphere confined by external pressure (Ebert 1955; Bonnor 1956; McCrea 1957).

**Thermal pressure:**

\[ M_{BE} \approx \frac{\sigma_{\text{th}}^4}{G^{3/2} P_{\text{th},0}^{1/2}} \approx \frac{\sigma_{\text{th}}^3}{G^{3/2} \rho_0^{1/2}} \approx 10 M_{\text{sun}} \left( \frac{n}{200 \text{ cm}^{-3}} \right)^{-1/2} \left( \frac{T}{10 \text{ K}} \right)^{3/2} \approx \frac{M_J}{2.47} \]

**Dynamic pressure of turbulence (shocks --> nonlinear density jump):**

\[ M_{BE,t} \approx \frac{\sigma_{\text{th}}^4}{G^{3/2} P_{\text{dyn,0}}^{1/2}} \approx \frac{\sigma_{\text{th}}^3}{G^{3/2} \rho_0^{1/2}} \left( \frac{\sigma_{\text{th}}}{\sigma_v} \right) = \frac{M_{BE}}{M_s} \approx 0.4 M_{\text{sun}} \quad (\text{for } M_s = 25) \]

(Notice that \( M_{BE,t} \sim n^{-1/2} T^2 \sigma_v^{-1} \))
3) Rate of star formation:

**Thermal energy**

\[ u_0 \gg C_S \implies E_{k,0} \gg E_{th} \]

Isothermal shocks create a complex filamentary density structure.

**Gravitational energy**

\[ \frac{E_k}{E_g} \sim \frac{u^2}{(\rho L)^2}, \quad u \sim L^{1/2}, \quad \frac{E_{k,0}}{E_{g,0}} \sim 1 \implies \frac{E_k(L)}{E_g(L)} = \left( \frac{L}{L_0} \right)^{-1} \]

The turbulence can prevent the gravitational collapse.

*Star formation occurs only where the density is enhanced and the turbulence is dissipated, few % of the total mass.*
Supersonic turbulence is ubiquitous and energetically dominant in star-forming regions.

*How do we study its role in the process of star formation?*

Two different numerical approaches......
1. Brute-force approach: AMR simulations of star-formation

5 pc $\rightarrow$ 0.5AU, $512^3$ $\rightarrow$ $(2 \times 10^6)^3$
2. Idealized experiments of supersonic turbulence

Statistics of turbulence (universal) --> Statistical theory of star formation

Experiment setup:

- Isothermal E.O.S.
- Periodic B.C.
- Uniform I.C. (rho, B)
- Random I.C. (u)
- Random acceleration (1 < k < 2)
- No gravity
- Up to 2,048^3 (or larger with AMR)
- The flow is relaxed for several $t_{dyn}$ before computing statistics

Euler Codes:

- PPM (Colella and Woodward 1984)
- PPML (Popov and Ustyugov 2007, 2008)
- Stagger (Nordlund)
$1000^3$ HD, Mach=10, Stagger Code
$1000^3$ ideal MHD, Mach=10, Stagger Code
Lognormal PDF of gas density

Nordlund and Padoan (1999):

\[ \sigma_{\rho} \approx M_S / 2 \quad \sigma_{\ln\rho}^2 \approx \ln\left(1 + M_S^2 / 4\right) \]

Consistent with observations (Alyssa Goodman et al. 2008)
Power-law velocity power spectrum:

Padoan et al. (2007):

\[ E(k) \propto k^{-1.9} \]

Kolmogorov: \( k^{-5/3} \)
Burgers: \( k^{-2} \)

Is there an energy cascade in supersonic turbulence?

Supersonic turbulence as **inertial motions** ending into **oblique shocks**:

- \( u_\perp \) is dissipated by the shock
- \( u_\parallel \) goes into postshock shear

\[ \frac{E_C}{E_S} \approx \frac{\epsilon_C}{\epsilon_S} \approx \frac{1}{2} \]

So there is a solenoidal cascade, but the dissipative flow geometry is primarily sheets (postshock regions), not filaments (vortices).
Energy cascade in incompressible turbulence

Kolmogorov (1941): \[ \delta u^2 \left( \frac{\delta u}{l} \right) = \text{constant} \Rightarrow \delta u^3 \propto l \Rightarrow \delta u^p \propto l^{p/3} \]

What are the scaling exponents in supersonic turbulence?

Kritsuk et al. (2007): 1,024³ and 2,048³ PPM simulations:

2nd order SFs: \( \zeta_2^\parallel = 0.95, \zeta_2^\perp = 0.98 \)

3rd order SFs: \( \zeta_3^\parallel = 1.26, \zeta_3^\perp = 1.29 \)
Energy cascade in supersonic turbulence

Lighthill (1955): \[ \rho \delta u^2 \left( \frac{\delta u}{l} \right) = \text{constant} \Rightarrow \rho \delta u^3 \propto l \]

\[ \nu \equiv \rho^{1/3} u \Rightarrow \delta v^p = \delta \left( \rho^{1/3} u \right)^p \propto l^{p/3} \]

Kolmogorov scaling for \( \nu \): \[ \Sigma(k) \sim k^{-1.7} \]
Velocity $u, v \equiv \rho^{1/3} u$, and $w \equiv \rho^{1/2} u$

Kolmogorov scaling for $v$: $S_3(\ell) \equiv \langle |\delta v|^3 \rangle \sim \ell$
Structure function exponents of $\nu \equiv \rho^{1/3} u$
Summary

- Supersonic turbulence can explain masses and formation rate of stars.

- A statistical theory of star formation can be derived from the statistics of supersonic turbulence.

- The pdf of gas density is a Lognormal and its standard deviation is a function of the rms Mach number.

- The energy spectrum is a power law, with slope $\sim 1.9$, and $E_s/E_c \sim 2$.

- The “energy cascade” concept applies to supersonic turbulence, in the sense that the average kinetic energy density rate does not depend on scale.

- The Log-Poisson intermittency model works well in supersonic turbulence, and its parameters (scaling exponent and dimension of the most dissipative structures) have the correct physical meaning.