The Size Effect Revisited

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Total Return

We measure time in quarters. In quarter t, stock has end-of-quarter price $S(t)$ and pays dividends $D(t)$

Total return is from price increase and dividends:

$$Q(t) := \ln \frac{S(t) + D(t)}{S(t-1)}.$$

Same for a mutual fund or an exchange-traded fund (ETF):

VTSMX: Vanguard Total Stock Market Index Fund

SPY: SPDR S&P 500 ETF

IYY: iShares Dow Jones ETF

Equity Premium

A 3-month Treasury bill has rate $r(t-1)$ at end of quarter $t-1$

Invest 1 at end of quarter $t-1$, get $1 + r(t-1)/4$ at end of quarter t

This provides total return

$$R(t) = \ln\left(1 + \frac{r(t-1)}{4}\right)$$

Equity premium: the different between stock and bond returns

$$P(t) = Q(t) - R(t)$$

Alpha and Beta

Benchmark: Standard & Poor 500 index, equity premium P_0

Any stock or portfolio with equity premium P: Regress

$$P(t) = \alpha + \beta P_0(t) + \varepsilon(t)$$

with residuals $\varepsilon(t)$ having mean 0 and variance σ^2

α: excess return
β: market exposure

Standard & Poor Funds

BlackRock iShares S&P ETFs:

IJH (S&P 400 Mid-Cap), IJR (S&P 600 Small-Cap)

Benchmark: IVV (S&P 500 Large-Cap)

Mid-cap: $\alpha = 0.0053$, $\beta = 1.069$, $\sigma = 0.0304$, $R^2 = 0.894$

Small-cap: $\alpha = 0.0071$, $\beta = 1.087$, $\sigma = 0.0395$, $R^2 = 0.837$

We can reject $\beta = 1$, but not $\alpha = 0$

Regression explains almost all signal

Shapiro-Wilk normality test for residuals is passed

Implications for Asset Allocation

Recall again:

$$Q(t) - R(t) = \alpha + \beta(Q_0(t) - R(t)) + \varepsilon(t)$$

If $\alpha = 0$, $\beta = 1.05$ for small-cap $Q(t)$ and large-cap $Q_0(t)$, then:

$$Q(t) = 1.05Q_0(t) - 0.05R(t) + \varepsilon(t)$$

Buy small stocks = short T-bills + buy large stocks

Morningstar Funds

BlackRock iShares Morningstar ETFs:

JKG Mid-Cap, JKJ Small-Cap

Benchmark: JKD Large-Cap

Mid-cap: $\alpha = 0$, $\beta = 1.107$, $\sigma = 0.0339$, $R^2 = 0.858$

Small-cap: $\alpha = 0$, $\beta = 1.207$, $\sigma = 0.0431$, $R^2 = 0.816$

We can reject $\beta = 1$

Shapiro-Wilk normality test for residuals is passed

Morningstar Box

Type/Size	Blend	Growth	Value
Large	JKD	JKE	JKF
Mid	JKG	JKH	JKI
Small	JKJ	JKK	JKL

Value = Stocks with low prices relative to fundamentals (earnings, dividends, book price); Growth = Stocks with price growth potential, high prices relative to fundamentals

Regress equity premium for Mid row or Small row upon Large row

$T = 171$ quarters, Shapiro-Wilk test passed

Morningstar Box: Results

CI $=$ 95% confidence interval

Mid-cap vs Large-cap: $\alpha = 0.00019$, CI $[-0.005, 0.005]$, $\beta = 1.117$, CI $[1.054, 1.180]$, $\sigma = 0.0323$, $R^2 = 88.4\%$

Small-cap vs Large-cap: $\alpha = -0.0027$, CI $[-0.009, 0.004]$, $\beta = 1.1636$, CI $[1.078, 1.249]$, $\sigma = 0.0438$, $R^2 = 81.1\%$

Summary: No excess return α, but additional market exposure β, and regression again explains almost all signal

We can do similar a box for iShares S&P funds

Vanguard Funds

Benchmark: VFINX Vanguard 500 Index Fund
Target: NAESX Vanguard Small-Cap Index Fund
Risk-free: VMFXX Vanguard Federal Money Market Fund

Dynamic returns: Dividends are reinvested the day they were collected

$T = 152$ quarters, Q3 1981 – Q2 2019

$p = 0.578$ for Shapiro-Wilk test, residuals are normal

$R^2 = 81\%$, $\alpha = -0.0083$, $\beta = 1.2719$

We can reject both $\alpha = 0$ and $\beta = 1$

Foreign Equity

Invesco mutual funds:
QIVAX total stock market
OSMAX small-cap stocks
For risk-free asset, take VMFXX Vanguard money market fund

Results: Residuals fail Shapiro-Wilk normality test

Reason: Different countries have different short-term interest rates

Random Portfolios: Construction

S&P 500 constituent stocks as of July 7, 2019

Q3 1989 – Q2 2019, $T = 120$ quarters

Beginning: 240 stocks, end: 500 stocks

Every quarter, generate a random portfolio, uniformly distributed weights on the simplex $\{\pi_i \geq 0, \sum \pi_i = 1\}$

Benchmark: Equally-weighted portfolio, corrects for survivor bias

Random Portfolios: Results

$P_\pi(t)$ = equity premium for portfolio π
$P_0(t)$ = equity premium for equally-weighted portfolio

$$V_\pi(t) = \ln C_\pi(t) - \ln \overline{C}(t)$$

$$P_\pi(t) = \alpha_0 + \alpha_1 V_\pi(t) + (\beta_0 + \beta_1 V_\pi(t)) P_0(t) + \varepsilon(t)$$

Residuals are not normal, $R^2 = 99\%$, $\sigma = 0.0082$

Point estimates:
$\alpha_0 = 0.0002$, $\alpha_1 = -0.0001$, $\beta_0 = 0.9826$, $\beta_1 = -0.0152$

We are most interested in β_1: Decrease in weighted market cap of π by 10 adds $\ln(10) \cdot 0.0152 = 0.035$ to market exposure β

Future Research

Do longer time steps for simulated portfolios to see whether normality of residuals is restored

Try for various sectors: Utilities, REITs

Try delisted stocks, to get all 500 stocks or all existing stocks at every quarter: See whether the result changes

Thank You!