Divergence between Indirect and Direct Seasonal Adjustment when using the Contribution to Growth method

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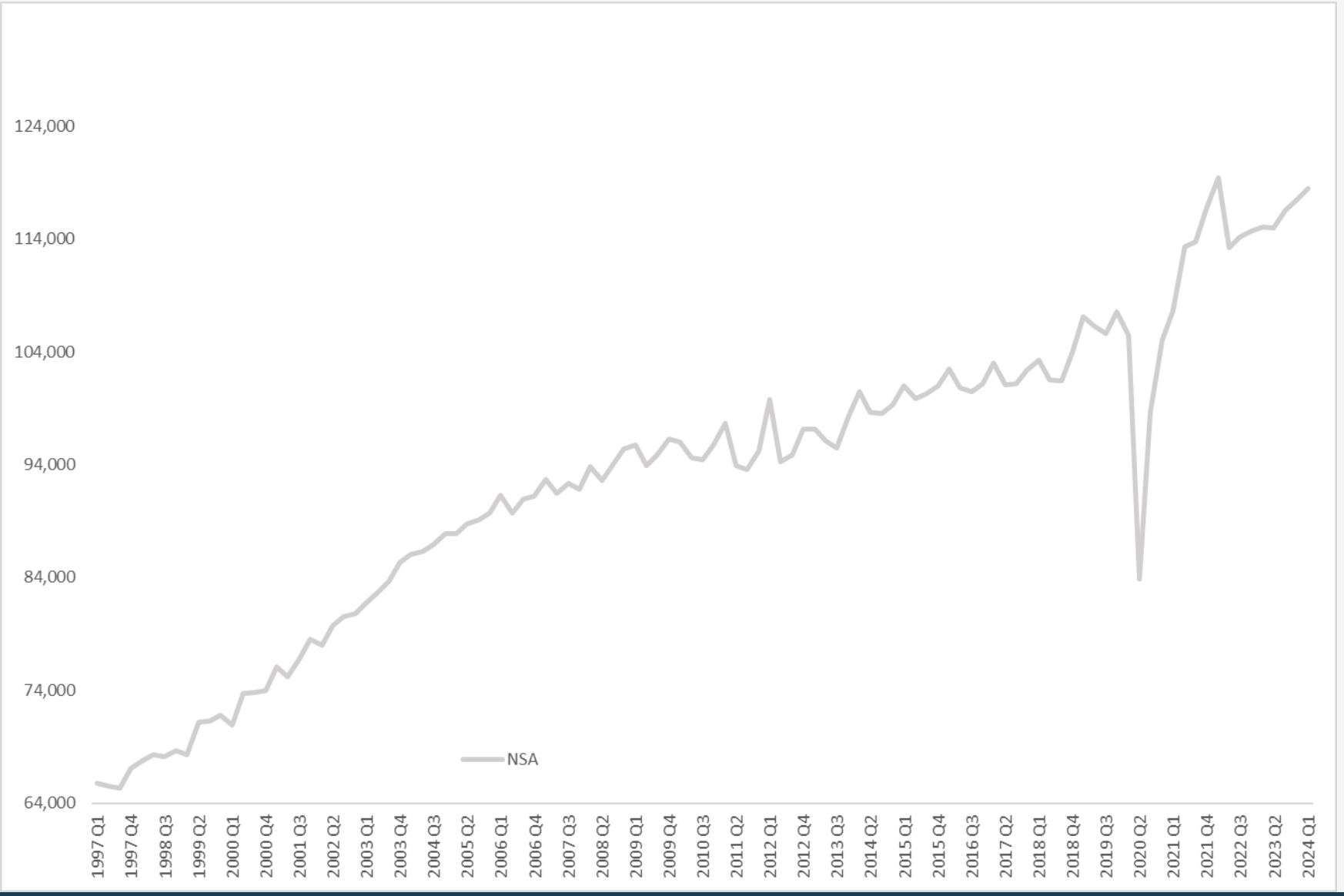
Head of Time Series Analysis Branch
Methodology and Quality

Census Bureau Seasonal Adjustment Practitioners Workshop
12 September 2024

The issue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2024 Q1

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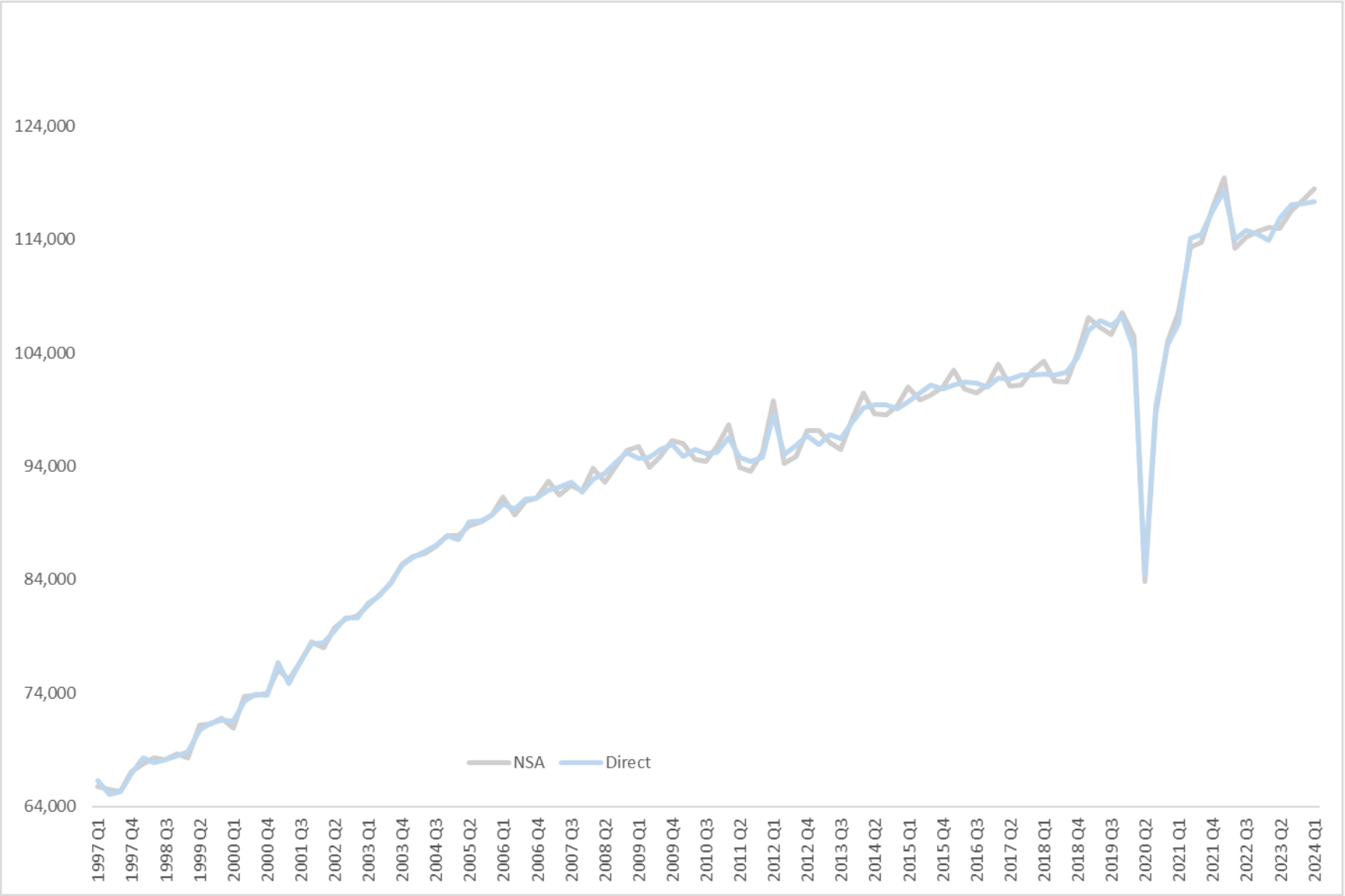
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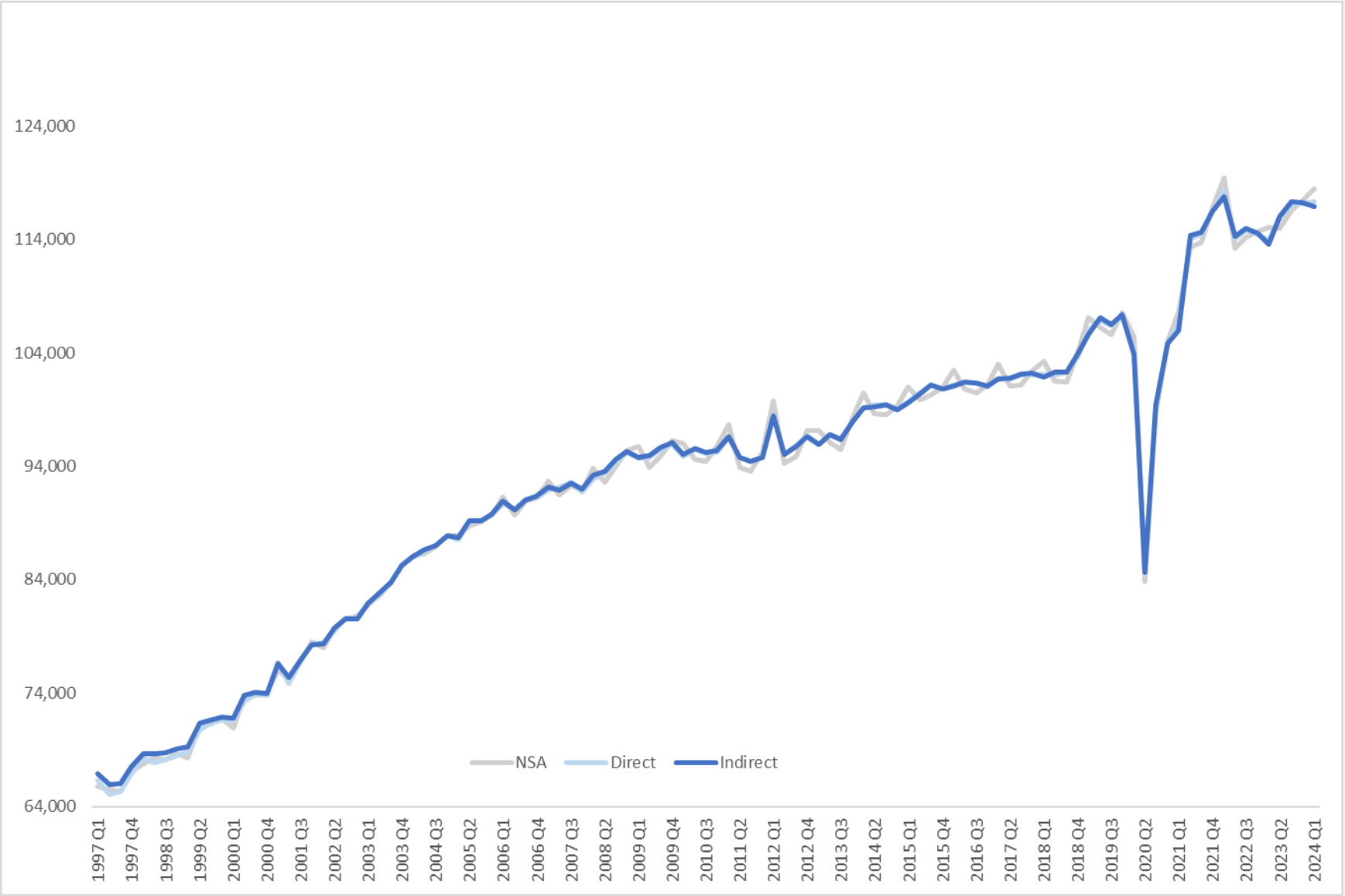
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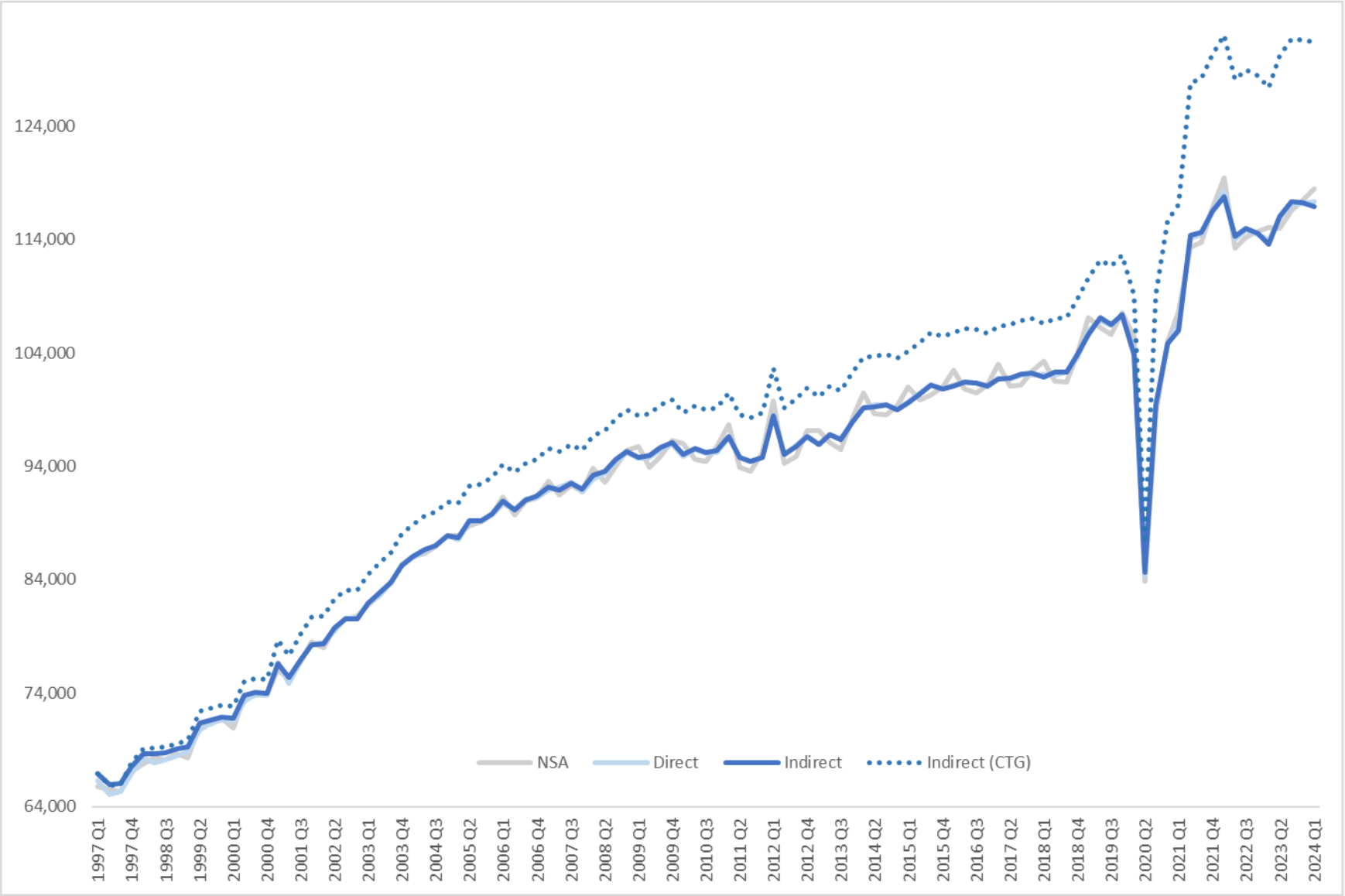

- Divergence between non-seasonally adjusted (NSA) and seasonally adjusted (SA) when using Contribution-to-Growth (CTG)

What is the CTG method?

t	T	A	B	Percentage growth		Weights		Contribution to Growth		CTG	Index
1	300	100	200								100
2	315	110	205								

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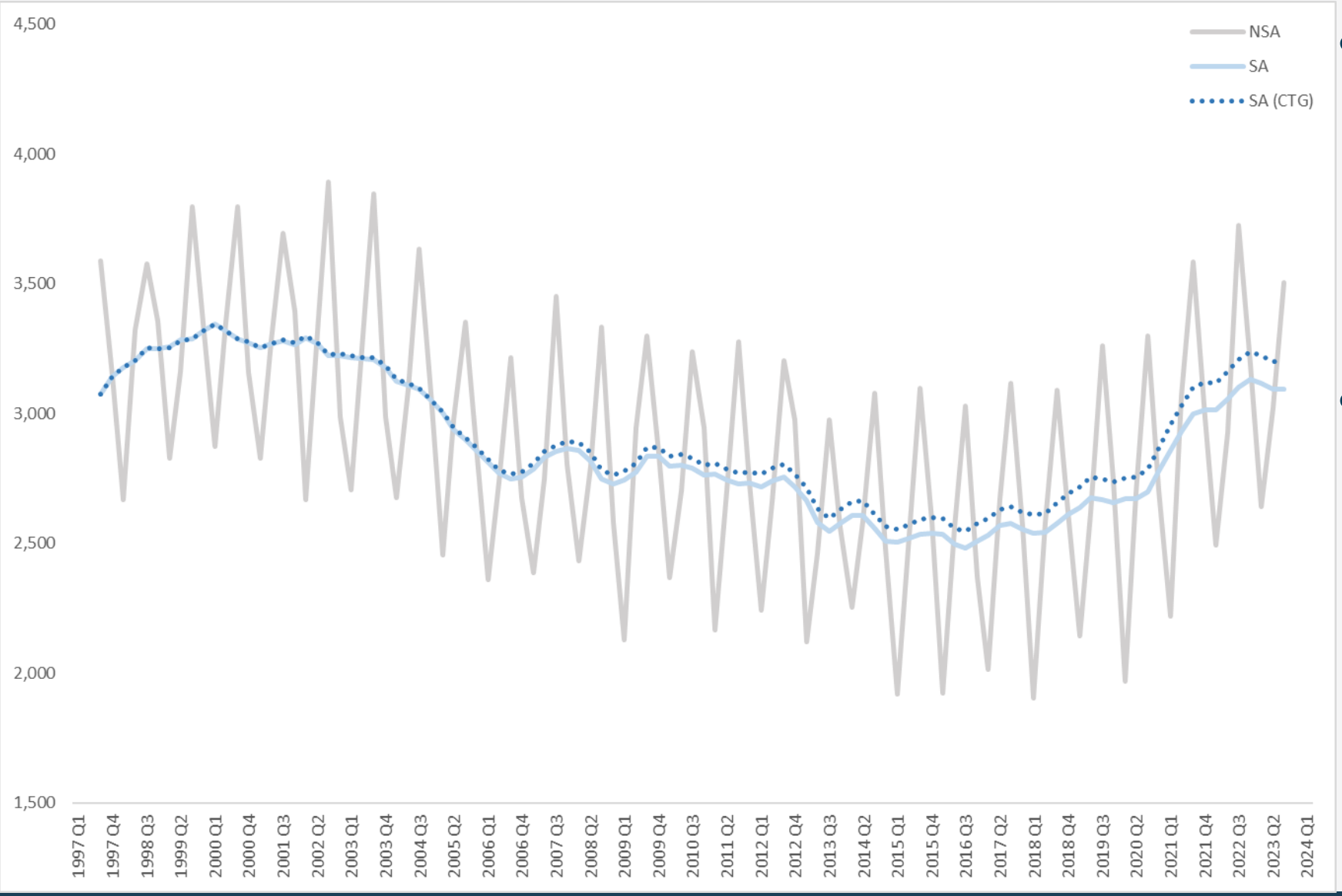
1. Calculate growth of components.
2. Calculate weight of components.
3. Calculate contribution to growth.
4. Aggregate.
5. Create index.

Research questions

- Is the divergence caused by the bespoke seasonal adjustment models?

Research questions

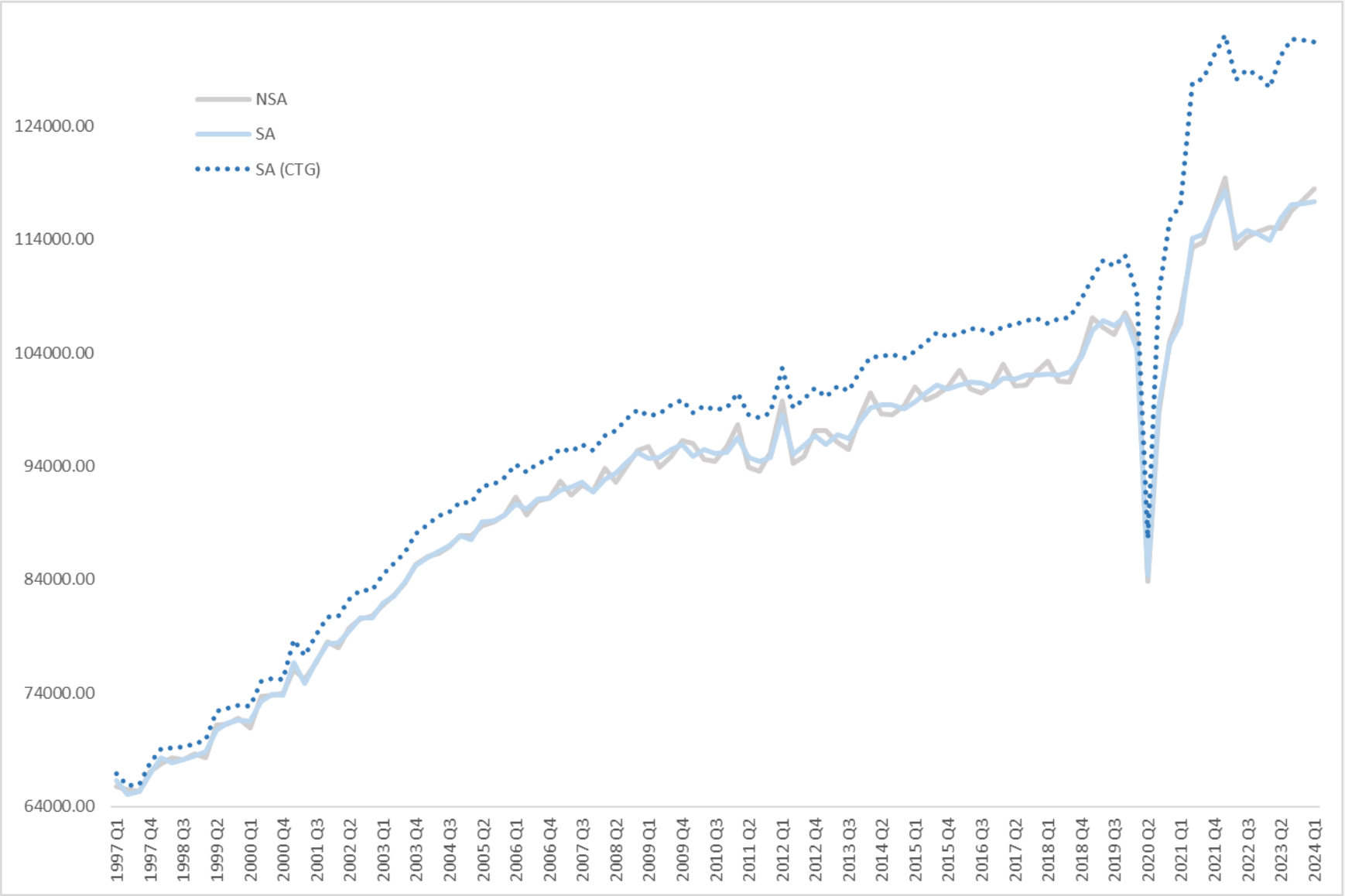


- Is the divergence caused by the bespoke seasonal adjustment models?
- No, the same divergence occurs with simple SA applied to simulated data.

Research questions

- Is the divergence caused by using indices?

Research questions

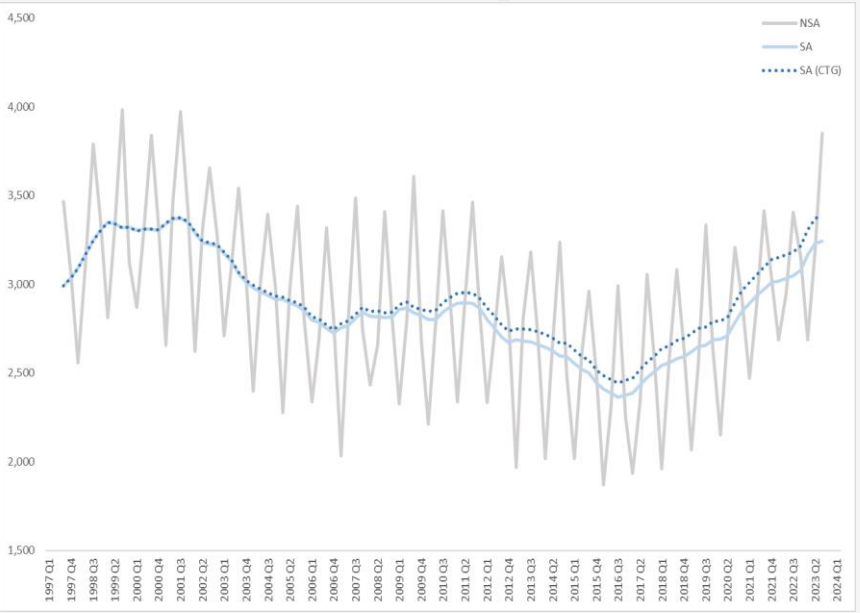


- Is the divergence caused by using indices?
- No, the same divergence occurs with simple SA applied to non-index series.

Research questions

- Is the divergence caused by specific data?

Research questions



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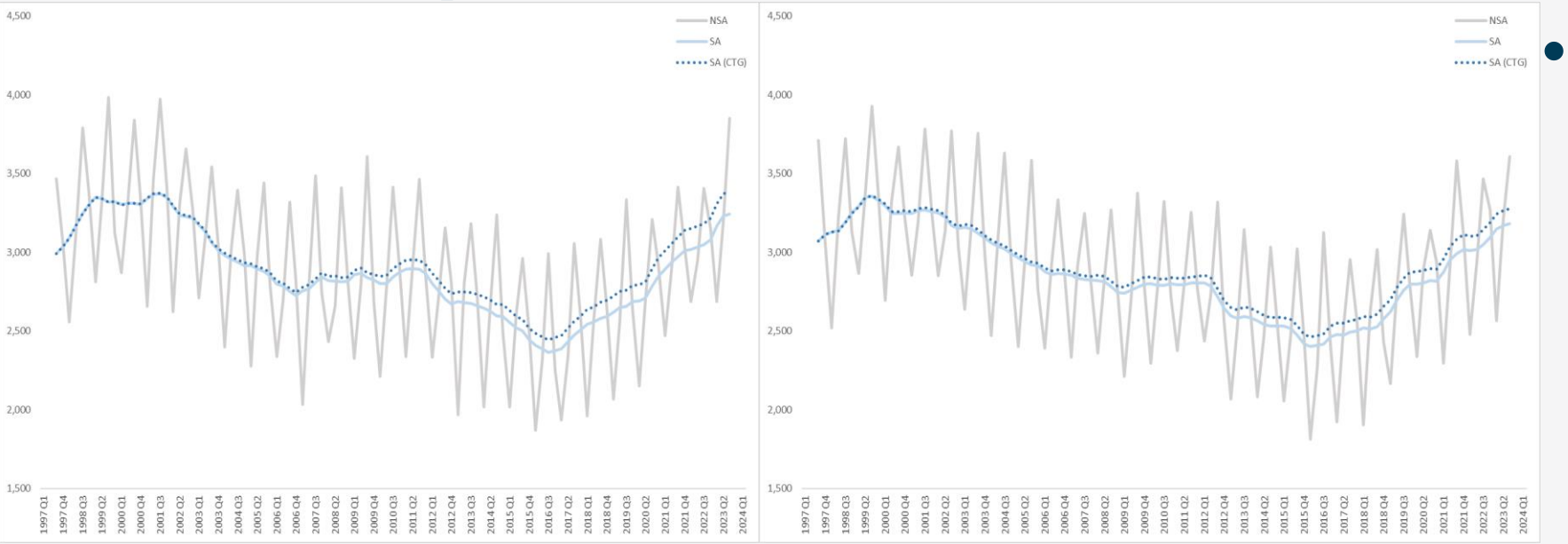

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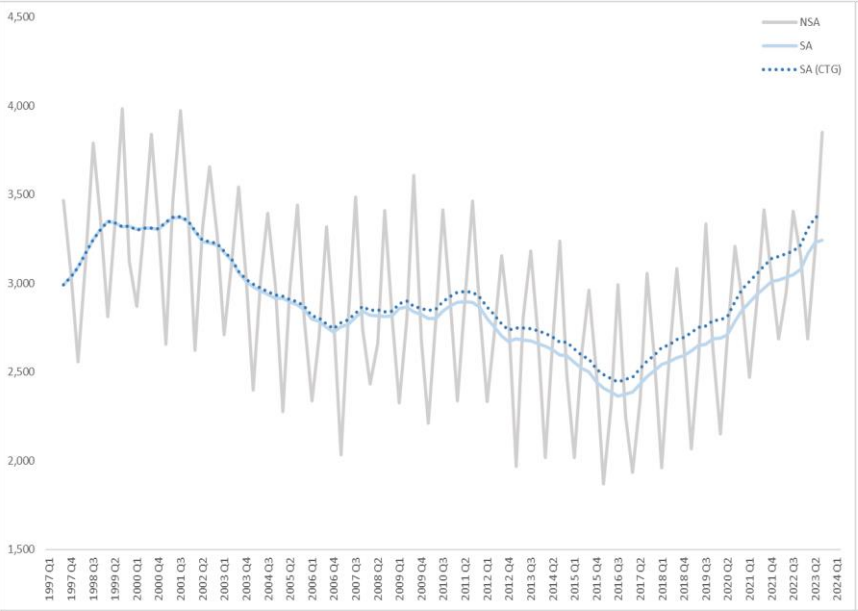

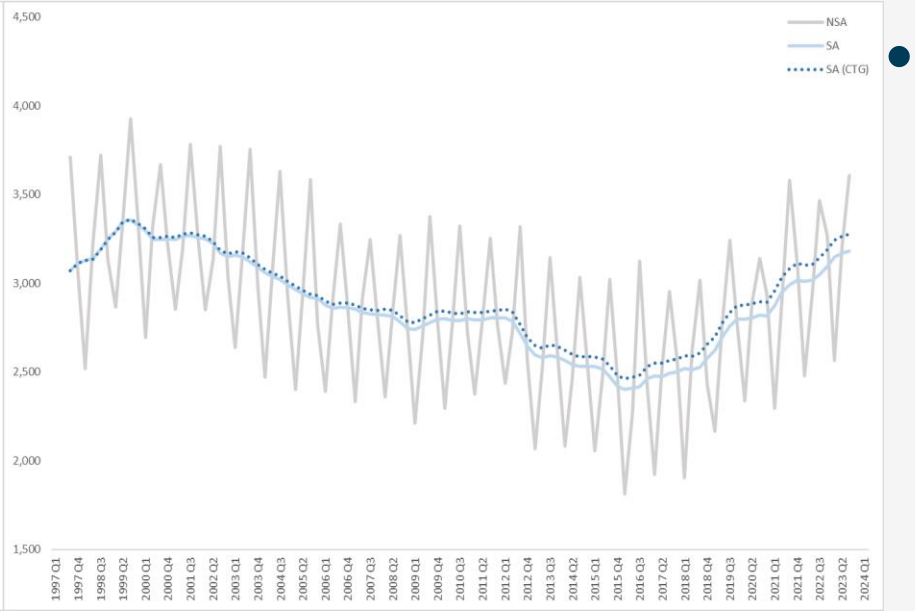

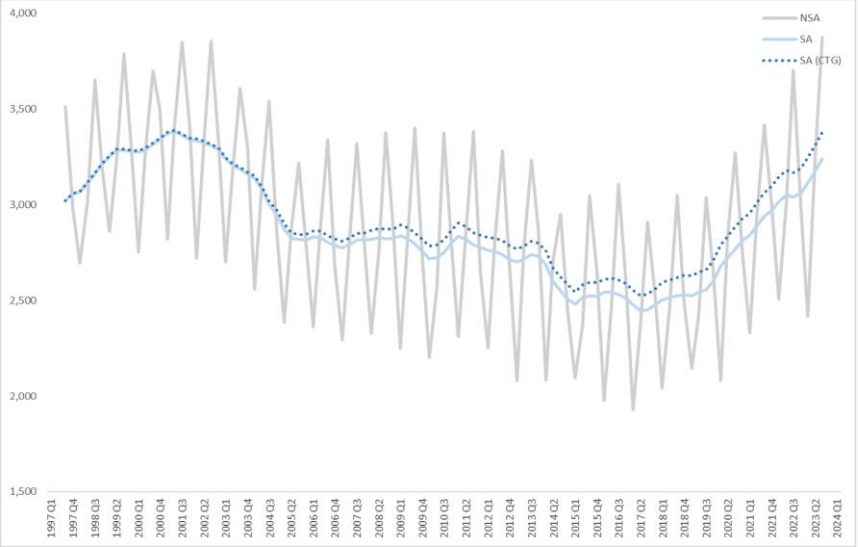

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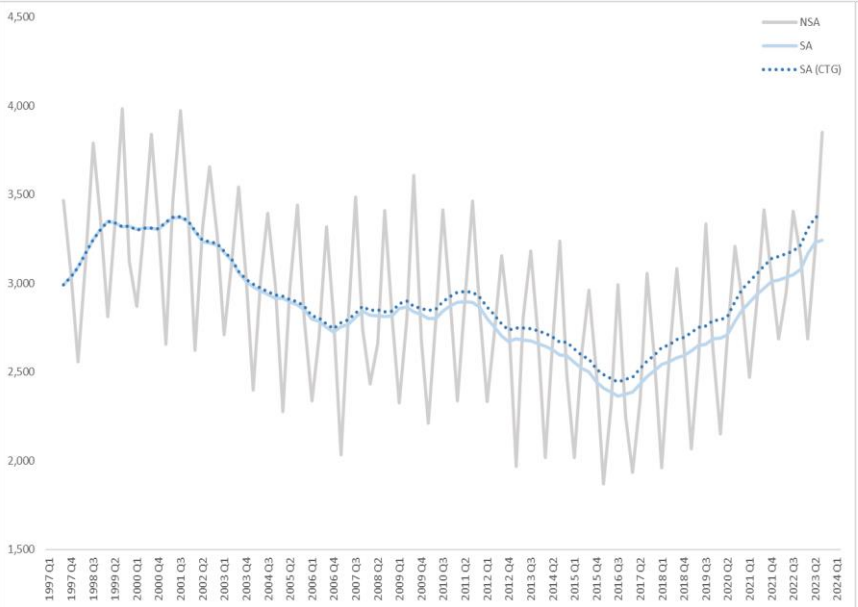
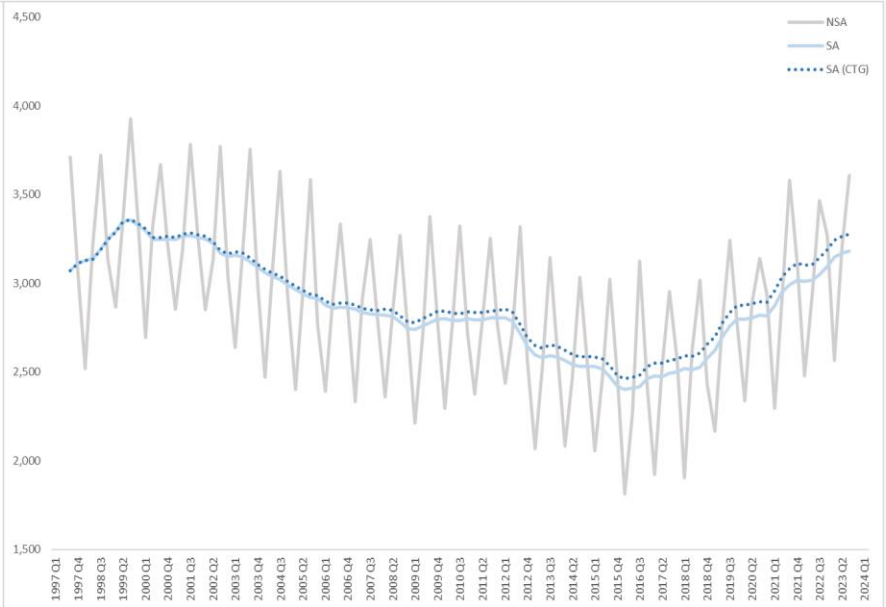
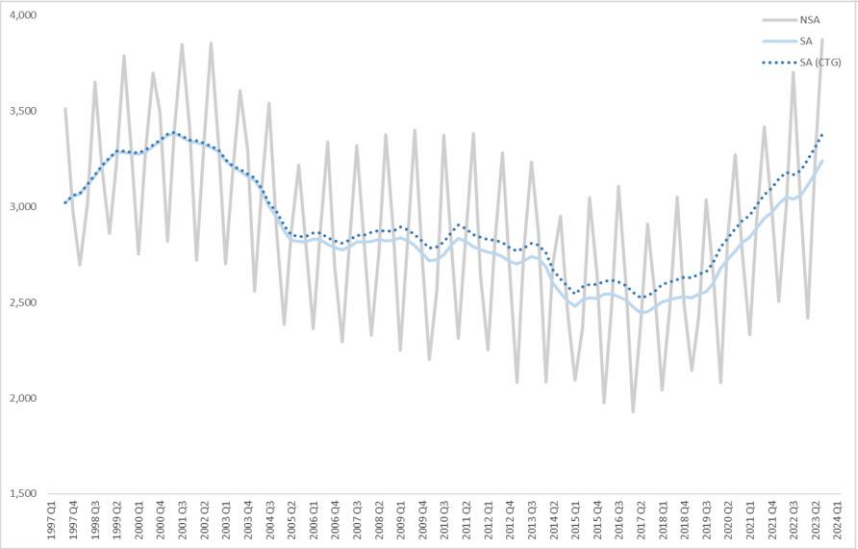
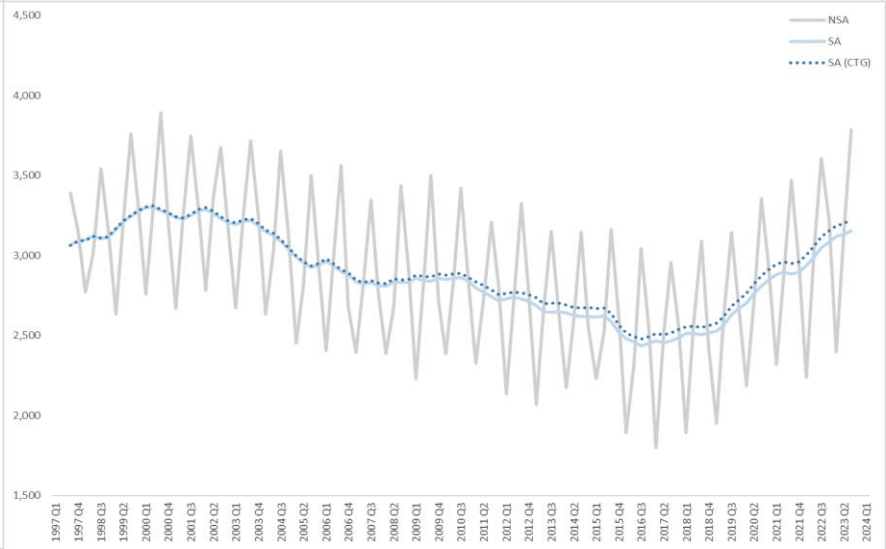






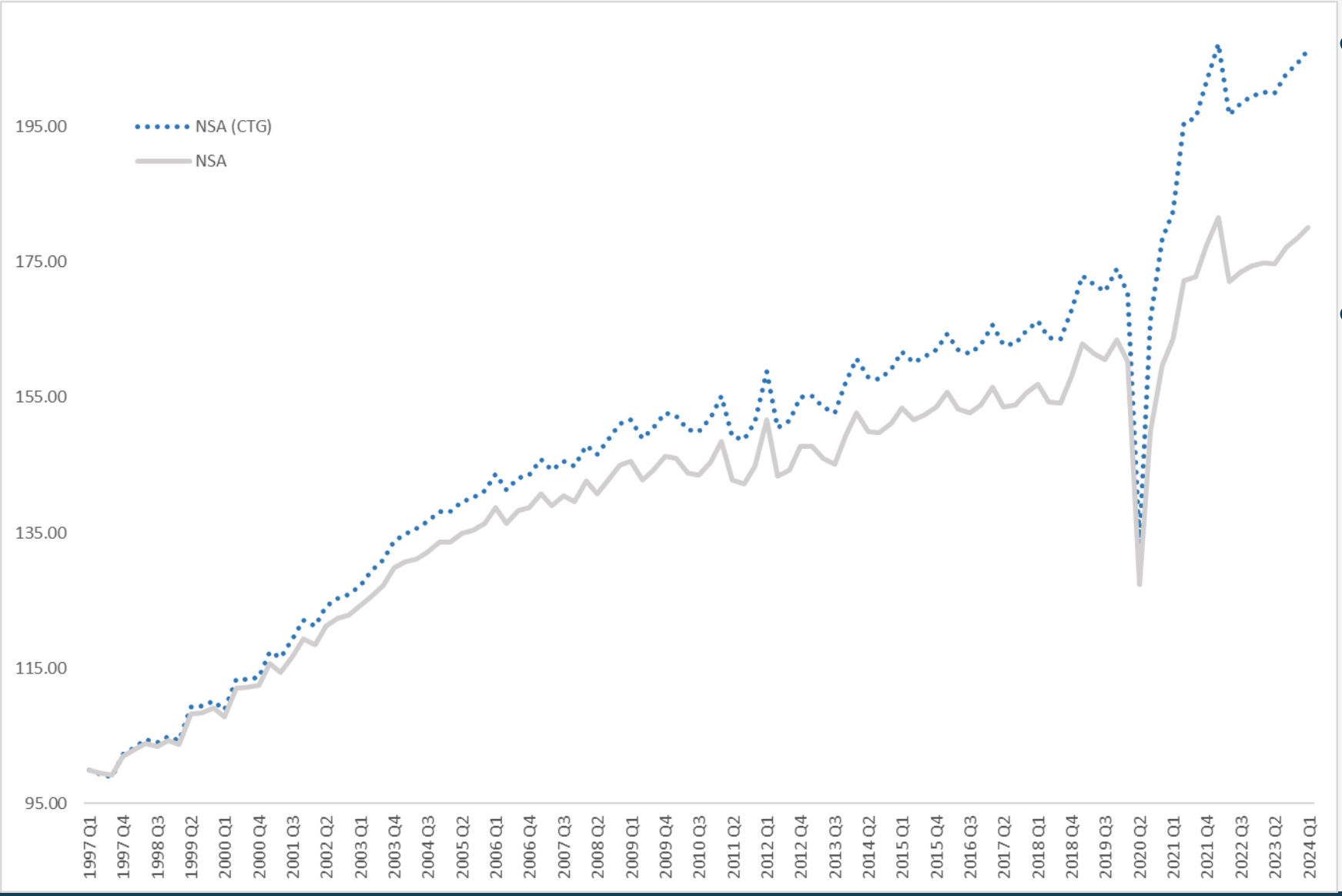


- Is the divergence caused by specific data?
- No, the same divergence occurs with simulated data.

Research questions

- Is the divergence caused by seasonal adjustment?

Research questions

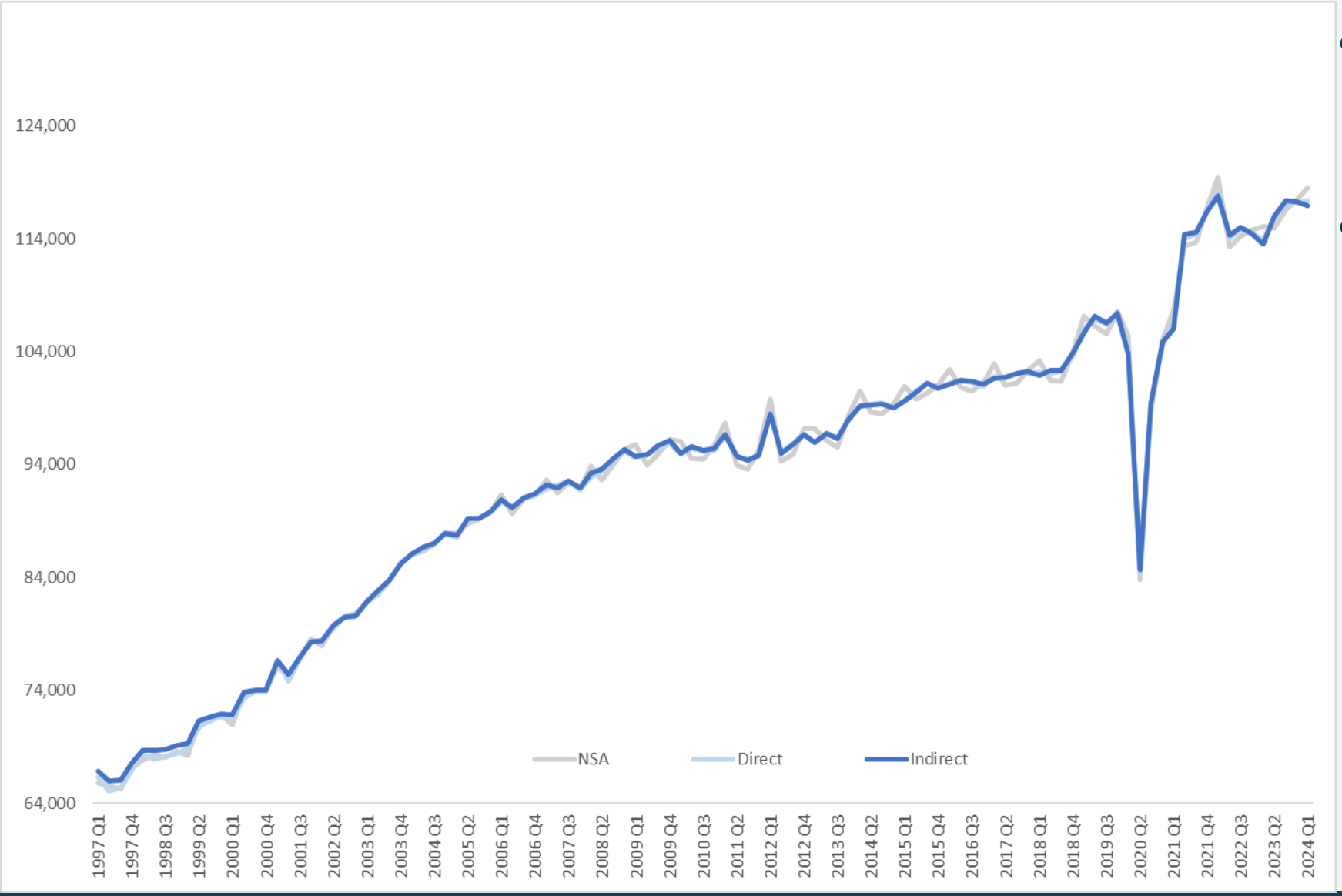


- Is the divergence caused by seasonal adjustment?
- No, the divergence is present when CTG is applied to NSA data.

Research questions

- Is the divergence caused by CTG?

Research questions



- Is the divergence caused by CTG?
- Yes, the same divergence disappears when CTG is not used.

Research questions

$T_t = A_t + B_t$

- How does CTG cause the divergence?

Research questions

$$T_t = A_t + B_t \qquad g_{A,t} = \frac{A_t - A_{t-1}}{A_{t-1}}$$

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$$= T_{CTG,\tilde{c},t-2}\left(1 + c_{T,t-1} + d_{t-1}\right)\left(1 + c_{T,t} + d_t\right) = \cdots.$$

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$$T_t = A_t + B_t \qquad g_{A,t} = \frac{A_t - A_{t-1}}{A_{t-1}} \qquad w_{A,t} = \frac{A_{t-1}}{T_{t-1}}$$

$$\widetilde{w}_{A,t} = \frac{A_t + A_{t+1} + A_{t+3} + A_{t+4}}{T_t + T_{t+1} + T_{t+2} + T_{t+3}} \qquad \widetilde{w}_{A,t} = w_{A,t} + \delta_{A,t}$$

$$c_{A,t} = g_{A,t} w_{A,t} \qquad \tilde{c}_{A,t} = g_{A,t} \widetilde{w}_{A,t} = g_{A,t} w_{A,t} + g_{A,t} \delta_{A,t}$$

$$c_{T,t} = c_{A,t} + c_{B,t}$$

$$\tilde{c}_{T,t} = \tilde{c}_{A,t} + \tilde{c}_{B,t} = c_{A,t} + c_{B,t} + g_{A,t}\delta_{A,t} + g_{B,t}\delta_{B,t}$$

$$d_t = g_{A,t}\delta_{A,t} + g_{B,t}\delta_{B,t}.$$

$$T_{CTG,\tilde{c},t} = T_{CTG,\tilde{c},t-1}\left(1 + \tilde{c}_{T,t}\right) = T_{CTG,\tilde{c},t-1}\left(1 + c_{T,t} + d_t\right)$$

$$= T_{CTG,\tilde{c},t-2}\left(1 + c_{T,t-1} + d_{t-1}\right)\left(1 + c_{T,t} + d_t\right) = \cdots.$$

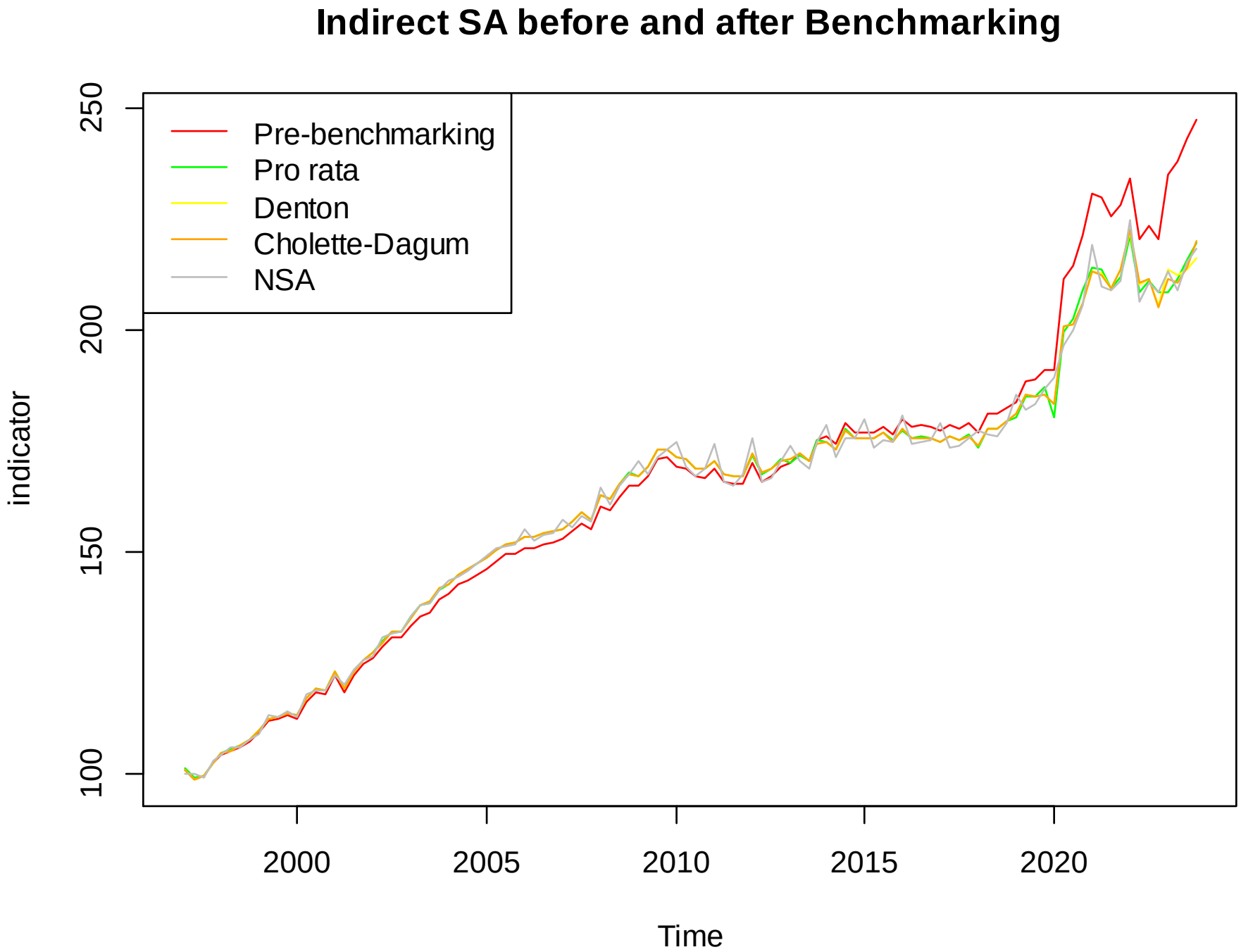
- How does CTG cause the divergence?
- CTG causes a discrepancy by applying annual NSA weights to quarterly SA data. CTG then compounds this discrepancy over time to cause the divergence.

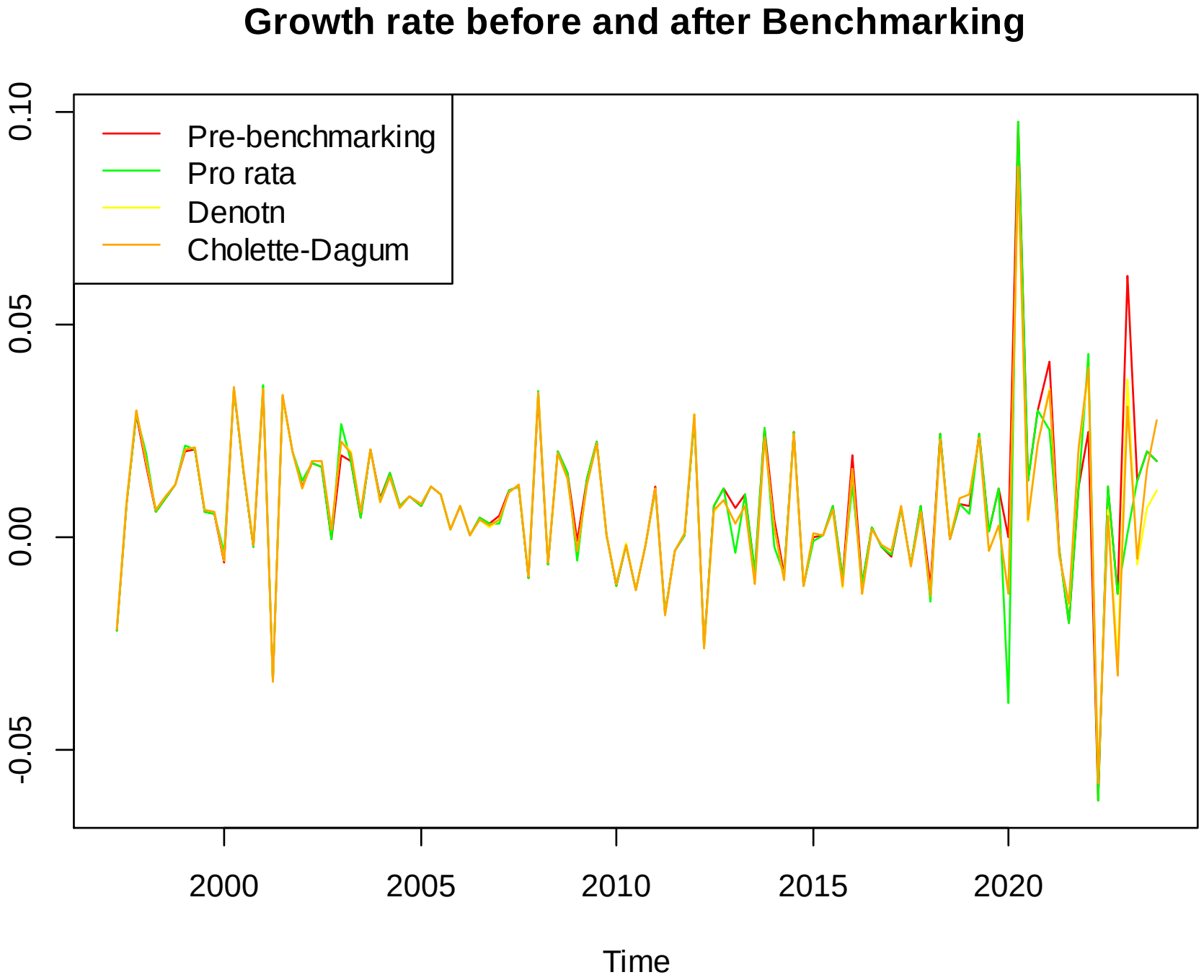
Solution Options

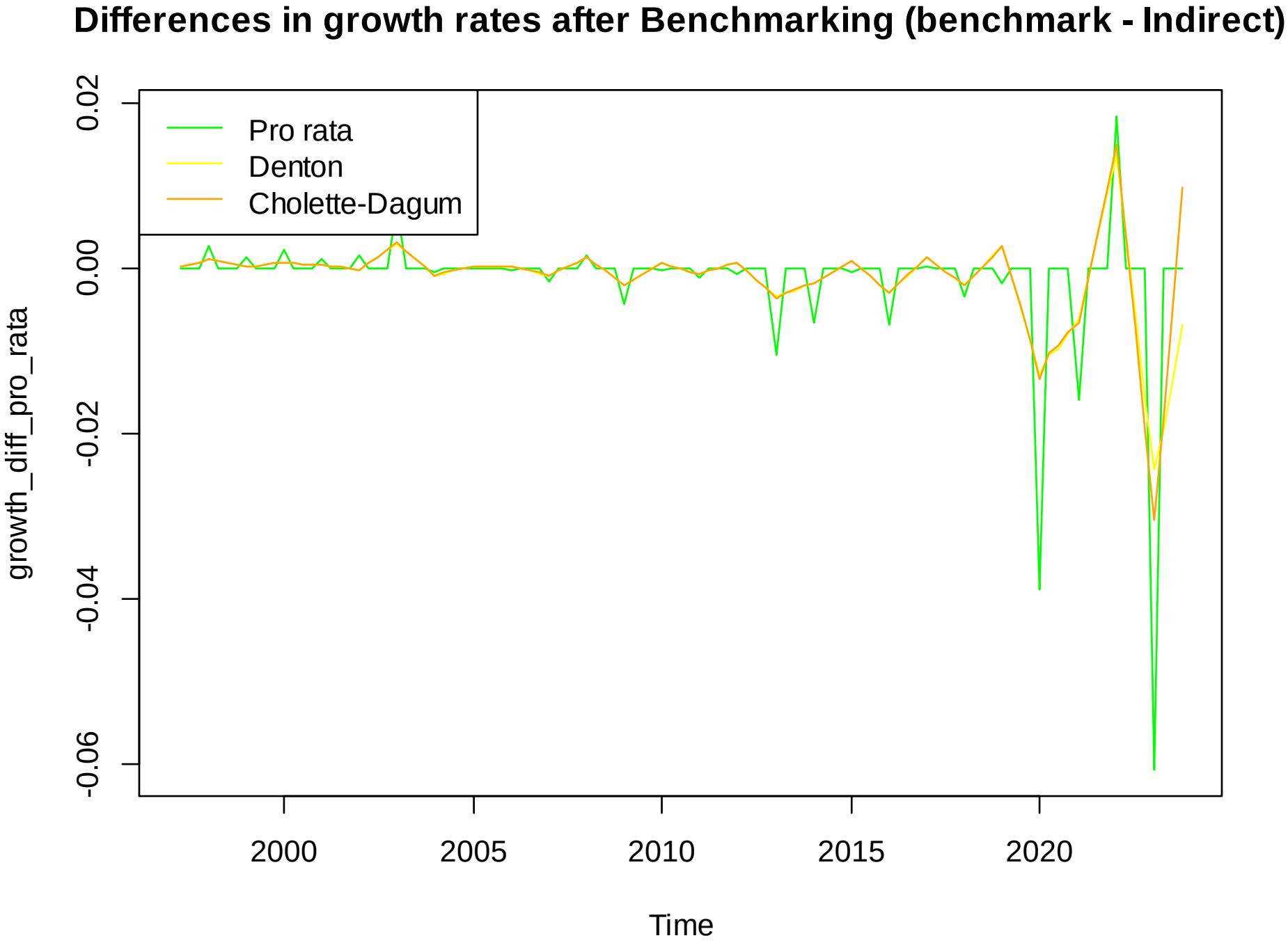
Option	Pros	Cons
Do nothing.	No change.	Unacceptable divergence between SA and NSA.

Solution Options

Option	Pros	Cons
Do nothing.	No change.	Unacceptable divergence between SA and NSA.
Benchmarking.		


Indirect SA before and after Benchmarking
Pre-benchmarking
Pro rata
Denton
Cholette-Dagum
NSA
indicator
100
150
200
250
2000
2005
2010
2015
2020
Time


Growth rate before and after Benchmarking
Pre-benchmarking
Pro rata
Denotn
Cholette-Dagum
0.10
0.05
0.00
-0.05
2000
2005
2010
2015
2020
Time


Differences in growth rates after Benchmarking (benchmark - Indirect)
Pro rata
Denton
Cholette-Dagum
growth_diff_pro_rata
0.02
0.00
-0.02
-0.04
-0.06
2000
2005
2010
2015
2020
Time

4.2 Dealing with large discrepancies

DESCRIPTION

In benchmarking the assumption is made that the differences between the indicator and the benchmarks are small. In this case, all methods based on movement preservation perform well and will yield very similar results. It is important, therefore, that this assumption is always checked.

It is not easy to define what constitutes a large discrepancy. A practical threshold value for most series will be somewhere between 5 and 10%. However, it really depends on the characteristics of the time series and how much effort enhancing the quality of a time series is worth.

Large discrepancies are a sign that something is wrong with either the indicator or the benchmark series and always warrant an investigation. If the discrepancies are systematic and large, you are in the realm of temporal disaggregation and an approach from the previous chapter should be used. If

Eurostat (2018) *ESS guidelines on temporal disaggregation, benchmarking and reconciliation*. Available at: https://ec.europa.eu/eurostat/web/products-manuals-and-guidelines/-/ks-06-18-355 (Accessed 31 July 2024).

Solution Options

Option	Pros	Cons
Do nothing.	No change.	Unacceptable divergence between SA and NSA.
Benchmarking.	Removes symptoms of divergence.	Treats the symptom, does not remove the cause. Increasingly large adjustments. Contrary to international best practice. The ‘story’ may become benchmarking.

Solution Options

Option	Pros	Cons
Do nothing.	No change.	Unacceptable divergence between SA and NSA.
Benchmarking.	Removes symptoms of divergence.	Treats the symptom, does not remove the cause. Increasingly large adjustments. Contrary to international best practice. The ‘story’ may become benchmarking.
Drop CTG method.	Cure divergence at source.	Substantial change to production and publication of business area.

Solution Options

Option	Pros	Cons
Do nothing.	No change.	Unacceptable divergence between SA and NSA.
Benchmarking.	Removes symptoms of divergence.	Treats the symptom, does not remove the cause. Increasingly large adjustments. Contrary to international best practice. The ‘story’ may become benchmarking.
Drop CTG method.	Cure divergence at source.	Substantial change to production and publication of business area.
Drop CTG and use CVM.	Cure divergence at source.	Substantial change to production and publication of business area.

Solution Options

Option	Pros	Cons
Do nothing.	No change.	Unacceptable divergence between SA and NSA.
Benchmarking.	Removes symptoms of divergence.	Treats the symptom, does not remove the cause. Increasingly large adjustments. Contrary to international best practice. The ‘story’ may become benchmarking.
Drop CTG method.	Cure divergence at source.	Substantial change to production and publication of business area.
Drop CTG and use CVM.	Cure divergence at source.	Substantial change to production and publication of business area.
Direct SA.	Avoids divergence in published SA series. Easily implemented.	Does not remove divergence in business area processing. Lack of additivity*. *Currently lacking additivity because using indices and not publishing weights, using CVM series, and non-additivity in original data due to different sources.