# Effects of Different Temporary Change Decay Rates in Monthly Retail Sales Time Series

Eric Valentine
Economic Statistical Methods Division
U.S. Census Bureau

# Disclaimer

Any opinions and conclusions expressed herein are those of the author and do not reflect the views of the U.S. Census Bureau. The Census Bureau has reviewed the data product for unauthorized disclosure of confidential information and has approved the disclosure avoidance practices applied. (Approval ID: CBDRB-FY23-001).

# Outline

Background on Retail Sales Data

Temporary Change (TC) Regressor

Phase 1 Research: Analysis of RegARIMA (Regression + Autoregressive Integrated Moving Average) Models at Different TC Decay Rates

Phase 2 Research: Analysis of RegARIMA Models at Different TC Decay Rates, with Multiple Regression Sets

Decay Rate as a Continuous Parameter

# Monthly Retail Trade Survey (MRTS)

Provides current estimates of sales and inventories at retail and food services stores, and inventories held by retail stores

Survey is authorized by Title 13, U.S. Code and provides for voluntary responses

Surveys about 13,000 retail businesses with paid employees, which is supplemented by estimates for nonemployers, new employers, and missed employers via benchmarking

Sample drawn from the Census Bureau's business register and is stratified by major kind of business and estimated sales

Data users from government, academic, and business communities (e.g. Bureau of Labor Statistics and Bureau of Economic Analysis)

# Monthly Retail Trade Survey (MRTS), contd

Investigation focuses on Monthly Retail and Food Services sales

Not adjusted and seasonally adjusted series

- 65 published not adjusted series
- 38 published seasonally adjusted series

Annual Review

- Team of reviewers
- Pandemic effects made reviews more difficult for 2021 and 2022
- Temporary Change (TC) Regressor

# Temporary Change Regressor in RegARIMA Models

Opportunities and growing interest in using temporary change regressors

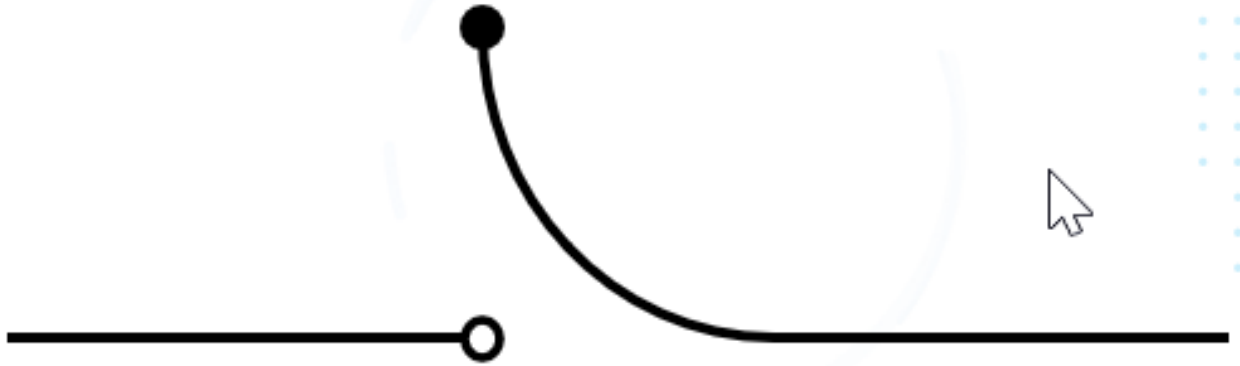
Is there a TC decay rate that is better suited for retail sales than the default 0.7?

Can we identify better TC decay rates?

# Temporary Change (TC) Regression Variable

TC at $t_0$

$$TC_t^{(t_0)} = \begin{cases} 0 & for\ \ t < t_0 \\ \alpha^{t-t_0} & for\ \ t \geq t_0 \end{cases}$$



where $\alpha$ is the rate of decay back to the previous level, $0 < \alpha < 1$ (default: 0.7 for monthly and 0.343 for quarterly series)

# Phase 1 Research: Analysis of RegARIMA Models at Different TC Decay Rates

65 retail sales time series modeled in X-13ARIMA-SEATS (X-13)

- January 2002 to June 2021
- Automatic model: ARIMA, outliers, trading day, Easter
- Allowed for mixed models, max orders of 2 for the nonseasonal ARIMA, and max order of 1 for the seasonal ARIMA
- Decay rates: 0.2, 0.5, 0.7, 0.9

Two problems

- Incomplete data files
- Regression matrix singularity
  - Caused by pandemic outliers in April, May, and June of 2020
  - This occurred under all of the decay rates, except for 0.5

# Phase 1 Research: Analysis of RegARIMA Models at Different TC Decay Rates, contd

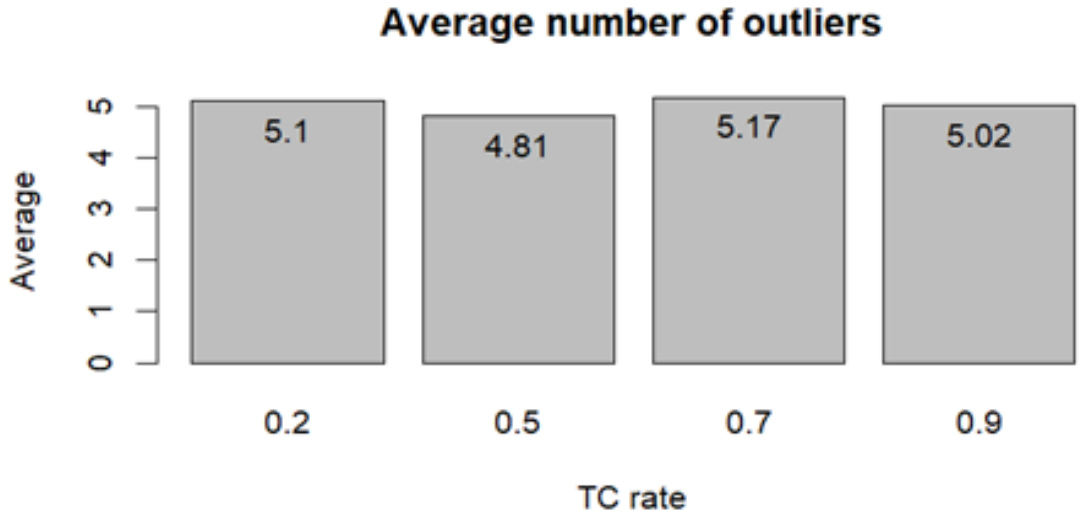
Removed the 13 time series that caused major modeling problems from the analysis

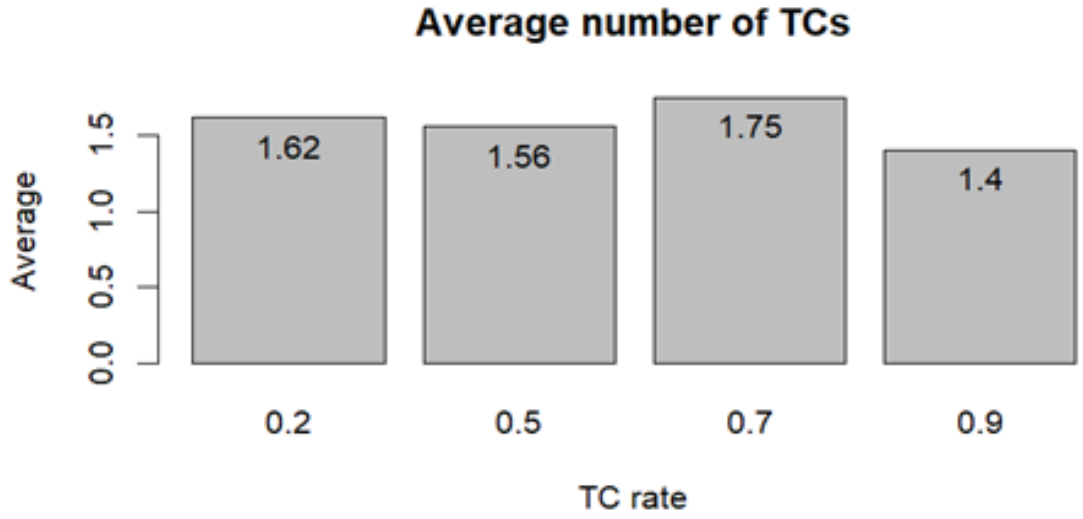
52 series remained for the analysis

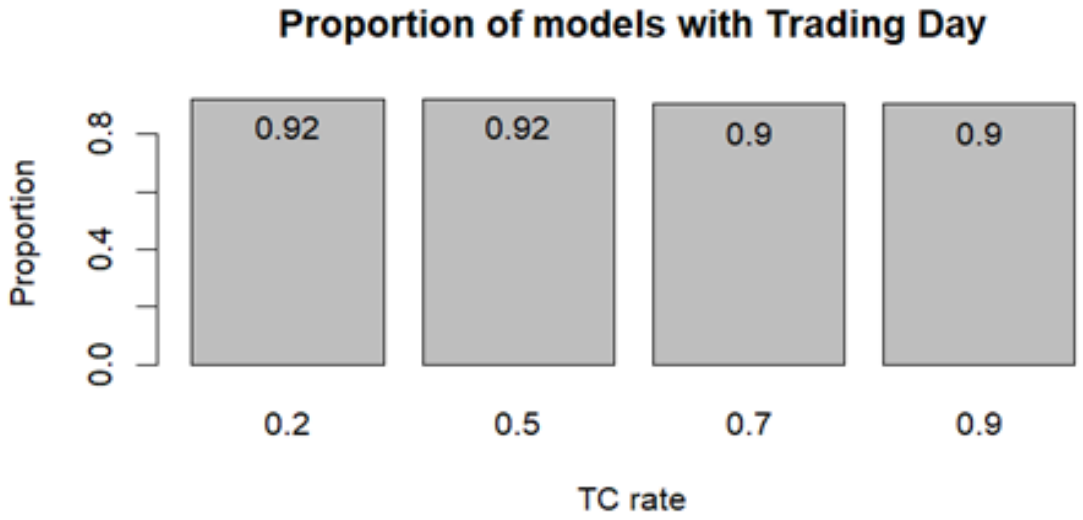
Evaluated the impacts of each TC decay rate on the regARIMA models by comparing some of the characteristics of the models at each decay rate

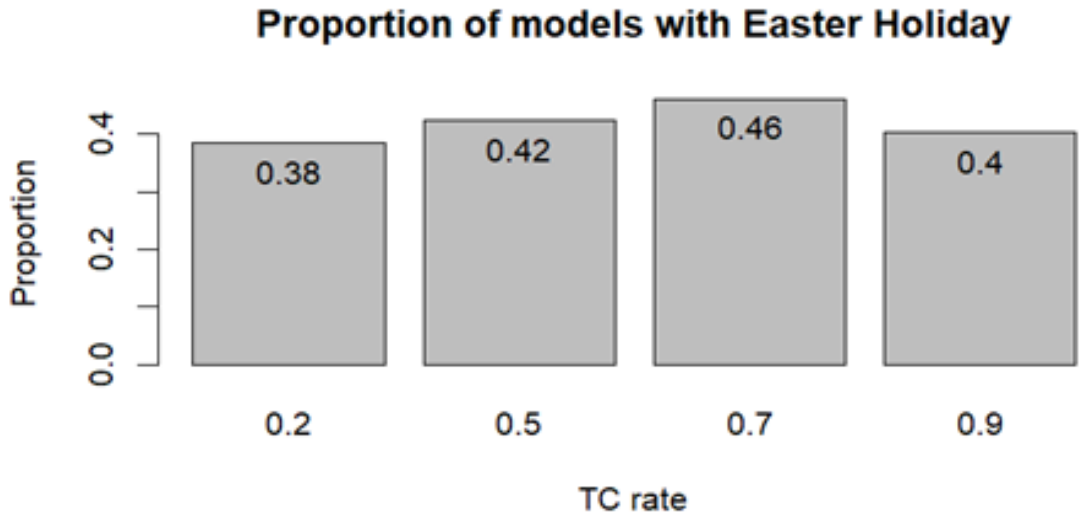
Compared proportions and averages of characteristics of the regARIMA models at each decay rate

- AR, MA, outliers, TCs, trading day, Easter
- Different regARIMA models


Average number of outliers
Average
0 1 2 3 4 5
5.1
4.81
5.17
5.02
0.2
0.5
0.7
0.9
TC rate


Average number of TCs
Average
0.0 0.5 1.0 1.5
1.62
1.56
1.75
1.4
0.2
0.5
0.7
0.9
TC rate


Proportion of models with Trading Day
Proportion
0.0 0.4 0.8
0.92
0.92
0.9
0.9
0.2
0.5
0.7
0.9
TC rate


Proportion of models with Easter Holiday
Proportion
0.0 0.2 0.4
0.38
0.42
0.46
0.4
0.2
0.5
0.7
0.9
TC rate

# Total Number of Significant Ljung Box Q-statistics (LBQs) and Box-Pierce Q-statistics (BPQs)

*Table 1. Total number of significant LBQs across all models at each decay rate and the percent of time series that had significant LBQs.* **Source:** Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). **Date Retrieved:** 9/13/2021

| Rate | Total LBQs | Percent of time series with at least one LBQ failure |
|---|---|---|
| 0.2 | 110 | 38.5% |
| 0.5 | 105 | 28.8% |
| 0.7 | 94 | 30.8% |
| 0.9 | 101 | 34.6% |

*Table 2. Total number of significant BPQs across all models at each decay rate and the percent of time series that had significant BPQs.* **Source:** Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). **Date Retrieved:** 9/13/2021

| Rate | Total BPQs | Percent of time series with at least one BPQ failure |
|---|---|---|
| 0.2 | 90 | 36.5% |
| 0.5 | 86 | 25.0% |
| 0.7 | 71 | 26.9% |
| 0.9 | 76 | 30.8% |

# Total Number of Significant Lags (1, 2, 3, 4, 6, 12) in the Sample Autocorrelation Functions (ACFs) and Partial Autocorrelation Functions (PACFs)

*Table 3. Total number of significant lags(1,2,3,4,6,12) in the sample ACFs at each decay rate and the percent of time series that had significant lags.* **Source:** Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). **Date Retrieved:** 9/13/2021

| Rate | Total significant lags | Percent of time series with at least one significant lag |
|---|---|---|
| 0.2 | 29 | 40.4% |
| 0.5 | 24 | 30.8% |
| 0.7 | 29 | 32.7% |
| 0.9 | 32 | 40.4% |

*Table 4. Total number of significant lags(1,2,3,4,6,12) in the sample PACFs at each decay rate and the percent of time series that had significant lags.* **Source:** Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). **Date Retrieved:** 9/13/2021

| Rate | Total significant lags | Percent of time series with at least one significant lag |
|---|---|---|
| 0.2 | 42 | 42.3% |
| 0.5 | 26 | 30.8% |
| 0.7 | 28 | 32.7% |
| 0.9 | 30 | 40.4% |

# Phase 2 Research: Analysis of RegARIMA Models at Different TC Decay Rates, with Multiple Regression Sets

ARMA coefficient cross correlations were not available for testing differences

Using a union of outliers was not a feasible approach

- Many different outliers
- Models were very different

Take the set of outliers that I got for a given TC decay rate (like the default of 0.7), fix those outliers, and then rerun estimation with the other values of the decay rate and with no further outlier detection

- Each set of outliers is called a “regression set”
- Used the ARIMA model that was automatically identified with the rate used
- Models with no TC regressors were not included in this analysis

Compared AICC, model innovation variance, within-sample forecast error, LBQs, BPQs, ACF lags, and PACF lags

# Percentage of Lowest Akaike's Information Criterion Corrected for Sample Size (AICC) by Rate

Table 5. Percentage of lowest AICCs by rate.

Source: Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). Date Retrieved: 9/13/2021

| Regression set | Top rate | Series that detected a TC | Percent |
|---|---|---|---|
| 0.2 regression set | 0.2 | 47 | 63.8% |
| 0.5 regression set | 0.5 | 43 | 86.0% |
| 0.7 regression set | 0.7 | 44 | 59.1% |
| 0.9 regression set | 0.9 | 42 | 54.8% |

# Percentage of Lowest Model Innovation Variance by rate

Table 6. Percentage of lowest model innovation variance by rate.

Source: Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). Date Retrieved: 9/13/2021

| Regression set | Top rate | Series that detected a TC | Percent |
|---|---|---|---|
| 0.2 regression set | 0.2 | 47 | 68.1% |
| 0.5 regression set | 0.5 | 43 | 83.7% |
| 0.7 regression set | 0.7 | 44 | 52.3% |
| 0.9 regression set | 0.9 | 42 | 50.0% |

# Counts of Lowest Average Within-Sample Forecast Error

Table 7. Counts of lowest average within-sample forecast error.

Source: Monthly Retail Trade, U.S. Census Bureau (census.gov/retail/). Date Retrieved: 9/13/2021

| Regression set | Rate=0.2 | Rate=0.5 | Rate=0.7 | Rate=0.9 |
|---|---|---|---|---|
| 0.2 regression set | 9 | 12 | 15 | 11 |
| 0.5 regression set | 9 | 12 | 9 | 13 |
| 0.7 regression set | 13 | 9 | 11 | 11 |
| 0.9 regression set | 13 | 10 | 5 | 14 |

# More Diagnostic Analysis

Average number of significant LBQs per series

- The fewest number of significant LBQs occurred when rate 0.7 was used with the 0.7 regression set

Average number of significant BPQs per series

- The fewest number of significant BPQs occurred when rate 0.7 was used with the 0.7 regression set

Average number of significant ACF lags (1,2,3,4,6,12) per series

- The fewest number of significant lags in the sample ACF per series occurred when rate 0.5 was used with the 0.5 regression set

Average number of significant PACF lags (1,2,3,4,6,12) per series

- The fewest number of significant lags in the sample PACF per series occurred when rate 0.7 was used with the 0.9 regression set

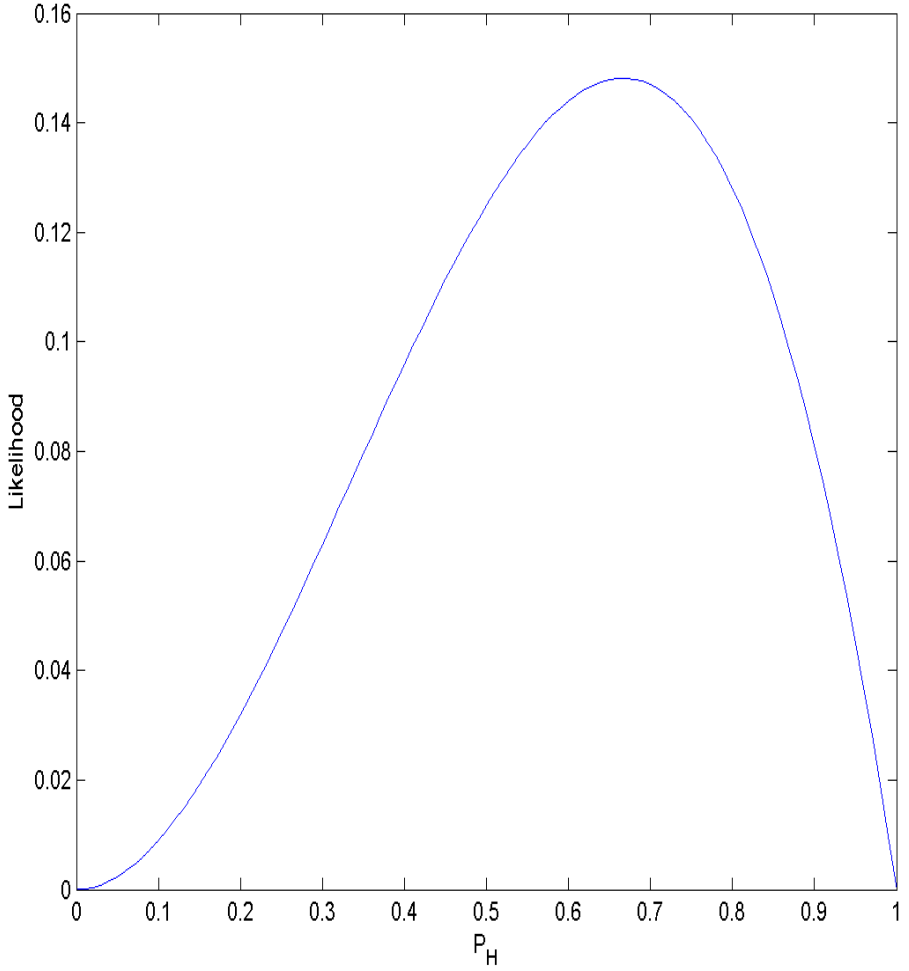
# Decay Rate as a Continuous Parameter

Statistical tests to compare the number of outliers detected under different TC decay rates

- Bill Bell offered the perspective of viewing the TC decay rate as a continuous parameter

Mimic this by using X-13 to estimate the models for

- Decay rates 0.1, 0.2, . . . , 0.9
- Pick the value that maximizes the likelihood for each set of regressors for each series
- 51 series
- Automated using the seasonal package in R

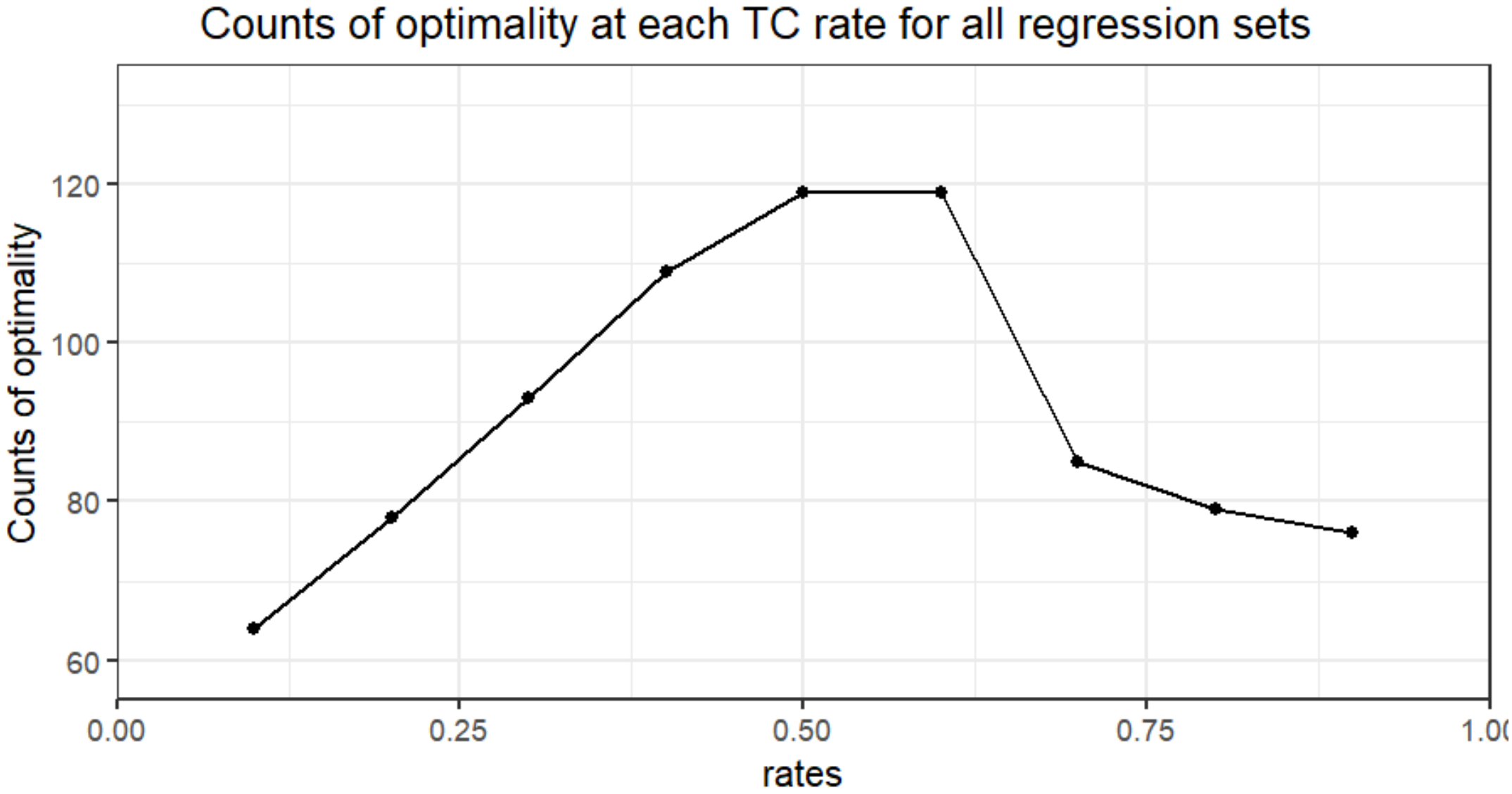



By Stephen.scott.webb - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=3791184

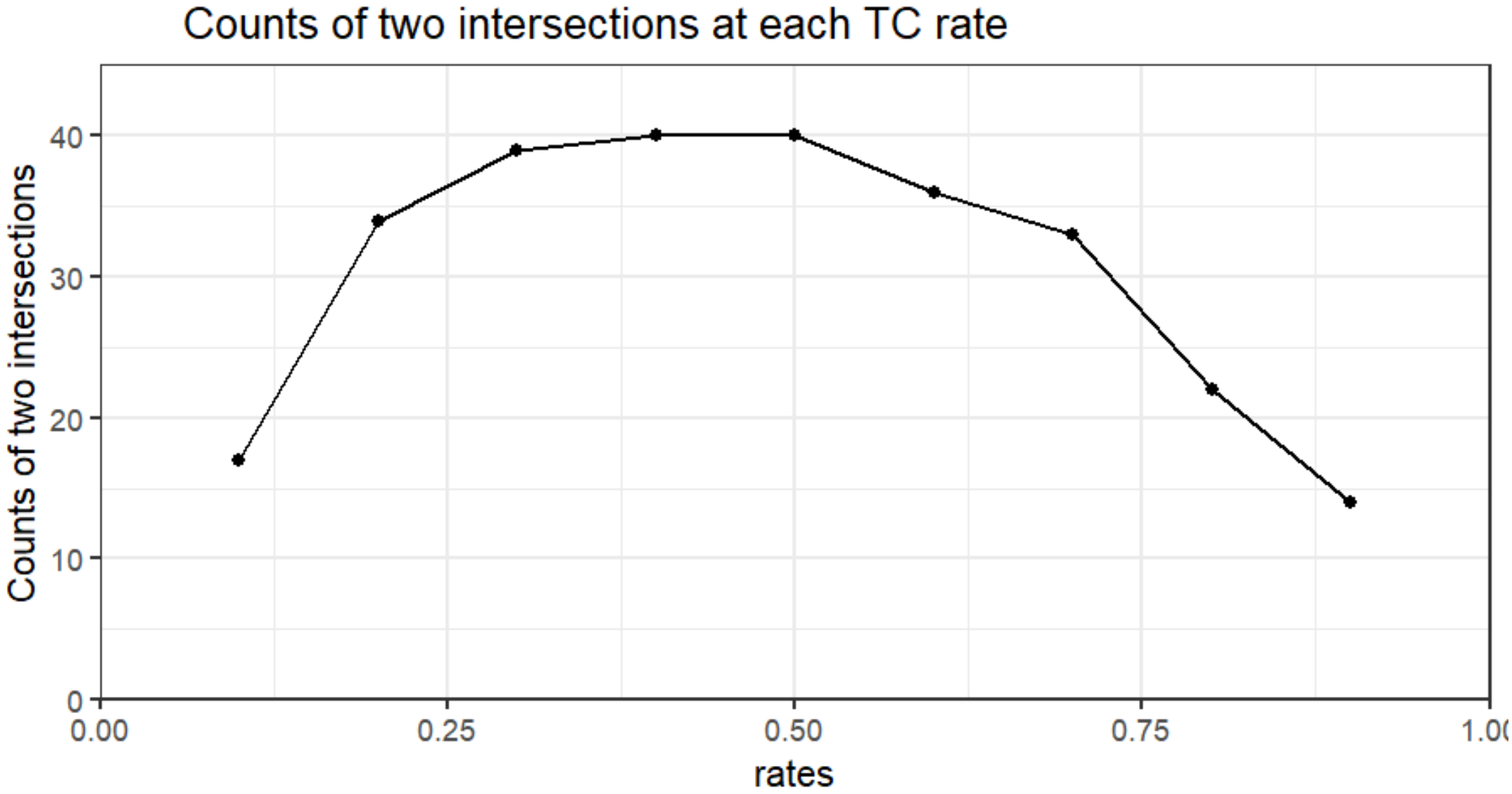
# Decay Rate as a Continuous Parameter

Confidence intervals for the decay rates

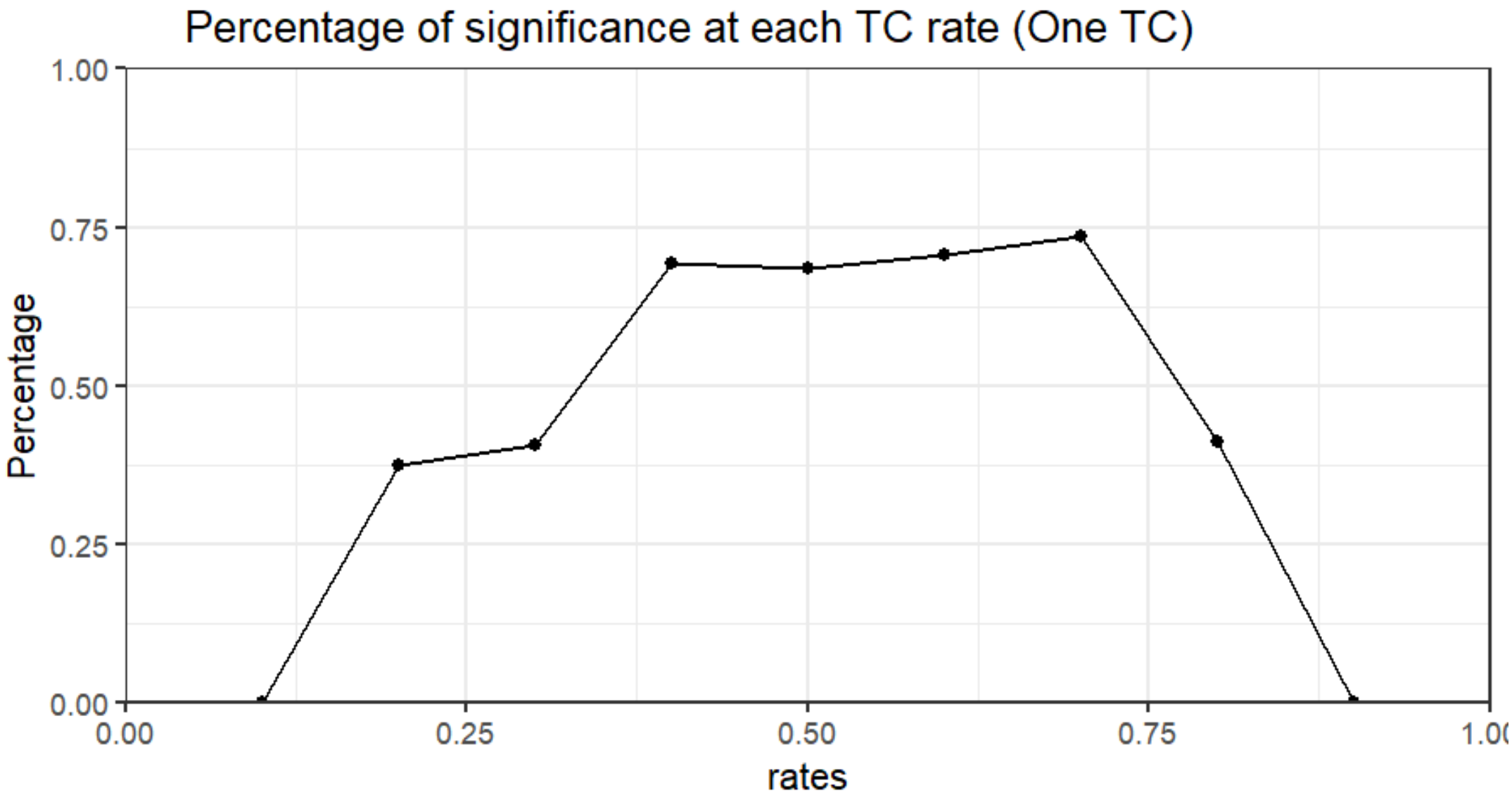
- Plot of the log-likelihood as a function of the decay rate
- 95% confidence intervals for each set of regressors for each series
  - Horizontal line that is 1.92 below the maximum log-likelihood value
  - 3.84 is the 5% critical value for the $\chi^2(1)$ distribution, which is the distribution for the likelihood ratio test statistic (which is -2 times the log of the likelihood ratio) for testing a single parameter
  - The intersection of the log-likelihood function and the horizontal line yields the lower and upper limits of the confidence interval for the decay rate
  - The values within the interval are the null hypothesis values that would not be rejected by the likelihood ratio test




United States Census Bureau




United States® Census Bureau




United States® Census Bureau

# Conclusions

Decay rate 0.5 may be very useful for U.S. retail sales time series

- Fewer outliers
- No instances of regression matrix singularity in the automatic models
- Leads to fewer significant lags in the sample ACF and PACF
- Lower AICCs
- Lower innovation variances
- Log-likelihood maximization

Findings only relate to U.S. retail sales time series

- Retail Sales data from other countries may yield different results
- Other surveys may not yield similar results
- Similar methods could be used for other surveys

# Future Work

Letting some or all TCs have their own decay rate (Bill Bell)

(E.g.) For series with two TCs, try optimizing the decay rate first over the most significant TC, then fixing that rate and optimizing the rate for the other TC.

Use more detailed values for the decay rates

# Acknowledgments

The author thanks Joe Barth, Laura James, Kathleen McDonald-Johnson, Evelyn Lewis, Laura Bechtel, and Kim Moore for careful readings of drafts of this paper. Also, special thanks to Bill Bell for suggestions about additional approaches for future work.

# Contact

Eric.Valentine@census.gov