

# Lemna

Laboratoire d'Économie et de  
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Institut national de la statistique  
et des études économiques

Mesurer pour comprendre

## Trend-cycle extraction and moving average manipulations in R with the rjdfilters package

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June 8th - June 9th

INSEE, LEMNA (French Statistician)

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1. Introduction

2. Moving averages

3. Asymmetric filters

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# Introduction

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*Moving average* are ubiquitous in trend-cycle extraction and seasonal adjustment (e.g. : X-13ARIMA) :

$$M_{\theta}(X_t) = \sum_{k=-p}^{+f} \theta_k X_{t+k}$$

# Introduction

---

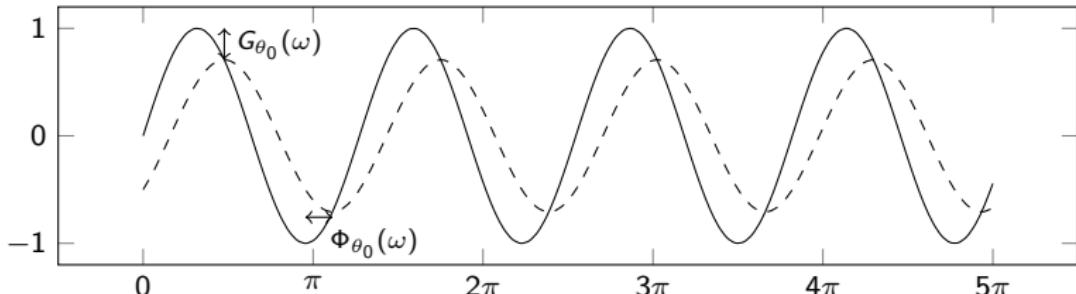
**Moving average** are ubiquitous in trend-cycle extraction and seasonal adjustment (e.g. : X-13ARIMA) :

$$M_\theta(X_t) = \sum_{k=-p}^{+f} \theta_k X_{t+k}$$

Applying  $M_\theta$  to  $X_t = e^{-i\omega t}$  will have two effects:

$$M_\theta X_t = \sum_{k=-p}^{+f} \theta_k e^{-i\omega(t+k)} = \left( \sum_{k=-p}^{+f} \theta_k e^{-i\omega k} \right) \cdot X_t = G_\theta(\omega) e^{-i\Phi_\theta(\omega)} X_t$$

1. Multiply the level by  $G_\theta(\omega)$  (*gain*)
2. phase-shift  $\Phi_\theta(\omega)/\omega$  which directly affects the detection of turning points



# Local polynomial filters

---

Hypothesis :  $y_t = \mu_t + \varepsilon_t$  with  $\varepsilon_t \stackrel{i.i.d}{\sim} \mathcal{N}(0, \sigma^2)$

$\mu_t$  locally approximated by a polynomial of degree  $d$ :

$$\forall j \in [-h, h] : y_{t+j} = m_{t+j} + \varepsilon_{t+j}, \quad m_{t+j} = \sum_{i=0}^d \beta_i j^i$$

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Estimation with WLS ( $K=\text{weights}=kernels$ ):  $\hat{\beta} = (X' K X)^{-1} X' K y$  and

$$\hat{m}_t = \hat{\beta}_0 = w'y = \sum_{j=-h}^h w_j y_{t-j} \rightarrow \text{equivalent to a symmetric moving average}$$

→ Arithmetic mean with  $K = 1$  and  $d = 0$  or 1.

→ Henderson filter with  $d = 2$  or 3 and specific kernel

# What already exists in ? (1)

In terms of smoothing, in  you can:

1. `stats::filter(., method= "recursive", sides = 2)`: symmetric MA ( $p$  even)

$$y_t = f_1 x_{t+\lceil(p-1)/2\rceil} + \cdots + f_p x_{t+\lceil(p-1)/2\rceil-(p-1)}$$

or `stats::filter(., method= "recursive", sides = 1)`:  
real-time asymmetric MA

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- ☞ possible to add 0 to create general asymmetric filters but endpoints won't be well treated.

# What already exists in R? (2)

---

Local polynomial models

4. KernSmooth::locpoly() local polynomial with gaussian kernel
5. locfit::locfit() local polynomial with tricube, rectangular, triweight, triangular, epanechnikov, bisquare, gaussian
6. stats::loess() tricube kernel

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Seasonal adjustment: seasonal, RJDemetra, x12: run X-13 but tricky to isolate one MA

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Seasonal adjustment: seasonal, RJDemetra, x12: run X-13 but tricky to isolate one MA

- ➔ No way to easily manipulate asymmetric moving averages, analyse their properties (gain, phase-shift)
- ➔ No way to create SA MA: Henderson, Musgrave, Macurves, etc.

## rjdfilters (1)

---

rjdfilters:  package based on the  libraries of JDemetra+ 3.0

Allows to:

- easily create/combine/apply moving averages `moving_average()`
- study the properties of the MA: plot coefficients (`plot_coef()`), gain (`plot_gain()`), phase-shift (`plot_phase()`) and different statics (`diagnostic_matrix()`)

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- trend-cycle extraction with different methods to treat endpoints:
  - `lp_filter()` local polynomial filters of Proietti and Luati (2008) (including Musgrave): Henderson, Uniform, biweight, Trapezoidal, Triweight, Tricube, "Gaussian", Triangular, Parabolic (= Epanechnikov)
  - `rkhs_filter()` Reproducing Kernel Hilbert Space (RKHS) of Dagum and Bianconcini (2008) with same kernels
  - `fst_filter()` FST approach of Grun-Rehomme, Guggemos, and Ladiray (2018)
  - `dfa_filter()` derivation of AST approach of Wildi and McElroy (2019)

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  - `dfa_filter()` derivation of AST approach of Wildi and McElroy (2019)
- change the filter used in X-11 for TC extraction

## rjdfilters (2)

---

Available at [palatej/rjdfilters](#)

Development version [AQLT/rjdfilters](#)

```
# rjdfilters depends on rjd3toolkit
remotes::install_github("palatej/rjd3toolkit")
remotes::install_github("AQLT/rjdfilters")
```

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# Create moving average moving\_average() (1)

(Recall:  $B^i X_t = X_{t-p}$  and  $F^i X_t = X_{t+p}$ )

```
library(rjdfilters)
m1 = moving_average(rep(1,3), lags = 1); m1 # Forward MA

## [1] "F + F^2 + F^3"
m2 = moving_average(rep(1,3), lags = -1); m2 # centered MA

## [1] "B + 1,0000 + F"
m1 + m2

## [1] "B + 1,0000 + 2,0000 F + F^2 + F^3"
m1 - m2

## [1] "- B - 1,0000 + F^2 + F^3"
m1 * m2

## [1] "1,0000 + 2,0000 F + 3,0000 F^2 + 2,0000 F^3 + F^4"
```

## Create moving average moving\_average() (2)

Can be used to create all the MA of X-11:

```
e1 <- moving_average(rep(1,12), lags = -6)
e1 <- e1/sum(e1)
e2 <- moving_average(rep(1/12, 12), lags = -5)
# used to have the 1rst estimate of the trend
tc_1 <- M2X12 <- (e1 + e2)/2
coef(M2X12) |> round(3)

##   t-6   t-5   t-4   t-3   t-2   t-1     t    t+1   t+2   t+3
## 0.042 0.083 0.083 0.083 0.083 0.083 0.083 0.083 0.083 0.083
##   t+4   t+5   t+6
## 0.083 0.083 0.042

si_1 <- 1 - tc_1
M3 <- moving_average(rep(1/3, 3), lags = -1)
M3X3 <- M3 * M3
# M3X3 moving average applied to each month
coef(M3X3) |> round(3)

##   t-2   t-1     t   t+1   t+2
## 0.111 0.222 0.333 0.222 0.111
```

## Create moving average moving\_average() (3)

```
M3X3_seasonal <- to_seasonal(M3X3, 12)
coef(M3X3_seasonal) |> round(3)
```

```
## t-24  t-23  t-22  t-21  t-20  t-19  t-18  t-17  t-16  t-15
## 0.111 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## t-14  t-13  t-12  t-11  t-10  t-9   t-8   t-7   t-6   t-5
## 0.000 0.000 0.222 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## t-4   t-3   t-2   t-1   t    t+1  t+2  t+3  t+4  t+5
## 0.000 0.000 0.000 0.000 0.333 0.000 0.000 0.000 0.000 0.000
## t+6   t+7   t+8   t+9   t+10  t+11  t+12  t+13  t+14  t+15
## 0.000 0.000 0.000 0.000 0.000 0.000 0.222 0.000 0.000 0.000
## t+16  t+17  t+18  t+19  t+20  t+21  t+22  t+23  t+24
## 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.111
```

# Create moving average moving\_average() (4)

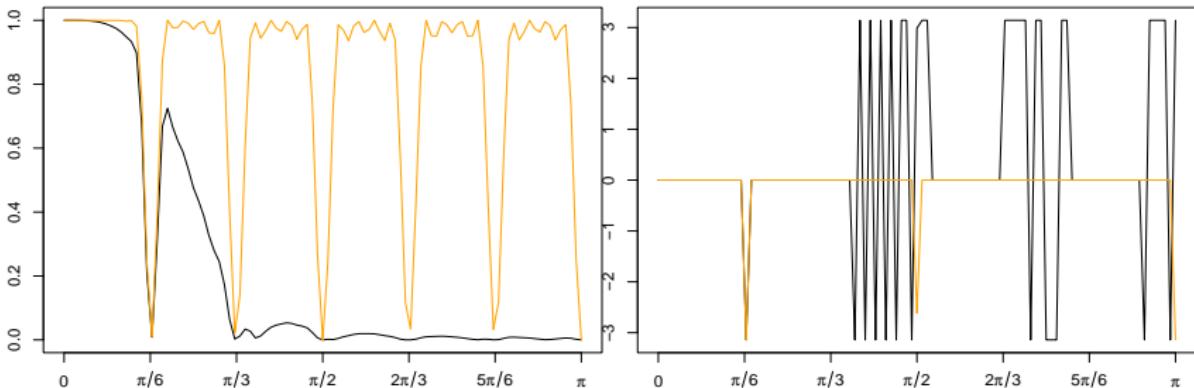
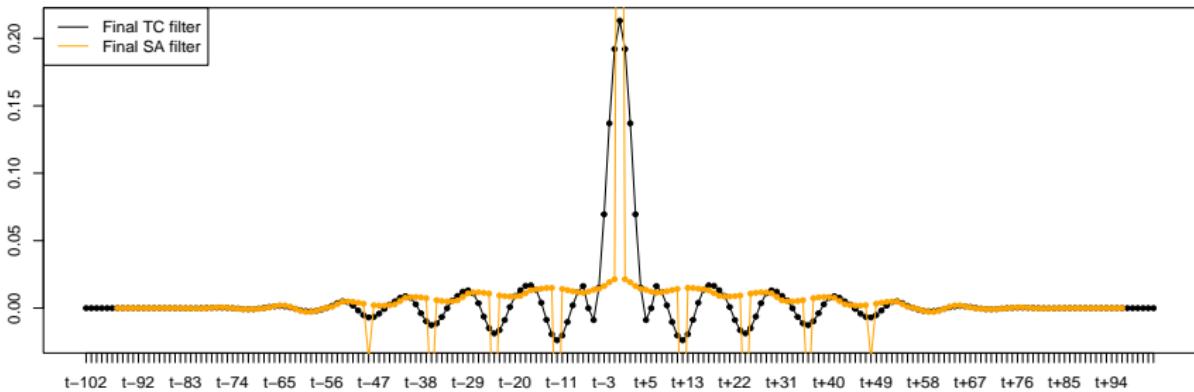
```
s_1 = M3X3_seasonal * si_1
s_1_norm = (1 - M2X12) * s_1
sa_1 <- 1 - s_1_norm
henderson_mm = moving_average(lp_filter(horizon = 6)$
                                filters.coef[, "q=6"],
                                lags = -6)
tc_2 <- henderson_mm * sa_1
si_2 <- 1 - tc_2
M5 <- moving_average(rep(1/5, 5), lags = -2)
M5X5_seasonal <- to_seasonal(M5 * M5, 12)
s_2 = M5X5_seasonal * si_2
s_2_norm = (1 - M2X12) * s_2
sa_2 <- 1 - s_2_norm
tc_f <- henderson_mm * sa_2
```

# Create moving average moving\_average() (5)

```
par(mai = c(0.3, 0.3, 0.2, 0))
layout(matrix(c(1,1,2,3), 2, 2, byrow = TRUE))

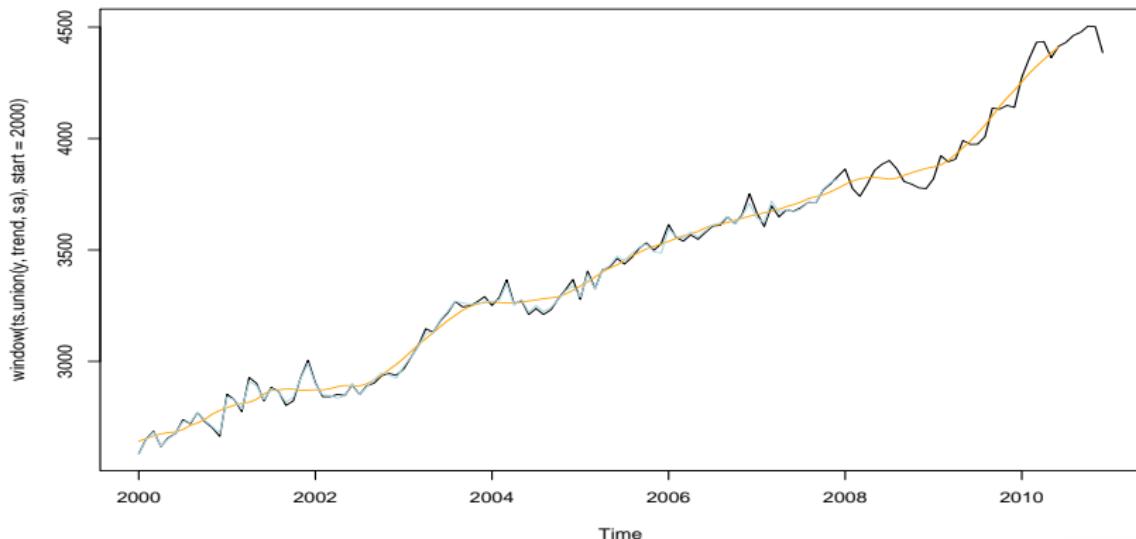
plot_coef(tc_f);plot_coef(sa_2, col = "orange", add = TRUE)
legend("topleft",
       legend = c("Final TC filter", "Final SA filter"),
       col= c("black", "orange"), lty = 1)
plot_gain(tc_f);plot_gain(sa_2, col = "orange", add = TRUE)
plot_phase(tc_f);plot_phase(sa_2, col = "orange", add = TRUE)
```

# Create moving average moving\_average() (6)



# Apply a moving average

```
y <- retailsa$AllOtherGenMerchandiseStores  
trend <- y * tc_1; sa <- y * sa_1  
plot(window(ts.union(y, trend, sa), start = 2000),  
     plot.type = "single",  
     col = c("black", "orange", "lightblue"))
```



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## 3. Asymmetric filters

### 3.1 Current X-13ARIMA

### 3.2 Local polynomials

### 3.3 Linear Filters and Reproducing Kernel Hilbert Space (RKHS)

### 3.4 Minimization under constraints: FST approach

### 3.5 One example: US retail sales (log)

## 4. Conclusion

# Trend-cycle extraction in X-13ARIMA

---

Idea closed to the following:

1. Series extended by an ARIMA model
2. Trend-cycle extraction with the Henderson filter:
  - a. Selection of bandwidth with the I-C ratio
  - b. Musgrave filter for the last points of the extended series

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  2. Trend-cycle extraction with the Henderson filter:
    - a. Selection of bandwidth with the I-C ratio
    - b. Musgrave filter for the last points of the extended series
- ⇒ Equivalent to the use of asymmetric MA with coefficients optimized to minimize the one-step-ahead forecast error
- ⇒ Different technics could be used

# Local polynomial filters

---

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Estimation with WLS ( $K=\text{weights}=kernels$ ):  $\hat{\beta} = (X' K X)^{-1} X' K y$  and

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- ⇒ Arithmetic mean with  $K = 1$  and  $p = 0/1$ .
- ⇒ Henderson filter with  $d = 3$  and specific kernel

## Asymmetric filters: rjdfilters::lp\_filter()

Several solutions:

1. Same method with less data (DAF)  $\iff$  Minimize revisions under same polynomial constraints (reproduce cubic trend)
-  **no bias but lots of variance**

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 **no bias but lots of variance**

2. Minimization of revisions filter under polynomial constraints:

2.1 *Linear-Constant* (LC):  $y_t$  linear and  $v$  reproduce constant trends (*Musgrave filter*)

2.2 *Quadratic-Linear* (QL):  $y_t$  quadratic and  $v$  reproduce linear trends

2.3 *Cubic-Quadratic* (CQ):  $y_t$  cubic and  $v$  reproduce quadratic trends

 Asymmetric filters  $v$  linked to “IC-Ratio” rjdfilters::ic\_ratio()

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 Asymmetric filters  $v$  linked to “IC-Ratio” rjdfilters::ic\_ratio()



simple models with easy interpretation



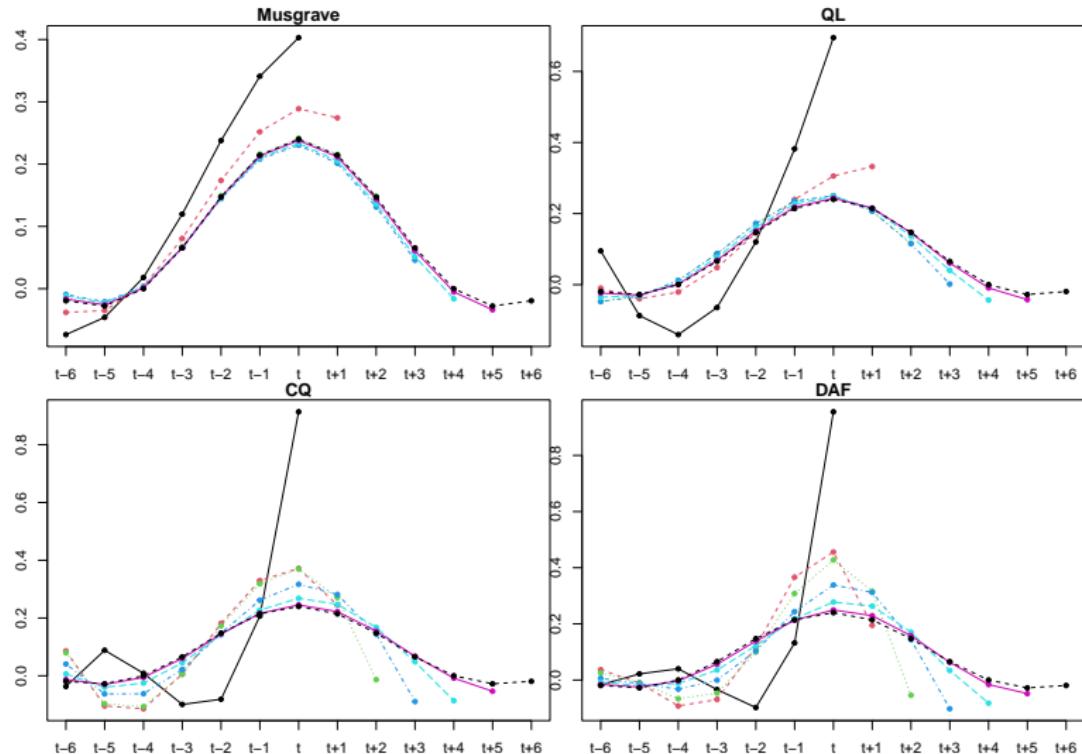
Timeliness not controlled  method extended in  
rjdfilters::lp\_filter()

## Example (1)

```
par(mai = c(0.3, 0.3, 0.2, 0))
layout(matrix(c(1,2,3,4), 2, byrow = TRUE))

lp_filter(endpoints = "LC") |>
  plot_coef(q = 0:6, main = "Musgrave", zeroAsNa = TRUE)
lp_filter(endpoints = "QL") |>
  plot_coef(q = 0:6, main = "QL", zeroAsNa = TRUE)
lp_filter(endpoints = "CQ") |>
  plot_coef(q = 0:6, main = "CQ", zeroAsNa = TRUE)
lp_filter(endpoints = "DAF") |>
  plot_coef(q = 0:6, main = "DAF", zeroAsNa = TRUE)
```

## Example (2)



## RKHS filters: rjdfilters::rkhs\_filter()

- RKHS theory used to approximate Henderson filter
- With  $K_p$  the **kernel function**, the symmetric filter:

$$\forall j \in [-h, h] : w_j = \frac{K_p(j/b)}{\sum_{i=-h}^h K_p(i/b)}$$

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$$\forall j \in [-h, q] : w_{a,j} = \frac{K_p(j/b)}{\sum_{i=-h}^q K_p(i/b)}$$

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- For asymmetric filters:
- $$\forall j \in [-h, q] : w_{a,j} = \frac{K_p(j/b)}{\sum_{i=-h}^q K_p(i/b)}$$
- ⇒  $b$  chosen by optimization, e.g. minimizing revisions linked to phase-shift:

$$b_{q,\varphi} = \min_{b_q} \int_0^{2\pi/12} \rho_s(\lambda) \rho_\theta(\lambda) \sin^2 \left( \frac{\varphi_\theta(\omega)}{2} \right) d\omega$$

# Asymmetric filters

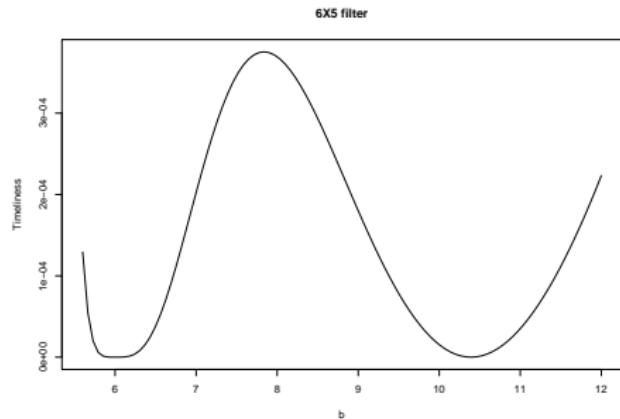


several local extremum

```
fun <- rkhs_optimization_fun(horizon = 6,
                             leads = 5, degree = 3,
                             asymmetricCriterion = "Timeliness")
plot(fun, 5.6, 12, xlab = "b",
     ylab = "Timeliness", main = "6X5 filter")
```



Generalizable to  
create filters that could  
be applied to irregular  
frequency series



```
rkhs_optimal_bw()
```

```
##      q=0      q=1      q=2      q=3      q=4      q=5
## 6.0000 6.0000 6.3875 8.1500 9.3500 6.0000
```

## FST approach: rjdfilters::fst\_filter()

Minimization of a weighted sum of 3 criteria under polynomial constraints:

$$\begin{cases} \min_{\theta} & J(\theta) = \alpha F_g(\theta) + \beta S_g(\theta) + \gamma T_g(\theta) \\ \text{s.c.} & C\theta = a \end{cases}$$

$$\begin{cases} F_g(\theta) = \sum_{k=-p}^{+f} \theta_k^2 & \text{Fidelity (variance reduction ratio, Bongard)} \\ S_g(\theta) = \sum_j (\nabla^d \theta_j)^2 & d = 3 \text{ Smoothness (Henderson criterion)} \\ T_g(\theta) = \int_0^{\omega_2} \rho_\theta(\omega)^2 \sin(\varphi_\theta(\omega))^2 d\omega & \text{Timeliness (phase-shift)} \end{cases}$$

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-  Unique solution
-  Asymmetric filters independent of data and symmetric filter
-  Non-normalized weights

# How to apply a filter (1)

rjdfilters::jfilter() to apply a filter with asymmetric MA

```
y <- retailsa$AllOtherGenMerchandiseStores
sc <- henderson(y, length = 13, musgrave = FALSE)
icr <- ic_ratio(y, sc)
icr

## [1] 2.414569

daf <- lp_filter(horizon = 6, ic = icr, endpoints = "DAF")$filters.coef
round(daf, 3)

##          q=0      q=1      q=2      q=3      q=4      q=5      q=6
## t-6 -0.017  0.037  0.025  0.006 -0.008 -0.016 -0.019
## t-5  0.022 -0.011 -0.009 -0.013 -0.020 -0.025 -0.028
## t-4  0.040 -0.092 -0.066 -0.032 -0.012 -0.003  0.000
## t-3 -0.034 -0.069 -0.047  0.000  0.036  0.056  0.065
## t-2 -0.098  0.118  0.100  0.104  0.123  0.139  0.147
## t-1  0.132  0.366  0.308  0.244  0.218  0.213  0.214
## t    0.955  0.456  0.428  0.339  0.278  0.249  0.240
## t+1  0.000  0.195  0.316  0.312  0.263  0.229  0.214
## t+2  0.000  0.000 -0.054  0.144  0.170  0.158  0.147
```

## How to apply a filter (2)

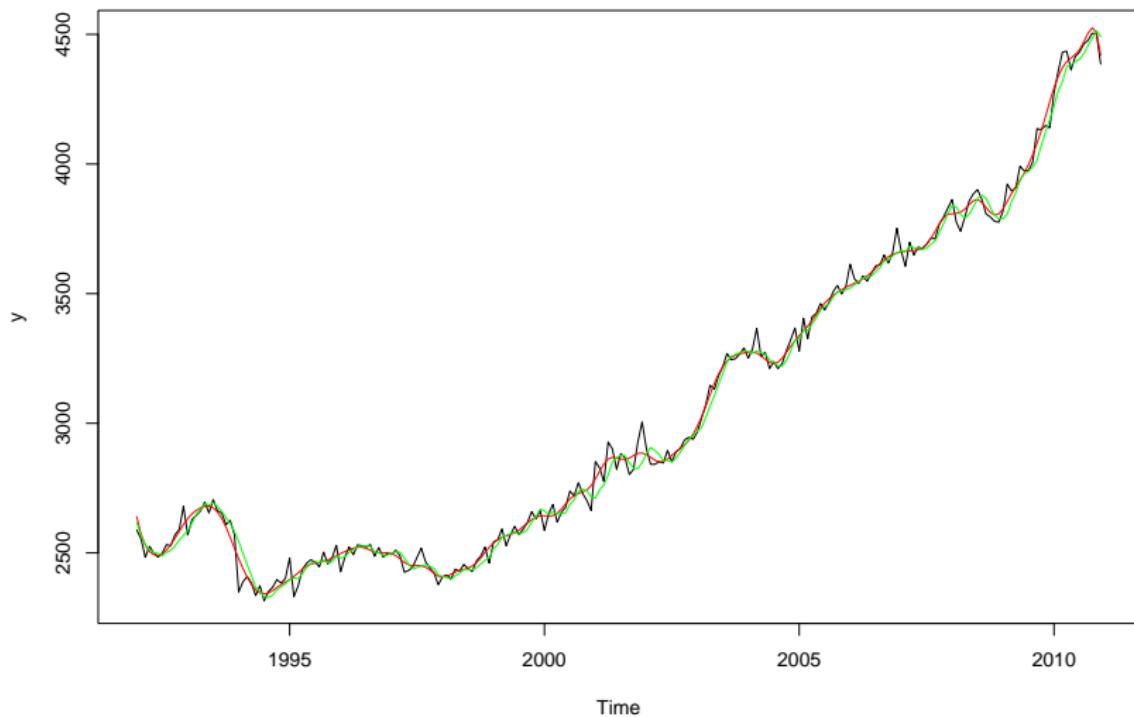
```
## t+3  0.000  0.000  0.000 -0.102  0.034  0.063  0.065
## t+4  0.000  0.000  0.000  0.000 -0.082 -0.016  0.000
## t+5  0.000  0.000  0.000  0.000  0.000 -0.048 -0.028
## t+6  0.000  0.000  0.000  0.000  0.000  0.000 -0.019
trend <- jfilter(y, daf)
```

rjdfilters::x11() to change the filters in X-11

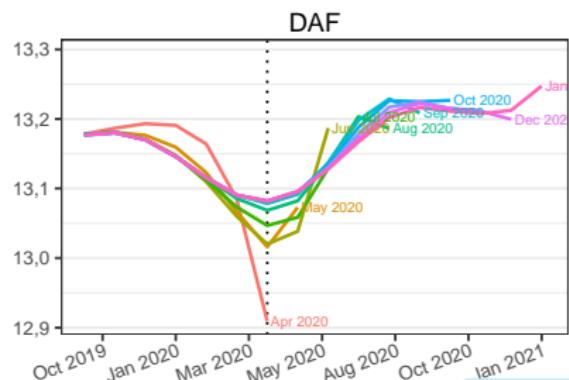
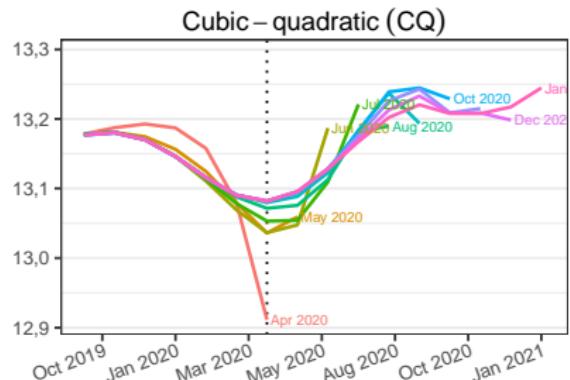
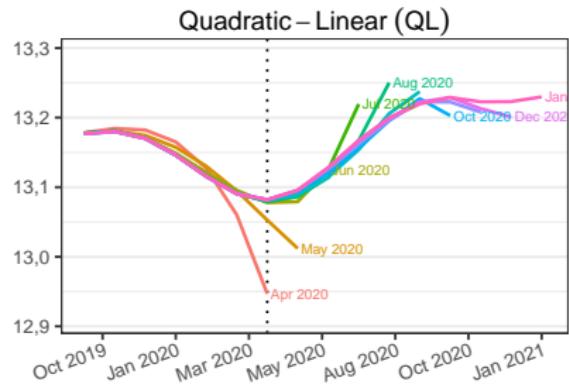
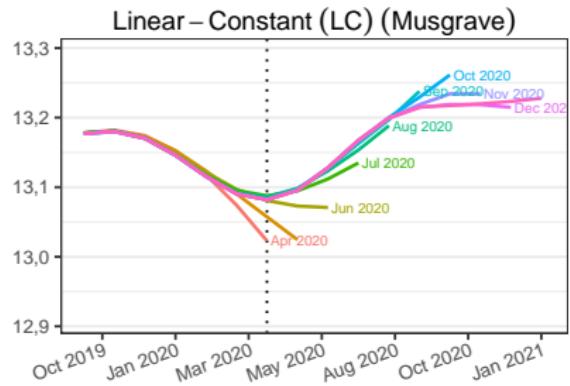
```
decomp_daf = x11(y, trend.coefs = daf)
daf_modif <- daf
# We change the final filter to a asymmetric filter
daf_modif[, "q=6"] <- lp_filter(endpoints = "LC")$filters.coef[, "q=0"]
decomp_daf_modif = x11(y, trend.coefs = daf_modif)

plot(y)
lines(decomp_daf$decomposition[, "t"], col = "red")
lines(decomp_daf_modif$decomposition[, "t"], col = "green")
```

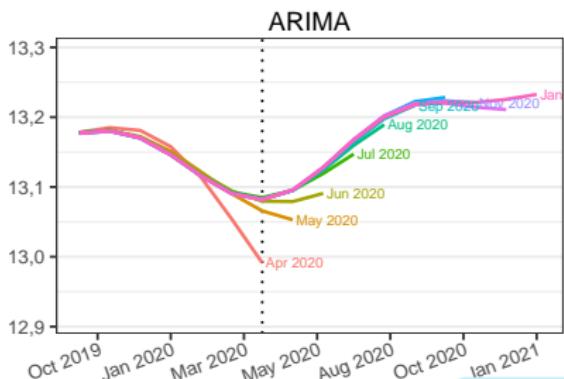
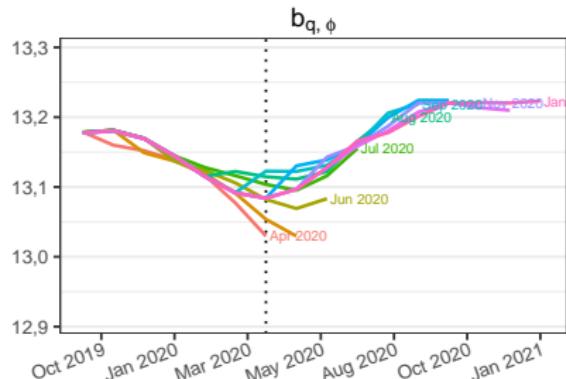
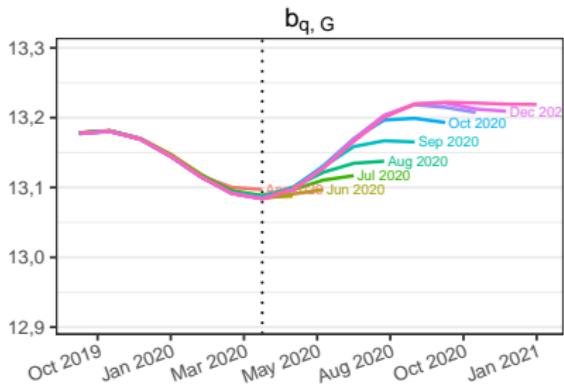
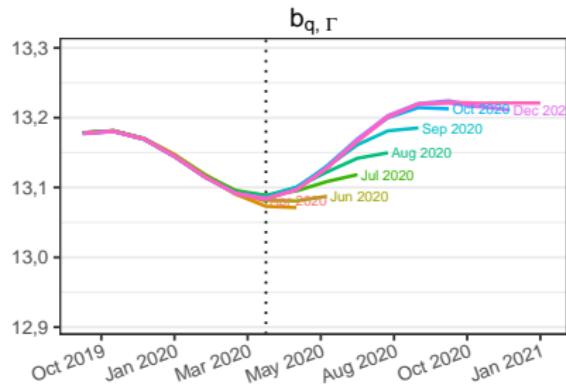
## How to apply a filter (3)



# Successive trend-cycle estimates (1)



## Successive trend-cycle estimates (2)



## Implicit forecast: `rjdfilters::implicit_forecast()`

$w^q$  used when  $q$  future values are known and  $\forall i > q, w_i^q = 0$ :

$$\forall q, \underbrace{\sum_{i=-h}^0 v_i y_i + \sum_{i=1}^h v_i y_i^*}_{\text{smoothing by } v \text{ of the extended data}} = \underbrace{\sum_{i=-h}^0 w_i^q y_i + \sum_{i=1}^h w_i^q y_i^*}_{\text{smoothing by } w^q \text{ of the extended data}}$$

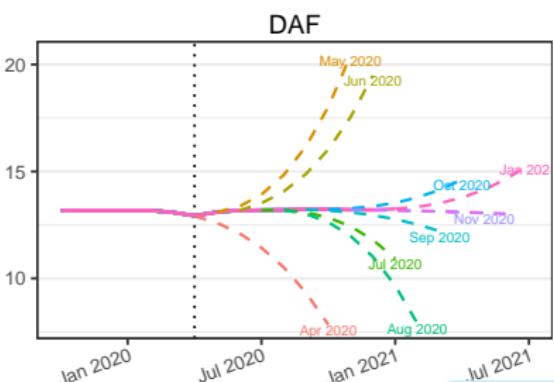
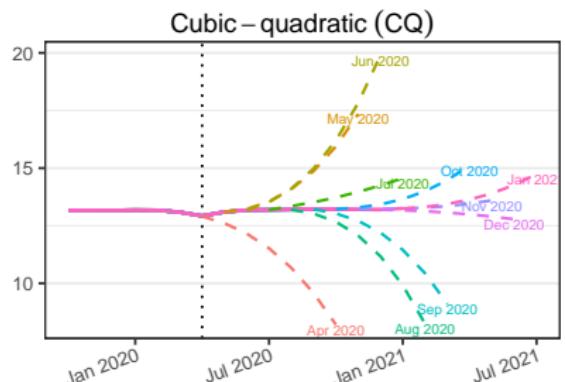
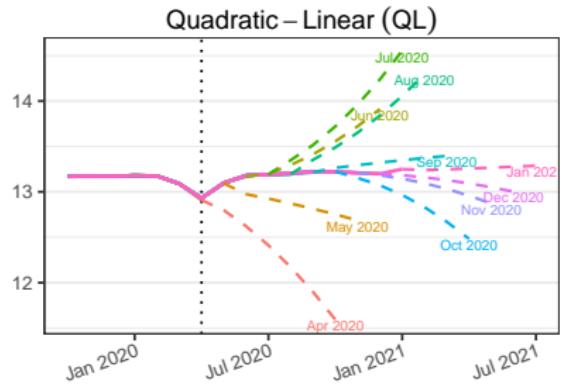
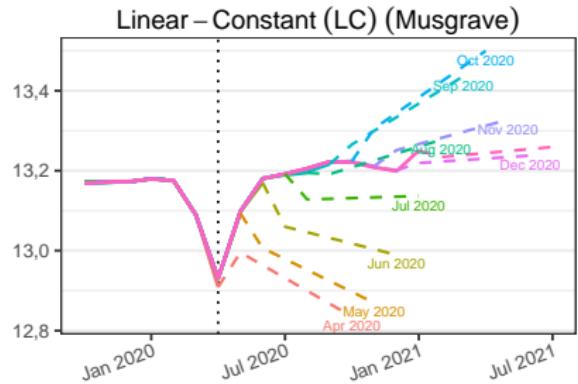
Which is equivalent to:

$$\forall q, \sum_{i=1}^h (v_i - w_i^q) y_i^* = \sum_{i=-h}^0 (w_i^q - v_i) y_i.$$

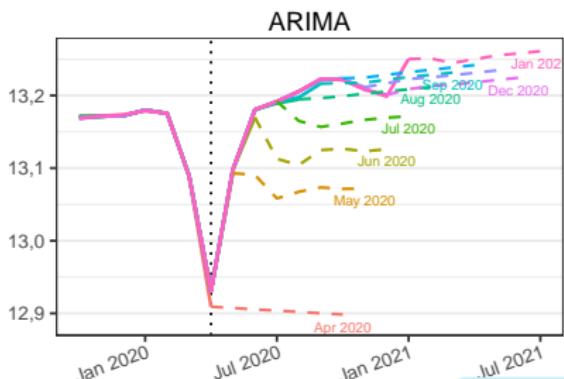
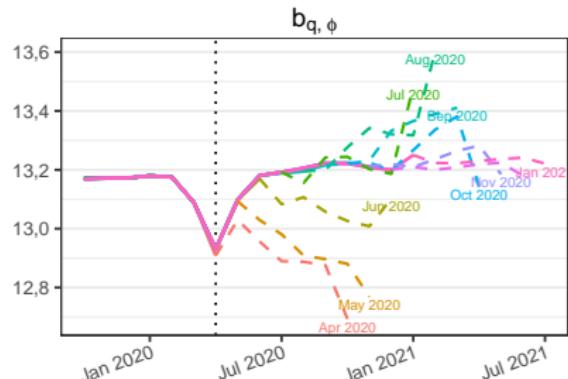
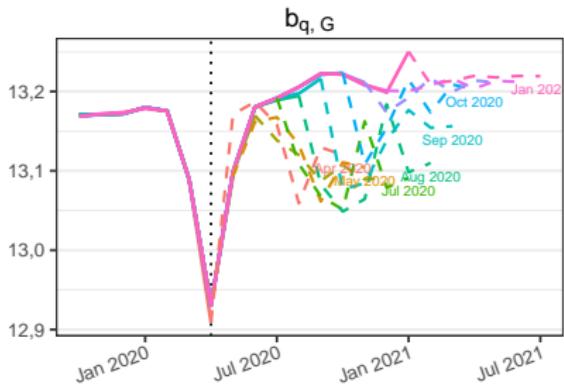
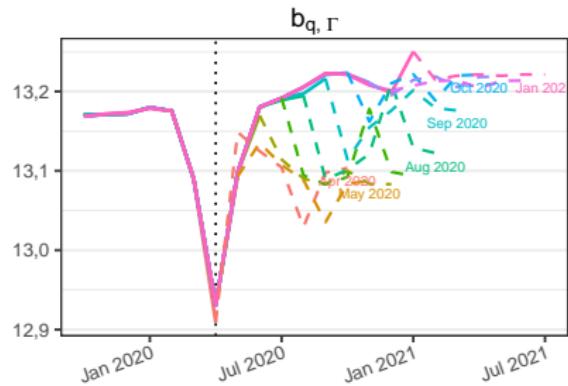
In matrix:

$$\begin{pmatrix} v_1 & v_2 & \cdots & v_h \\ v_1 - w_1^1 & v_2 & \cdots & v_h \\ \vdots & \vdots & \ddots & \vdots \\ v_1 - w_1^{h-1} & v_2 - w_2^{h-1} & \cdots & v_h \end{pmatrix} \begin{pmatrix} y_1^* \\ \vdots \\ y_h^* \end{pmatrix} = \begin{pmatrix} w_{-h}^0 - v_{-h} & w_{-(h-1)}^0 - v_{-(h-1)} & \cdots & w_0^0 - v_0 \\ w_{-h}^1 - v_{-h} & w_{-(h-1)}^1 - v_{-(h-1)} & \cdots & w_0^1 - v_0 \\ \vdots & \vdots & \ddots & \vdots \\ w_{-h}^{h-1} - v_{-h} & w_{-(h-1)}^{h-1} - v_{-(h-1)} & \cdots & w_0^{h-1} - v_0 \end{pmatrix} \begin{pmatrix} y_{-h} \\ \vdots \\ y_0 \end{pmatrix} \quad (1)$$

# Implicit forecasts (1)



## Implicit forecasts (2)



# Contents

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# Conclusion

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With `rjdfilters` you can already:

- create and manipulate moving averages
- used different technics for real-time trend-cycle estimates
- custom X-11 filters

# Conclusion

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With `rjdfilters` you can already:

- create and manipulate moving averages
- used different technics for real-time trend-cycle estimates
- custom X-11 filters

What might be implemented in the future:

- Class on finiteFilters (central filter with asymmetric filters) to easily combined them
- Bandwidth selection methods: IC ratios, CV, AIC, etc.
- More ideas?

Thank you for your attention

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Package :

 [palatej/rjdfilters](#)

Development version  [AQLT/rjdfilters](#)