The Effects of Seasonal Heteroskedasticity on Trend Estimation and Seasonal Adjustment

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Disclaimer: The views expressed in this paper are those of the author and not necessarily those of the Census Bureau.

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Seasonal heteroskedasticity

- Defined by regular changes in variability over calendar year

- Contrasts with the usual seasonality - in levels

- Previous work: Bell (2004) and Proietti (2004) introduce two different forms of model

  Bell (2004): Airline plus Seasonal Noise

  Proietti (2004): Season-Specific

- Tripodis and Penzer (2007) analyze case of one season having different variance

- Trimbur and Bell (2012) introduce test based on three different Time Series models

Relevance of Seasonal Heteroskedasticity

- **Numerous Applications to Economic Time Series** (Construction activity, Industrial Production, Retail sales)

- Strong Effects for **Monthly Construction Indicators**

  Source: unpredictable impact of severe weather during the winter

  Construction sector important share of economic activity

  Sometimes used as leading indicator for total output

- Also relevant for many Production-Related Economic Time Series

  Example 1: effect of "model change-over" and maintenance shutdown in July/August on **Motor Vehicles Production**

  Example 2: effect of winter temperatures on **Utilities Output** (uncertainty in heating demand)

Trimbur and Bell (2012): Test for Seasonal Heteroskedasticity

- **Tests for Presence of Seasonal Heteroskedasticity**, developed using three models (tables of critical values provided in article)

- Form of Heteroskedasticity: **high-low variance classification**

  multiple seasons allowed in each group (more general than single-season)

  parsimonious approximation for dominant source

  easy to interpret: baseline variation and **extra variation during certain months** (e.g. in winter from unpredictable weather effects on construction)

- **Algorithm** to determine **high-low variance groups**

- **Likelihood ratio test statistic**, asymptotic and **finite sample**

  distribution related to mixture of chi-squared

Trimbur and Bell (2012): Empirical findings and further work

- Application to **Monthly Construction indicators** [U.S. housing starts and building permits in four Census regions (NE, MW, S, W)]

  "Month" = "Season" in what follows

  Intuitively, **source of seasonal heteroskedasticity: winter-related** excess variability

  Algorithm confirmed winter grouping for high-variance months for NE, **MW**, S

  Test results: very strong seasonal heteroskedasticity in NE, **MW**

  Model comparisons: seasonally heteroskedastic models strongly preferred in NE, MW

- Given the clear presence of seasonal heteroskedasticity:

  How much does it matter – for estimating trends and for adjusting for seasonality?

Seasonal heteroskedasticity: Trends and Seasonal Adjustment - Aims

- Examine **impact of using Time Series Models for Seasonal Heteroskedasticity**

- **Trends**

  response to movements related to Seasonal Heteroskedasticity

  appropriately weight observations around higher variability seasons

- Seasonality

  distinguish seasonal levels vs. seasonal variability

  contour of Seasonality over time: stability, reaction to seasonal variability

  Expanded **Adjustment for Seasonal Heteroskedasticity**

Plan

- Consider **different Time Series Models**

- **Generalize to Heteroskedastic** Form

- Examine Signal Extraction of **Trends and Seasonally Adjusted Series**

- Make three types of comparisons:

  Across **homoskedastic** models

  Across **heteroskedastic** models

  Homoskedastic vs. Heteroskedastic models

Homoskedastic models

- seasonal, nonstationary time series

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \qquad \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2), \qquad t = 1, ..., T$$

- μ_t stochastic trend (long-term, low-frequency)

- γ_t stochastic seasonal (repetitive and predictable over calendar year, gradual and modest variation over long periods)

- ε_t stationary irregular (absorbs remainder, measurement error or special "one-off" influences)

Approaches:

1) express each of $\{\mu_t, \gamma_t, \varepsilon_t\}$ explicitly, take sum

2) express model for y_t, define μ_t and γ_t in terms of this model

Structural time series model

Specify as

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \qquad \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2), \qquad t = 1, ..., T$$

with μ_t a Local Linear Trend:

$$\mu_{t+1} = \mu_t + \beta_t + \eta_t, \qquad \eta_t \sim WN(0, \sigma_\eta^2)$$

$$\beta_{t+1} = \beta_t + \zeta_t, \quad \zeta_t \sim WN(0, \sigma_\zeta^2)$$

γ_t Trig-1 Seasonal (decompostion into stochastic cycles at the seasonal frequencies)

ε_t homoskedastic irregular

SARIMA model

- Let a seasonal nonstationary Y_t follow the "airline model" (Box-Jenkins, 1976) :

$$(1-L)(1-L^{12})Y_t = (1-\theta L)(1-\theta_s L^{12})a_t, \quad a_t \sim WN(0, \sigma_a^2),$$

- standard homoskedastic model for observed process

- Trend and Seasonal defined by **Canonical Decomposition** of Hillmer-Tiao (1982), which effectively **removes maximum degree of irregular** movements:

  $\mu_t^{can} = F_\mu^{can}(L)Y_t$

  $\gamma_t^{can} = F_\gamma^{can}(L)Y_t$

  This definition gives certain SARIMA processes $\{\mu_t^{can}, \gamma_t^{can}, \varepsilon_t^{can}\}$ with

  $\varepsilon_t = \varepsilon_t^{can} \sim WN(0, \sigma^2_{\varepsilon,can})$

  such that $\sigma^2_{\varepsilon,can}$ is maximized. Tends to produce **smooth component series** when implemented.

Definition of Seasonal Noise

- A **Seasonal Noise process** $\varepsilon_{s,t}$ is defined by

$$E[\varepsilon_{s,t}] = 0, \quad E[\varepsilon_{s,t}^2] = \sigma_{\varepsilon,s}^2(j(t)), \qquad t = 1, ..., T$$

  where $j(t)$ is the season at time t:

  $j(t) = 1 + (t-1) \operatorname{mod} S$

so $j \in \{1, 2, ..., S\}$, where S is # seasons per year, e.g. $S = 12$ for monthly data.

For identification: impose $\underset{j}{min}\{\sigma_{\varepsilon,s}^2(j)\} = 0$

- So $\varepsilon_{s,t}$ represents additional variability during certain seasons

  With two variance groups, $\sigma_{\varepsilon,s}^2(j) = 0$ in low-variance season, and $\sigma_{\varepsilon,s}^2(j) = \sigma_{\varepsilon,s}^2$ in high-variance season

Seasonal Noise

- For any homoskedastic model, generalize the irregular component

$$\varepsilon_t \Rightarrow \varepsilon_{n,t} + \varepsilon_{s,t}, \quad \varepsilon_{n,t} \sim WN(0, \sigma^2_{\varepsilon,n})$$

- $\varepsilon_{n,t}$ **homoskedastic (**"nonsystematic" or "non-seasonal" noise, pure randomness, no patterns in variance)

- $\varepsilon_{s,t}$ **heteroskedastic (**excess variability during certain months of the year)

- **Seasonal Noise**

  flexibility in modelling: expand any standard model with homoskedastic noise

  convenient for signal extraction, noise easily kept separate from trends and other signals

Airline-plus-seasonal-noise model

- Bell (2004): let Y_t follow "airline model" (Box-Jenkins, 1976) :

$$y_t = Y_t + \varepsilon_t^s$$
$$(1-L)(1-L^{12})Y_t = (1-\theta L)(1-\theta_s L^{12})a_t, \quad a_t \sim WN(0, \sigma_a^2),$$

- Baseline variation: $\varepsilon_{n,t} = \varepsilon_t^{can} \sim WN(0, \sigma^2_{\varepsilon,can})$, the canonical irregular from Y_t

- Extra variation: ε_t^s

  Total irregular variance alternates between $\sigma^2_{\varepsilon,n}$ and $(\sigma^2_{\varepsilon,n} + \sigma^2_{\varepsilon,s})$

- STSM - replace ε_t with $\varepsilon_{n,t} + \varepsilon_{s,t}$

Parameters

Parameters estimated by ML.

Likelihood via Kalman Filter for each θ.

Find MLE's $\widehat{\theta}^{Hom}$ for $M(Hom)$, and $\widehat{\theta}^{Het}$ for $M(Het)$.

Trend Estimation - Weights on Observations

The extracted trends are

$\widehat{\mu}_t^{Hom} = E[\mu_t | Y_T, \widehat{\theta}^{Hom}, M(Hom)] = \sum_k w_k^{Hom} y_{t+k}$

$\widehat{\mu}_t^{Het} = E[\mu_t | Y_T, \widehat{\theta}^{Het}, M(Het)] = \sum_k w_k^{Het} y_{t+k}$

w_j^{Hom}, $j = 0, \pm 1, \pm 2, ...$ is a weighting function

$w_j^{Hom\prime}s$ and w_j^{Het}'s computed via Koopman-Harvey (2002) [Alternative: McElroy (2008)]

w_k^{Hom}'s independent of season; $w_k^{Het}(j)$'s depend on season j

Gaussian disturbances: MMSE

General WN disturbances: MMSE within linear class

Application: Monthly time series of U.S. Housing Starts

Data: Total US Housing Starts in Midwest, Unadjusted (in Logarithms)

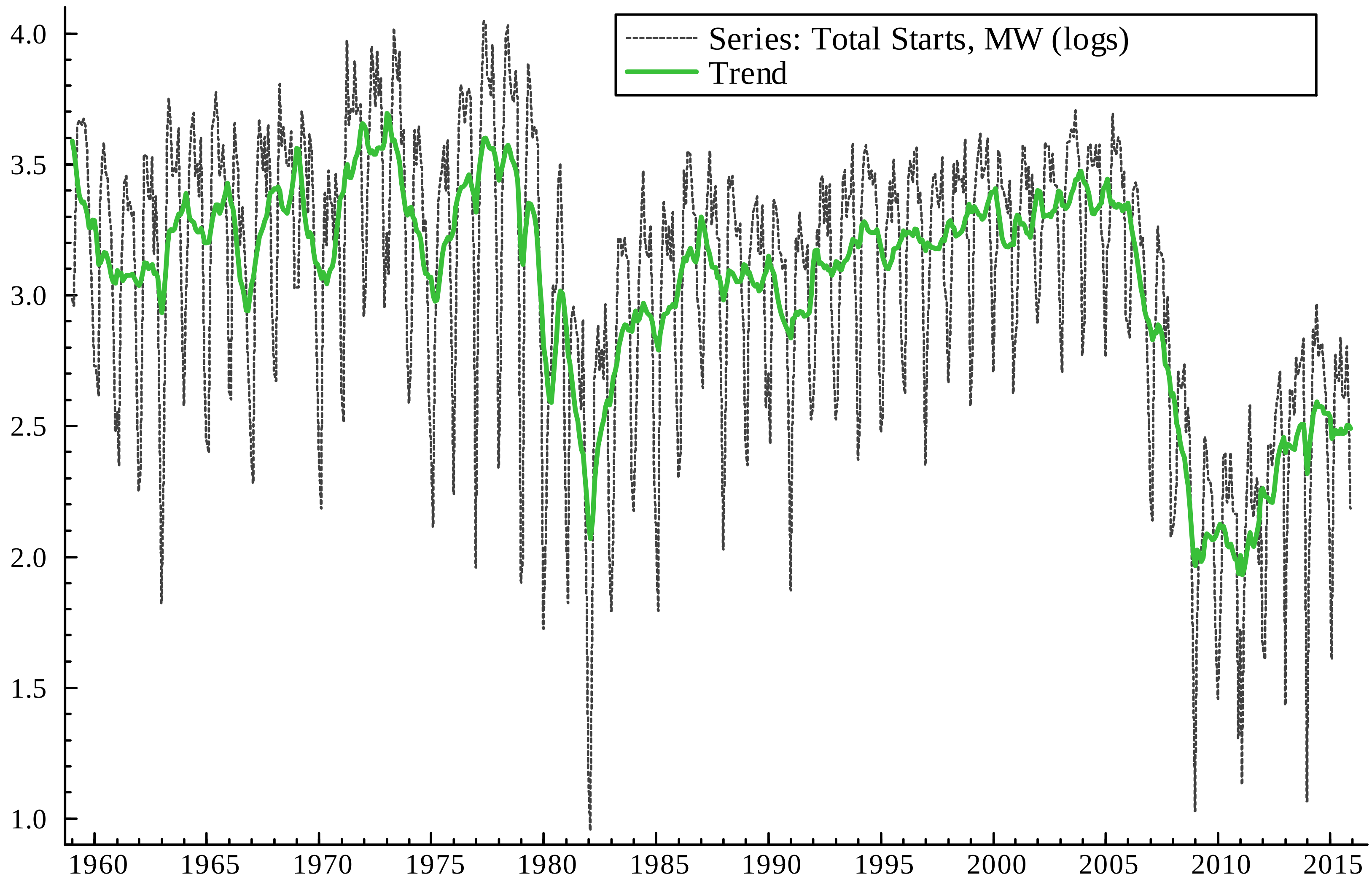
Sample: Jan. 1959 to Dec. 2015

Severe weather in **winter months** can impede construction; these effects are most pronounced and unpredictable in the MW region.

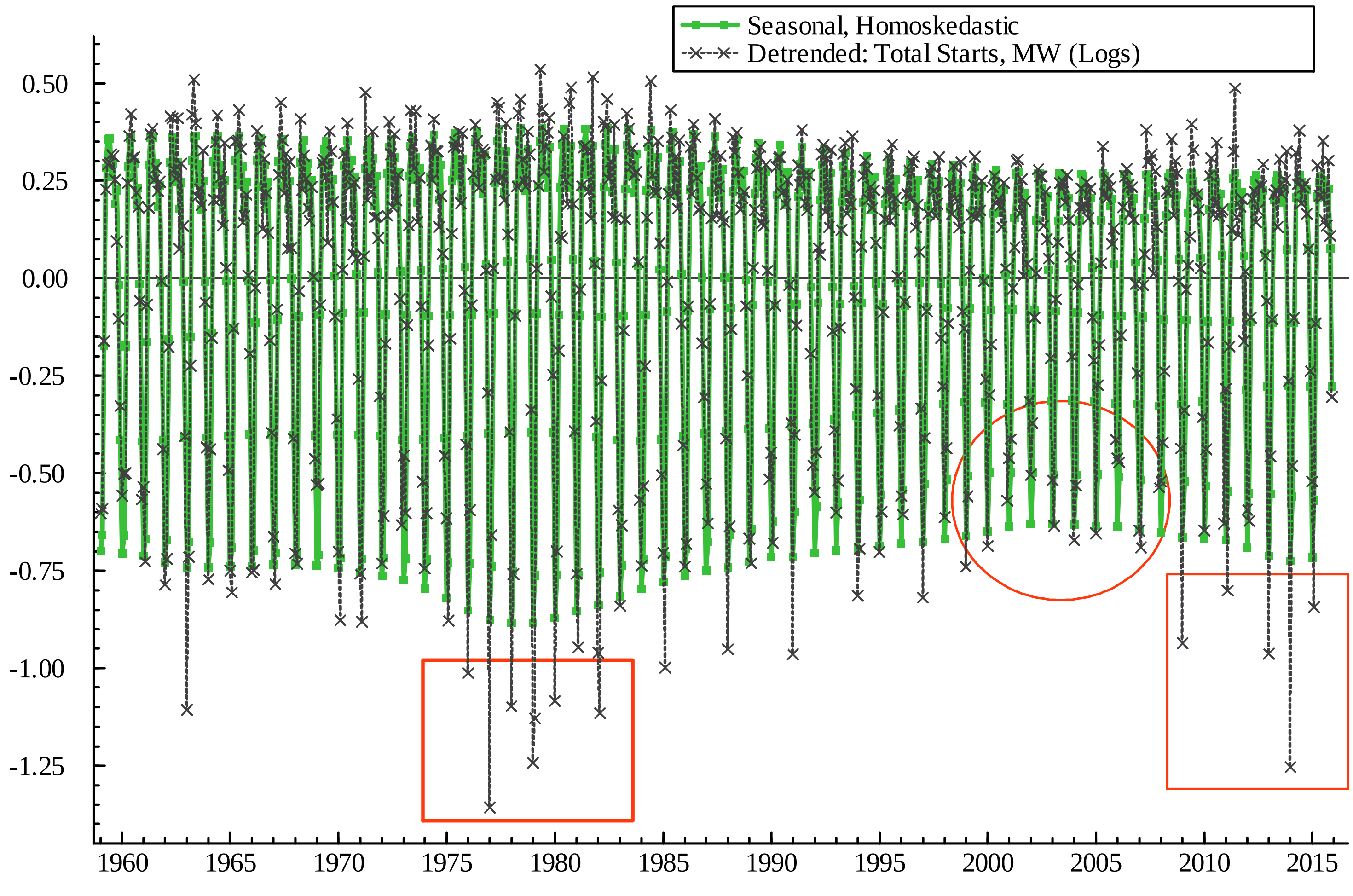
This increases degree of uncertainty in **{Dec., Jan., Feb.}, the high-variance group.**

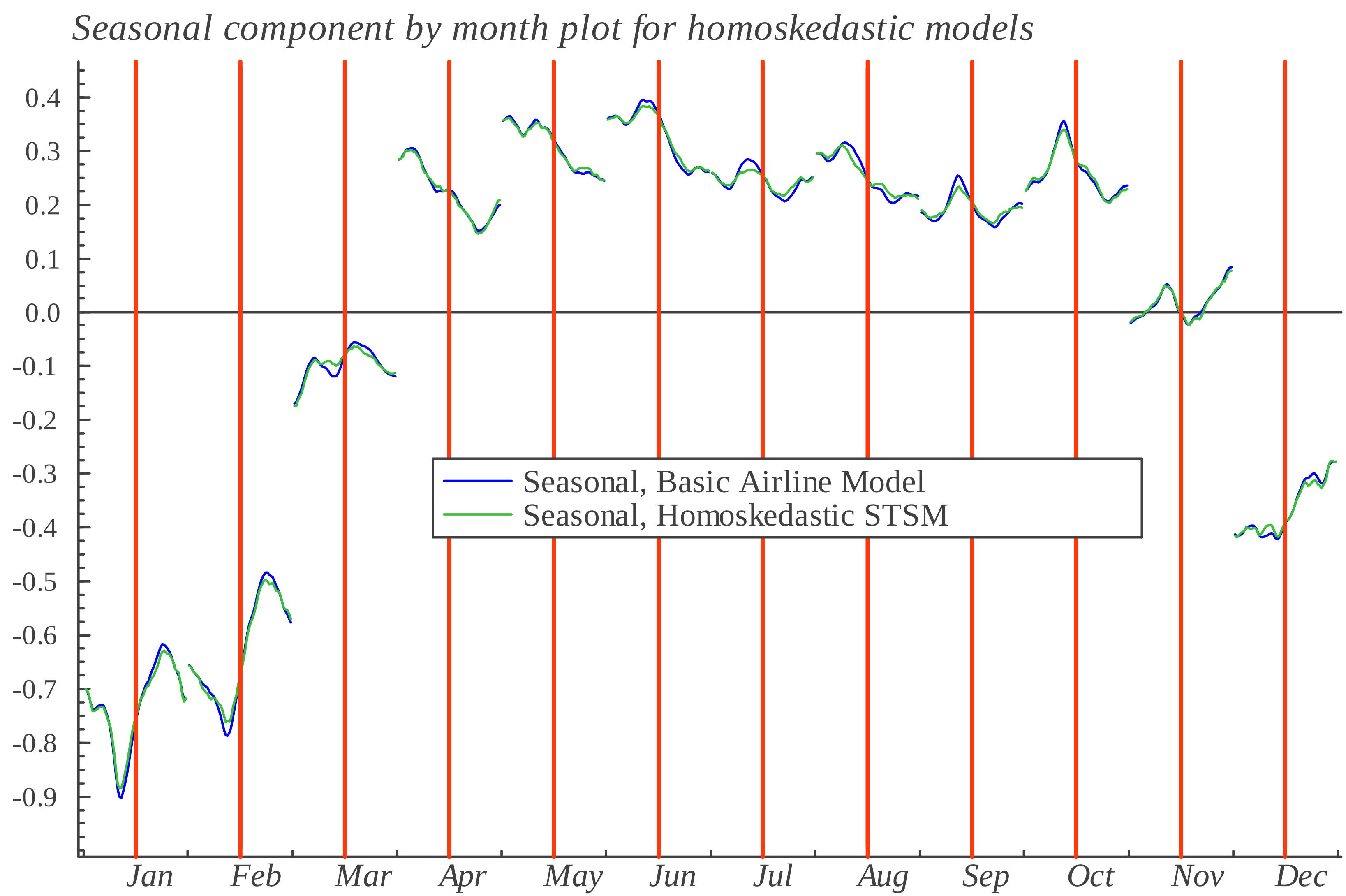
Then $\varepsilon_{s,t}$ represents excess winter-related variability over baseline.

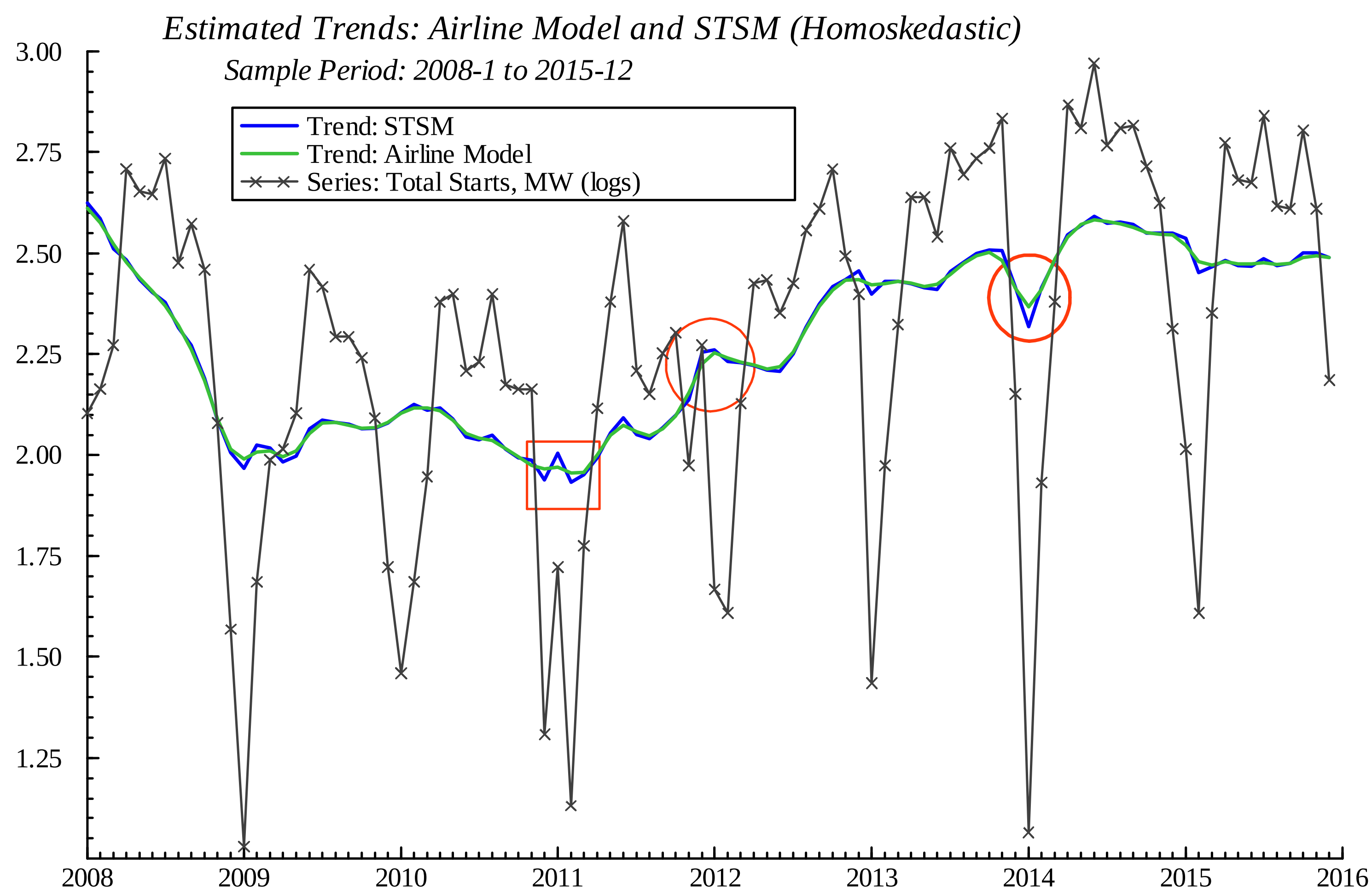
Computations with Ox language [Doornik (2009)] and SsfPack package [Koopman et. al. (2012)]



Series: Total Starts, MW (logs)
Trend
4.0
3.5
3.0
2.5
2.0
1.5
1.0
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015


Seasonal, Homoskedastic
Detrended: Total Starts, MW (Logs)
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
-1.25
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015


Seasonal component by month plot for homoskedastic models
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
Seasonal, Basic Airline Model
Seasonal, Homoskedastic STSM
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec


Estimated Trends: Airline Model and STSM (Homoskedastic)
Sample Period: 2008-1 to 2015-12
Trend: STSM
Trend: Airline Model
Series: Total Starts, MW (logs)
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
2008
2009
2010
2011
2012
2013
2014
2015
2016

Homoskedastic models: STSM vs. Airline comparisons and results

- **Comparisons**

  Seasonals very close. Slight difference in Jan-Feb. Therefore, Seasonally Adjusted series also very close.

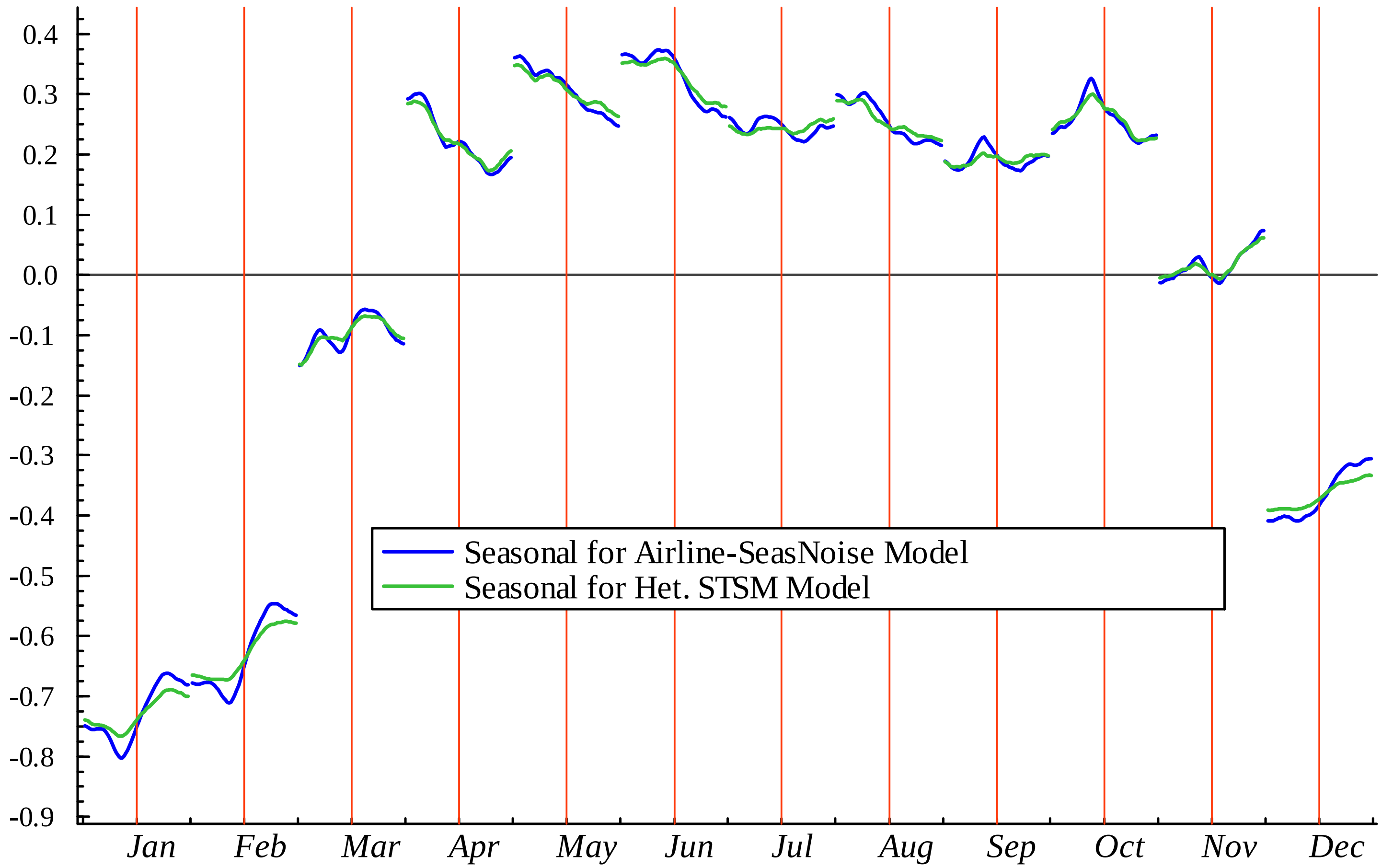
  Trends mostly track same level - canonical one smoother. Somewhat more irregular removed by canonical trend and seasonal.

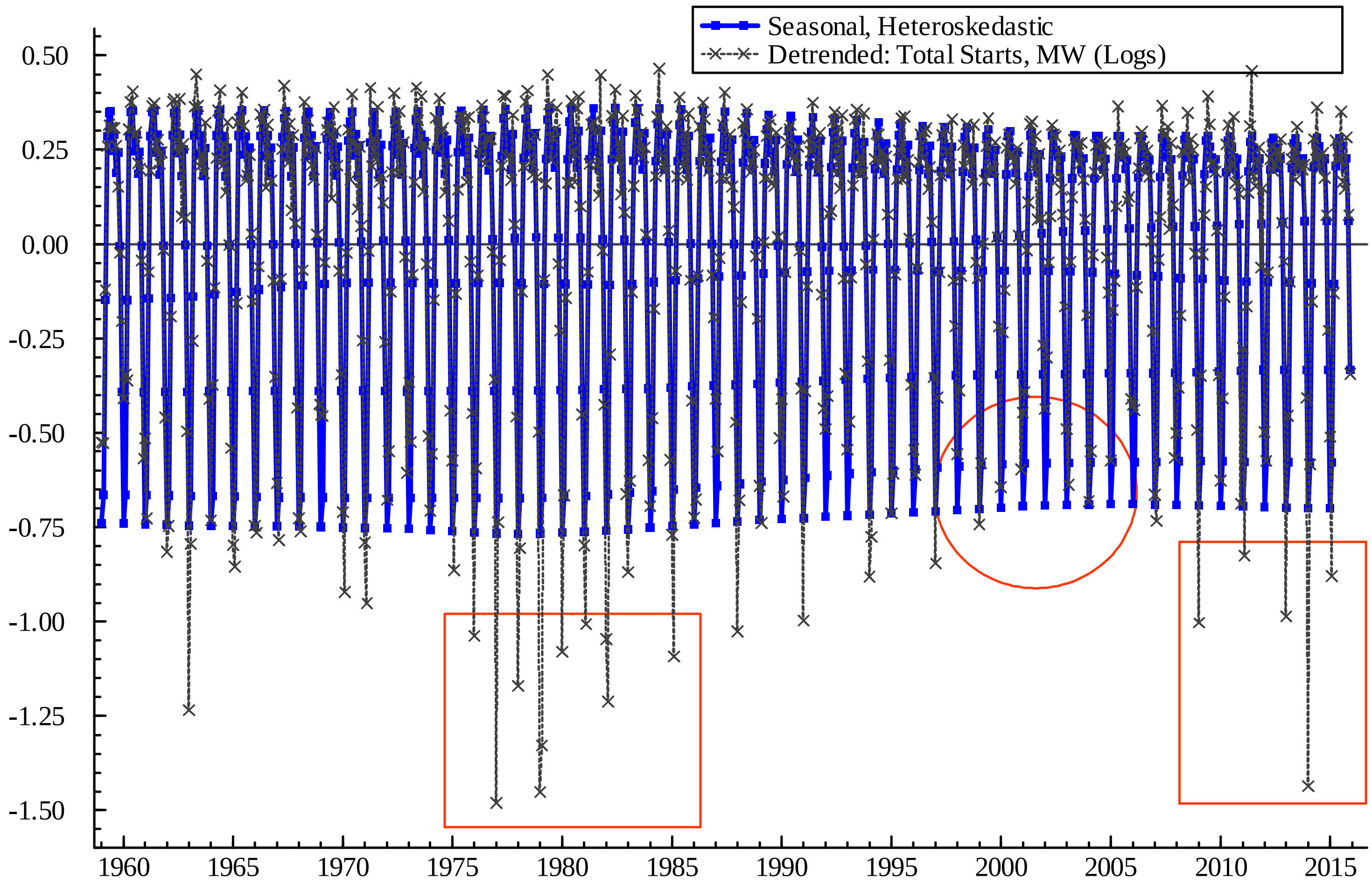
- **Overall results**

  Seasonal influenced by temporary movements around winter – individual observations unusually low or high relative to ongoing trend-seasonal.

  Trend shows significant short-term reactions to surprisingly low or high observation around turn of year.

Seasonal-by-month plot: Airline-SeasNoise model and Heteroskedastic STSM





Seasonal, Heteroskedastic
Detrended: Total Starts, MW (Logs)
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
-1.25
-1.50
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Heteroskedastic models: STSM vs. Airline-SeasNoise comparisons and results

- **Comparisons**

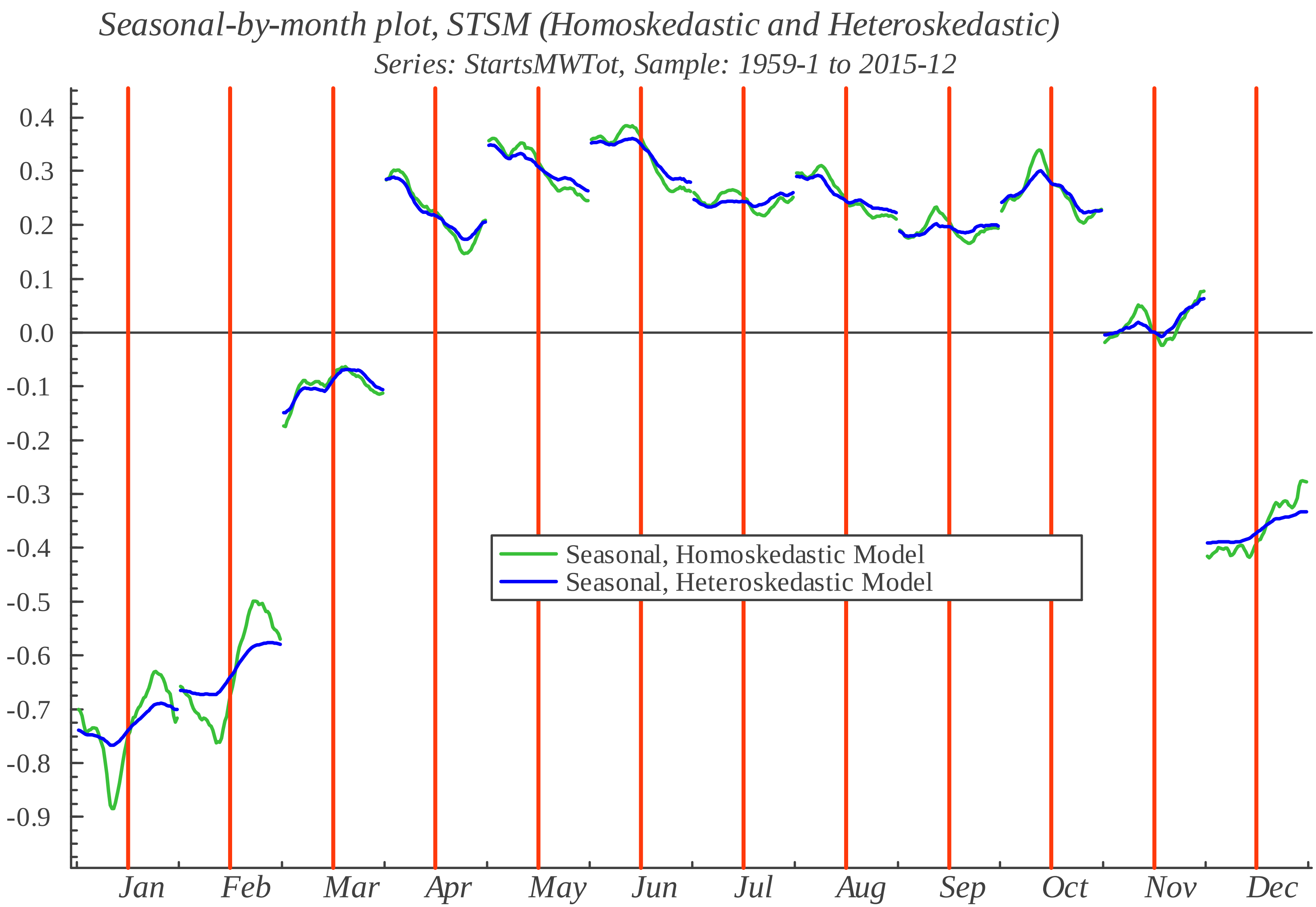
  Seasonals fairly close, though noticeable differences in Jan-Feb, when STSM seasonals vary by less. Therefore, Seasonally Adjusted series mostly similar, depending on sample period.

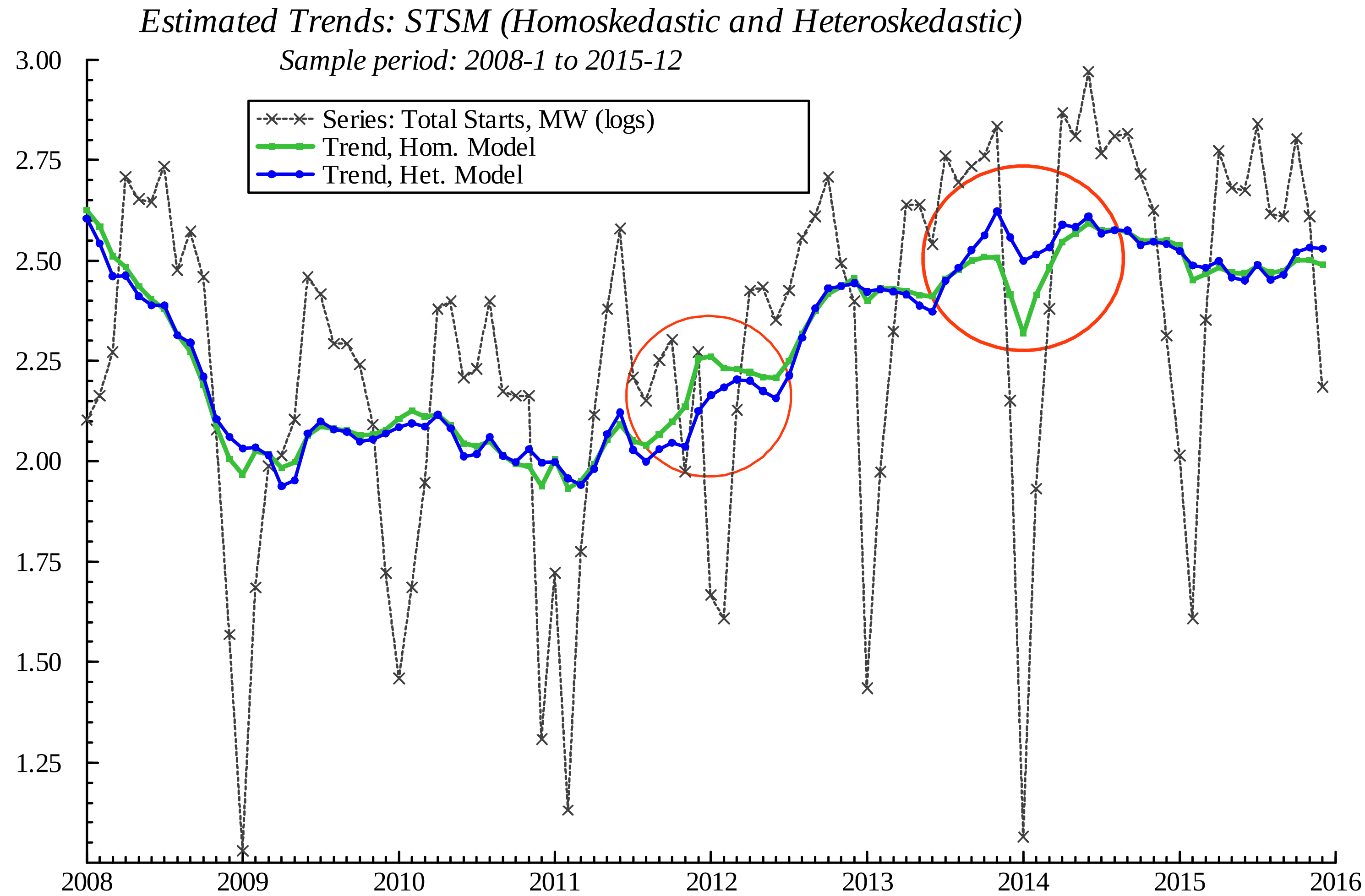
  Trends generally move together - canonical one again smoother.

- **Overall results**

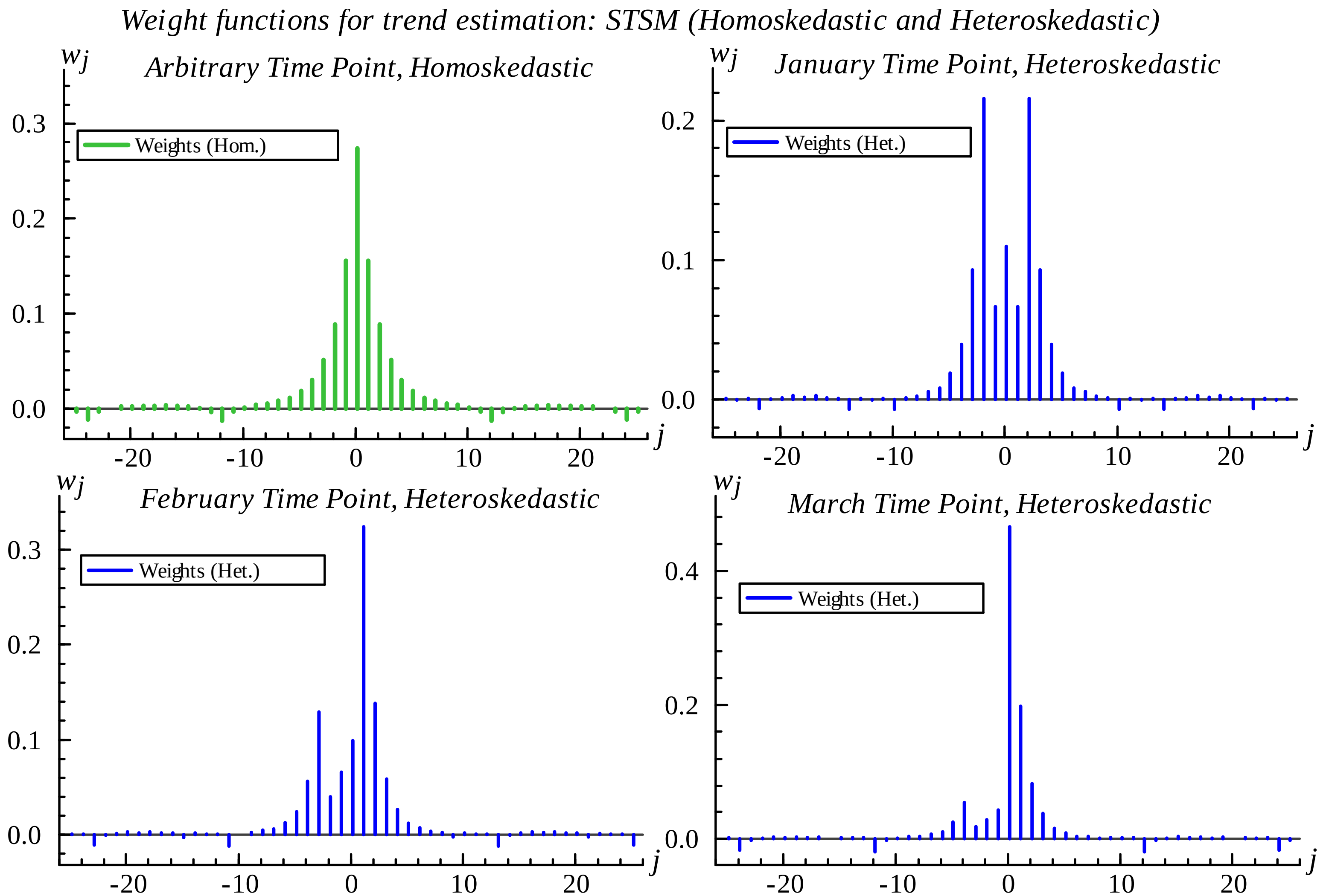
  Seasonal robust to the temporary movements around winter – individual observations unusually low or high relative to ongoing trend-seasonal – over the sample period.

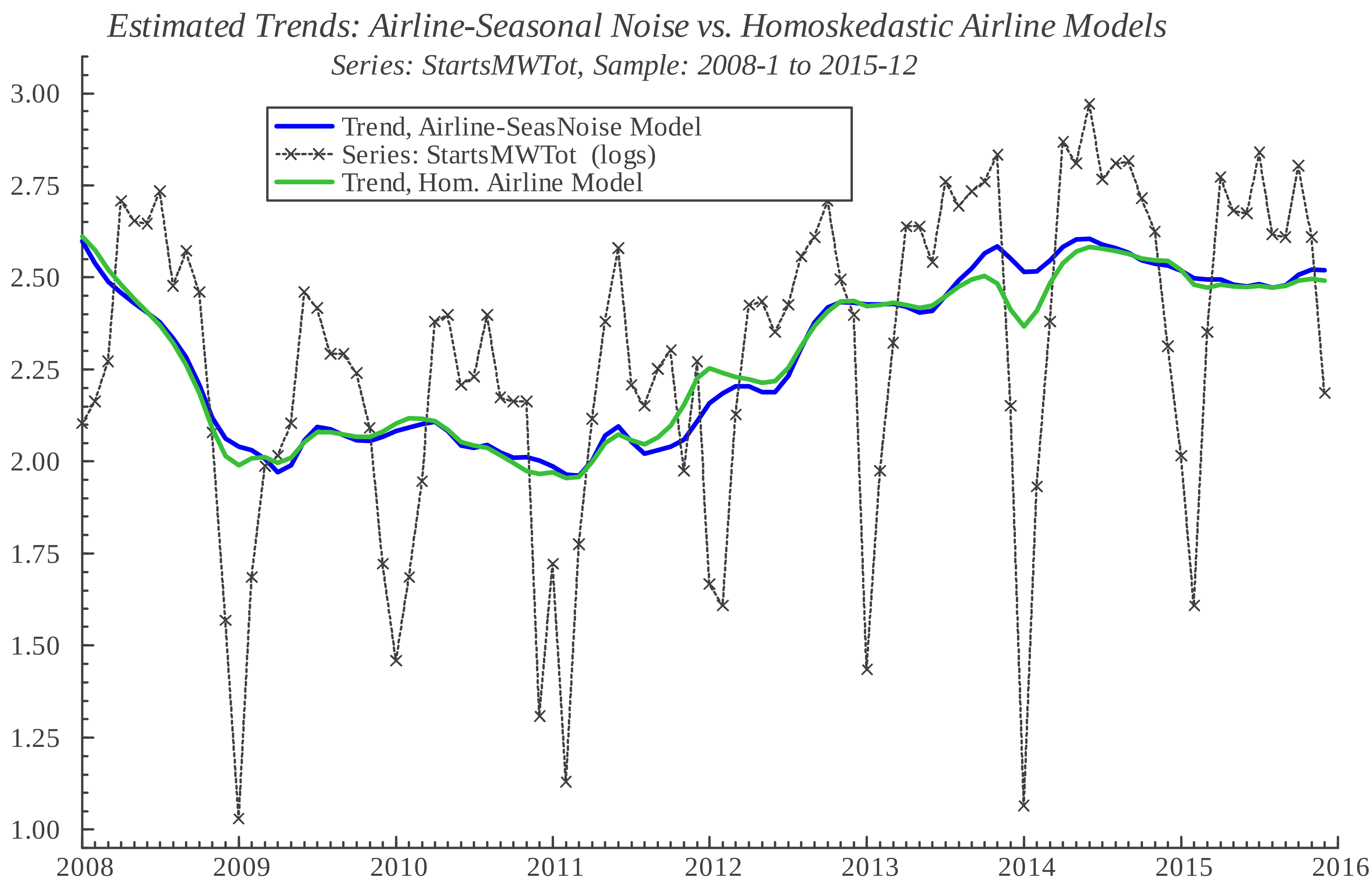
  Trend less reactive to temporary movements around winter (shown below).

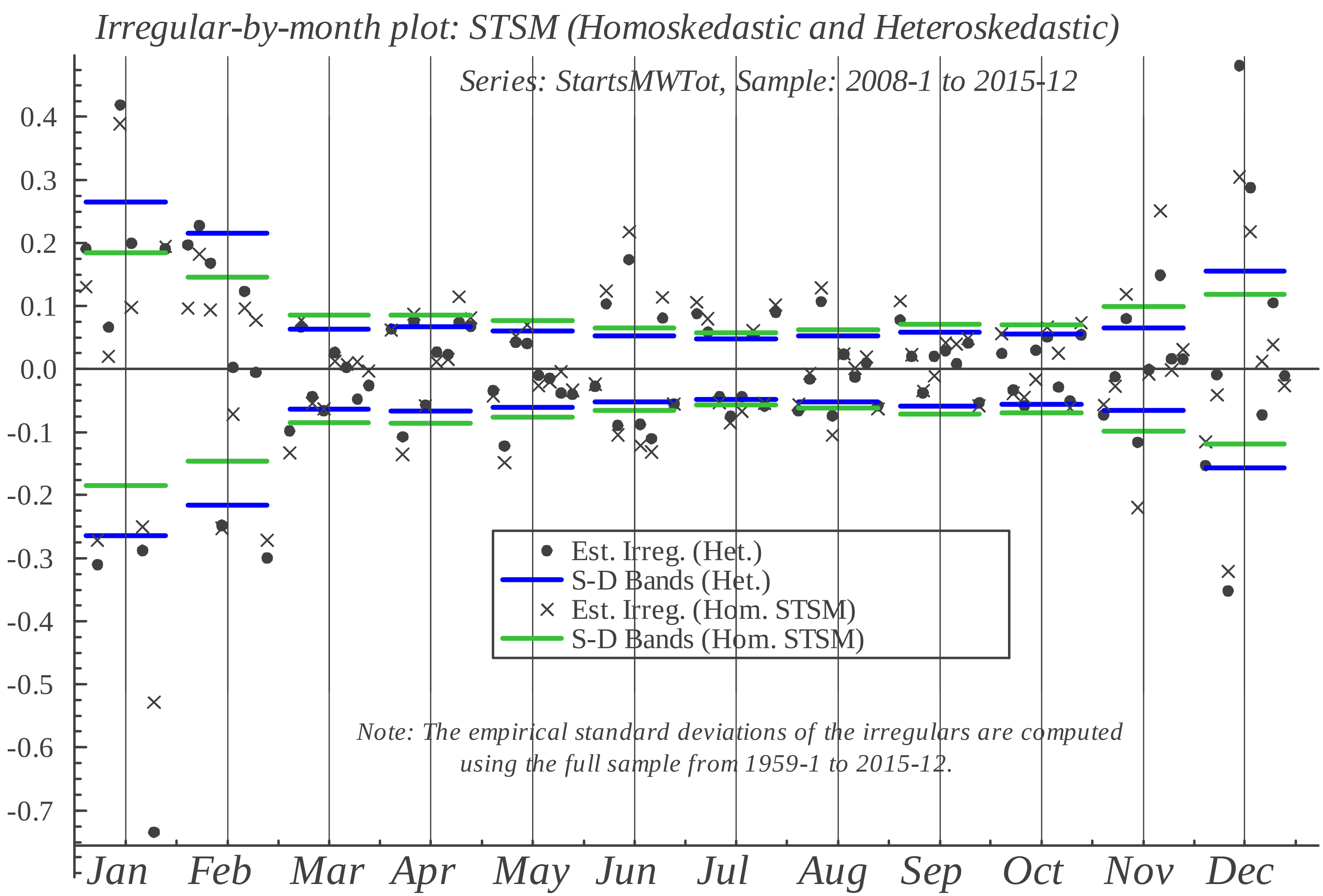

Seasonal-by-month plot, STSM (Homoskedastic and Heteroskedastic)
Series: StartsMWTot, Sample: 1959-1 to 2015-12
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
Seasonal, Homoskedastic Model
Seasonal, Heteroskedastic Model
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec

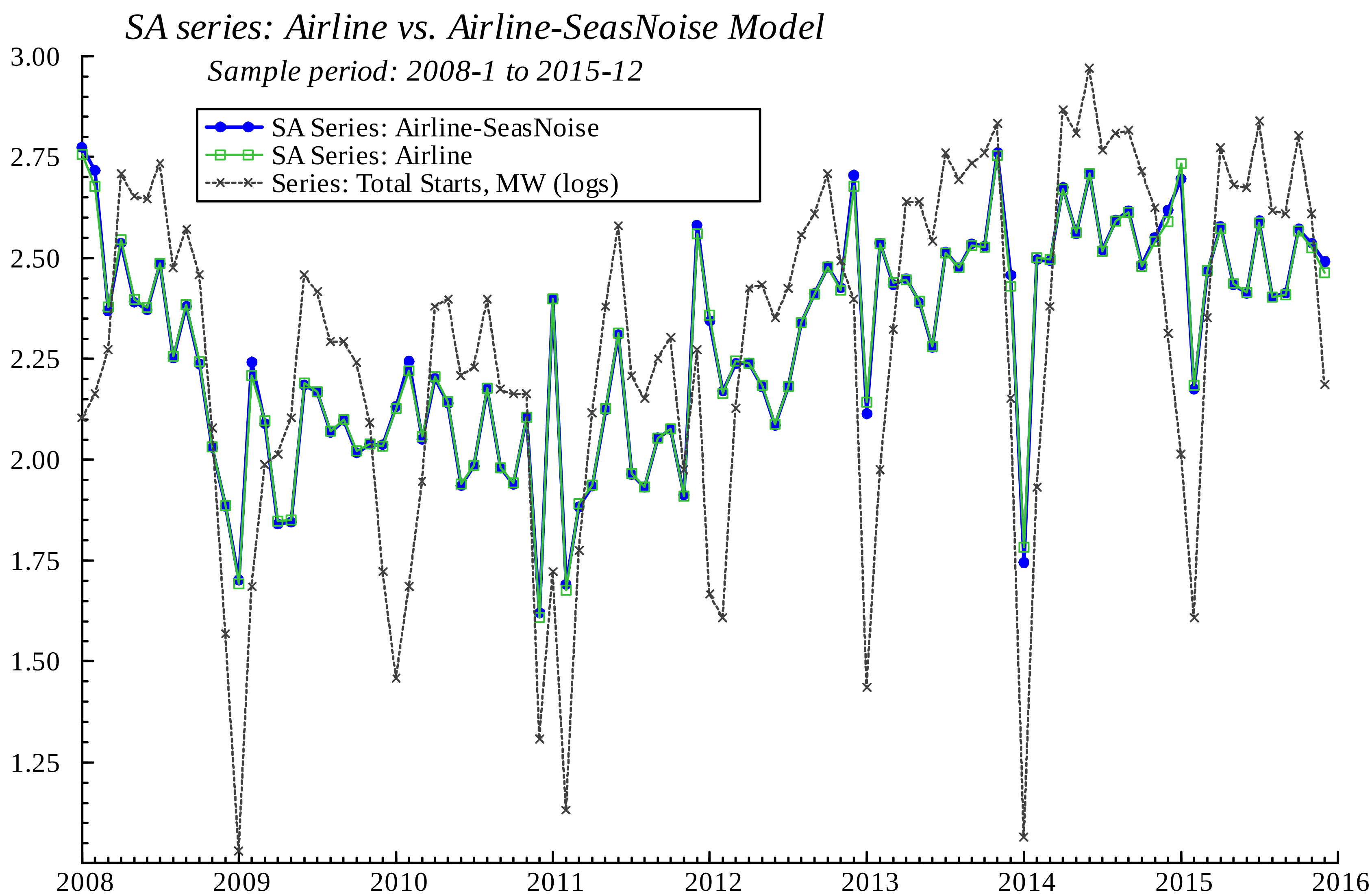


Estimated Trends: STSM (Homoskedastic and Heteroskedastic)
Sample period: 2008-1 to 2015-12
Series: Total Starts, MW (logs)
Trend, Hom. Model
Trend, Het. Model
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
2008
2009
2010
2011
2012
2013
2014
2015
2016

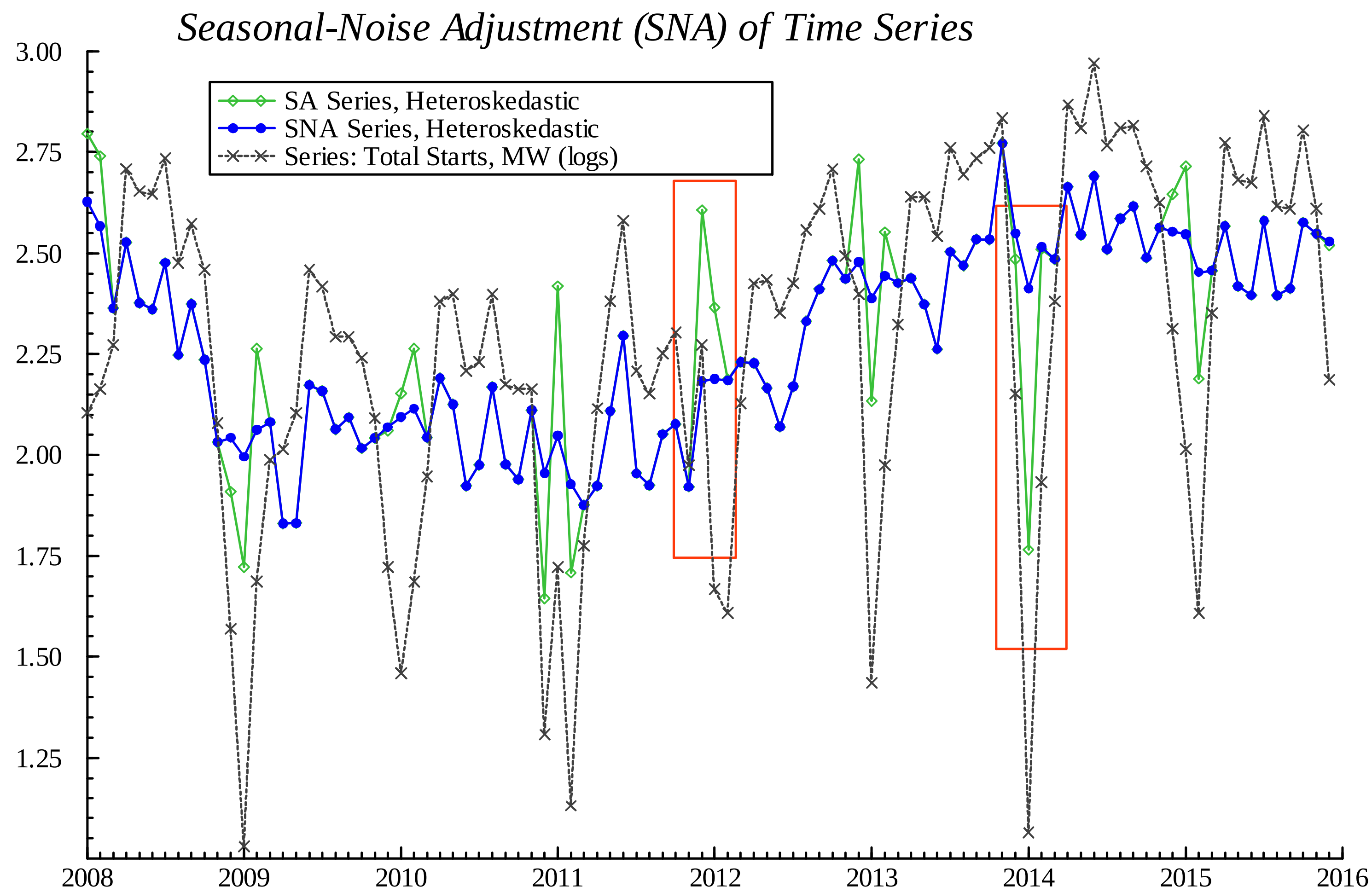

Weight functions for trend estimation: STSM (Homoskedastic and Heteroskedastic)
Arbitrary Time Point, Homoskedastic
w_j
Weights (Hom.)
0.3
0.2
0.1
0.0
-20
-10
0
10
20
j
January Time Point, Heteroskedastic
w_j
Weights (Het.)
0.2
0.1
0.0
-20
-10
0
10
20
j
February Time Point, Heteroskedastic
w_j
Weights (Het.)
0.3
0.2
0.1
0.0
-20
-10
0
10
20
j
March Time Point, Heteroskedastic
w_j
Weights (Het.)
0.4
0.2
0.0
-20
-10
0
10
20
j


Estimated Trends: Airline-Seasonal Noise vs. Homoskedastic Airline Models
Series: StartsMWTot, Sample: 2008-1 to 2015-12
Trend, Airline-SeasNoise Model
Series: StartsMWTot (logs)
Trend, Hom. Airline Model
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
2008
2009
2010
2011
2012
2013
2014
2015
2016


Irregular-by-month plot: STSM (Homoskedastic and Heteroskedastic)
Series: StartsMWTot, Sample: 2008-1 to 2015-12
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Est. Irreg. (Het.)
S-D Bands (Het.)
Est. Irreg. (Hom. STSM)
S-D Bands (Hom. STSM)
Note: The empirical standard deviations of the irregulars are computed
using the full sample from 1959-1 to 2015-12.



SA series: Airline vs. Airline-SeasNoise Model
Sample period: 2008-1 to 2015-12
SA Series: Airline-SeasNoise
SA Series: Airline
Series: Total Starts, MW (logs)
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
2008
2009
2010
2011
2012
2013
2014
2015
2016



Seasonal-Noise Adjustment (SNA) of Time Series
SA Series, Heteroskedastic
SNA Series, Heteroskedastic
Series: Total Starts, MW (logs)
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
2008
2009
2010
2011
2012
2013
2014
2015
2016

Results for comparisons between homoskedastic and heteroskedastic models

Heteroskedastic Trends far less influenced by strong seasonal noise.

Heteroskedastic Seasonals less reactive to occurrences of seasonal noise over the sample period.

Seasonal factors for a standard Seasonal Adjustment **more stable over time.**

Heteroskedastic models allow us to effectively remove temporary bursts of seasonal noise from long-term trend and regular seasonal patterns.

Empirical findings

- Different homoskedastic models yield similar results, and likewise for different heteroskedastic models

- Heteroskedastic models produce results highly distinct from homoskedastic ones

    Especially in trend movements

    Very clear in filter weights – the downweighting of high-variability months by heteroskedastic filters allows them to discount large temporary shocks associated with seasonal noise

    Shift noise out of trend and into the new seasonal noise component

    Also, seasonals show more plausible variation (modest, nearly monotonic contraction in trough for construction time series affected by winter weather)

Conclusions

- Models for seasonal heteroskedasticity, basis for estimating and extracting seasonal noise, the extra irregular in high-variability months.

- Makes substantial difference to trend, producing robustness to (temporary) seasonal noise.

- Distinguish between smooth seasonal and seasonal noise.

- More stable adjustment over long sample.

- Option of Seasonal-Noise Adjustment (SNA)

Further work

- Explore properties of SNA series, e.g. do they perform significantly different than SA series as leading indicators

- Examine forecasting ability and characeteristics of heteroskedastic vs. homoskedastic models

- Application of seasonally heteroskedastic models to other Time Series

- Investigate more general trend-cycle or seasonal models

- Study effects of using canonical definition of trend - for instance, "canonicalize" the STSM trend