Detecting Seasonality in Seasonally Adjusted Monthly Time Series

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Special Features of Seasonally Adjusted Series

With additive or log-additive direct seasonal adjustment (approximately with multiplicative X-11 adjustments), **SA removes stable seasonality**, see Bell (2011). Includes that defined by each calendar month's **average seasonal effect**.

- Residual seasonality thus **changes over time** and is **weaker** than seasonality in unadjusted series.
- Most diagnostics should be calculated for a **subspan** of the (SA or Irr) series, **not the full series**.
- Maravall's (2012) QS diagnostic for **positive seasonal-lag autocorrelation** is especially appropriate: Stationarized SA's and Irr's usually have a **negative** lag 12 autocorrelation, see e.g., McElroy (2011), Findley, Lytras and Maravall (2016).

Stationarized: Differenced, log transformed as needed for stationarity (required for diagnostics)

Seasonality Diagnostics

Three types of diagnostics:

i. **Frequency domain** (spectrum, Maravall F-T, periodogram, JD+ periodogram sum)

ii. **Regression**

iii. **Positive seasonal autocorrelation**

Diagnostics for a Specified SA or Irr Interval of Length *n*

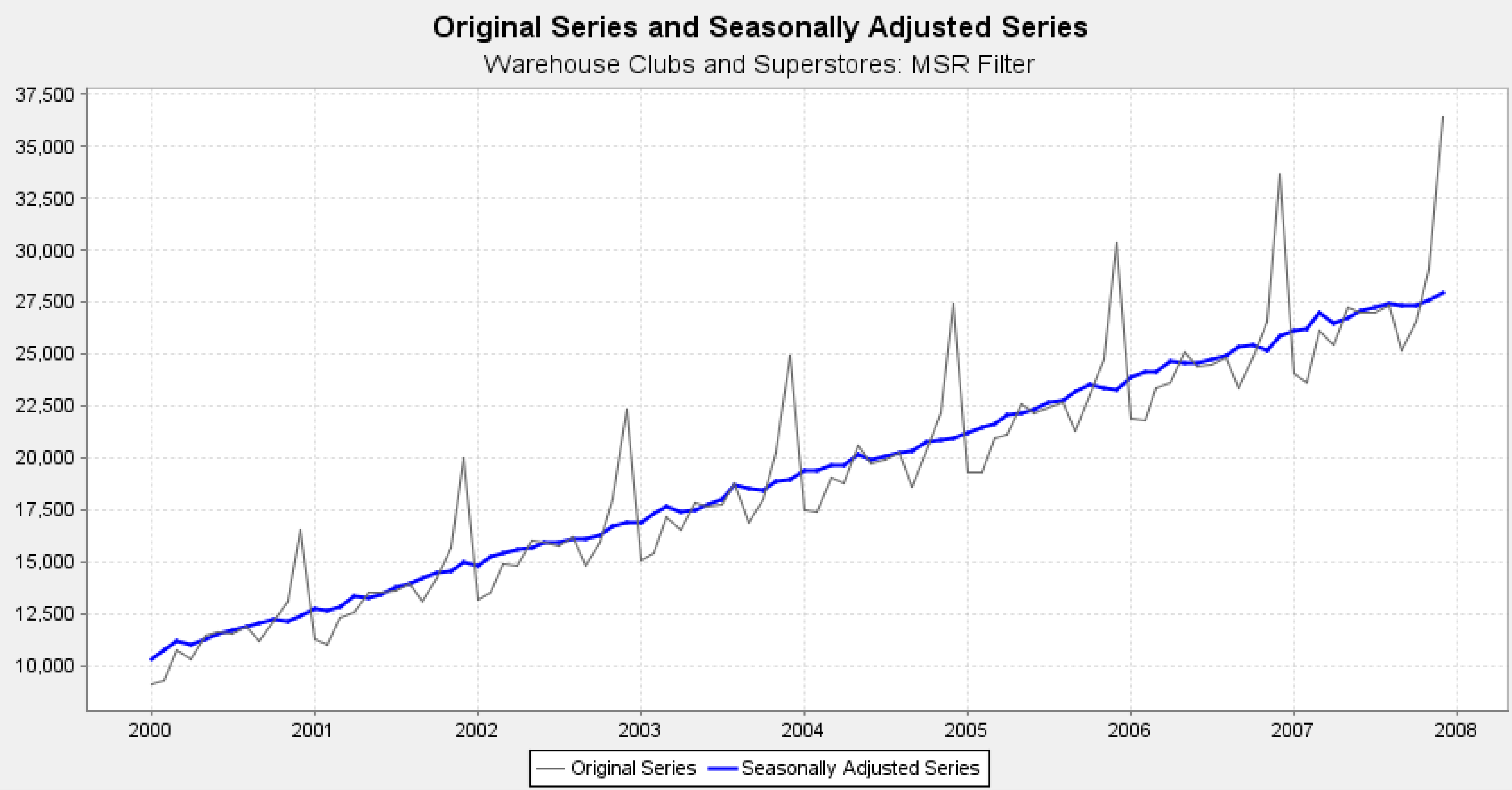
(Length *n* + *h*, -12<*h*<12 for Periodogram Sum OLS Diagnostic)

Seasonality Diagnostic	**Criterion**
AR(30) spectrum seas. freq. peaks	Visual significance (V.S.)
Periodogram (pdg) seas. freq. peaks	V.S.
Maravall "Tukey" spectrum ratios	Quasi-$F^{M\text{-}T}$
JD+ Periodogram sum F-test	OLS F^{fs}
Stable-seasonal regression	GLS F^{M}
Positive seas. autocorrelation QS	Quasi-$\chi^2(2)$

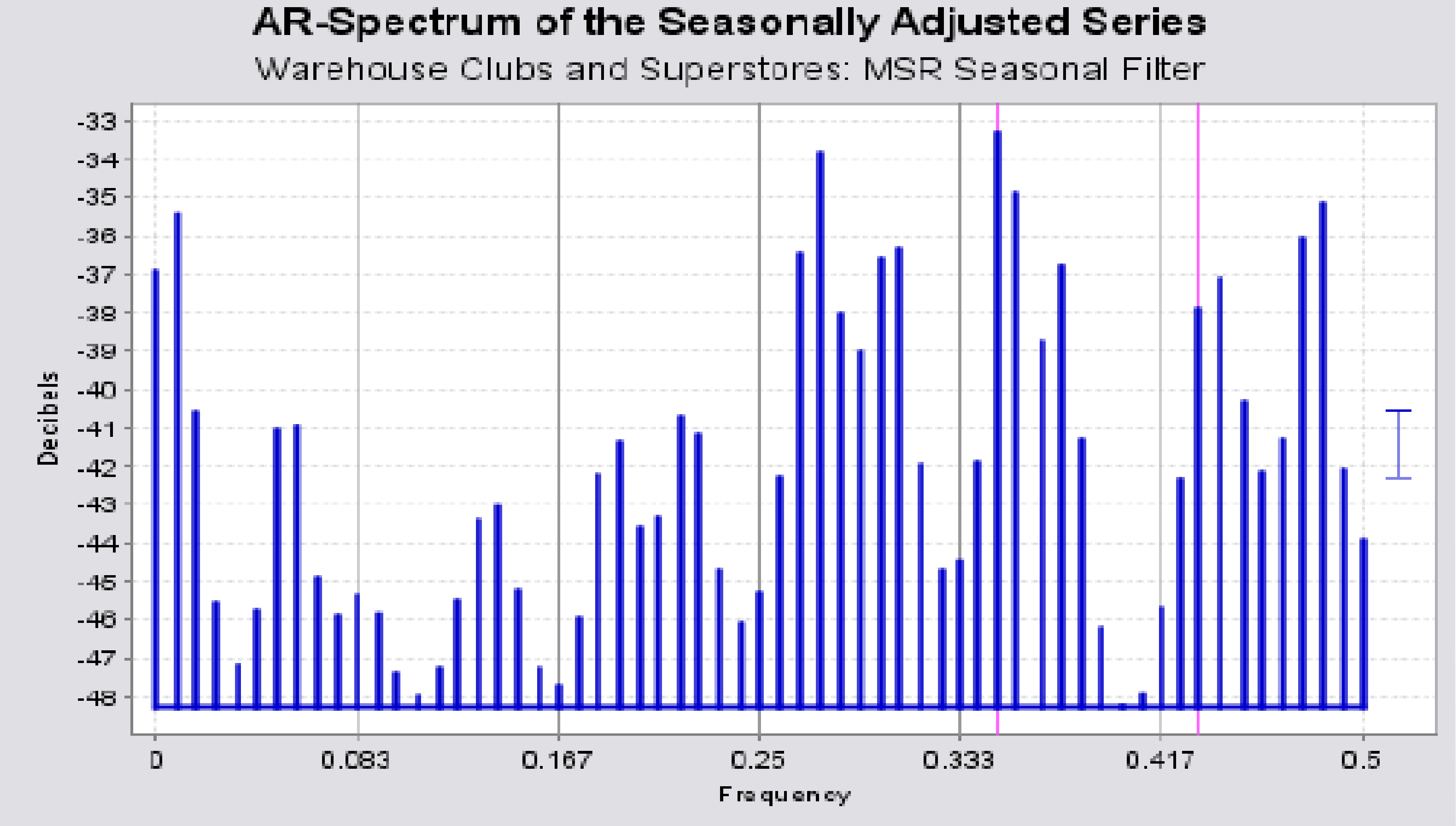
(Quasi- for simulation-based approximation)

The Study

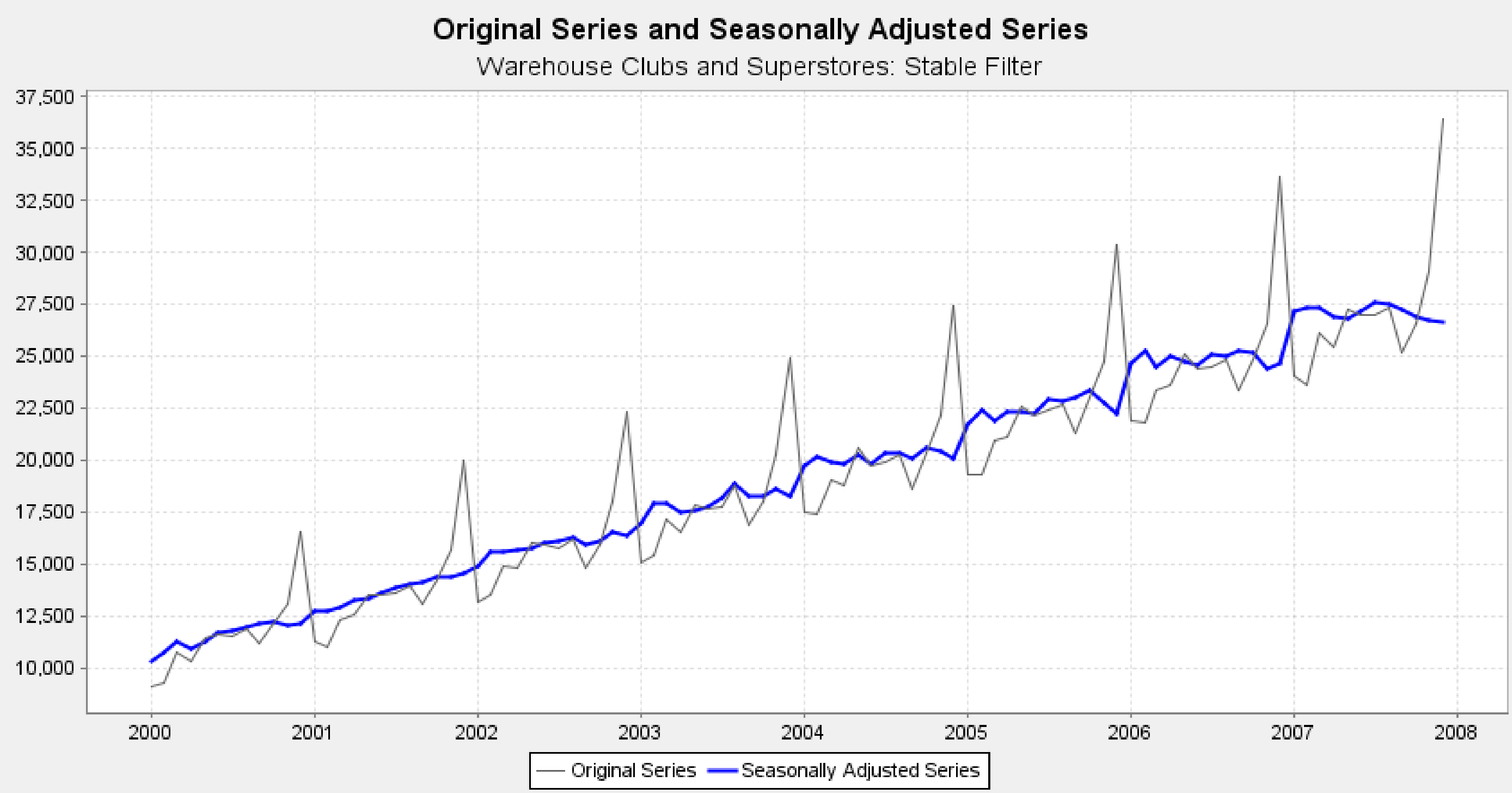
- We compare diagnostics' residual seasonality detections after **stable-seasonal SA** of sixteen 1992 –2007 **U.S. Retail Trade Survey** series.
- All series preadjusted for calendar and outlier effects (i.e. X-11 B1 series). X-11 filter for series is either s3x3 or s3x5.
- See Findley and Lytras (2016) for details.



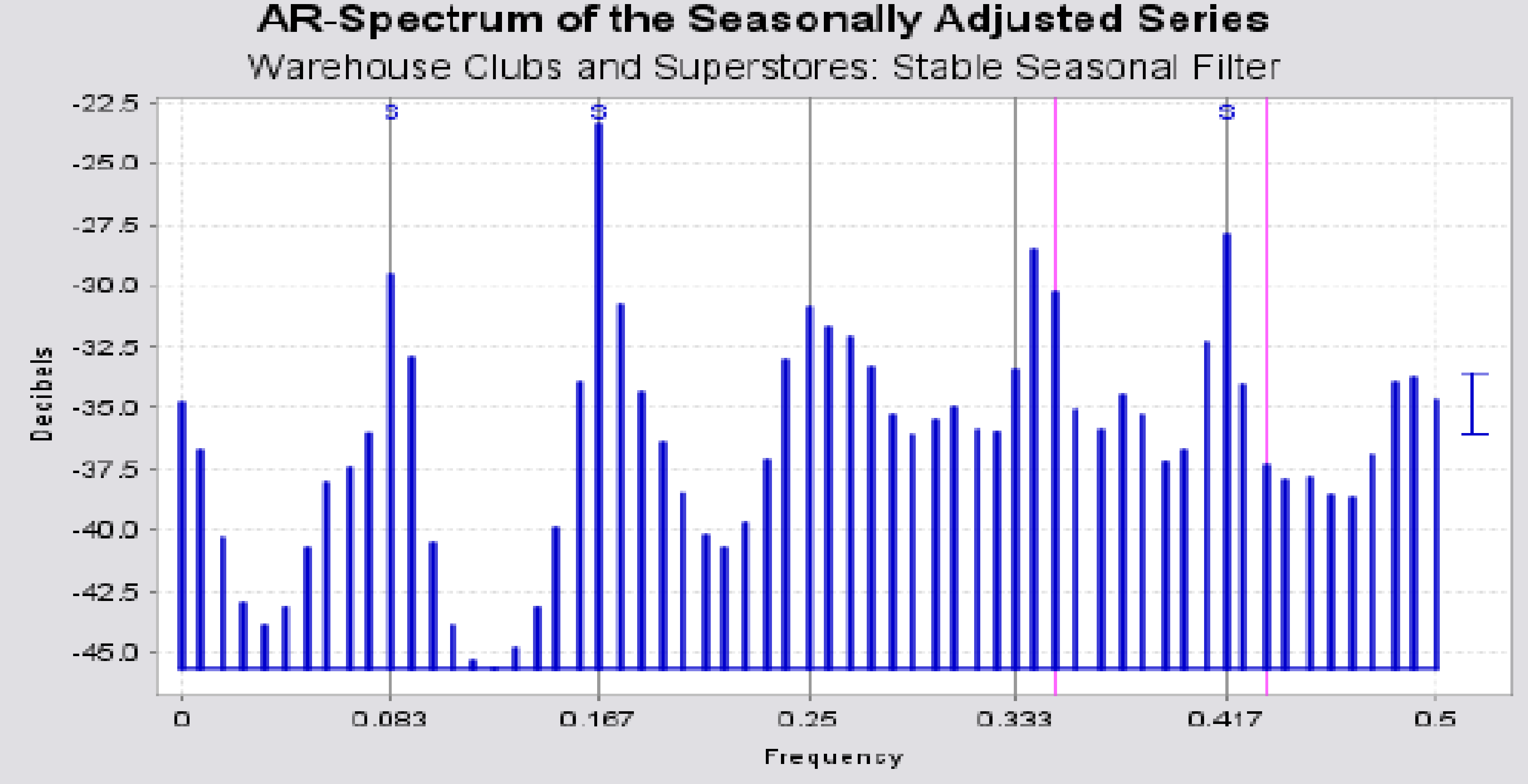
Last 8 years of series and its adjustment using **MSR filter (s3x5)**



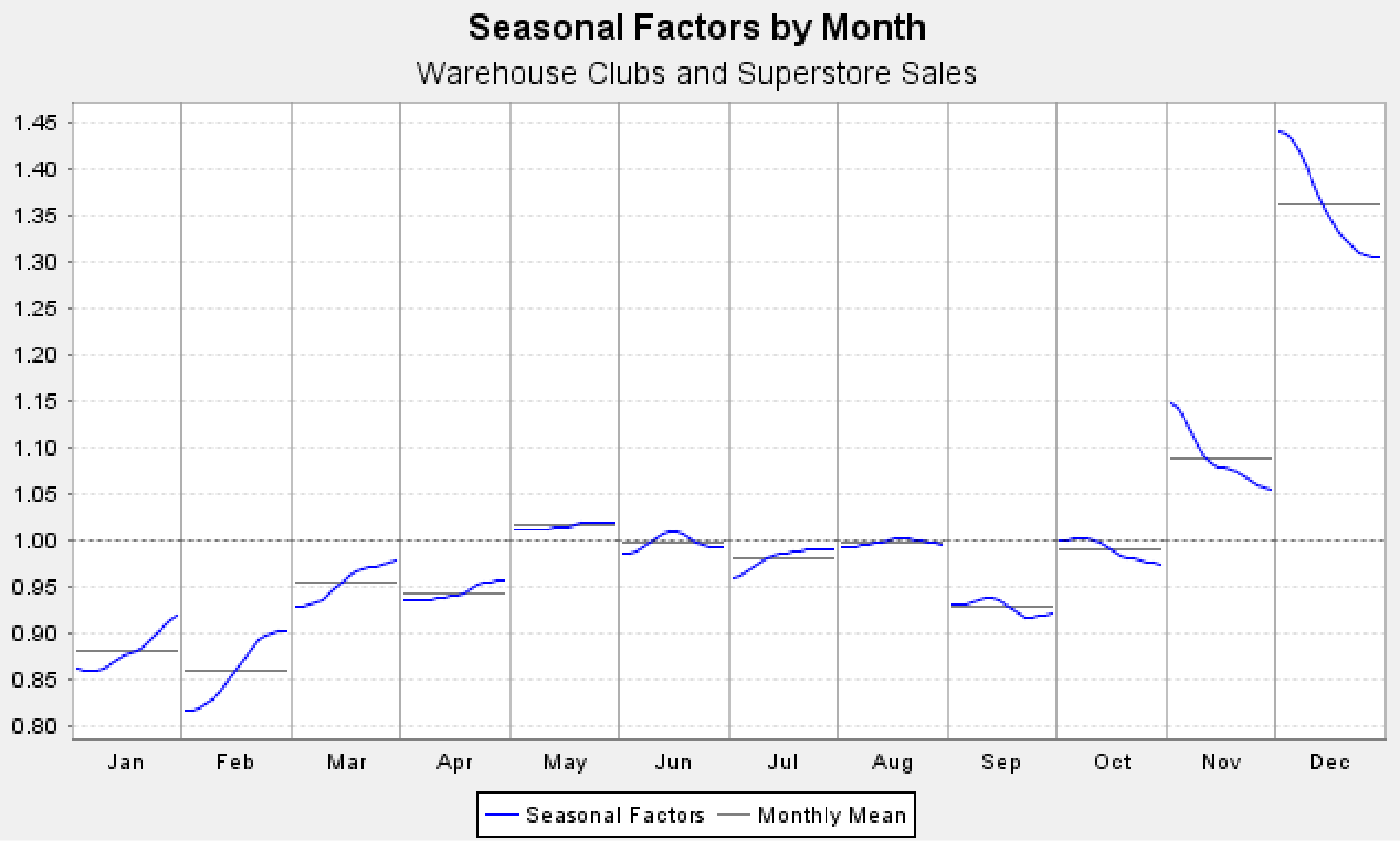
The arspec over the last 8 years of the series adjusted with the automatic X-11 filter has no seasonal peaks (nor does *pdg*, not shown).



Last 8 years of series with **stable-factor** seasonal adjustment



Residual seasonality is found in the last 8 years of the **stable-factor SA** by the AR spectrum: v.s. peaks at the **first**, **second**, and **fifth** seasonal frequencies. (*pdg*, not shown, has only v.s. **second** seasonal frequency peak.)

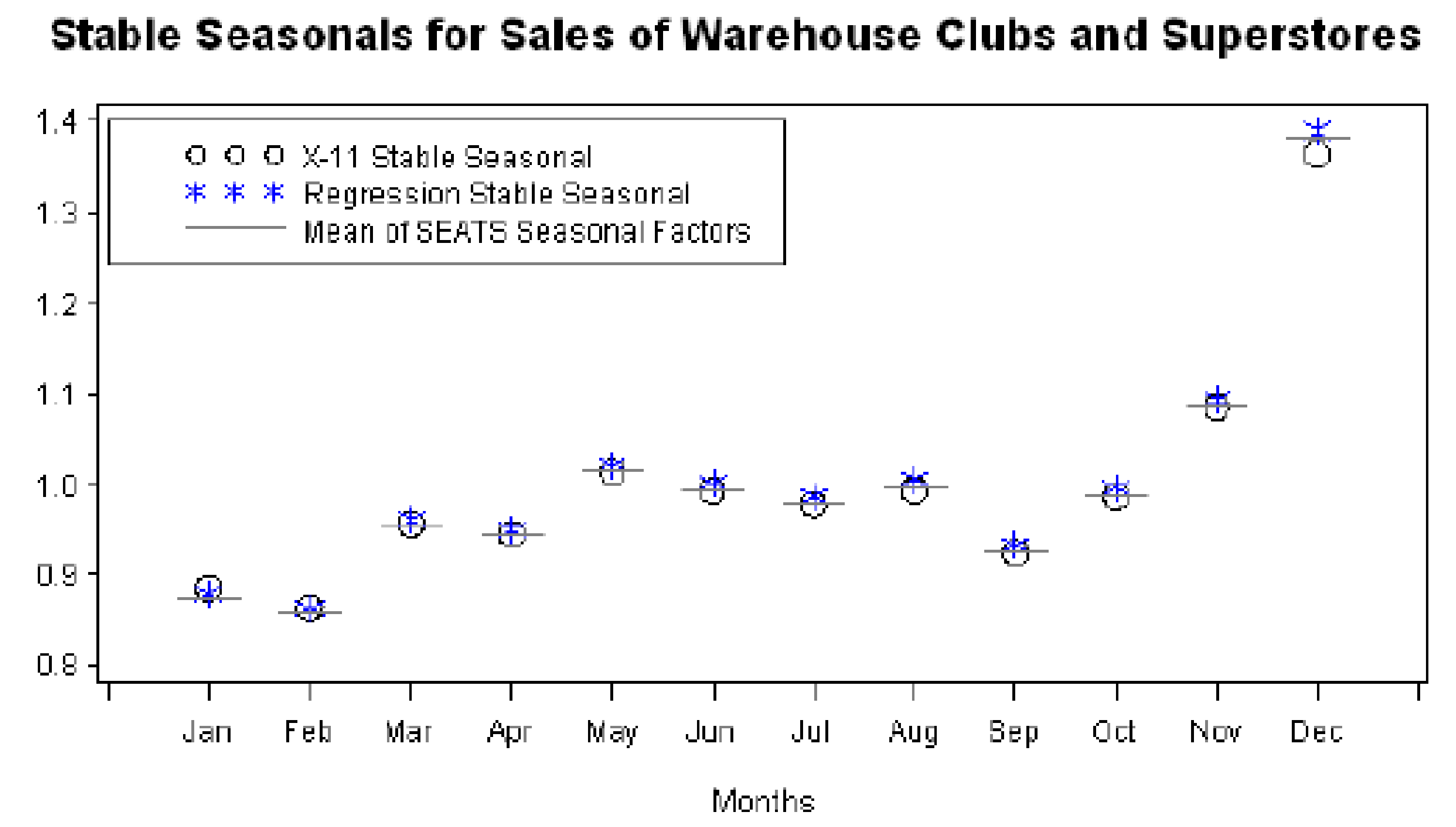


Multiplicative MSR **s3x5** seasonal factors by calendar month for Sales of Warehouse Clubs and Superstores, January 1992-December 2007. The horizontal lines are the calendar month factor sample means for the 16 years.

Table 1: Stable Regression t-Statistics of the Final 8-year Span: $\hat{a}_j$

($|t_j| > 2$ in bold)

j	1	2	3	4	5	6	7	8	9	10	11	12
t_j	1.77	**4.78**	**3.24**	0.97	0.60	0.46	1.56	1.24	-1.11	**-2.04**	**-4.39**	**-7.01**



The exponentiated GLS **stable seasonal coefficient estimates** $\exp(\hat{a}_j)$ are very close to the **calendar month sample means** of both the log-additive SEATS seasonal factors and the multiplicative X-11 stable seasonal factors

Maravall's QS Statistic for Positive Seasonal Autocorrelation

With *s* denoting the seasonal period, *s* = 12 here, the focus is on the signs of the sample seasonal autocorrelations $\mathbf{r_i}$**, i = s, 2s** of the **stationarized data** x_1, … , x_n.

$$R_i = \begin{cases} r_i, \text{if } r_i > 0 \\ 0, \text{if } r_i \le 0 \end{cases}$$

When $\mathbf{R_s}$ **= 0**, set **QS = 0**. When $\mathbf{R_s}$ **> 0**, set

$$QS = n(n+2)\left\{\frac{{R_s}^2}{n-s} + \frac{{R_{2s}}^2}{n-2s}\right\}$$

NAICS Codes for the Series

44000 Retail and food services sales, total
44300 Electronics and appliance stores (s3x3)
44312 Computer and software stores
44400 Building materials and garden equipment and supplies dealers
44510 Grocery stores
44800 Clothing and clothing accessory stores (s3x3)
44811 Men's clothing stores
44812 Women's clothing stores (s3x3)
44820 Shoe stores (s3x3)
45100 Sporting goods, hobby, book, and music stores
45200 General merchandise stores (s3x3)
45210 Department stores – excluding leased departments (s3x3)
45291 Warehouse clubs and superstores
45400 Nonstore retailers (s3x3)
45410 Electronic shopping and mail-order houses (s3x3)
72200 Food services and drinking places

Table 2: Strong and Weak Detections

Diagnostic	Strong Detection	Weak Detection
arspec	2 v.s. peaks	1 v.s. peak
pdg	2 v.s. peaks	1 v.s. peak
M-T	One $F_{.01}$ peak (*) or two $F_{.05}$ peaks	One $F_{.05}$ peak
F^{fs}	.01 significance	.05 significance
Stable-seas. GLS F^{M}	.01 significance	.05 significance
QS	.01 significance	--

Maravall (2012) also defined **Strong M-T peak**, indicated by * in Table 5.
Maravall's detection criteria is never weaker than Table 2's.

A Tentative Action Criterion

Requires Action by Adjustor: Two strong detections, esp. if from different diagnostic types.

Examples where **action is optional** (hence dropped from Table 5):

44510 Grocery stores (only one SA strong)

72200 Food services and drinking places (only one Irr strong)

(Seasonal pattern changes are expected to be slow in both series.)

All Spans Detection Summaries

Table 3: Detections (Strong Detections) in the Stable-SA Series

span	*arspec*	*pdg*	*M-T*	$F^{fs}_{.05}$ ($F^{fs}_{.01}$)	$F^{M}_{.05}$ ($F^{M}_{.01}$)	$QS_{.01}$
8yr	16 (9)	10 (2)	12 (8)	14 (10)	14 (11)	(9)
10yr	15 (9)	9 (0)	12 (7)	8 (3)	10 (7)	(10)
12yr	13 (4)	7 (0)	2 (0)	1 (0)	0 (0)	(10)
16yr	14 (9)	0 (0)	0 (0)	0 (0)	0 (0)	(12)

Table 4: Detections (Strong Detections) in the E3 Irregulars

span	*arspec*	*pdg*	*M-T*	$F^{fs}_{.05}$ ($F^{fs}_{.01}$)	$F^{M}_{.05}$ ($F^{M}_{.01}$)	$QS_{.01}$
8yr	14 (9)	10 (2)	12 (7)	14 (12)	14 (13)	(12)
10yr	14 (6)	5 (0)	4 (1)	12 (10)	6 (4)	(12)
12yr	13 (3)	5 (0)	2 (1)	1 (1)	0 (0)	(14)
16yr	12 (5)	1 (1)	0 (0)	0 (0)	0 (0)	(14)

Some Considerations

- 8 year detections are important; most users are interested in recent SA span.

- Is the adjusted span the published SA span?

Possible Consistency Checks For a Significant Seasonal Peak

Monthly **seasonal frequencies**: **j/12** cycles/month,
j = 1, 2, 3, 4, 5, 6

For a given series, is a significant seasonal peak significant for **2+ diagnostics**?

Or both for **SA and Irr** with the **same diagnostic**?

Or for **same diagnostic with an aggregate or subaggregate**?

Table 5: Last 8 Year Spectrum Diagnostics

NAICS	*arspec* SA	*arspec* Irr	*M-T* SA	*M-T* Irr
44000	(1,2,4)	(2,4)	2	(2*)
44300	(2,4)	(2,4)	2	6
44312	3	(1,3)	(3*)	3
44400	2	(1,2)	2	-
44800	(1,2,3,4)	(1,2,3,4)	-	2
44811	(1,2)	2	(1*,2*)	(1,2*)
44812	(1.3)	(1,3)	-	-
44820	4	-	(1,2)	2
45100	4	(1,4)	-	2
45200	(1,2)	2	(2*)	(2*)
45210	2	2	(1,4)	(1,2,4)
45291	(1,2,5)	(2,5)	(1,2*)	(2*)
45400	2	2	(2,3)	(2,3*)
45410	(1,2)	(1,2)	(1,2*)	(2*,3)
Totals	16 (9)	14 (9)	12 (8)	12 (7)

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