History of probability (Part 1) From Pre-history to 1600. Gerolamo Cardano (1501-1576)

Any history of probability starts before written records, with archeological discoveries of dice-like objects. These were used in board games, in gambling, and in religious ceremonies (prediction of fortunes, for example). It seems obvious that just by experience, and without any formal mathematics, players all over the world had an empirical sense of the likelihood of various outcomes.

An open question is what was the ancient sense of "pure luck." What was the driving force behind the outcomes on the dice? In old games many of the outcomes on dice were named after gods. What role did the gods play?

The history of probability theory traces the path from the empirical observation that some outcomes are more likely than others to the assignment of numbers to such likelihoods, and then to the establishment of rules for calculating with these numbers. It took a surprisingly long time to get to what we clearly recognize as our definition of probability. Any introductory probability text today starts with a definition something like this:

> Suppose an event E can happen in *r* ways out of a total of *n* possible equally likely ways.
>
> Then the probability of occurrence of the event (called its success) is denoted by $P(E) = \frac{r}{n}$.

We don't see that formal definition in math papers until the 1700s. (See last page for the introduction to Abraham de Moivre's book of 1718.)

The oldest "dice" we know of were made from animal bones, usually the heel bone of a sheep or goat. The Greeks later called these *astragali*, and we still sometimes refer to dice as "bones." [Figure 1] The astragali had six sides and were not symmetrical. Two of the sides were so small that the dice never really landed on them. The other 4 sides themselves were not equally likely.

The next phase was to file those unusable sides down to get a cube with six useable and equally likely sides. The sides could be marked with scratches or holes for values. They did not necessarily have the values in the same arrangement that we have now, where opposite sides add to seven.

By the time we reach 600 BCE we have very fine dice, made from bone, clay, or metal all over the world.

The Egyptian painting in Figure 2 is from about 1250 BCE. It shows a woman throwing a die for a board game called senet. The Greek urn which dates from about 550 BCE depicts the soldiers, Achilles and Ajax, playing with dice.

Heel bone *astragali* about 5th century BCE

**Greek Bone die IV-II century B.C.E.
Opposite side's values: 6-2; 5-3; 4-1**[1]

Die from India about 2000 BCE. Note the 6 is not opposite the 1.

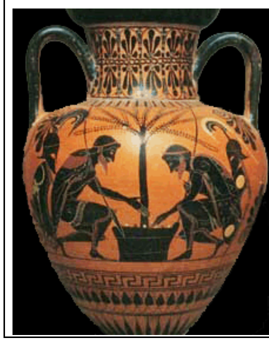
20-sided die from Egypt dated between 2nd century BCE to 4th century CE. Made of serpentine, a hard stone.

Figure 1

In games at this time one important outcome was called the Venus throw, throwing four dice and having all four faces be different. This was a fairly unusual result. In 44 BCE the Roman philosopher, Cicero, in his book, *De Divinatione* (*Concerning Divination*), had two people arguing over the role of the goddess Venus in causing this outcome. (Divination is using random occurrences to predict the future.)

Figure 2

Painting in tomb of Queen Nefertari (1295–1255 BCE)



Urn: Museum of Fine Arts, Boston

> **Nothing is so unpredictable as a throw of the dice, and yet every man who plays often will at some time or other make a Venus-cast: now and then indeed he will make it twice and even thrice in succession. Are we going to be so feeble-minded then as to aver that such a thing happened by the personal intervention of Venus rather than by pure luck?** (Cicero, quoted by David, p25)

Dicing was so much a part of Roman culture that the historian Suetonius (1st Century CE) reports that Julius Caesar said, in what has become one of history's most famous quotations, "The die is cast (*Alea iacta est)*" when in 49 BCE he led his army south across the Rubicon river from Gaul to take over Rome.

The fall of the Roman Empire after about 500 years together with the simultaneous growth of Christianity left learning mainly in the hands of the clerics of the Church. Though many of the great Greek and Latin works of mathematics were preserved in monasteries throughout the Middle Ages, there were no new texts about probability. Not that the monks were immune from gambling. There is evidence that many were addicted to dice. In the 10th century Bishop Wibold, of Cambrai (France), invented a game to be played with three dice. The monks were encouraged to play the game, whose title translated to English is something like "A game suitable for monks or priests." When three dice are rolled, there are 56 different possible outcomes. Each outcome was associated with a virtue, and assigned a point value from 3 to 18. Figure 3 shows a few of the outcomes and their virtues.

Outcome	Virtue	Points
I I I	Charity	3
I I II	Faith	4
I I III	Hope	5
I I IV	Justice	6
I I V	Prudence	7
I I VI	Temperance	8
I II II	Courage	5
I II III	Peace	6
I II IV	Chastity	7
I II V	Pity	8
I II VI	Obedience	9

Source:www.angelfire.com/space/tarot/wibold.html

Figure 3

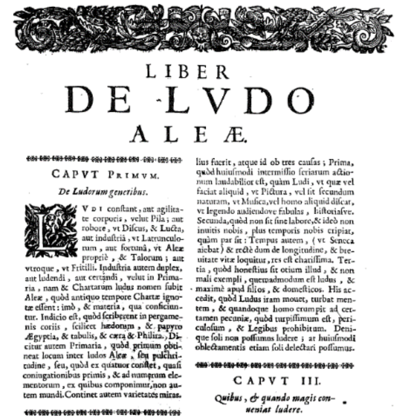
It took hundreds of years for a theoretical mathematics of probability to develop. One reason for the slow progress was the lack of good numeric and algebraic symbolism. Progress in the west had to wait at least until the 8th century, after Islam had been well established, and Baghdad had become a major center of learning. Scholars in Baghdad took on the task of studying and translating the preserved great books from Ancient Greece and from India. This is how the west learned our system for writing numbers. The contributions of the Arabs to arithmetic, algebra, and trigonometry are significant and well documented, but there was little mention of problems related to probability.

Starting about 1500 in Italy, well into the Italian Renaissance with its burst of interest in the old texts, we begin to see mathematicians in the west writing papers on probability.

Gerolamo Cardano (1501-1576), was a great Italian Renaissance "brain," best known in mathematics for his role in the solution of cubic equations. He was one of the first to describe how to compute odds in gambling. His book ***Liber de Ludo Aleae*** (***The Book of Games of Dice***), was put together piece-meal over several years around 1526, but was not published until 1663, long after his death. This text, written in Latin, is often considered the *first book that describes rules for computing mathematical probability*. It was not specifically a mathematics textbook, but rather a manual for gamblers.

Portrait of Cardano on display at the School of Mathematics and Statistics, University of St Andrews.

Figure 4



LIBER
DE LVDO
ALEÆ.

CAPVT PRIMVM.

De Ludorum generibus.

LVDI constant, aut agilitate corporis, velut Pila; aut robore, vt Discus, & Lucta, aut industriâ, vt Latrunculorum, aut fortunâ, vt Aleæ propriè, & Talorum; aut vtroque, vt Fritilli. Industria autem duplex, aut ludendi, aut certandi, velut in Primaria, nam & Chartarum ludus nomen subit Aleæ, quòd antiquo tempore Chartæ ignotæ essent: imò, & materia, qua conficiuntur. Indicio est, quòd scriberent in pergamenis coriis, scilicet hædorum, & papyro Ægyptia, & tabulis, & cæra & Philira. Dicitur autem Primaria, quòd primum obtineat locum inter ludos Aleæ, seu pulchritudine, seu, quòd ex quatuor constet, quasi coniugationibus primis, & ad numerum elementorum, ex quibus componimur, non autem mundi. Continet autem varietates miras.

lius fuerit, atque id ob tres causas; Prima, quòd huiusmodi intermissio feriarum actionum laudabilior est, quàm Ludi, vt quæ vel faciat aliquid, vt Pictura, vel sit secundum naturam, vt Musica, vel homo aliquid discat, vt legendo audiendove fabulas, historiasve. Secunda, quòd non sit sine labore, & ideò non inuitis nobis, plus temporis nobis eripiat, quàm par sit: Tempus autem, (vt Seneca aiebat) & rectè dum de longitudine, & breuitate vitæ loquitur, res est charissima. Tertia, quòd honestius sit otium illud, & non mali exempli, quemadmodum est ludus, & maximè apud filios, & domesticos. His accedit, quòd Ludus iram mouet, turbat mentem, & quandoque homo erumpit ad certamen pecuniæ, quòd turpissimum est, periculosum, & Legibus prohibitum. Denique soli non possumus ludere; at huiusmodi oblectamentis etiam soli delectari possumus.

CAPVT III.

Quibus, & quando magis conueniat ludere.

First page of Cardano's Book

This excerpt from Cardano's book shows the awkwardness of writing with Roman numerals. Also, note that the style of explanation is sentence after sentence. There is no symbolism for equations. By comparing the English translation to the Latin you can see he is working with 91, 125, 8281 and 15,625.

Thus, if it is necessary for someone that he should throw an ace twice, then you know that the throws favorable for it are 91 in number, and the remainder is 125: so we multiply each of these numbers by itself and get 8281 and 15,625, and the odds are about 2 to l. Thus, if he should wager double, he will contend under an unfair condition, although in the opinion of some the condition of the one offering double stakes would be better.	Velut alicui necessarium est, vt iaciat vnum bis : tunc tu scis numero eius esse nonaginta vnum, & residuum centum, & viginti quinque; ducemus ergo singulos in se, & fient viij. cclxxxi. & xv. D.cxxv. & est proportio fermè dupla. Si ergo duplum posuerit, iniqua conditione certabit, quamuis ex quadam opinione videatur conditio melior certantis duplo pignore.

Cardano was a fascinating character whose autobiography is still fun to read. He was a doctor, an astrologer, and a mathematician. This combination was not unusual for a professor of mathematics. In fact, the mathematics was considered the tool for understanding how the stars influenced people's health. No matter what his professional status, Cardano was a wild man and a serious gambler:

> **I was ever hot-tempered, single minded and given to women. From these cardinal tendencies there proceed truculence of temper, wrangling, obstinacy, rudeness of carriage, anger and an inordinate desire for revenge in respect of any wrong done to me.**
>
> **…**
>
> **During many years - for more than 40 years at the chessboards and 25 years at dice - I have played not off and on but, as I am ashamed to say, every day.** (from Cardano's autobiography)

Cardano gives what some would consider the first rule for mathematical probability. You can see that it is phrased in terms of *odds*, where you compare the number of successes to the number of failures (not to the total number of possible outcomes – which he calls the "whole circuit"). This odds framework is more useful in gambling to determine a fair bet, one where both parties have the same risk in betting.

> **So there is one general rule, namely, that we should consider the whole circuit, and the number of those casts which represents in how many ways the favorable result can occur, and compare that number to the rest of the circuit, and according to that proportion should the mutual wagers be laid so that one may contend on equal terms.** (from *The Book of Games of Dice*)

Experienced gamblers like Cardano had a very good sense of how likely certain outcomes were. Even so, a comparison of certain of these outcomes was "iffy." If two outcomes had almost the same rate of occurrence, people might disagree. It would have been useful to be able to prove which outcome was more likely, but a mathematically based system of agreed rules for such calculations was not quite developed.

Exercises:

1. (Version of the Venus cast) You have four 4-sided dice. If you throw them what is the probability that none will show the same face? Suppose they were four 6-sided dice?

2. Confirm there are 56 different outcomes with 3 dice.

3. In the Cardano excerpt, try matching the English to the Latin to see how he wrote those numbers in Latin. Notice what an improvement the Hindu-Arabic system of numeration is. Hint: Our letter "u" is a "v" in Latin. So when you see vnum that is unum, which means "one." The translator calls that an "ace."

4. Suppose there is a game where the probability of success is ¾ per play. That means the *odds* in favor of success are 3 to 1. Confirm this by the definition of odds. Now suppose you play this game twice.
a. What is the probability of two successes in a row?
b. What are the odds in favor of two successes in a row?

5. a. Show that the probability of throwing *at least* one 1 in a toss of 3 dice is 91/216.
b. Show that the *odds against* throwing at least one 1 in a toss of three dice are 125 to 91 or, as a ratio, 125/91. Hint: "At least one" is the opposite of "none."
c. Show that the probability of throwing at least one 1 in two tosses of 3 dice is simply $(91/216)^2$.
d. Show that the odds against throwing at least one 1 in two tosses of 3 dice are *not* $(125/91)^2$.

6. Here is a problem Cardano worked on. How many throws of a fair die do we need in order to have an even chance of at least one six? Cardano thought it was 3, but he was wrong. What is the right answer?

Sources:

Games, Gods, and Gambling, F.N. David, 1962, Charles Griffin & Company

Image Credits: Greek urn: http://classical-inquiries.chs.harvard.edu/a-roll-of-the-dice-for-ajax/

Egyptian board game: https://en.wikipedia.org/wiki/Senet

Cardano portrait: https://en.wikipedia.org/wiki/Gerolamo_Cardano

Cardano's autobiography: *The Book of My Life*, Dover, 1962

Thanks to Herb Weisberg for pointing out that de Moivre's 1718 *Doctrine of Chances* has a clear definition of probability. Here is the beginning of the introduction.

The DOCTRINE OF CHANCES:

INTRODUCTION.

THE Probability of an Event is greater, or leſs, according to the number of Chances by which it may Happen, compar'd with the number of all the Chances, by which it may either Happen or Fail.

Thus, If an Event has 3 Chances to Happen, and 2 to Fail; the Probability of its Happening may be eſtimated to be $\frac{3}{5}$, and the Probability of its Failing $\frac{2}{5}$.

Therefore, if the Probability of Happening and Failing are added together, the Sum will always be equal to Unity.

By *A. De Moivre.* F. R. S.

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https://play.google.com/store/books/details/Abraham_de_Moivre_The_Doctrine_of_Chances?id=3EPac6QpbuMC