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Validity of Cubic Law for Fluid Flow in a Deformable Rock Fracture

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The validity of the cubic law for laminar flow of fluids through open fractures consisting of parallel planar plates has been established by others over a wide range of conditions with apertures ranging down to a minimum of 0.2 μm. The law may be given in simplified form by $Q/\Delta h = C(2b)^3$, where Q is the flow rate, Δh is the difference in hydraulic head, C is a constant that depends on the flow geometry and fluid properties, and $2b$ is the fracture aperture. The validity of this law for flow in a closed fracture where the surfaces are in contact and the aperture is being decreased under stress has been investigated at room temperature by using homogeneous samples of granite, basalt, and marble. Tension fractures were artificially induced, and the laboratory setup used radial as well as straight flow geometries. Apertures ranged from 250 down to 4 μm, which was the minimum size that could be attained under a normal stress of 20 MPa. The cubic law was found to be valid whether the fracture surfaces were held open or were being closed under stress, and the results are not dependent on rock type. Permeability was uniquely defined by fracture aperture and was independent of the stress history used in these investigations. The effects of deviations from the ideal parallel plate concept only cause an apparent reduction in flow and may be incorporated into the cubic law by replacing C by C/f. The factor f varied from 1.04 to 1.65 in these investigations. The model of a fracture that is being closed under normal stress is visualized as being controlled by the strength of the asperities that are in contact. These contact areas are able to withstand significant stresses while maintaining space for fluids to continue to flow as the fracture aperture decreases. The controlling factor is the magnitude of the aperture, and since flow depends on $(2b)^3$, a slight change in aperture evidently can easily dominate any other change in the geometry of the flow field. Thus one does not see any noticeable shift in the correlations of our experimental results in passing from a condition where the fracture surfaces were held open to one where the surfaces were being closed under stress.

INTRODUCTION

The problem of laminar flow in a viscous incompressible fluid in a fracture has been studied by many workers starting with *Boussinesq* [1868]. *Lomize* [1951], *Polubarinova-Kochina* [1962], *Snow* [1965], *Romm* [1966], *Louis* [1969], and *Bear* [1972] are only a few of the investigators that have derived the basic equations describing flow through a fracture. If the flow is laminar and one adopts the analogy of parallel planar plates to represent the fracture surfaces, these workers have shown that the hydraulic conductivity of a fracture with an aperture $2b$ is given by (see notation section for definitions of all terms)

$$K_f = (2b)^2 \rho g / 12\mu \tag{1}$$

If the flow is steady and isothermal, the flux per unit drop in head can be developed from Darcy's law and may be written in simplified form as

$$Q/\Delta h = C(2b)^3 \tag{2}$$

where C is a constant, which in the case of radial flow is given by

$$C = \left(\frac{2\pi}{\ln (r_e/r_w)}\right)\left(\frac{\rho g}{12\mu}\right) \tag{3}$$

and in the case of straight flow by

$$C = \left(\frac{W}{L}\right)\left(\frac{\rho g}{12\mu}\right) \tag{4}$$

Equation (2) is the basis for what is often called the 'cubic law' for flow in a fracture. One must bear in mind that this equation has been derived for an 'open' fracture; i.e., the planar surfaces remain parallel, and thus are not in contact at any point.

The law for flow in fractures can be generalized in terms of the Reynolds number Re and the friction factor Ψ [*Lomize*, 1951; *Romm*, 1966; *Louis*, 1969]. These workers have shown that if one introduces the factor D equal to 4 times the hydraulic radius, then

$$Re = Dv\rho/\mu \tag{5}$$

$$\Psi = \frac{D}{(v^2/2g)}(\nabla h) \tag{6}$$

The cubic law for laminar flow in an open fracture then reduces to the simple relationship

$$\Psi = 96/Re \tag{7}$$

The first comprehensive work on flow through open fractures was by *Lomize* [1951]. He used parallel glass plates and demonstrated the validity of the cubic law as long as the flow was laminar. He also investigated the effects of changing the fracture walls from smooth to rough and, finally, to models with different fracture shapes. He introduced the concept of defining the roughness ϵ in terms of the absolute height of the asperities and developed the empirical equation

$$\Psi = \frac{96}{Re}\left[1 + 6.0\left(\frac{\epsilon}{2b}\right)^{1.5}\right] \tag{8}$$

which is valid for $\epsilon/2b > 0.065$. *Lomize* [1951] also considered the effect of flow through fractures with planar but nonparallel (converging or diverging) sides.

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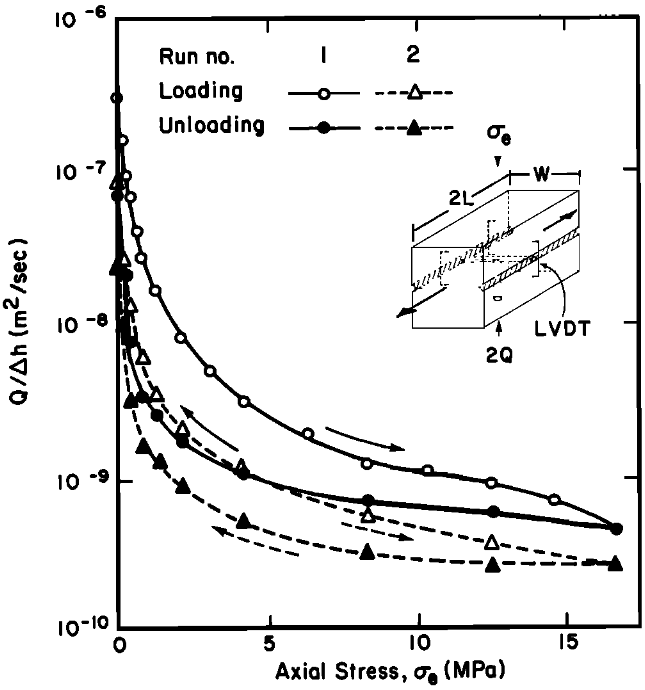

Fig. 1. Effect of cyclic loading on permeability of tension fracture in granite with straight flow [after *Iwai*, 1976].

We shall simplify (8) by rewriting the equation in the form

$$\Psi = \frac{96}{Re}f \tag{9}$$

where f is a factor that accounts for deviations from the ideal conditions that were assumed in deriving (7). In the case of roughness, $f \geq 1$. The cubic law then becomes

$$\frac{Q}{\Delta h} = \frac{C}{f}(2b)^3 \tag{10}$$

Lomize [1951] developed a flow regime chart with several semiempirical flow laws to take into account the effects of roughness and turbulent flow in open fractures. *Louis* [1969] has independently performed experiments similar to Lomize's and reached essentially the same conclusions. A number of other workers [*Baker*, 1955; *Huitt*, 1956; *Maini*, 1971; *Parrish*, 1963; *Rayneau*, 1972; *Rissler*, 1978] have also investigated the effects of roughness on flow in open fractures.

In his comprehensive treatise on flow in fractured rocks, *Romm* [1966] presented the results of very careful laboratory studies on flow phenomena in fine (10–100 μm) and superfine (0.25–4.3 μm) fractures. His superfine fractures were made of optically smooth glass and were carefully constructed so as to be open fractures. He demonstrated the validity of the cubic law for laminar flow in both fine and superfine fractures by using various fluids and also verified the presence of a boundary layer 0.015 μm in thickness in some cases. He concluded that laminar flow in a fracture obeys the cubic law at least down to apertures of 0.2 μm. The critical Reynolds number at the transition from laminar to turbulent flow was found to be 2400. This same critical value has also been reported by *Lomize* [1951], and *Louis* [1969] reports the value to be 2300.

Sharp [1970] performed flow tests in the laboratory with a natural rock fracture in a hard granite porphyry whose matrix permeability was extremely small. He started the flow tests from a so-called 'closed' condition which was achieved by placing the fracture in a horizontal position so that it would 'close' under the weight of the upper half of the rock sample. Water flow through the fracture in this closed condition indicated that the aperture was about 270 μm, and flow tests were then conducted by jacking the fracture open until a maximum aperture of 1540 μm was reached. *Sharp* [1970] and *Sharp and Maini* [1972] proposed an empirical flow law for natural fractures that was based on the "effective" aperture, which is the difference in the opening from the initial condition, and the net flow rate obtained from a calculation of the measured flow rate minus that observed under the initial closed condition. They dispute the validity of the cubic law and have suggested that for their particular conditions the exponent in (2) should be 2. In reviewing these results, *Gale* [1975] has pointed out that the cubic law is still valid if one correlates their flow rates corresponding to the apertures that were actually present.

All of these investigations have been concerned with open fractures, and, of course, one will encounter many situations in the field where the fractures are not open. Usually, fracture surfaces have some degree of contact, and the effective aperture will depend upon the normal stresses acting across the discontinuity. Under these conditions, flow is still possible, but since part of the flow path is now blocked by asperities, will the cubic law still hold? This is an important question in many fields where flow through fracture systems must be considered. It is of critical importance in the underground isolation of radioactive waste where one is concerned with the problem of investigating nearly impermeable rocks. To our knowledge the validity of the cubic law under these conditions has not been investigated. We have therefore undertaken a laboratory investigation of this problem by using fractures created in three different rock types [*Iwai*, 1976].

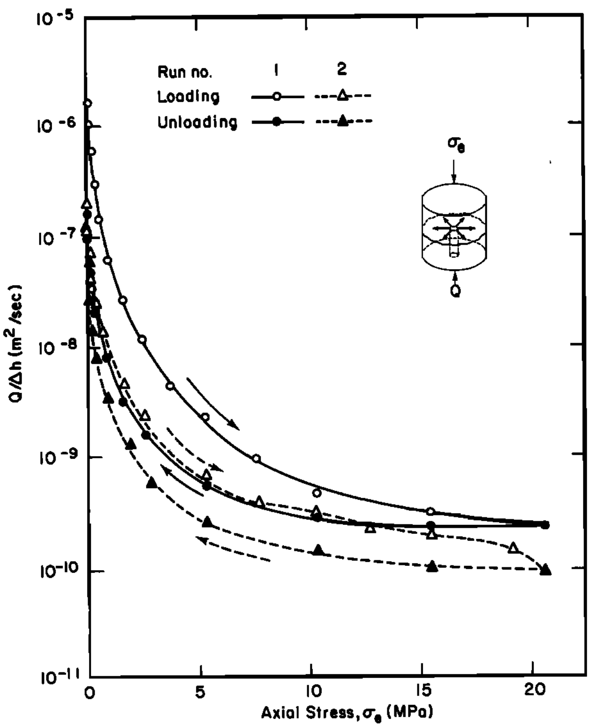

Fig. 2. Effect of cyclic loading on permeability of tension fracture in granite with radial flow [after *Iwai*, 1976].

LABORATORY PROCEDURES

Homogeneous intact samples of basalt, granite, and marble were chosen for investigation. All samples came from quarries in California and have matrix permeabilities that are so low that for the purposes of this study they can be neglected. The samples were prepared with radial and straight flow geometries for the flowing fluid, which was filtered water at ambient conditions. Details of the sample preparation and laboratory procedures are given by *Iwai* [1976] and will only be summarized here.

For the radial flow work, cylinders of intact rock with a diameter of 0.15 m were diamond-cored from each rock sample in lengths ranging from 0.17 to 0.21 m. A horizontal fracture was created in each sample by using a modified form of the 'Brazilian' loading method. In general, this method produced an excellent fracture surface that was essentially orthogonal to the cylindrical axis. A center hole 0.022 m in diameter provided access for outward radial flow of water.

A Riehle testing machine provided axial loads up to 20 MPa. The loading piston of this testing machine was also attached to the upper half of the radial flow samples so that when the piston was raised sufficiently, an open fracture (no points of contact between the two surfaces) of known aperture could be obtained. Three linear variable differential transducers (LVDT), placed 120° apart and mounted so as to straddle the fracture, were capable of detecting aperture changes as small as 0.4 μm.

For the straight flow work a horizontal tension fracture was created in a rectangular block cut from the granite sample with width = 0.121 m, length = 0.207 m, and height = 0.155 m. By sealing both sides of the fracture along the 0.207 m length of the block and keeping the ends open a straight flow field was established. A groove across the middle of the fracture plane provided for even distribution of water across the width of the fracture. A 0.08-m hole drilled into this groove from below provided the inlet connection for water to reach the fracture. Two LVDT's straddled the fracture on one side of the rectangular block, and one LVDT was similarly mounted on the opposite side (see Figure 1).

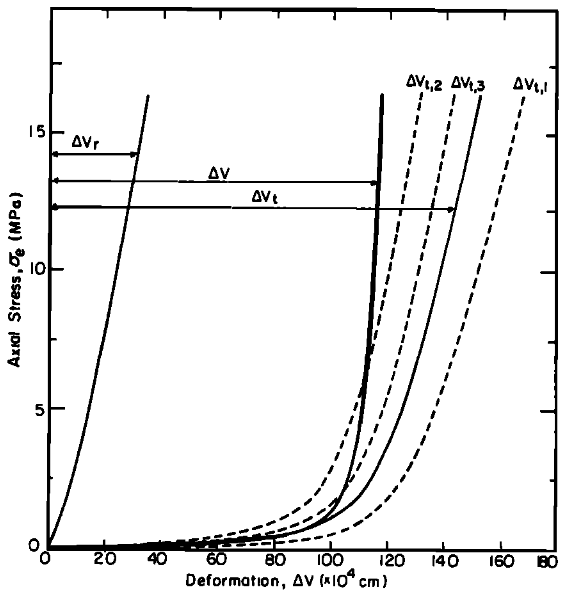

Fig. 3. Mechanical properties of fractured granite sample used in straight flow model [after *Iwai*, 1976].

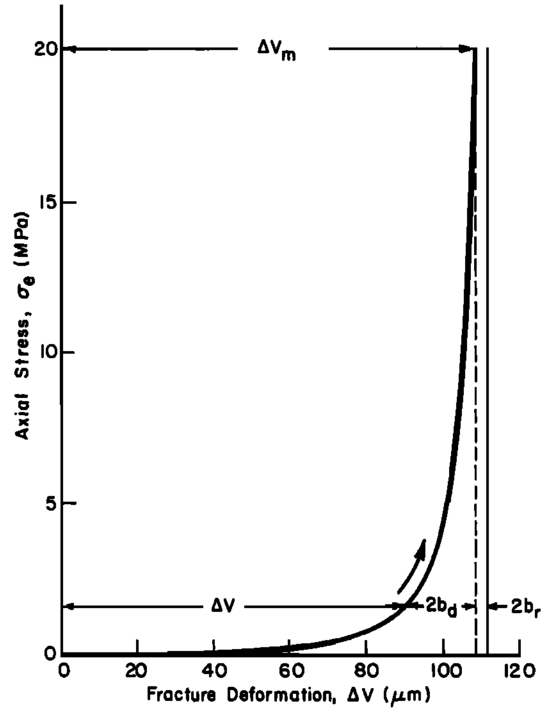

Fig. 4. Mechanical properties of fracture used in determining changes in aperture with stress.

Before testing, the flow system was carefully checked for leaks with the new tension fracture subjected to the first loading cycle with maximum load conditions. During the second loading cycle a series of flow measurements were made for run 1, both during loading to maximum stress and also during unloading. The fracture was next loaded and unloaded twice to the same maximum stress without making flow measurements. On the fifth loading cycle, run 2 data were collected. Runs 1 and 2 used a head difference of $\Delta h \simeq 20$ m of water. After these high pressure experiments, run 3 was carried out using $\Delta h \simeq 0.5$ m of water to reduce any tendency for the development of turbulent conditions.

One of the most critical problems was that of determining the fracture aperture. Figure 1 shows an example of flow rates as measured with the straight flow granite sample when normal stresses up to 17 MPa were employed in an attempt to close the fracture. The effect of repeated loading cycles was to reduce the flow rate per unit head difference, $Q/\Delta h$, to a minimum of 5.33×10^{-10} m²/s, but the fracture could not be closed completely. Assuming that (2) is valid for a nearly closed fracture such as this the residual aperture was computed to be 6.7 μm under these conditions. Figure 2 shows similar results for flow rates obtained on a tension fracture in the same granite with radial flow.

When deformations across the fractures were measured as a function of stress, a highly nonlinear behavior was observed. This is shown on Figure 3 for the first loading cycle with the straight flow granite sample. The $\Delta V_{t,1}$ is the total deformation as measured on one side, and $\Delta V_{t,2}$ and $\Delta V_{t,3}$ are the total deformations as measured on the opposite side (see Figure 1). Knowing ΔV_r, which was measured on an intact sample of granite, the net deformation of the fracture was computed from

$$\Delta V = \frac{1}{2}\left[\Delta V_{t,1} + \frac{\Delta V_{t,2} + \Delta V_{t,3}}{2}\right] - \Delta V_r \qquad (11)$$

TABLE 1. Results of Least Squares Fit for Parameters n and $2b_r$

Sample	Run	Fitted n	Residual Aperture Fitted, μm	Residual Aperture Calculated,* μm
Granite†	1	3.04	9.0	7.9
	2	3.03	6.7	6.7
	3	3.01	11.6	11.4
Granite‡	1	3.07	5.1	4.4
	2	3.04	4.0	3.2
	3	3.06	13.1	10.9
Basalt‡	1	3.08	10.5	10.0
	2	3.10	10.8	10.4
	3	3.02	9.1	9.8
Marble‡	1	3.06	2.5	4.0
	2	3.06	2.2	4.0
	3	3.01	18.2	18.1

* Calculated from equation (2).
† With straight flow.
‡ With radial flow.

TABLE 2. Results of Least Squares Fit for Parameters of f and $2b_r$

Sample	Run	Fitted f	Residual Aperture Fitted, μm	Residual Aperture Calculated,* μm
Granite†	1	1.21	8.8	7.9
	2	1.15	6.6	6.7
	3	1.04	11.6	11.4
Granite‡	1	1.49	4.8	4.4
	2	1.29	3.8	3.2
	3	1.32	12.4	10.9
Basalt‡	1	1.45	9.8	10.0
	2	1.65	9.9	10.4
	3	1.10	8.7	9.8
Marble‡	1	1.36	2.2	4.0
	2	1.36	1.8	4.0
	3	1.05	18.2	18.1

* Calculated from equation (2).
† With straight flow.
‡ With radial flow.

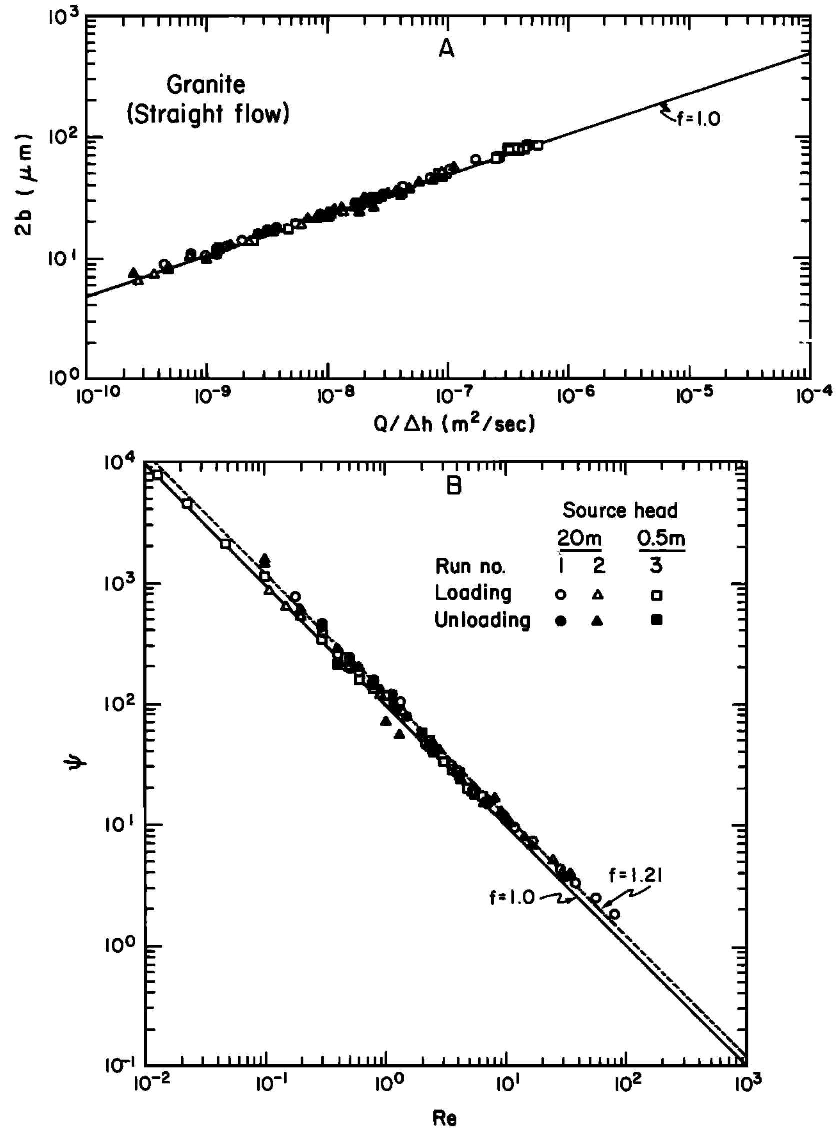

Fig. 5. Comparison of experimental results for straight flow through tension fracture in granite with cubic law.

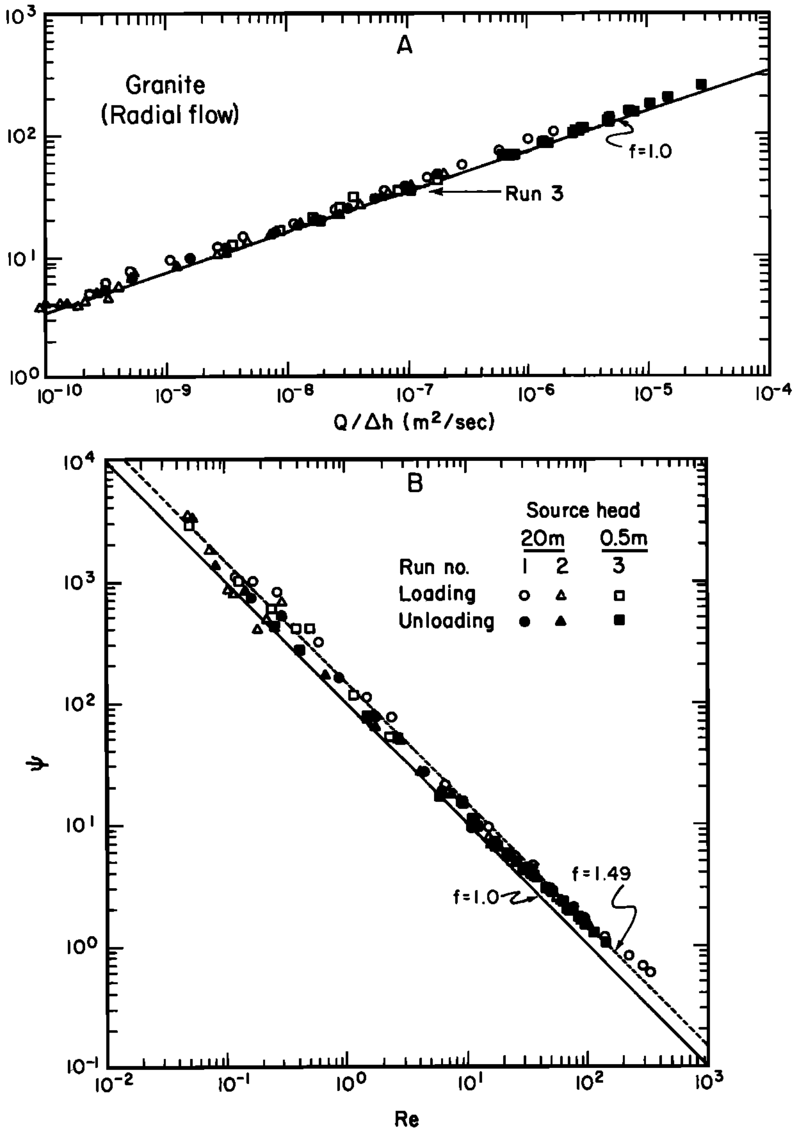

Fig. 6. Comparison of experimental results for radial flow through tension fracture in granite with cubic law. In run 3, fracture surfaces were no longer in contact during unloading when aperture exceeded value indicated by arrow.

The apparent aperture at any stress level, $2b_d$, could then be determined by reference to the maximum fracture deformation ΔV_m (see Figure 4), using

$$2b_d = \Delta V_m - \Delta V \tag{12}$$

Deformations were carefully measured during loading and unloading cycles so that (12) could be used regardless of the effects of hysteresis which are clearly evident on Figures 1 and 2. The same procedure was used to calculate ΔV for the radial flow samples except that a simple average of the ΔV_i values was used in this case because the LVDT's were equally spaced around the circumference.

DISCUSSION OF RESULTS

At any given stress the experimentally determined quantities were the apparent aperture $2b_d$, the flow rate Q, and the difference in hydraulic head Δh. For a given aperture, Q was observed to be proportional to Δh under the experimental conditions [*Iwai*, 1976], and thus Darcy's law holds. Therefore we have a unique functional relation between $Q/\Delta h$ and $2b_d$.

The apparent aperture could not be used directly in (2) or (10) to check the validity of the cubic law because the measured flow rates depend on the true aperture $2b$, which could not be measured directly. As shown on Figure 4, the true aperture was taken as the sum of the apparent and residual values, i.e.,

$$2b = 2b_d + 2b_r \tag{13}$$

One might wish to estimate the residual value using (2), but this would assume that the cubic law holds for all apertures, especially as they diminish at large stresses.

To test the validity of the cubic law, we must treat $2b_r$ as an unknown parameter common to all data points. As a first approach, we assumed that the factor f could be set to unity and rearranged (10) to

$$Q/\Delta h = C(2b_d + 2b_r)^n \tag{14}$$

The unknown parameters n and $2b_r$ were then determined by minimizing

$$\chi^2 = \sum_i \left\{ \log \frac{Q}{\Delta h} - \log \left[C(2b_d + 2b_r)^n \right] \right\}^2 \omega \tag{15}$$

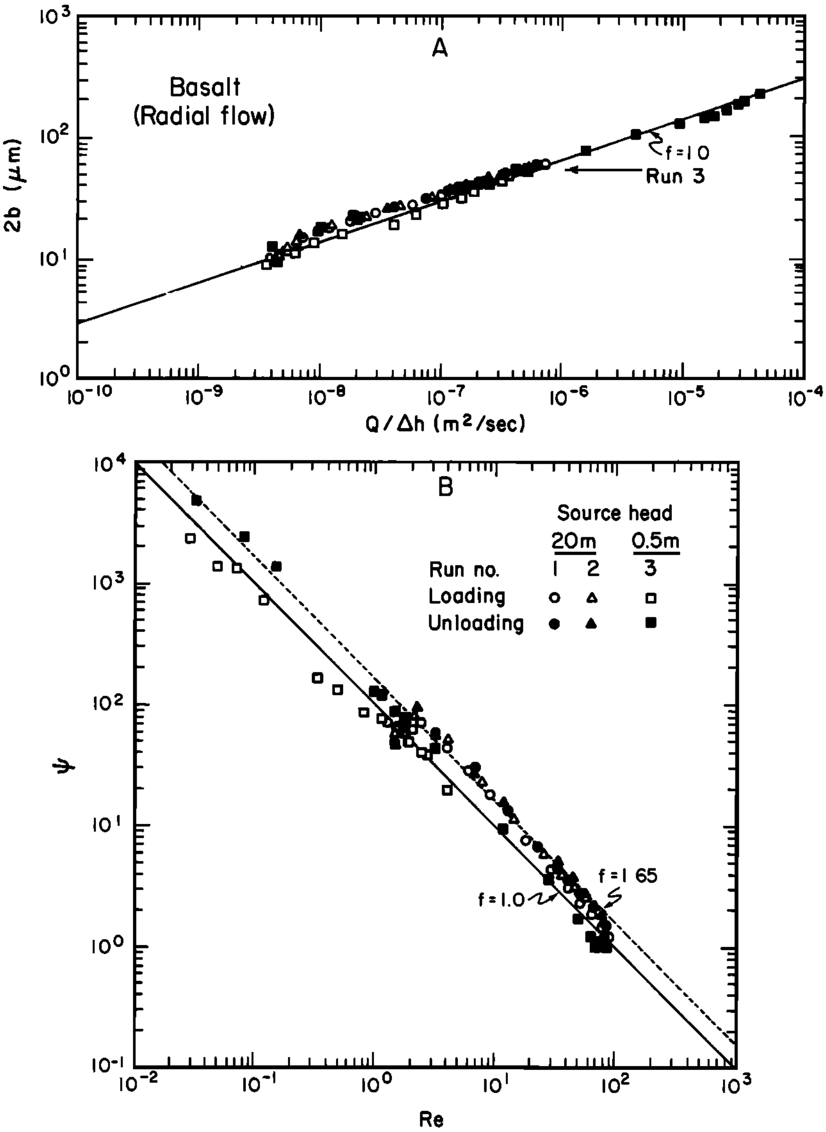

Fig. 7. Comparison of experimental results for radial flow through tension fracture in basalt with cubic law. In run 3, fracture surfaces were no longer in contact during unloading when aperture exceeded value indicated by arrow.

over all data points i from one flow run, including measurements during first the loading and then the unloading cycles (Figures 1 and 2). The symbol ω in (15) is a weighting factor that took into account the fact that the values of log $Q/\Delta h$ were not evenly spread. For a given data point i,

$$\omega = \tfrac{1}{2}|\log (Q/\Delta h)_{i+1} - \log (Q/\Delta h)_{i-1}| \qquad (16)$$

is used. The results of the least squares fit are not very sensitive to this weighting factor if the data points are fairly evenly spaced. The distribution of the data points for different runs will be shown later in Figures 5–8.

The results of the least squares fit for the data using (15) are given in Table 1. The table also includes values for the residual apertures that were calculated directly, assuming (2) is valid. Note that these results are not significantly different from those obtained by the least squares fit. The reason is quite evident; the factor $n \cong 3$. The cubic law is valid for these fracture flow conditions.

This is a significant result because the data on Figures 1 and 2 show evidence of hysteresis and permanent set. For example, during the first loading cycle for the radial flow tests in granite (Figure 2) a maximum displacement of $\Delta V_m = 108.2$ μm was obtained, and upon unloading the deformations stopped with $\Delta V = 41.4$ μm. The cubic law seems to hold regardless of the loading path and no matter how often the loading process is repeated. Permeability is uniquely defined by the fracture aperture, and one could predict changes due to stress as long as there are no effects of shear movement or weathering.

From a theoretical standpoint the value of n should be 3, and in the results in Table 1 it is apparent that experimental results differ by no more than 3% from the theoretical value. Note also that all values are greater than 3, which suggests that some other systematic factor is contributing to this difference from theory. We therefore concluded that we were justified in adopting $n = 3$ and should reexamine the experimental data to determine the importance of the factor f.

To do this, we recast (14) in the form

$$\frac{Q}{\Delta h} = \frac{C}{f}(2b_a + 2b_r)^3 \qquad (17)$$

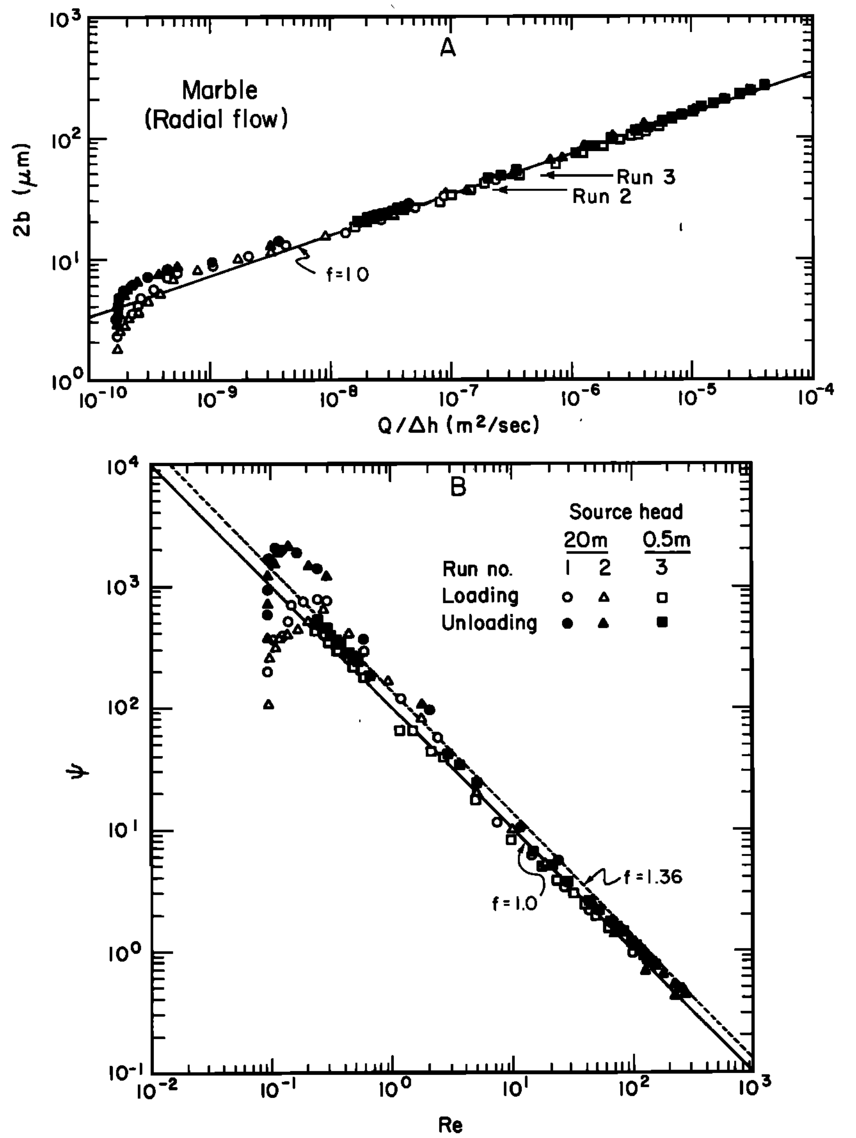

Fig. 8. Comparison of experimental results for radial flow through tension fracture in marble with cubic law. In runs 2 and 3, fracture surfaces were no longer in contact during unloading when aperture exceeded value indicated by arrow.

The unknown parameters f and $2b_r$ were then determined by minimizing

$$\chi^2 = \sum_i \left\{ \log \frac{Q}{\Delta h} - \log \left[\frac{C}{f} (2b_a + 2b_r)^3 \right] \right\}^2 \omega \qquad (18)$$

over all data points in the same fashion as previously. The results are given in Table 2. We see immediately that the least squares results for the residual aperture are somewhat closer to the values that were calculated directly from (2) than is the case in Table 1. We also see that reasonable values are obtained for the factor f and that they are all greater than unity, as predicted by (8).

Having determined values for the residual aperture at maximum normal stress, it was then possible to examine how well the experimental data agree with the cubic law over the range of flow rates employed. Figure 5 shows results for the granite sample with straight flow (see Figure 1). Figure 5*a* presents the data in accordance with (10), where the solid line is for the special case of $f = 1.0$. For comparison, Figure 5*b* presents the data in accordance with (9), which is the method used by *Lomize* [1951], *Romm* [1966], and *Louis* [1969]. The effect of changing f from 1.0 to 1.21, which is the range of values shown in Table 2, is indicated. Note in Table 2 that the values of f generally decreased in progressing from run 1 to run 3. This may be an indication that the fractures became progressively better mated during the cyclic loading.

Figure 6 shows results for the same granite but with radial flow (see Figure 2). Note that the radial flow data deviate somewhat more from the ideal cubic law ($f = 1.0$) than the straight flow data shown in Figure 5. This may be a manifestation that the effects of roughness are more complicated in a radial flow field, as is indicated by the fact that the maximum value of f reached 1.49 (see Table 2).

In a radial flow system the Reynolds number is difficult to define because velocities decrease by a factor of 7 from the inner radius to the outer boundary. The values of *Re* for radial flow samples were calculated by using the inner radius and are therefore maximum values. Note that in run 3, data were also collected during the unloading cycle after the fracture surfaces were truly open; i.e., the surfaces were no longer in contact. One sees good agreement with the cubic law over a range of apertures from 4 to 250 μm, regardless of whether the fracture was open or closed.

Figure 7 shows results for the basalt sample with radial flow. Here the maximum deviation from the ideal cubic law (f = 1.0) was obtained, and this is indicated by the dashed line for f = 1.65 in Figure 7*b*. The basalt fracture surfaces were more undulating than in the case of the granite and marble samples, and one experimental difficulty arose when a small chip of rock broke off around the center borehole. We assumed that r_w should be increased from 0.011 m to an estimated value of 0.02 m, but this may not have been the only effect on the flow field.

Figure 8 shows results for the marble sample with radial flow. Experimental difficulties at low flow rates during runs 1 and 2 caused the marked deviations from the cubic law, as apertures decreased below 10 μm. Some subtle changes in mating of the fracture surfaces also occurred between runs 2 and 3. In run 2 the initial aperture was 25.5 μm, but during the unloading cycle the aperture was opened. Upon reseating in preparation for run 3 the initial aperture was found to be 123.0 μm. Note in Table 2 that f was found to be 1.36 in run 2 and 1.05 in run 3. Despite these difficulties with both the basalt and marble samples, it is apparent in Figures 7 and 8 that good agreement with the cubic law has been obtained.

The manner in which a fracture closes as the surfaces are forced together under stress raises some important issues. We visualize the process as being controlled by the strength of asperities that can withstand significant stresses while maintaining space for fluids to continue to flow as the fracture aperture decreases. An idealization of a fracture in the open and closed positions is illustrated in Figure 9. Asperities of height ϵ are visualized as coming together in a perfect union that provides fixed areas of contact. These areas of contact then deform elastically such that the contact area is not changed significantly, at least not under the relatively low stress levels employed in this investigation. The only significant change that takes place is that of the closing of the aperture.

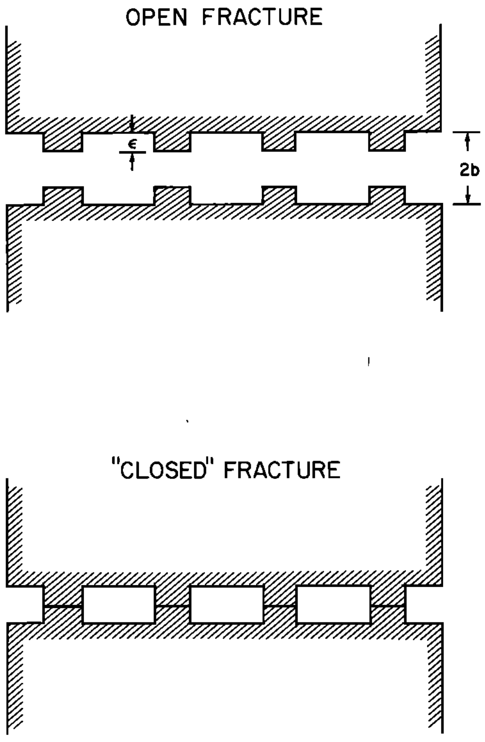

Fig. 9. Idealized fracture showing the mating of asperities, as the fracture surfaces are closed under stress.

As we have seen above, there is a very strong dependence between flow and size of the aperture. As a fracture closes, however, the flow paths are certainly far more complex than the idealization portrayed in Figure 9. The flow may actually occur through an intricate assembly of interconnected tubes of varying cross section and separated by the local asperities. In any event the overall geometrical effect can still be represented by a measurement of $2b$. Since flow per unit head depends on $(2b)^3$, a slight change in aperture evidently can easily dominate any other change in the geometry of the flow field. Thus one does not see any noticeable shift in correlations of the experimental results in passing from an open to a so-called closed fracture.

Much more work is needed in understanding the mechanical and hydraulic behavior of a fracture that is deforming under stress. This study has been concerned with fractures under normal stress, whereas the general situation in the field will involve both normal and shear deformations. The actual physical process that takes place during the deformation of fracture surfaces and the effect this behavior has on the fluid flow process must be understood. There is also a problem of a potential size effect that has been observed in determining the hydraulic properties of fractures in rock specimens of different dimensions [*Witherspoon et al.*, 1979]. An understanding of contact area behavior in fractures will be important in analyzing this problem. The present work was carried out with ambient temperatures, and the flow sensitivity of fractures to very slight changes in aperture indicates that thermal expansion is yet another factor that needs investigation in order to understand fracture behavior at elevated temperatures.

CONCLUSIONS

The results of this laboratory investigation on tension fractures that were artificially induced in homogeneous samples of granite, basalt, and marble have clearly shown that the cubic law for fluid flow in a fracture, which is given by $Q/\Delta h = (C/f)(2b)^3$, is valid. The investigations included radial and straight flow geometries and covered apertures ranging from 250 down to 4 μm and normal stresses up to 20 MPa. The cubic law was found to hold whether the fractures were open or closed, and the results are not dependent on rock type. The cubic law seems to hold regardless of the loading path and no matter how often the loading process is repeated. Permeability is uniquely defined by the fracture aperture, and one could predict changes due to stress as long as the fracture surfaces are not affected by shear movements or weathering. The effects of deviations from the ideal parallel plate concept only cause an apparent reduction in flow and are taken care of by the factor f. In this investigation, f varied from 1.04 to 1.65.

We visualize the process of a fracture that is closing under normal stress as being controlled by the strength of the asperities that are in contact. These contact areas are able to withstand significant stresses while maintaining space for fluids to continue to flow as the fracture aperture decreases. The controlling factor is the magnitude of the aperture, and since flow per unit head depends on $(2b)^3$, a slight change in aperture evidently can easily dominate any other change in the geometry of the flow field. Thus one does not see any noticeable shift in correlations of the experimental results in passing from an open fracture to one where the surfaces are being closed under stress. Much more work is needed in understanding the mechanical and hydraulic behavior of a deformable rock fracture.

NOTATION

- b aperture half width, L.
- b_a apparent aperture half width, L.
- b_r residual aperture half width, L.
- C proportional constant in cubic law, $1/LT$.
- D hydraulic diameter ($= 4b$), L.
- f fracture surface characteristic factor.
- g acceleration of gravity, L/T^2.
- h hydraulic head, L.
- K_f fracture hydraulic conductivity, L/T.
- L length of fracture, L.
- n exponent in fracture flow law.
- Q flow rate, L^3/T.
- Re Reynolds number.
- r_e outer radius, L.
- r_w well bore radius, L.
- V fracture deformation, L.
- V_m maximum fracture deformation, L.
- V_r rock deformation, L.
- V_t total deformation, L.
- v flow velocity, L/T.
- W width of fracture, L.
- ϵ height of asperities, L.
- ρ fluid density, M/L^3.
- Ψ friction factor.
- σ_e axial stress, M/LT^2.
- μ flow viscosity, M/LT.
- ω weighting factor.

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